# ABOUT THE MODEL OF THE CONTROL SYSTEM IN CONDITIONS OF INACCURATE INFORMATION AND ITS APPLICATION TO ONE ECONOMIC PROBLEM

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***https://doi.org/10.5281/zenodo.7575089***

***Abstract.*** *The paper considers a mathematical model of a control system under conditions of inaccurate information about the initial data and the parameter of external influences. The model is presented as a controlled differential inclusion with a parameter. A problem of optimal control with a non-smooth terminal functional is studied. Necessary and sufficient optimality conditions are obtained. A general scheme - an algorithm for using these optimality conditions - is given. The application of the obtained results to the economic problem of the optimal distribution of investments is shown.*

***Kerwords:*** *mathematical model, control system, information inaccuracy, optimal control, non-smooth functional, optimality conditions, optimal distribution of investments.*

## Introduction

Mathematical models of processes and objects from the economy and technology are very diverse in form and content. Mathematical methods are widely used to make informed decisions when planning the comprehensive development of industries, regions, enterprises, consumers, and naturally, they serve the interests of various parties involved in economic relations. With the help of a modern method of mathematical modeling and computational experiment, it is possible to conduct active scientific, theoretical and practical research aimed at automating control and making optimal decisions.

At present, economic systems are distinguished by the variety and scale of control parameters and influences, which, as usual, have material and financial content, as well as by the large influence of the factor of random changes and risk (additional funds for raw materials and material resources, investments in various forms, new economic indicators, market price changes, etc.). Many management tasks in the field of economics are implemented in the form of dynamic optimization models with different objective functions and many restrictions on state and control variables [1,2,3]. To describe the behavior of dynamic control systems, various classes of differential equations are used, including differential equations with multivalued right-hand side - differential inclusions.

Differential inclusions, which proved to be an effective mathematical apparatus of research [4], deserve special attention in the theory of optimal control. To date, the theory of differential inclusions has been developing in various directions, controlled differential inclusions are being studied, both with delay and without delay, as well as their discrete analogs [5–20]. Differential inclusions with a control parameter play an important role as a model of control systems under conditions of data inaccuracy - informational limitations. They are also of interest in the study of problems of differential games, which are a mathematical model of conflict situations, which is characteristic of the relations of a market economy.

**Materials and methods of research**

An important class of dynamic models of control systems under conditions of information constraints is controlled differential inclusions with a parameter [7,8,12]. In the study of such models, various problems arise for controlling the ensemble of trajectories of a dynamical system. Basically, these problems belong to the class of non-smooth optimal control problems.

Let’s consider the control system described by the differential inclusion

$$\frac{dx}{dt} \in A(t)x + b(t,u,v), x(t_0) \in X_0, u \in U(v), v \in V, t \geq t_0, \tag{1}$$

where $x = x(t) \in R^n$, $A(t)$– given $n \times n$- matrix, $b(t,u,v) \subset \mathrm{R}^n$, $U \subset R^m$, $V \subset R^k$. In contrast to the control influence $u = u(t), t \in [t_0, t_1]$, parameter $v$ represents as an additional factor, the consideration of which is associated with the purpose of management. It can also be considered that it is a parameter of some external influences or a parameter of permissible changes in the structure of the control system, the identification of which is required in the control process.

It should be noted that model (1) describes, in particular, the process of controlling a linear system from the form

$$\frac{dx}{dt} = A(t)x + B(t,u)v + q(t), u \in U(v), \mathrm{v} \in \mathrm{V} \tag{2}$$

where $B(t,u)$– $n \times k$ -matrix, external influences $q = q(t)$ are priori unknown, there is no statistical description and only the set of possible values is known: $q(t) \in Q$ , $Q \subset R^n$.

We will assume that the following assumptions are fulfilled for the control system (1):

1) elements of the matrix $A(t)$ are continuous on $T = [t_0, t_1]$; 2) for any $t \in T, u \in U(v), v \in V$ the set of $b(t,u,v)$ is a non-empty compact from $R^n$; 3) multivalued mapping $(t,u,v) \to b(t,u,v)$ is contionuous; 4) $X_0 \subset R^n$, $U(v) \subset R^m$, $V \subset R^k$ – convex compacts.

Let be $U(T)$ – the set of all piecewise continuous controls $u = u(t), t \in T$, with the values from the set of $U(v) \subset R^m$; $X(t_1,u,v)$– the set of those points of the state of space $R^n$, which are reachable by absolutely continuous trajectories at time $t_1$. The quality of system control (1) will be estimated by the terminal functional of the form

$$J(u,v) = \max\left\{\min_{p \in P}(p,\xi) : \xi \in X(t_1,u,v)\right\}, \tag{3}$$

where $P$ – compact $R^n$. The goal of the control is to minimize the functional (3), i.e. the minimax problem is considered:

$$\max\left\{\min_{p \in P}(p,\xi) : \xi \in X(t_1,u,v)\right\} \to \min, u \in U, v \in V. \tag{4}$$

A characteristic feature of this control task is determined by the non-smoothness (non-differentiability) of the minimized functional. Such problems often arise in cases where the goal of control is to obtain a guaranteed result under conditions of inaccurate information regarding the initial data and perturbation parameters.

Let us study the control system (1) using the results of the theory of differential inclusions, multivalued mappings, and convex analysis [4]. The main goal of the study is to obtain necessary and sufficient optimality conditions in problem (4). It is also planned to study the issue of developing a general scheme - an algorithm for solving problem (4), based on the use of optimality conditions. We will discuss the application of the obtained results on the example of a dynamic problem of the optimal distribution of investments between industries. The work develops research [7,8,13,17].

**Main results**

Let: $F(t,\tau)$ be fundamental matrix of solutions to the equation $\frac{dx}{dt}=A(t)x,\ F(\tau,\tau)=E$; $C(D,\psi)=\max_{d\in D}(d,\psi)$ – support function of a compact $D\subset R^n$. It can be shown that functional (3) satisfies the representation:

$$J(u,v)=\min_{p\in coP}[C(F(t_1,t_0)X_0,p)+\int_{t_0}^{t_1}C(F(t_1,t)b(t,u(t),v),p)dt], \quad (5)$$

where $coY-$ convex hull of a compact $P\subset R^n$. Using formula (5), we obtain the following necessary optimality condition in problem (4).

**Theorem 1.** *If $u^*(t), t\in T,$ – optimal control and $v^*$ – the optimal value of the parameter in the task (4), then there is a global minimum point $p^*\in coP$ of the function*

$$\eta(p)=C(F(t_1,t_0)X_0,p)+\min_{v\in V}\int_{t_0}^{t_1}\min_{u\in U(v)}C(F(t_1,t)b(t,u,v),p)dt, \quad (6)$$

*such that the following equalities hold*:

$$\min_{v\in V}\int_{t_0}^{t_1}\min_{u\in U(v)}C(F(t_1,t)b(t,u,v),p^*)dt=\int_{t_0}^{t_1}\min_{u\in U(v^*)}C(F(t_1,t)b(t,u),v^*),y^*)dt, \quad (7)$$

$$C(F(t_1,t)b(t,u^*(t),v^*),p^*)=\min_{u\in U(v^*)}C(F(t_1,t)b(t,u,v^*),p^*),\ t\in T. \quad (8)$$

Note that these necessary optimality conditions are also sufficient optimality conditions in the problem under consideration, i.e., the following is true.

**Theorem 2.** *The pair $(u^*(\cdot),v^*)$ constitutes a solution to problem (4) if and only if there is a global minimum point $p^*\in coP$ of the function $\eta(p)$ and conditions (7),(8) are satisfied.*

Based on the results obtained, we can propose the following general scheme - an algorithm for applying optimality conditions for solving the minimax problem (4).

Algorithm***:***

***Step 1***. *To calculate the fundamental matrix $F(t,\tau)$ of the system $\dot{x}=A(t)x,\ F(\tau,\tau)=E$.*

***Step 2***. *To calculate support functions $C(F(t_1,t_0)X_0,\psi), C(F(t_1,t)b(t,u,v),\psi)$.*

***Step 3.*** *To compute function $\eta(p)$, defined by the formula (6).*

***Step 4.*** *To calculate minimum point $p^*\in coP$ of the function $\eta(p)$.*

***Step 5.****To find a point $v^*\in V$, satisfying the condition (7).*

***Step 6.*** *To find the function $u^*(t)$ from the condition (8).*

After performing the operations of these steps, we get a pair $(u^*(t), v^*)$, which is the solution to the problem (4).

In this algorithm, steps 3 and 4 are key steps. In the case when the function $\eta(p)$ is determined by an analytically explicit formula, step 4 represents as a solution to the non-linear programming problem. Under certain conditions on the right side of the system (1), in step 4, the problem of convex programming is solved. Steps 5 and 6 also represent a non-linear programming problem, and in step 6 such a problem is solved depending on the parameter $t \in T$.

The operations of each step of this algorithm can be refined and specified taking into account the conditions on the parameters of the problem. To ensure the practicality of the algorithm, it is necessary to use numerical optimization methods and computer technologies.

**The discussion of the results**

The considered system (1) is of interest from the point of view of application in studies of various control processes with inaccurate information that arise in economics and technology.

As an example, let's consider the application of the considered model and the results obtained to the problem of optimal distribution of investments between industries. Let there be industries in which investments are supposed to be made to ensure their growth over a certain period of time $T = [t_0, t_1]$. Coefficients $a_i$ of annual disposal of fixed production assets $\mathrm{K}_i(t)$ of each of the industry $i = \overline{1,n}$ are known. To ensure the growth of production assets, it is planned to invest $R_i(t)$ in the year $t \in T$ in each of the industry $i = \overline{1,n}$. We assume that the initial state of fixed production assets and the total volume of the allocated investment are known. Then the dynamic system of distribution of capital investments between industries can be represented by the following model [3]:

$$\frac{d\mathrm{K}_i(t)}{dt} = -a_i \mathrm{K}_i(t) + R_i(t), t \in T, \mathrm{K}_i(t_0) = \mathrm{K}_i^0, \sum_{i=1}^{n} R_i(t) \le M .$$

Now let's make some changes to this model. It is planned to invest in the amount $R_i(t) = v_i u(t)$ every year $t \in T$ in each of the industry $i = \overline{1,n}$, while on the coefficients $v_i$ acceptable limits: $0 < \underline{v}_i \le v_i \le \overline{v}_i$ are set. In the system under consideration, we will take into account the factors of the influence of external parameters on the process of growth of production assets with the attraction of investments. As such factors, it should be noted the change in prices in the market for purchased materials, resources and new equipment. However, these factors are distinguished by their indefinite nature of change in the considered period of time, and therefore, one can only use the forecast of their admissible sets of values: $q_i(t) \in [\underline{q}_i, \overline{q}_i]$. As a result, we obtain an investment management system in which there are external perturbations, i.e. system

$$\frac{d\mathrm{K}_i(t)}{dt} = -a_i \mathrm{K}_i(t) + v_i u(t) + q_i(t), t \in T, \mathrm{K}_i(t_0) = \mathrm{K}_i^0, \sum_{i=1}^{n} v_i u(t) \le M .$$

Since the total amount of input investments is limited by the condition $\sum_{i=1}^{n} v_i u(t) \le M$, then $u(t) \in U(v) = [0, m(v)], m(v) = M / \sum_{i=1}^{n} v_i$. The goal of system management is to maximize the total states of the production assets of industries at the end of the period. To evaluate this criterion, one can use the terminal functional

$$J(u,v)=\sum_{i=1}^{n}\max_{\gamma_i\in[\underline{\gamma}_i,\bar{\gamma}_i]}\min_{q_i\in[\underline{q}_i,\bar{q}_i]}\gamma_i K_i(t_1,u,v_i,q_i).$$

Then the problem of optimal distribution of investments is reduced to the following problem:

$$\sum_{i=1}^{n}\min_{q_i\in[\underline{q}_i,\bar{q}_i]}\max_{\gamma_i\in[\underline{\gamma}_i,\bar{\gamma}_i]}\gamma_i K_i(t_1,u,v_i,q_i)\to\max, u\in[0,m(v)], v_i\in[\underline{v}_i,\bar{v}_i].$$

This problem is equivalent to a problem of the form (4):

$$\sum_{i=1}^{n}\max_{q_i\in[\underline{q}_i,\bar{q}_i]}\min_{p_i\in[\underline{p}_i,\bar{p}_i]}p_i K_i(t_1,u,v_i,q_i)\to\min, u\in[0,m(v)], v_i\in[\underline{v}_i,\bar{v}_i],$$ где

$\underline{p}_i=-\bar{\gamma}_i,\ \bar{p}_i=-\underline{\gamma}_i$.

To solve this problem, we apply the algorithm proposed above. Fundamental matrix of the system $\frac{dK_i(t)}{dt}=-a_iK_i(t), t\in T$, has a form:

$$F(t,\tau)=\begin{pmatrix} e^{-a_1(t-\tau)} & 0 & \dots & 0\\ 0 & e^{-a_2(t-\tau)} & \dots & 0\\ \dots & \dots & \dots & \dots\\ 0 & 0 & \dots & e^{-a_n(t-\tau)}\end{pmatrix}.$$

Considering that the initial set consists of one point

$$K^0=(K_1^0,K_2^0,...,K_n^0), \text{ и } b(t,u,v)=\{b: b=(b_1,...,b_n), b_i=v_iu+q_i(t), q_i(t)\in[\underline{q}_i,\bar{q}_i]\},$$

we define a function $\eta(p)$ of the form (6). We have:

$$\eta(p)=\sum_{i=1}^{n}K_i^0e^{-a_i(t_1-t_0)}p_i+\min_{v_i\in[\underline{v}_i,\bar{v}_i]}\frac{M}{\sum_{s=1}^{n}v_s}\sum_{i=1}^{n}(e^{-a_i(t_1-t_0)}-1)v_ip_i+\sum_{i=1}^{n}\underline{q}_i(e^{-a_i(t_i-t_0)}-1)p_i=$$

$$=\sum_{i=1}^{n}\{\underline{q}_i-(K_i^0+\underline{q}_i)e^{-a_i(t_1-t_0)}\}\gamma_i+\min_{v_i\in[\underline{v}_i,\bar{v}_i]}\frac{M}{\sum_{s=1}^{n}v_s}\sum_{i=1}^{n}(1-e^{-a_i(t_1-t_0)})v_i\gamma_i.$$

Let $\underline{q}_i-(K_i^0+\underline{q}_i)e^{-a_i(t_1-t_0)}\ge 0, i=\overline{1,n}$. Then it can be shown that the global minimum of the function $\eta(p)$ on $P=\{p=(p_1,...,p_n): \underline{p}_i\le p_i\le\bar{p}_i, i=\overline{1,n}\}$ is reached at the point $p^*=-(\bar{\gamma}_1,...,\bar{\gamma}_n)$. Now let's define optimal value $v^*$ of the parameter $v=(v_1,...,v_n)$, as a solution to a problem:

$$\frac{M}{\sum_{s=1}^{n}v_s}\sum_{i=1}^{n}(e^{-a_i(t_1-t_0)}-1)v_i\bar{\gamma}_i\to\min, \underline{v}_i\le v_i\le\bar{v}_i, i=\overline{1,n}.$$

Then we get the optimal distribution of investments:

$$R_i^*=\frac{M}{\sum_{s=1}^{n}v_s^*}v_i^*, i=\overline{1,n}.$$

**Conclusion**

The studied nonsmooth optimization problem (4) is of direct interest for further studies of similar models of optimal control problems. The results obtained can be generalized for analogous discrete dynamic control systems. And this allows the study of models of multi-step control processes, which take into account the limited and incomplete information on external uncontrolled parameters.

The proposed algorithm for solving the considered problem is a step-by-step application of necessary and sufficient optimality conditions. According to this scheme, the solution of an infinite-dimensional non-smooth optimization problem can be obtained after performing several steps of finite-dimensional optimization.

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