Weak Nonlocality in Elementary Particle Physics

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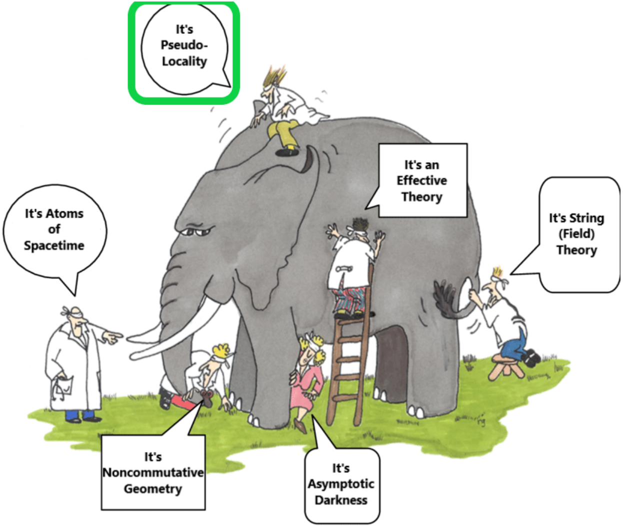
Weakly Nonlocal Theory ⇒ Interpolates btw UV Nonlocal Theory & IR Local QFT



1. Introduction – Weakly Nonlocal QFT

Weakly Nonlocal QFT

Nonlocal Form Factor $\gamma(z) \equiv$ Analytic Function with Regular Taylor Expansion in IR Limit $z = \Box = 0$

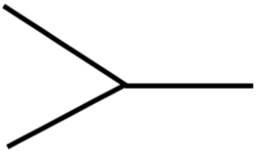
Selected Reviews on Weak Nonlocality:

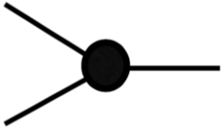
- Toy Scalar QFT's: [Buoninfante, Lambiase, Mazumdar, Nucl.Phys.B 944 (2019) 114646]
- Gravity & Gauge QFT's:
  - Tomboulis, Mod.Phys.Lett.A 30 (2015) 03n04, 1540005
  - Modesto, Rachwal, Nucl.Phys.B 900 (2015) 147-169
  - Modesto, Rachwal, Int.J.Mod.Phys.D 26 (2017) 11, 1730020
  - Bas i Beneito, Calcagni, Rachwal (2022), arXiv:2211.05606
- String Field Theory: [Erbin, Lect.Notes Phys. 980 (2021) 1-421]
- Nonlocal Black Holes: [Buoninfante, Giacchini, de Paula Netto (2022), arXiv:2211.03497]

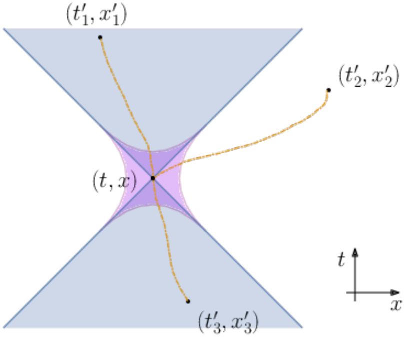
1. Introduction – Contents

2. Why Weak Nonlocality? – Archaeology

- End 1920s ⇒ Birth of Local QFT + Problem of ∞'s
- Beginning 1930s ⇒ 1st Attempts to Regularize UV ∞'s by Smearing Interactions
- Weak Nonlocality is Almost as Old as Local QFT → Book on Old Literature:
  [Namsrai, *Nonlocal QFT and Stochastic Quantum Mechanics*, Springer Dordrecht (1986)]
- BUT Clash btw UV Finiteness & Gauge Invariance!
  [Chretien, Peierls, Proc.Roy.Soc.Lond.A 223 (1954) 1155, 468-481]

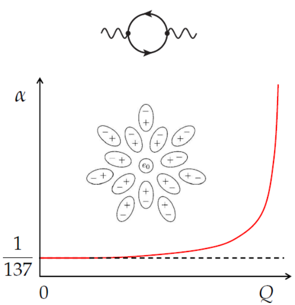


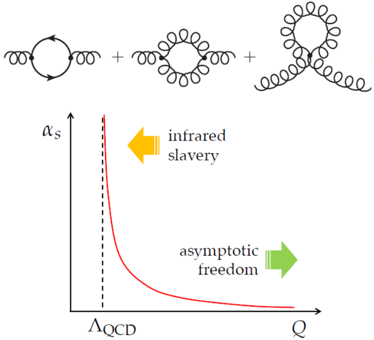


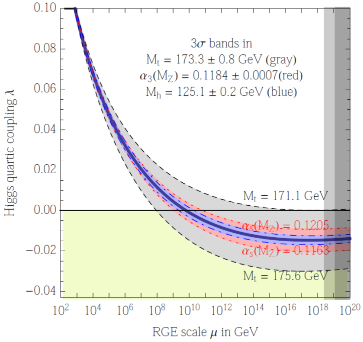


2. Why Weak Nonlocality? – Asymptotic Safety

- Local QFT ⇒ SM of Particle Physics
  - Lorentz-Poincaré Invariance (Special Relativity) + Unitarity (Quantum Mechanics)
  - Gauge Symmetries + Higgs Mechanism.
  - Perturbative Renormalizability ⇒ Finite Number of Free Parameters
- BUT Landau Poles + Higgs Instability ⇒ Not Asymptotically Safe!
  - ≠ Quantum Chromodynamics (QCD) ⇒ Asymptotic Freedom
  - Total Asymptotic Freedom [Giudice, Isidori, Salvio, Strumia, JHEP 02 (2015) 137]





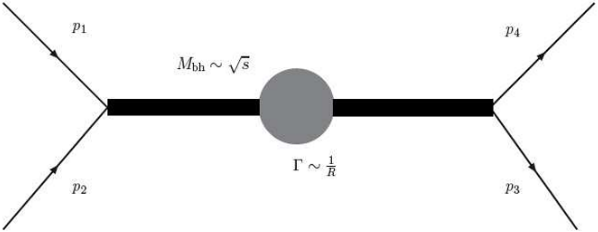


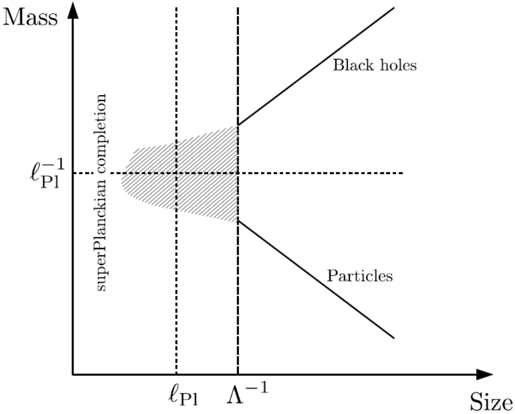
2. Why Weak Nonlocality? – Quantum Gravity

Quantum Gravity in Local QFT:

- Drawbacks for Perturbative Renormalizability + Unitarity!
- Black Holes: UV/IR Mixing + Information Paradox ⇒ Nonlocality? [Giddings, Porto, Phys.Rev.D 81 (2010) 025002]

⇒ Going Beyond Local QFT?
e.g. String (Field) Theory / Something Simpler?





3. Nonlocal Scalar Field

3. Nonlocal Scalar Field – Toy Model

- Nonlocal Toy Scalar Model:

$$S = \int d^4x \left[\frac{1}{2} \phi(x) \gamma(\Box) \phi(x) - V(\phi) \right]$$

- Weakly Nonlocal Form Factor:

$$\gamma(\Box) = \sum_{n=0}^{\infty} c_n \Box^n$$

- Nonlocality Manifest via Delocalization Kernel:

$$\phi(x)\gamma(\Box)\phi(x) = \int d^4y \phi(x) K(y-x) \phi(y)$$

- Euler-Lagrange Equation:

$$\frac{\delta S}{\delta \phi(x)} = 0 \quad \Longrightarrow \quad \gamma(\Box)\phi(x) - \frac{dV}{d\phi(x)} = 0$$

N.B: Free Field $\Rightarrow$ Local Theory

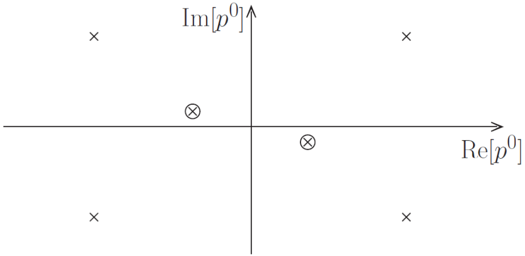
- Truncated Sum ⇒ Higher-Derivative Theory:

$$\mathcal{L}_\phi = -\frac{1}{2}\phi\left[\left(\frac{\Box}{M^2}\right)^2 + 1\right]\left(\Box + m^2\right)\phi - \lambda \sum_{n=4}^{N} \frac{c_n}{n!}\phi^n\,, \quad N \in \mathbb{N}$$

- Propagator with Complex Conjugate Poles:

$$iD(p^2, m^2, \epsilon) = \frac{iM^4}{(p^2 - m^2 + i\epsilon)((p^2)^2 + M^4)}$$

- Problems with Causality/Unitarity/Stability ⇒ No Consensus on Higher-Derivative Theories [Platania, JHEP 09 (2022) 167]



3. Nonlocal Scalar Field – Ghost Problem (2/2)

- No Ghosts in EFT's!!! Weinberg's Footnote [Weinberg, Phys.Rev.D 77 (2008) 123541]:

$$\mathcal{L} = -\frac{1}{2}[\partial_\mu\varphi\partial^\mu\varphi + m^2\varphi^2 + M^{-2}(\Box\varphi)^2] + J\varphi$$

where $M \gg m$ is some very large mass, and J is a c-number external current. We can easily find the connected part Γ of the vacuum persistance amplitude:

$$\Gamma = i\int d^4k \frac{|J(k)|^2}{k^2 + m^2 + k^4/M^2} \; .$$

If we took this result seriously, then we would conclude that in addition to the usual particle with mass $m + O(m^3/M^2)$, the theory contains an unphysical one particle state with mass $M + O(m^2/M)$. But if we regard $\mathcal{L}$ as just the first two terms in a power series in $1/M^2$, then we must treat the term $M^{-2}(\Box\varphi)^2$ as a first-order perturbation, so that the vacuum persistence amplitude is

$$\Gamma = i\int d^4k |J(k)|^2 \left[\frac{1}{k^2 + m^2} - \frac{k^4}{M^2(k^2+m^2)^2} + \ldots\right] ,$$

and the only pole is at $k^2 = -m^2$. This is just the same result for Γ that we would find if we were to eliminate the second time derivatives in the $O(M^{-2})$ term in $\mathcal{L}$ by using the field equation derived from the leading term in the Lagrangian

$$\Box\varphi = m^2\varphi - J \; .$$

In this case the effective Lagrangian becomes

$$\mathcal{L} = -\frac{1}{2}[\partial_\mu\varphi\partial^\mu\varphi + m^2\varphi^2 + m^4M^{-2}\varphi^2] + (1 + m^2/M^2)J\varphi - J^2/2M^2 \; .$$

Original Argument: [Simon, Phys.Rev.D 41 (1990) 3720]

3. Nonlocal Scalar Field – Ghost-Free Form Factors (1/5)

- Ghost-Free Infinite-Derivative Form Factor (Nonlocal Scale M):

$$\gamma(\Box) = (\Box - m^2)e^{H(\Box)}$$

$H(\Box)$ is an Entire Function with $H(0) = 1$; Euclidean Bare Propagator:

$$-\frac{e^{-H(-k^2)}}{k^2 + m^2}$$

- Wataghin Form Factor (String-Inspired) [Wataghin, Z.Phys. 88 (1934) 92-98]:

$$H_{\text{Wat}} = -\frac{\Box}{M^2}$$

- Krasnikov Form Factor [Krasnikov, Theor.Math.Phys. 73 (1987) 1184-1190]:

$$H_{\text{Kras}} = \frac{\Box^2}{M^4}$$

3. Nonlocal Scalar Field – Ghost-Free Form Factors (2/5)

- Kuz'min Form Factor [Kuz'min, Sov.J.Nucl.Phys. 50 (1989) 1011-1014]:

$$H_{\text{Kuz}}(\Box) = \log m(\Box) + \Gamma\left[0, m(\Box)\right] + \gamma_E \,, \quad m(\Box) = a\left(\frac{\Box}{M^2}\right)^{n_{deg}}$$

  - $m(\Box) \equiv$ Real Monomial ($a \in \mathbb{R}^*$) of Degree $n_{deg} \in \mathbb{N}^*$
  - Upper Incomplete Gamma Function / Exponential Integral:

$$\Gamma(0, z) = \int_z^{\infty} dx \frac{e^{-x}}{x}$$

  - Asymptotically Monomial in Conical Region:

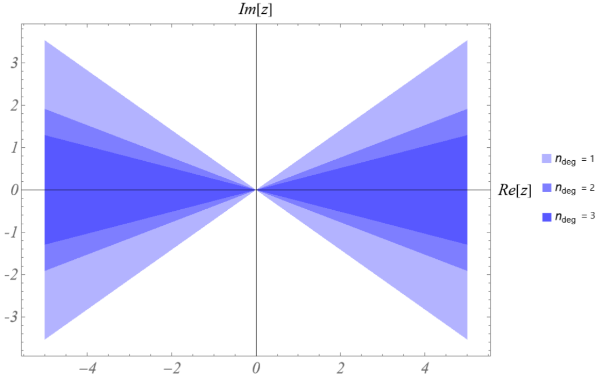
$$e^{\mathrm{H}(z)} \overset{|z|\to\infty}{\simeq} |z|^{n_{\text{deg}}} \qquad z \in \mathcal{C}, \quad -\theta < \arg(z) < \theta$$

- Tomboulis Form Factor: [Tomboulis (1997), arXiv:hep-th/9702146]

$$H_{\text{Tom}}(\Box) = \frac{1}{2}\left[\log p^2(\Box) + \Gamma\left[0, p^2(\Box)\right] + \gamma_E\right] \; ; \; \text{Polynomial } p(z) \text{ of Degree } n_{deg} \in \mathbb{N}^*$$

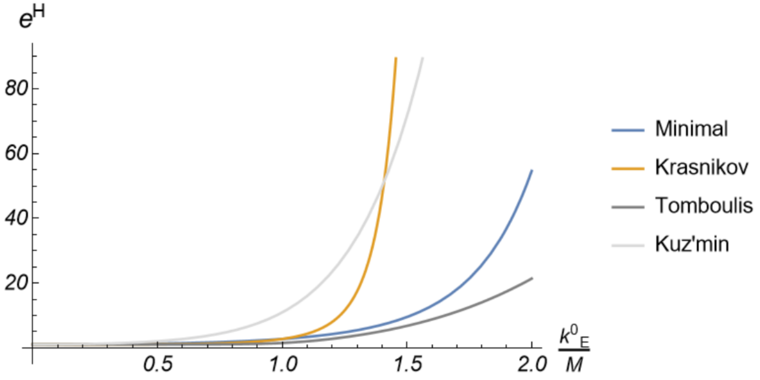
Asymptotically Polynomial in Conical Region:

$$\mathcal{C} = \{z| \quad -\theta < \arg(z) < \theta \quad \cup \quad \pi - \theta < \arg(z) < \pi + \theta\}, \qquad \theta = \frac{\pi}{4n_{\text{deg}}}$$



3. Nonlocal Scalar Field – Ghost-Free Form Factors (4/5)

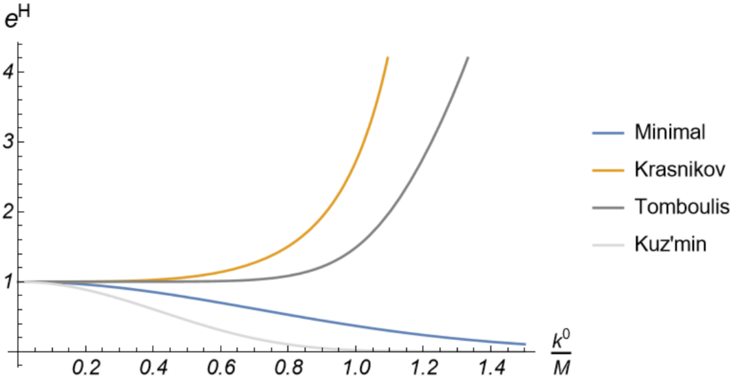
Form Factors vs Euclidean Energies k_E^0:



Minimal $\equiv$ Wataghin; Kuz'min ($n_{deg} = 3$); Tomboulis ($n_{deg} = 2$)

$\Rightarrow$ All Form Factors Dampen Propagators in UV!

Form Factors vs Minkowskian Energies k^0:

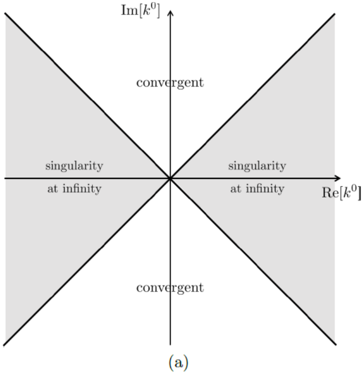


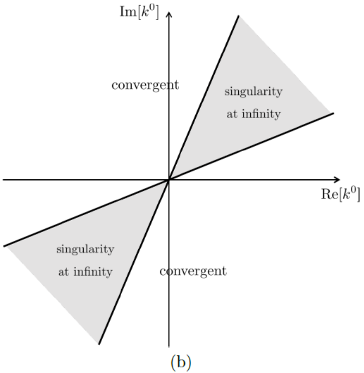
⇒ Wataghin (Minimal) & Kuz'min ($n_{deg} = 3$) Form Factors Make Propagators Blow Up in UV!

⇒ Only Krasnikov & Tomboulis ($n_{deg} = 2$) Form Factors Dampen Propagators in UV!

3. Nonlocal Scalar Field – Analyticity & Unitarity (1/2)

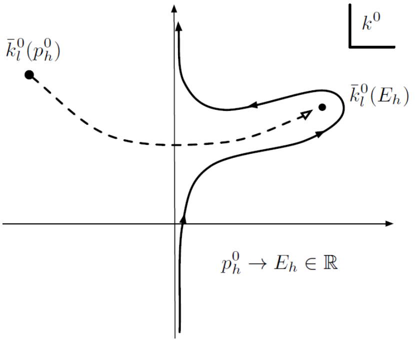
- Review: [Buoninfante, Phys.Rev.D 106 (2022) 12, 126028]
- Essential Singularity at Complex Infinity ⇒ Wick Rotation Not Defined!
- Form Factors Blow Up for Large Real Energies ⇒ Perturbative Unitarity Lost [Koshelev, Tokareva, Phys.Rev.D 104 (2021) 2, 025016]

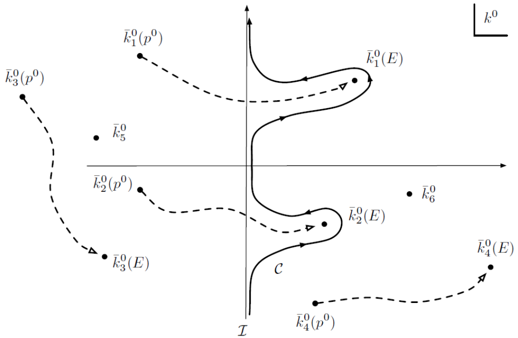




(a) Wataghin Form Factor $e^{-\Box}$ (b) Krasnikov Form Factor $e^{\Box^2}$

- Efimov Analytic Continuation ⇒ Only External Momenta (Euclidean Starting point) [Efimov, Sov.J.Nucl.Phys. 4 (1967) 2, 309-315]





- Cutkosky Rules [Briscese, Modesto, Phys.Rev.D 99 (2019) 10, 104043]:

$$\frac{i\, e^{-H(p^2)}}{p^2 - m^2 + i\epsilon} \quad \longrightarrow \quad (-2\pi i)\, \sigma(p^0)\, \delta(p^2 - m^2 + i\epsilon)$$

3. Nonlocal Scalar Field – Higgs Mechanism & Naturalness (1/2)

- Naturalness of Higgs Boson Mass [Biswas, Okada, Nucl.Phys.B 898 (2015) 113-131]
- Toy ϕ^4 Theory (Klein-Gordon Mass):

$$S = \int d^4x \left[-\frac{1}{2}\phi e^{\frac{\Box+m^2}{M^2}} (\Box + m^2)\phi - \frac{\lambda}{4!}\phi^4 \right]$$

- Radiative Corrections to ϕ-mass (External Momenta $p^2 = -m^2$):

$$\delta m^2 = i\Gamma_2 = \frac{\lambda}{32\pi^2} \left[e^{-\frac{m^2}{M^2}} + \left(\frac{m^2}{M^2} \right) Ei \left(-\frac{m^2}{M^2} \right) \right] M^2$$

$$M^2 \gg m^2 \quad \Rightarrow \quad \delta m^2 \simeq \frac{\lambda}{32\pi^2} M^2$$

- Terascale Fuzziness $\Rightarrow$ $M = \mathcal{O}(1)$ TeV $\Rightarrow$ Electroweak Scale Stabilized?

- Ghost-Free Higgs Mechanism Is Tricky [Hashi, Isono, Noumi, Shiu, Soler, JHEP 08 (2018) 064]
- Tachyon Field ⇒ Ghost-Free in Tachyonic Vacuum (Unstable):

$$S = \int d^4x \left[\frac{1}{2} \phi \, e^{-\frac{\Box}{M^2}} (\Box + \mu^2) \phi - \frac{1}{3} g \phi^3 \right] \quad \text{with} \quad \mu^2 > 0$$

- Spinless Boson ⇒ Not Ghost-Free in Physical Vacuum:

$$S = \int d^4x \left[\frac{1}{2} \varphi e^{-\frac{\Box}{M^2}} (\Box + \mu^2) \varphi - \mu^2 \varphi^2 - \frac{1}{3} g \varphi^3 \right] .$$

The propagator of φ reads

$$\Pi_\varphi(k) = \left[e^{\frac{k^2}{M^2}} (k^2 - \mu^2) + 2\mu^2 \right]^{-1} .$$

⇒ No Ghost-Free Spectrum Anymore!

4. Pseudolocality

4. Pseudolocality – Local Higher-Derivative Theories (1/4)

- Stelle Quadratic Gravity [Stelle, Phys.Rev.D 16 (1977) 953-969]:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right)$$

$\Rightarrow$ Renormalizable in $D = 4$ (Euclidean) BUT 1 Ghost

- Higher-Derivative Gravity [Asorey, Lopez, Shapiro, Int.J.Mod.Phys.A 12 (1997) 5711-5734]:

$$S_{local} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[R + \sum_{n=0}^{n_{\text{deg}}-1} R a_n \Box^n R + \sum_{n=0}^{n_{\text{deg}}-1} R_{\mu\nu} b_n \Box^n R^{\mu\nu} \right]$$

Superficial Degree of Divergence:

$$\omega(\mathcal{F}) = 4 - 2(n_{\text{deg}} - 1)(L - 1)$$

$n_{deg} > 3 \Rightarrow$ Only 1-loop UV $\infty \Rightarrow$ Super-Renormalizable BUT Ghosts.

- Possibility to get a UV Fixed Point [Fradkin, Tseytlin, Phys.Lett.B 104 (1981) 377-381]

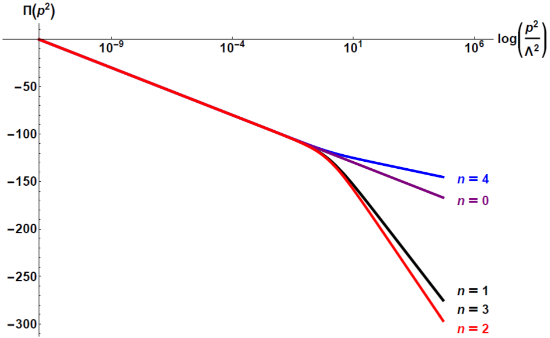
Local QFT of Gravity $\Rightarrow$ Ghost Problem!

- Eulidean Higher-Derivative Yang-Mills [Asorey, Falceto, Rachwal, JHEP 05 (2021) 075]:

$$S = \frac{1}{4g^2} \int d^4x \, F^a_{\mu\nu} F^{\mu\nu\,a} + \frac{1}{4g^2 \Lambda^{2n}} \int d^4x \, F^a_{\mu\nu} \, \Delta^n \, F^{\mu\nu\,a}$$

$$\Delta^{\nu\,b}_{\mu\,a} = -D^2 \delta^\nu_\mu \delta^b_a + 2 f^b{}_{ca} \, F_\mu{}^{\nu c}$$

- $2 \leq n \leq 4 \Rightarrow$ Superrenormalizable + Asymptotic Freedom (Only 1-loop UV ∞)



How to Get Rid of those Ghosts?

4. Pseudolocality – Nonlocal Exorcism (1/3)

4. Pseudolocality – Nonlocal Exorcism (2/3)

- Weakly Nonlocal QFT's with Ghost-Free Form Factor:

$$\boxed{\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4g^2_{\mathrm{YM}}}\left[\mathrm{tr}\, F e^{H(-\mathcal{D}^2_\Lambda)} F + \mathcal{V}_{\mathrm{YM}}\right]} \quad (\mathrm{GAUGE-THEORY})$$

$$\boxed{\mathcal{L}_{\mathrm{gr}} = -2\kappa_D^{-2}\sqrt{|g|}\left[R - \frac{1}{2}R\,\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R + R_{\mu\nu}\,\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R^{\mu\nu} + \mathcal{V}_{\mathrm{gr}}\right]} \quad (\mathrm{GRAVITY})$$

- Original Articles:
  - Kuz'min, Sov.J.Nucl.Phys. 50 (1989) 1011-1014
  - Tomboulis (1997), arXiv:hep-th/9702146
  - Modesto, Phys.Rev.D 86 (2012) 044005
  - Modesto, Rachwal, Nucl.Phys.B 889 (2014) 228-248
  - Modesto, Piva, Rachwal, Phys.Rev.D 94 (2016) 2, 025021

- Exponential Form Factors (Wataghin & Krasnikov) $\Rightarrow$ Violate Power Counting Theorem!

- Asymptotically Polynomial Form Factors (Kuz'min & Tomboulis) $\Rightarrow$ Superrenormalizability!

- Theory Interpolates btw 2 local QFT's (deep IR & deep UV) via a Nonlocal Window

4. Pseudolocality – Nonlocal Exorcism (3/3)

- Renormalization of Weakly Nonlocal Yang-Mills QFT [Tomboulis (1997), arXiv:hep-th/9702146]:

$$\mathcal{L}_R = -\frac{1}{2g^2}\mathrm{tr}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \frac{\alpha}{2}\mathrm{tr}\mathbf{F}_{\mu\nu}h(-\frac{\mathbf{D}^2}{\Lambda^2})\mathbf{F}^{\mu\nu} - \frac{1}{2g^2}(Z_3 - 1)\mathrm{tr}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}$$

- Ghost-Free Form Factor (α Is Not Renormalized):

$$\overline{h}(z) \equiv 1 + g^2 \alpha h(z)$$

- No Ghosts $\Rightarrow$ Renormalization Condition at Scale $\mu_0 \sim \Lambda$ with $\alpha = 1/g(\mu_0)^2$.
- Dressed Propagator $\Rightarrow$ ∞-Tower of CC Poles? [Shapiro, Phys.Lett.B 744 (2015) 67-73]:

$$-\,i\,\frac{e^{-H(k^2)}}{k^2\left(1+\beta_\alpha\, e^{-H(k^2)}\log(k^2/\mu_0^2)\right)}$$

- Naive Guess for Matter & Higgs Fields [Modesto, Rachwal, Nucl.Phys.B 900 (2015) 147-169]:

$$\sum_a^{N_f} \bar{\psi}_a\, i\not{D}_a e^{H(-\not{D}^2_{a,\Lambda})}\,\psi_a + (\mathcal{D}_\mu\Phi)^\dagger e^{H(-\mathcal{D}^2_\Lambda)}(\mathcal{D}^\mu\Phi) - \mu^2\Phi^\dagger e^{H(-\mathcal{D}^2_\Lambda)}\Phi - \lambda(\Phi^\dagger\Phi)^2$$

BUT Not Ghost-Free in Physical Vacuum! [Hashi, Isono, Noumi, Shiu, Soler, JHEP 08 (2018) 064]

4. Pseudolocality – Matter & Higgs Sectors

- Pseudolocal Recipe from a Mother Local QFT (Minimal Nonlocality):
  - Modesto (2021), arXiv:2103.04936
  - Modesto, Calcagni, JHEP 10 (2021) 169

$$
\begin{aligned}
S(\Phi_i) &= \int d^D x \sqrt{|g|} \left[\mathcal{L}_{\rm loc} + E_i \, F_{ij}(\Delta) \, E_j + \mathcal{V}(E) \right] , \\
S_{\rm loc} &= \int d^D x \sqrt{|g|} \, \mathcal{L}_{\rm loc} \, , \qquad \mathcal{L}_{\rm loc} = \frac{1}{2\kappa^2} R + \mathcal{L}_{\rm m} \, , \\
E_i(x) &:= \frac{\delta S_{\rm loc}}{\delta \Phi_i(x)} \, , \\
\Delta_{ki} &:= \frac{\delta E_i}{\delta \Phi_k} = \frac{\delta^2 S_{\rm loc}}{\delta \Phi_k \delta \Phi_i} \, ,
\end{aligned}
$$

$$
2 \Delta_{ik} F_{kj}(\Delta) \equiv \left[e^{\mathrm{H}(\Delta)} - 1 \right]_{ij} \, , \qquad \mathcal{E}_k = \left[e^{\mathrm{H}(\Delta)} \right]_{kj} E_j + O(E^2) = 0
$$

- Classical Solutions of Local Theory $\Rightarrow$ Solutions of Weakly Nonlocal Theory!
- Tree-Duality $\Rightarrow$ Perturbative Spectrum + Tree-Level Scat. Amplitudes $\equiv$ Mother Local QFT!
- Ghost-Free Higgs Mechanism: [Modesto, JHEP 06 (2021) 049]

5. Epilogue

5. Epilogue – Summary & Outlook

- Summary:
  - Intrinsic to Gravity + Inspired by String (Field) Theory
  - Weak Nonlocality ⇒ UV Softening Without Ghosts
  - Euclidean → Lorentzian Signature via Efimov Analytic Continuation
  - Gauge Interactions + Gravity ⇒ Asymptotically Polynomial Form Factors
  - Matter + Higgs ⇒ Tree-Duality BUT Detailed Analysis Still Missing...
- Outlook:
  - Which Notion of Causality in a Pseudolocal QFT?
  - Analyticity/Unitarity of Scattering Amplitudes at Loop-Level?
  - Consistency ⇒ Isolate a Unique Form Factor?
  - Other Consistent Weakly Nonlocal Theories? ⇒ Worldline Inversion Symmetry? $t \mapsto 1/t$ [Abel, Dondi, JHEP 07 (2019) 090] ⇒ UV/IR Duality?

$$\Pi(p^2) \;=\; \int_0^\infty dt e^{-T(t)(p^2+m^2)}$$

Modular Invariance in String Theory [Abel, Dienes, Phys.Rev.D 104 (2021) 12, 126032]: