Comparison of Different Ionosphere and Troposphere Models in Open SSR Correction Formats in Terms of Accuracy, Complexity, and Bandwidth

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Abstract

The GNSS State Space Representation (SSR) technology is widely accepted to be the most versatile approach for real-time GNSS corrections. It is employed in several commercial and scientific PPP and PPP-RTK services. Its main advantage over observation space representation (OSR) techniques (e.g., RTK or network RTK) is the intrinsic support for broadcast applications disseminating corrections to an unlimited number of users.

A complete set of SSR corrections consists of the five basic components: clock, orbit, bias, ionosphere, and troposphere corrections for the different GNSS, frequencies, and signals. In a classical OSR service, the lump-sum of these five basic components is computed by the service provider for the user position and sent to the user. This implies that a user does not need to know the underlying models used by the server. In contrast to OSR, an SSR user must compute the influences of the five SSR components itself. For that reason, SSR models are part of an SSR format documentation. The models chosen in different SSR formats are a compromise between target accuracy, complexity, required bandwidth, and computational workload of the rover.

In this conference contribution, we give an overview of different ionosphere and troposphere models used in different open SSR formats. The focus is on SSR formats supporting the high resolution atmospheric corrections (Compact SSR, SPARTN, SSRZ, 3GPP-LPP), but also formats with reduced message sets are addressed (IGS-SSR, RTCM-SSR). We motivate the frequently used multi-stage approach to separate atmospheric corrections into functional (spherical harmonics, polynomials) and residual parts. For the ionosphere, we compare different types of polynomials, vertical and slant TEC, and interpolation heights as well as the advantage of a sun-fixed coordinate frame. For the troposphere, we discuss the advantages and disadvantages of metric vs. relative and slant vs. zenith delay corrections, respectively, and This overview of different ionosphere and troposphere models in SSR formats is intended to help an SSR user to choose a suitable SSR service.

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State Space Representation SSR and Open Formats

SSR Basic Parameters

- SV **clock**
- SV **orbit**
- SV code **bias**
- SV phase **bias**
- **ionosphere**
- **troposphere**
- complete SSR model

State Space Representation SSR and Open Formats

SSR Basic Parameters	Multi-stage/ Scalabilty		RTCM-SSR	IGS SSR (1.0) 4076	SSRZ (1.1) RTCM Geo++ 4090.7	Compact SSR Melco	SPARTN (2.0) Sapcorda	3GPP-LPP (Release 16)	Galileo HAS
SV **clock**	high rate clock		available	available	available	available	available	available	available
	low rate clock		available	available	available				
SV **orbit**			available	available	available	available	available	available	available
SV code **bias**			available	available	available	available	available	available	available
SV phase **bias**			in preparation	available	available	available	available	available	available
ionosphere	global	VTEC	in preparation	available	available		available		
	global	STEC		in preparation	available				
	regional	STEC			available	available	available	available	
	residual	gridded			available	available	available	available	
troposphere	global		in preparation		in preparation				
	regional				available		available	available	
	residual	gridded			available	available	available	available	
complete SSR model			No	no	yes	yes	yes	yes	no

State Space Representation SSR and Open Formats

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SV phase **bias**			in preparation	available	available	available	available	available	available
ionosphere	global	VTEC					available		
	global	STEC							
	regional	STEC				available	available	available	
	residual	gridded				available	available	available	
troposphere	global								
	regional						available	available	
	residual	gridded				available	available	available	
complete SSR model			No	no	yes	yes	yes	yes	no

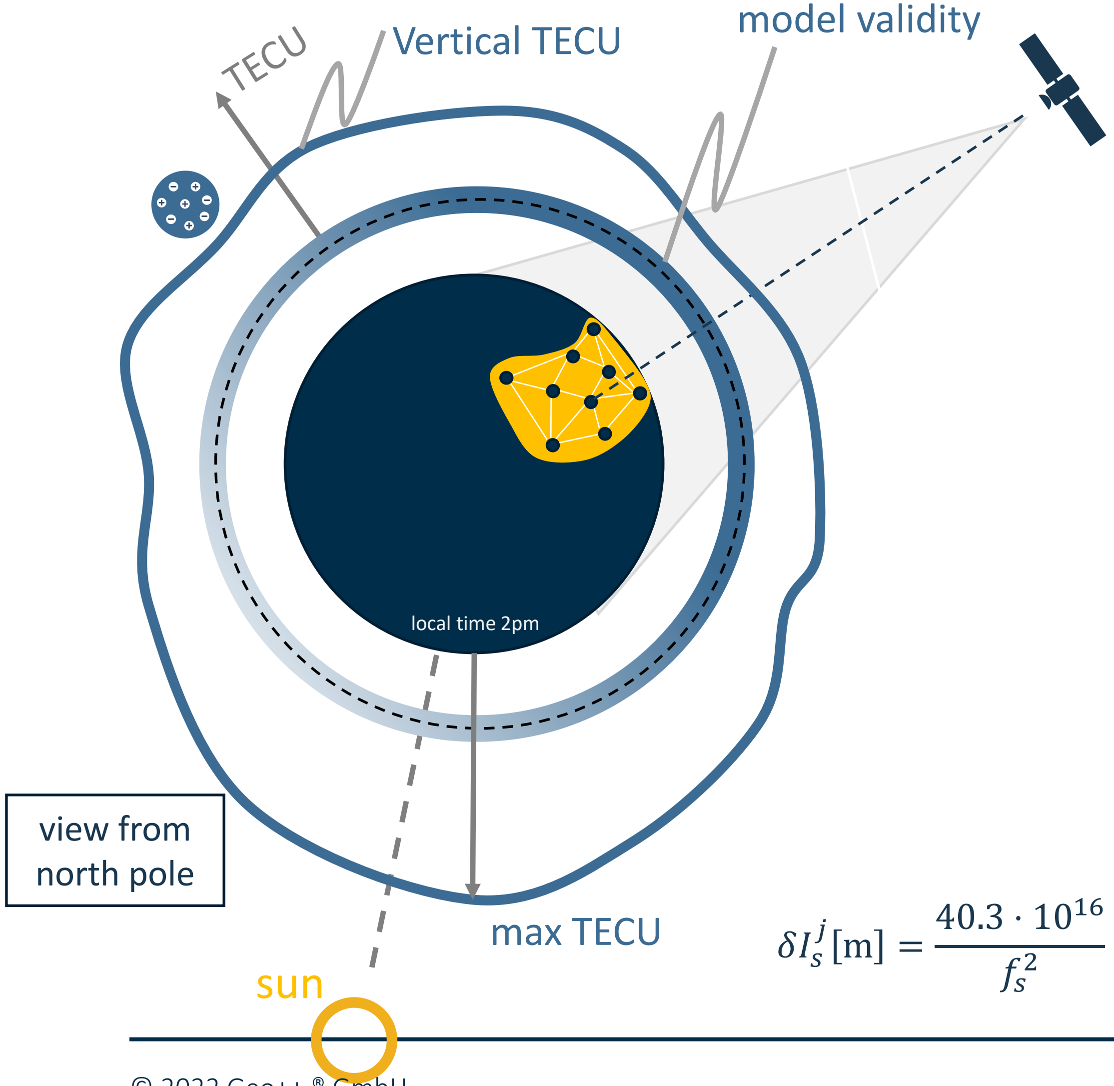
- Sun-fixed vs. Earth-fixed
- STEC vs. VTEC
- algebraic vs. Chebyshev polynomials

- metric δZTD vs. scale factor

Compromises of Service/Format Definitions

- used **SSR** format is **compromise** between
  - **service quality** (decimeter, centimeter, millimeter level)
    - how does resolution of SSR parameters affect the resulting positioning
    - atmospheric model stages (with different resolutions)
  - **memory requirements and computational workload** of the user receiver
    - target user with (low-cost) mass-market receivers need simple formats and models due to low computational power and limited memory
  - **bandwidth optimization** (broadcast or bi-directional communication)
    - usage of dynamic encoding (no fixed message size)
    - keep transmitted values small (e.g. transmit differences to a reference (model))
  - **availability**
    - chose SSR models to reduce variations of corrections,
      thus, a user can use older corrections if messages are lost

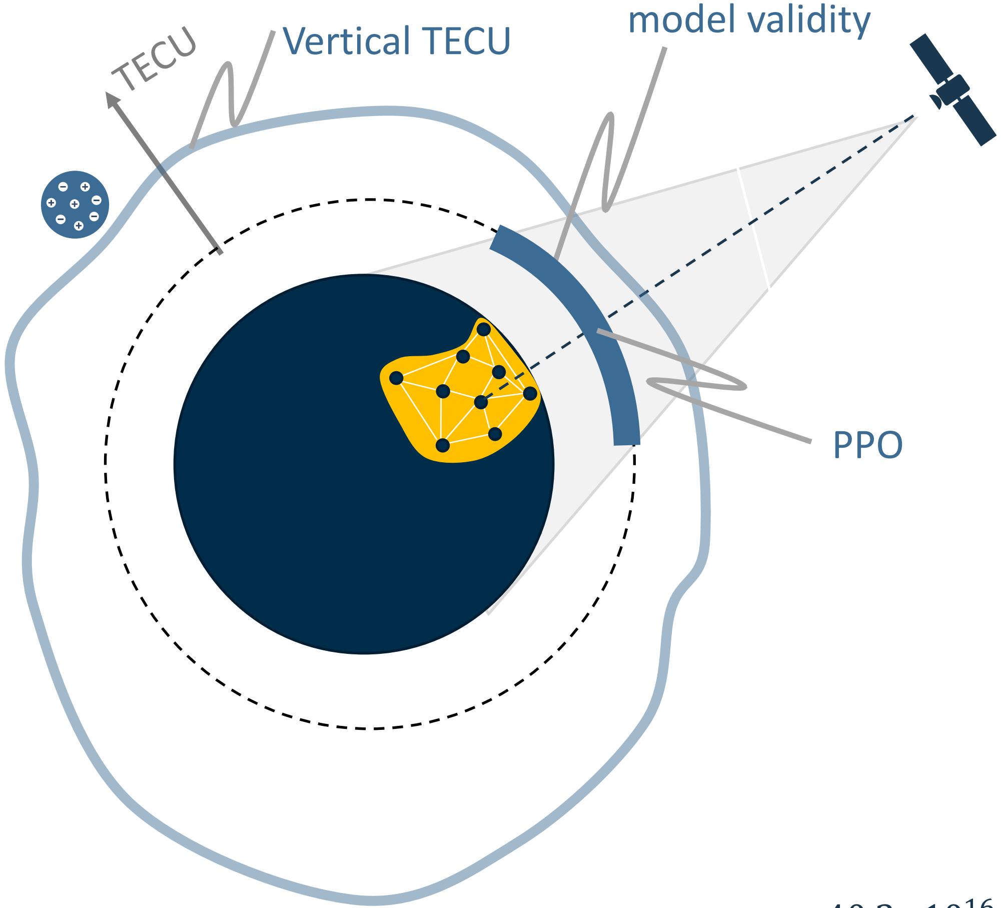
SSRZ – Ionosphere Stage Global VTEC (GVI) - 1



- **functional model** using **spherical harmonics** at **ionospheric layer** (close to iono physics)
- **satellite independent VTEC**
- ionosphere could be considered as quasi-stationary **in Sun-fixed frame** (maximum at 2pm local time) – effect of **Earth rotation** in coefficient estimation is **compensated** in this representation /frame
- **coefficients** retain their magnitudes
  → **low variation**
- model validity: network area

$$\delta I_s^j[\mathrm{m}] = \frac{40.3 \cdot 10^{16}}{f_s^2} sf \cdot (\boldsymbol{VTEC_{GVI}})$$

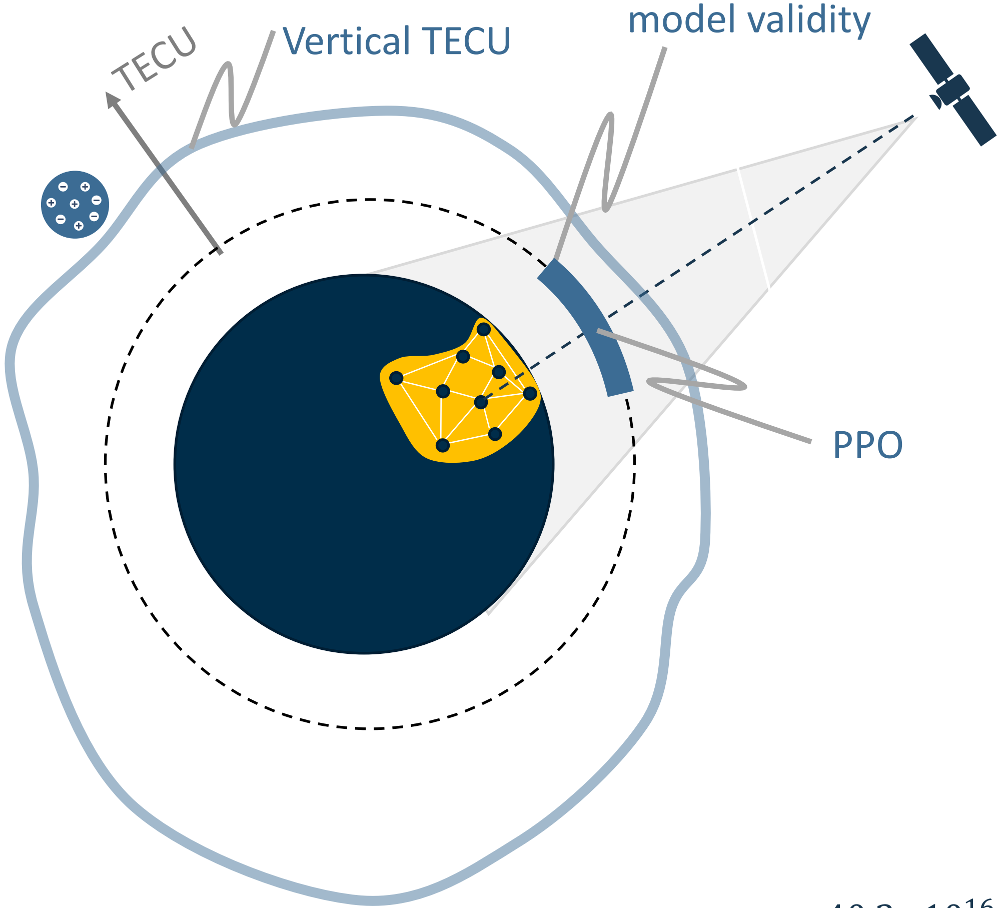
SSRZ – Ionosphere Stage Global STEC (GSI) - 2



- **functional model** using **Chebyshev polynomials** at **ionospheric layer**
- **satellite dependent STEC** (VTEC with **mapping function** sf)
- **compensation of Earth-rotation**: step-wise change of polynomial expansion point PPO: (fixed for ~2min) in **Sun-fixed** frame
- larger change of coefficients every 120s (better than typical SSR update rates of 1s or 30s)
- model validity: footprint of satellite

$$\delta I_s^j[\mathrm{m}] = \frac{40.3 \cdot 10^{16}}{f_s^2} sf \cdot (VTEC_{GVI} + \boldsymbol{VTEC_{GSI}})$$

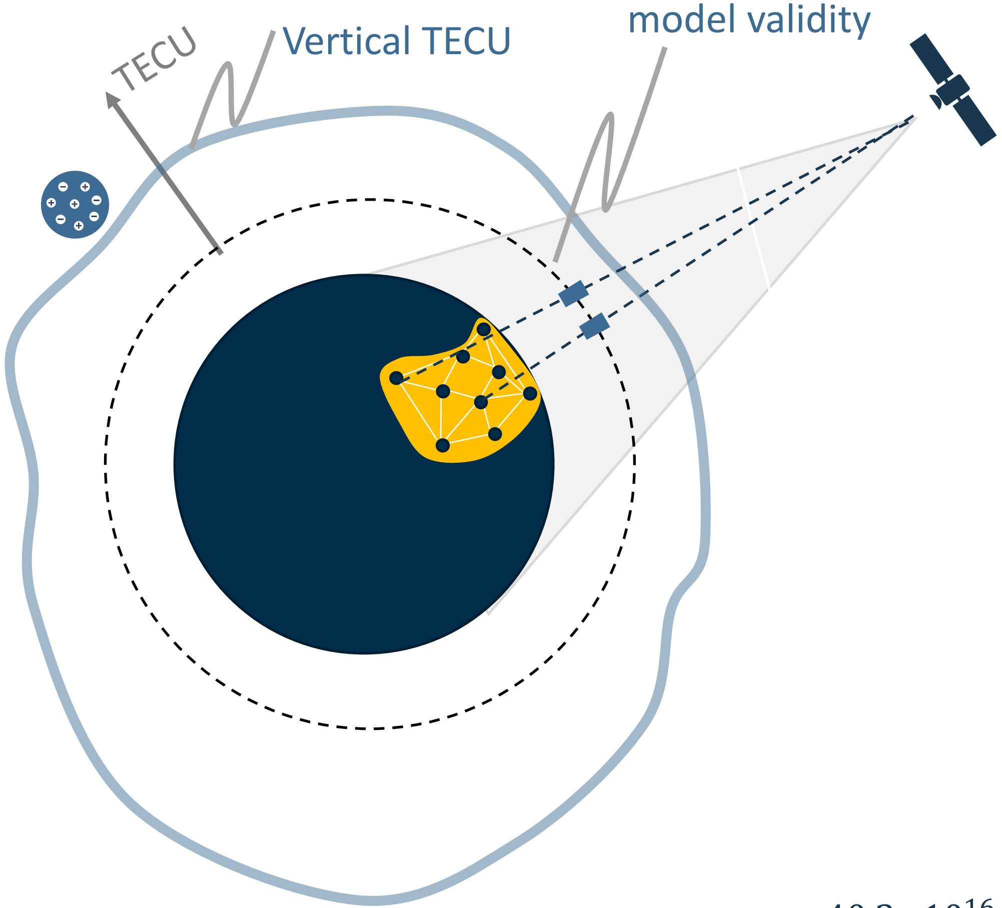
SSRZ – Ionosphere Stage Regional STEC (RSI) - 3



- **functional model** using **Chebyshev polynomials** at **ionospheric layer**
- **satellite dependent STEC** (VTEC with **mapping function** sf)
- **compensation of Earth-rotation**: step-wise change of polynomial expansion point PPO: (fixed for ~2min) in **Sun-fixed** frame
- larger change of coefficients every 120s (better than typical SSR update rates of 1s or 30s)
- model validity: limited area in ionospheric layer

$$\delta I_s^j[\mathrm{m}] = \frac{40.3 \cdot 10^{16}}{f_s^2} sf \cdot (VTEC_{GVI} + VTEC_{GSI} + \boldsymbol{VTEC_{RSI}})$$

SSRZ – Ionosphere Stage Gridded STEC (GRI) - 4

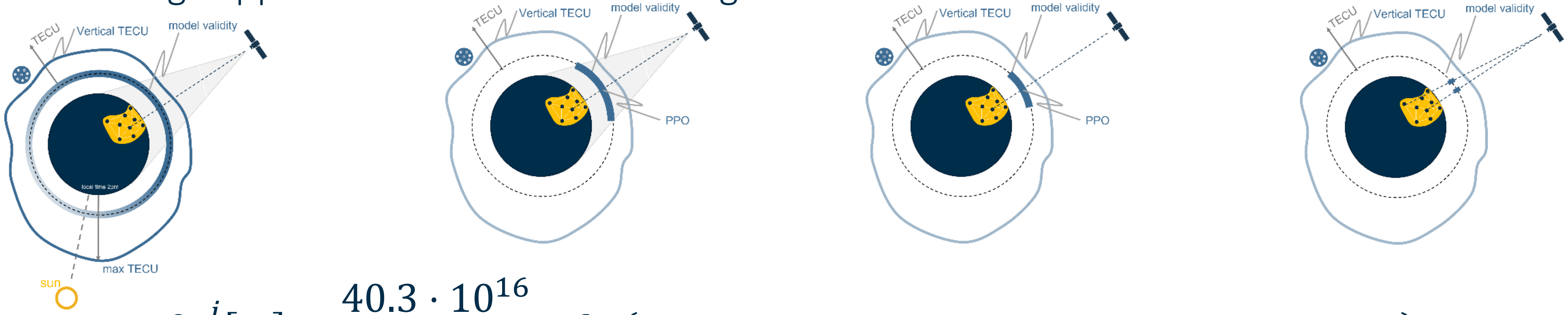


- **residual model** using **grids** and **ionospheric layer** (remaining non-functional residual)
- **satellite dependent STEC** (VTEC with **mapping function** sf
- grids in SSRZ are **natural grid** (grid points = station positions) → avoid double-interpolation error
- grid point height
- **interpolation** is done at **ionospheric layer**
- model validity: network area

$$\delta I_s^j[\mathrm{m}] = \frac{40.3 \cdot 10^{16}}{f_s^2} sf \cdot (VTEC_{GVI} + VTEC_{GSI} + VTEC_{RSI} + \boldsymbol{VTEC_{GRI}})$$

SSRZ – Ionospheric Stages and Service Scalability

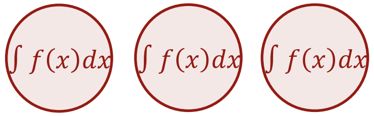
- multi-stage approach with consecutive stages



$$\delta I_s^j[\mathrm{m}] = \frac{40.3 \cdot 10^{16}}{f_s^2}\, sf \cdot (VTEC_{GVI} + VTEC_{GSI} + VTEC_{RSI} + VTEC_{GRI})$$

- corrections transmitted as vertical-mapped values
  → reduction of value variation
  → mapping function sf computed by rover
- stages can be omitted to offer service scalability

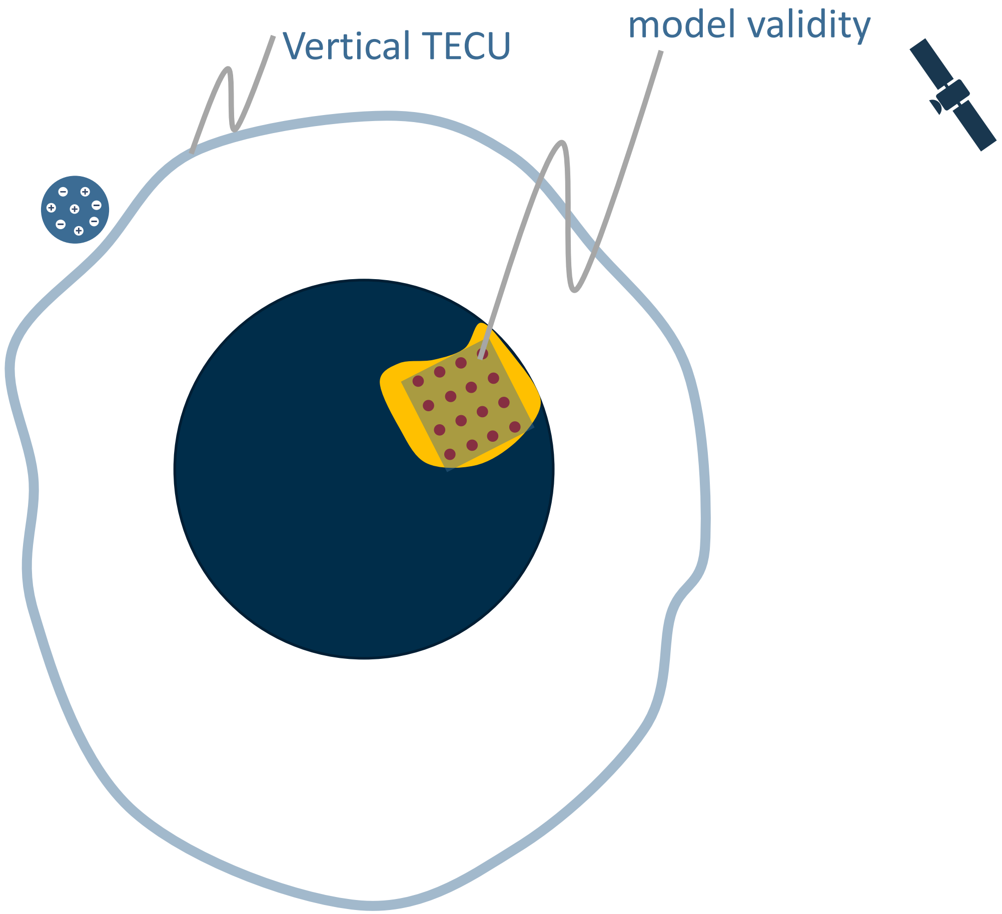
high-resolution corrections



Complex models in functional parts

low-varying coefficients, functional parts

SPARTN – Ionosphere HPAC using Dual-Stage - 1



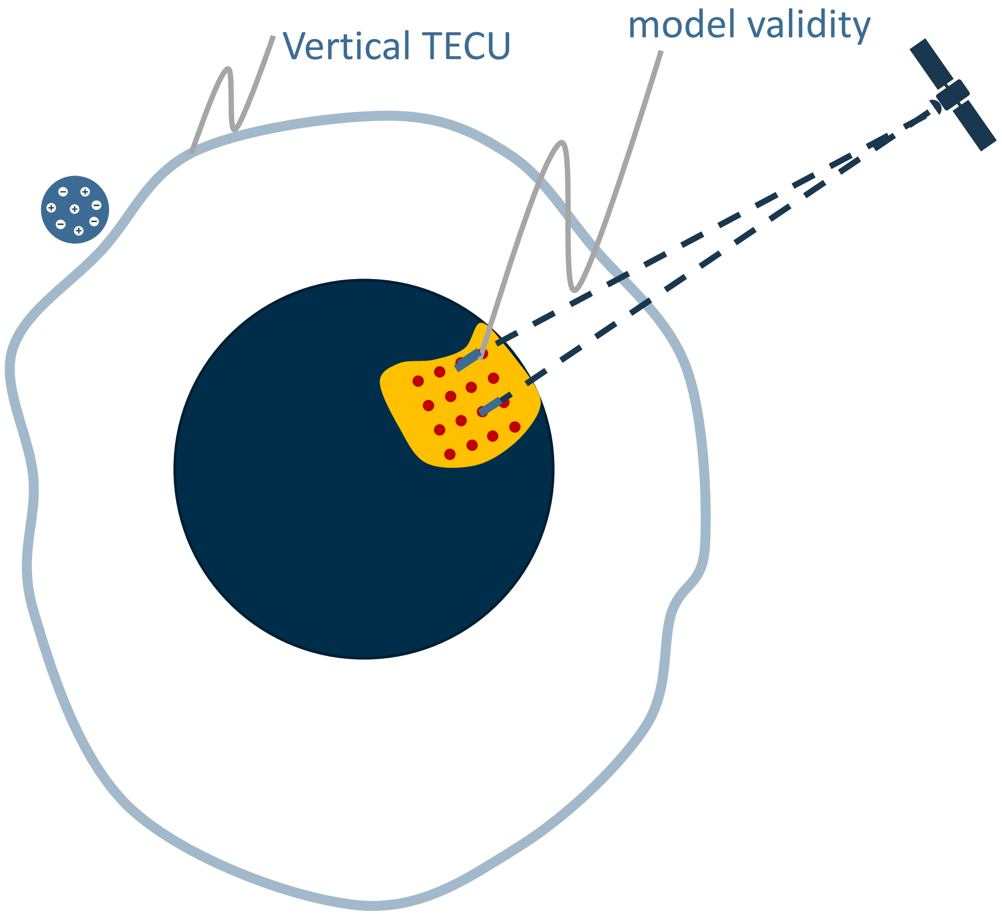
- **functional model** using **algebraic polynomials**
- **satellite dependent STEC** (without mapping function**)**
- based on **regular STEC grid** at **height zero**
- **Earth-fixed** frame
- grid centroid is polynomial expansion point

- model validity: grid

$$\delta I_s^j = \frac{40.3 \cdot 10^{16}}{f_s^2} (\boldsymbol{STEC_P})$$

High-Precision Atmospheric Corrections HPAC

SPARTN – Ionosphere HPAC using Dual-Stage - 2



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- **satellite dependent STEC** (without mapping function**)**
- based on **regular STEC grid** at **height zero**
- **Earth-fixed** frame

- model validity: grid

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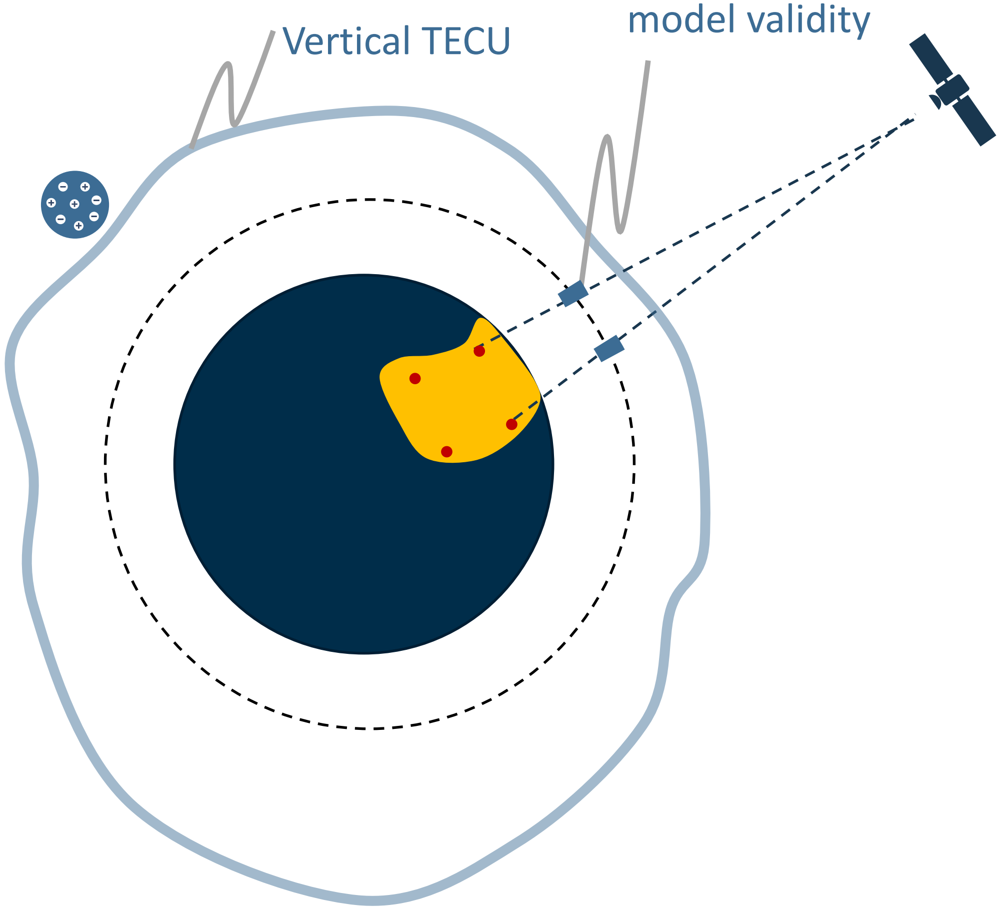
High-Precision Atmospheric Corrections HPAC

SPARTN – Ionosphere BPAC* using Single-Stage - 1



- **residual model** using grids at **ionospheric layer**
- **satellite independent VTEC**
- **Earth-fixed** frame
- **mean value subtracted** to reduce residuals
- regular grid (grid spacing BPAC>HPAC)
- **interpolation** is done at **ionospheric layer**

- model validity: network area

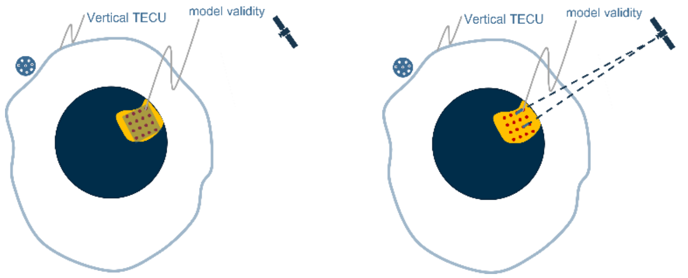
$$\delta I_s^j = \frac{40.3 \cdot 10^{16}}{f_s^2} sf(\boldsymbol{VTEC_G})$$

Basic-Precision Atmospheric Corrections BPAC

SPARTN – Iono Stages and Service Scalability

- **different services** with **different modeling**

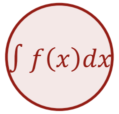
- SPARTN – **HPAC** using **Dual-Stage**



$$\delta I_s^j = \frac{40.3 \cdot 10^{16}}{f_s^2} (STEC_P + STEC_G)$$

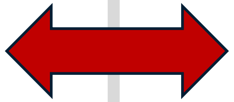
high-resolution corrections

polynomials

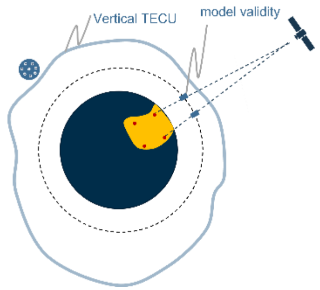


simple model (no sf, simple interpolation)

NOT consecutive



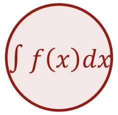
- SPARTN – **BPAC** using **Single-Stage**



$$\delta I_s^j = \frac{40.3 \cdot 10^{16}}{f_s^2} sf(VTEC_G)$$

high-resolution corrections

polynomials



simple model (computation of sf, no interpolation)

Multi-Stage for Troposphere Modelling

- **total ZTD** (zenith tropospheric delay)
  - **dry delay**: 90% of ZTD, easy to model based on temperature and pressure, applicable for large area, prediction ~mm level
  - **wet delay**: 10% of ZTD, large spatial and temporal variations, model accuracy ~cm level
- **height dependency**
  - reference station and user on different heights
  - total ZTD decreases with height

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- multi-stage is more complex
- **use different characteristics** of dry and wet component
- **functional model** using **polynomial** for **dry** delay
- **residual model** using grids for **wet** delay
- different concepts to handle **height dependency**
  - scale factor w.r.t to model ZTD
  - usage of metric ZTD correction

$$\delta STD(\phi,\lambda,h) = mf_{dry} \cdot \delta ZTD_{Poly,dry}(\phi,\lambda,h) + mf_{wet} \cdot \delta ZTD_{Grid,wet}(\phi,\lambda,h)$$

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SSRZ - Multi-Stage for Troposphere Modeling

- **functional model** using **Chebyshev polynomials** with expansion point are defined by networks
- use global troposphere models ZTD_{dry}^{model} and ZTD_{wet}^{model} (height is considered)
- **scale factor** w.r.t to model ZTD
  → $\delta t_{dry} = \frac{\delta ZTD_{Poly,dry}}{ZTD_{dry}^{model}}$ and $\delta t_{wet} = \frac{\delta ZTD_{Grid,wet}}{ZTD_{wet}^{model}}$

  → **height dependency** of ZTD is **intrinsically considered**
- SSRZ allows for **different stages per component**, e.g.:

$$\delta STD(\phi, \lambda, h) = mf_{dry} \cdot ZTD_{dry}^{model} \left(1 + \delta t_{dry}\right) + mf_{wet} \cdot ZTD_{wet}^{model} \left(1 + \delta t_{wet}\right)$$

- model validity: network area

SPARTN - Multi-Stage for Troposphere Modeling

- **functional model** using **algebraic polynomials** for **dry** and **wet** delay
- **mean value** (2.3m) **subtracted** to reduce size of $\delta ZTD_{Poly,dry}$
- **residual model** for **wet** delay optionally
- **height dependency** of ZTD **is not rigorously considered**
    - $\delta ZTD_{Poly,dry} = \delta ZTD_{Poly,dry}(h=0)$
    - $\delta ZTD_{wet} = \delta ZTD_{wet}(h=0)$

$$\delta STD(\phi,\lambda,h) = mf_{dry} \cdot [2.3 + \delta ZTD_{Poly,dry}(\phi,\lambda)] + mf_{wet}[\delta ZTD_{Poly,wet}(\phi,\lambda) + \delta ZTD_{Grid,wet}\ (\phi,\lambda)]$$

- model validity: grid

Conclusion

- **complete SSR corrections** for orbit, clock, bias, ionosphere, troposphere
- **SSR formats** are **compromise** between, e.g.,
  positioning accuracy, bandwidth, target user hardware
  → service requirements needed to compare SSR formats
- **atmospheric corrections** modeled as (functional) and residual/gridded **multi-stages**
- smart **model design** affecting service quality (e.g. ionospheric reference frame, layer)
- **actual height** should be considered for **tropospheric correction**
- same analysis of **other open SSR formats** meaningful

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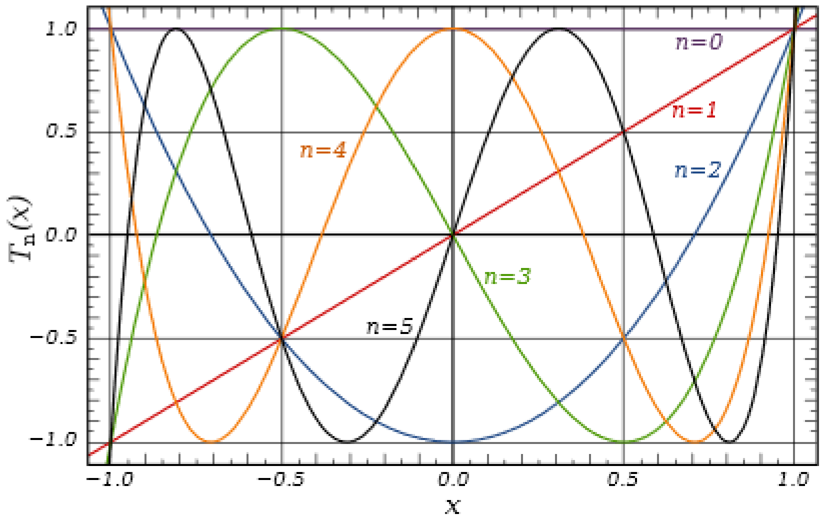
Thank you

Algebraic vs. Chebyshev Polynomials

- $f(\phi,\lambda)$ are commonly modeled as **algebraic** $P_n(x)$ or **Chebyshev polynomials** $T_n(x)$:
$$f(\phi,\lambda) = C_{00} + C_{10}P_1(\phi) + C_{01}P_1(\lambda) + C_{11}P_1(\phi)P_1(\lambda) + C_{20}P_2(\phi) + \ \dots$$
- if $|x| \leq 1$: $|P_n(x)| \leq 1$ and $|T_n(x)| \leq 1$
- In this range inaccuracy of $f(\phi,\lambda)$ based on the (limited) resolution of the coefficients ΔC_{ij} is
$$|\delta f| \lesssim \sqrt{2n+1}\,\max\left(\Delta C_{ij}\right)/2$$
- $|x| \leq 1$ for $P_n(x)$ is often not considered (e.g. SPARTN $\max(x) \sim 11$) →quality vanishes at edges; service quality becomes inhomogeneous
- for large networks higher-order $(\mathrm{n} \geq 2)$ $T_n(x)$, allow for homogenous representation in range $|x| \leq 1$

$P_0(x) = 1$ $\quad$ $T_0(x) = 1$
$P_1(x) = x$ $\quad$ $T_1(x) = x$
$P_2(x) = x^2$ $\quad$ $T_2(x) = 2x^2$-1
$P_3(x) = x^3$ $\quad$ $T_3(x) = 4x^3 - 3x$
… $\quad$ …
$P_n(x) = x^n$ $\quad$ $T_n(x) = 2x\,T_{n-1} - T_{n-2}$



[https://de.wikipedia.org/wiki/Tschebyschow-Polynom; 2022-09-05]