A new, general model for radial transport of plasma, angular momentum and energy in the magnetospheres of Jupiter and Saturn

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Abstract

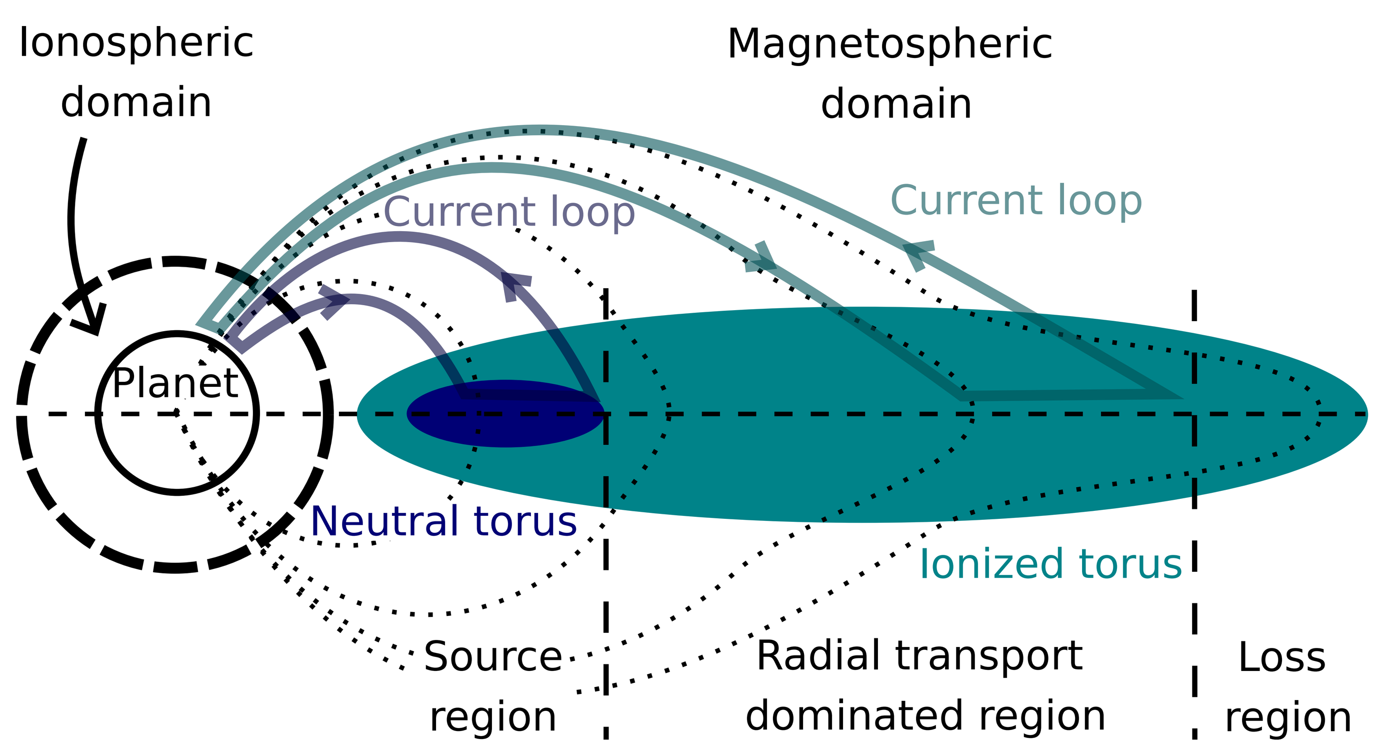
A self-consistent modelling of radial transport in the magnetospheres of gas giants requires a full description of the interactions between the plasma disk, the magnetosphere and the upper atmosphere. We introduce a new general model describing radial transport of plasma, angular momentum and energy in those magnetospheres, taking into account the thickness of the disk and extended source and sink regions. It can be applied to the analysis of the incoming Juno and future JUICE data.

I. Context and objectives

We aim at building a **self-consistent model of radial transport** in the magnetospheres of gas giants, combining two historical approaches of radial transport : the angular momentum transport based on magnetosphere-ionosphere coupling, and the interchange-instability-driven diffusion of mass and energy. The giant planet system is separated into two main domains :

- the ionospheric domain, to which only the ionospheric Ohm's law is applied
- the magnetospheric domain, described by
  - **ideal MHD equations**, including source (pick up) and loss terms
  - plasma state equations : quasi-neutrality and perfect gas law

Only the inner and middle magnetosphere is modeled, which allows us to assume **axisymmetry**. We base our derivation on the quasi-linear theory : all quantities are separated into an average value, indexed 0, and a departure from the mean state, labeled with δ.



II. Equilibrium

Considering only the 0^{th} order terms, we study the projection of the fundamental equations along a field line and in a meridian plane to derive the equilibrium state of the magnetosphere.

Projection along a field line : Distribution of the plasma, composed of four species i, two ion (light, heavy) and two electron (cold, warm) species :

$$\frac{\partial n_{i0}}{\partial s} = \left(\frac{Z_i e}{kT_i}E_{0\parallel} + \frac{m_i}{kT_i}\frac{\partial \Psi_g}{\partial s}\right) n_{i0} \qquad (1)$$

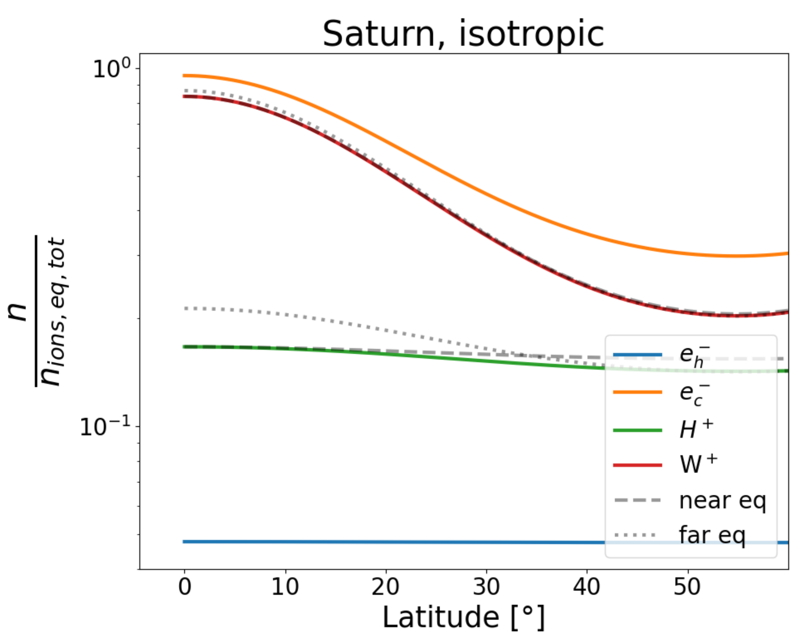
$$\left(\sum_i \frac{n_{i0}Z_i^2}{kT_i}\right) eE_{0\parallel} = -\left(\sum_i \frac{Z_i m_i n_{i0}}{kT_i}\right)\frac{\partial \Psi_g}{\partial s} \qquad (2)$$

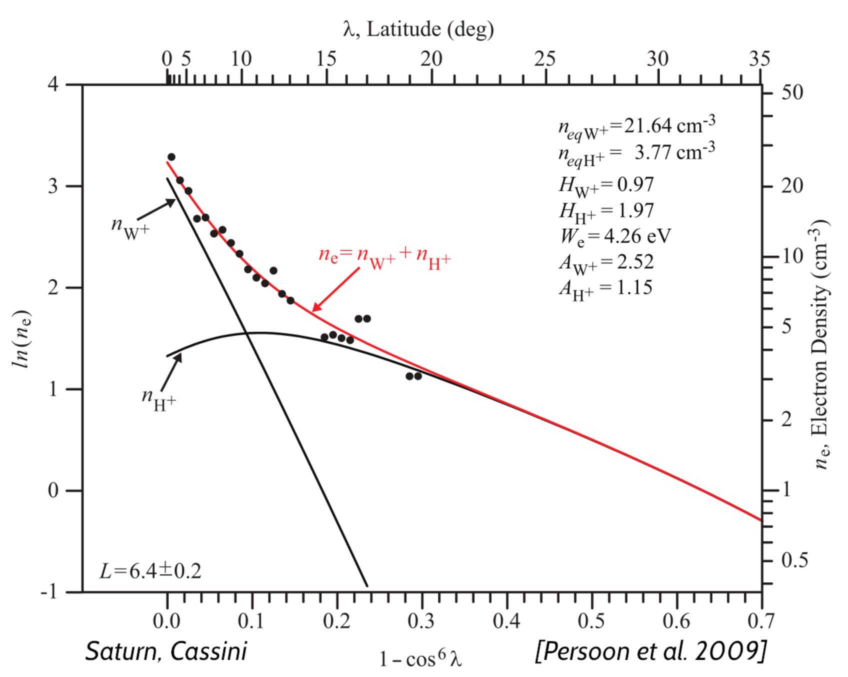
where Ψ_g is the combined gravitational and centrifugal potential.

Projection onto a meridian plane : Topology of the magnetic field

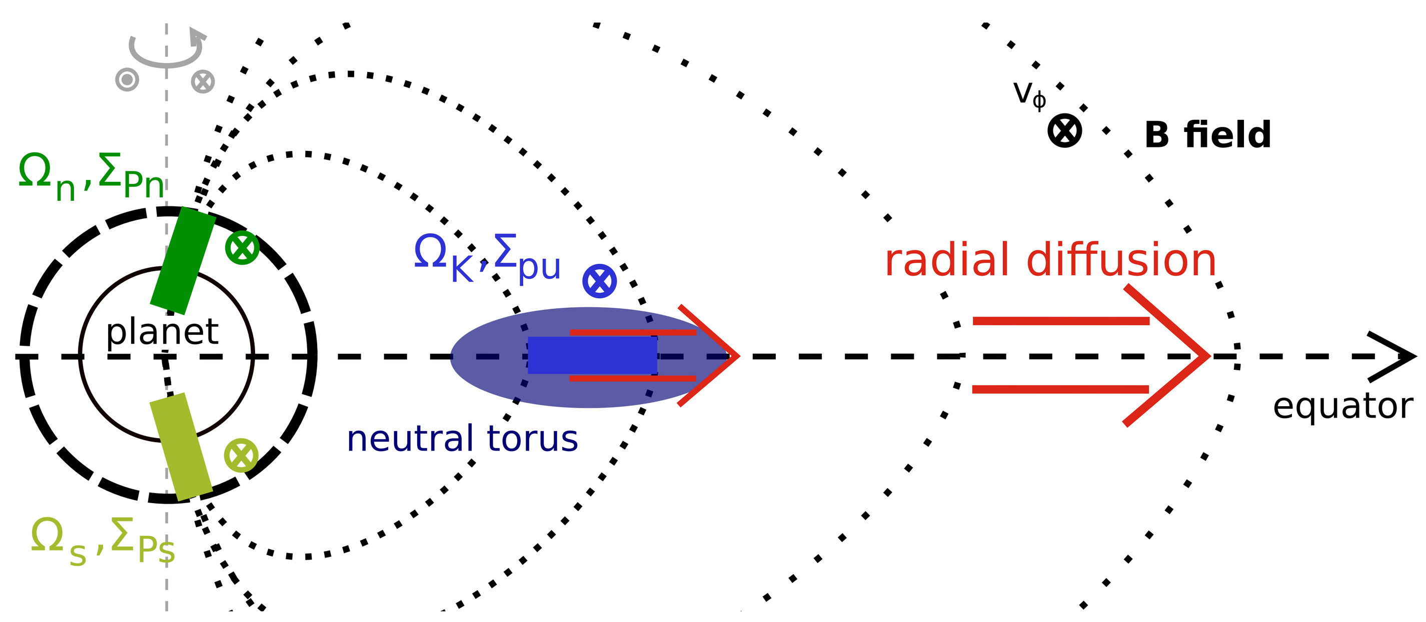
Grad-Shafranov equation : $\frac{\partial^2 \alpha}{\partial r^2} + \frac{1-\mu^2}{r}\frac{\partial^2 \alpha}{\partial \mu^2} = -g$, with the source fonction g expressed for **two ion species** k under the two plasma approximation [1]:

$$g = -\mu_0 \frac{R^2}{R_P^2} \sum_{k=l \text{ or } h} \left(\frac{\partial P_{k0,eq}}{\partial \alpha} + \frac{P_{k0,eq}R_P}{l_k^2 B_{0\,\theta,eq}}\right) \exp\left(\frac{R^2 - R_{eq}^2}{2l_k^2}\right) \qquad (3)$$





III.A. Transport : angular momentum



The equilibrium state is perturbed to second order to derive transport equations for the flux tube contents, based on the **interchange instability theory**. The azimuthal projection of the perturbed equations yields the transport of angular momentum **integrated over the thickness of the disk**, from **exchanges between the magnetosphere and the upper atmosphere**.

$$\dot{\mathcal{M}}_\perp \frac{\partial \Omega}{\partial \alpha} + 2\dot{\mathcal{M}}_{R/\alpha}\Omega = 4\pi R_P \alpha \sum_{k=n,s} \cos\theta_{i,k}\frac{\Sigma_{Pk}}{\sin(I_k)^2}(\Omega_{n,k} - \Omega) + 2\pi B_{0,eq}R_{eq}^2 \Sigma_{pu}(\Omega - \Omega_K) \qquad (4)$$

where $\dot{\mathcal{M}}_\perp = \int 2\pi \frac{R^2}{h_\alpha}\langle \delta\rho\delta v_\alpha \rangle \frac{ds}{B_0}$ and $\dot{\mathcal{M}}_{R/\alpha} = \int 2\pi R \langle \delta\rho \delta v_R \rangle \frac{ds}{B_0}$.

III.B. Thin disk : closed set of transport equations

Projecting the MHD equations onto the equatorial plane results in transport equations for the mass and energy contents of flux tubes. In the simplified case of a **thin disk**, a **closed set of transport equations** for the integrated tube contents is obtained, using results from the literature.

Generalized Hill-Pontius equation for the angular momentum [3], [4] :

$$\frac{R_{eq}}{2}\frac{\partial \Omega}{\partial R_{eq}} + \Omega = \sum_{k=n,s} \frac{2\pi R_P \alpha B_{0,eq}}{\dot{M}} \cos(\theta_{i,k}) \frac{\Sigma_{Pk}}{\sin(I_k)^2}(\Omega_{n,k} - \Omega) + \frac{2\pi R_{eq}^2 B_{0,eq}^2}{\dot{M}}\Sigma_{pu}(\Omega - \Omega_K) \qquad (5)$$

Diffusion of the mass content [5] :

$$\dot{M} = -2\pi R_P^5 \frac{D_{LL}}{R_{eq}^2}\frac{\partial}{\partial R_{eq}}\left(\frac{\rho_{0,eq}H}{mB_{0,eq}}\right) \qquad (6)$$

Diffusion of the energy content [6] :

$$R_{eq}^2 \frac{\partial}{\partial R_{eq}}\left(\frac{D_{LL}}{R_{eq}^2}\frac{\partial}{\partial R_{eq}}\left(2\pi R_P P_{0,eq} V_0^\gamma\right)\right) = -\mathcal{S}_{q0} + \mathcal{L}_{q,0} \qquad (7)$$

IV. Results and perspectives

1. We generalize the Hill-Pontius equation for angular momentum transport to the case of a thick disk including source regions of arbitrary distribution. The resulting equation allows one to study the effects of distributed plasma sources and sinks, including their temporal variability, and asymmetries between the two conjugate ionospheres.
2. We provide a closed set of transport equations for the integrated mass, angular momentum and energy flux tube contents in the limit of a thin equatorial disk. Our equations can be applied to the interpretation of past, present (Juno) and future observations (JUICE).

Further work will allow us to generalize this set of equations to the case of a **thick disk** and to **link transport equations to equilibrium equations**.

References

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