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Learning Trajectory Tracking for Underactuated Compliant Arms

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***Abstract*— Trajectory tracking is a classic control theory topic that has received in-depth research in the literature. However, dealing with compliant arms that is underactuated makes the issue more difficult. Compliant systems frequently exhibit difficult-to-model dynamics in addition to their underactuation. To prevent a severe modification of the robot elasticity, the feedback components should be limited. In this letter, we use an iterative learning controller to solve the trajectory tracking problem. The presented control law mixes feedforward and feedback terms. The feedforward component tracks the desired trajectory raising the robot to one equilibrium, and the feedback term stabilizes the equilibrium. We investigate the closed-loop stiffness variation. Finally, we simulate an underactuated compliant arm to verify the suggested technique.**

I. Introduction

Underactuated compliant robots include systems with springs at the joints level, i.e., articulated robots, or continuum arms [1]. The control problem is still unresolved despite the extensive work put into designing highly effective structures. One of the most intriguing robotics issues has always been trajectory tracking [2].

In the literature, model-based controllers such as partial feedback linearization [3] or Lyapunov controllers [4] solve the tracking problem. However, they need dependable model descriptions, though, as well as high gain feedback terms. Conversely, learning-based and model-free approaches, i.e., reinforcement learning [5], present fresh solutions for the tracking problem.

Due to the elasticity embedded in their structure, two main difficulties arise when controlling them. First, their dynamics are frequently complicated [6], making model descriptions unreliable. Second, high-gain feedback controllers also impair system compliance [7], [8]. For these reasons, it is not advised to apply traditional model-based control strategies with high-gain feedback terms. Learning-based approaches, on the other hand, appear to be a good alternative since they may be created to be mostly feedforward [9]. However, they are time-consuming and exclude a thorough analysis of the system characteristics, such as stability [2]. Therefore, trajectory tracking problems can be tackled using Iterative Learning Control (ILC), which enhances tracking performance by making use of the tracking error from previous iterations without needing precise robot modeling.

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ILC primarily relies on feedforward components, thus it maintains system elasticity. Additionally, ILC allows to formally conclude on the stability of any equilibria.

In this paper, assuming a fixed relative degree, we suggest an ILC law, which combines feedback and feedforward terms. The former is utilized to follow the desired trajectory, whilst the latter is an output feedback action. Relying on the Lyapunov indirect Theorem, we can prove the equilibrium stability. We provide a criterion that limits the separation between open and closed-loop stiffness to retain the robot's elastic behavior. Finally, we validate the methodology by simulating a two degrees of freedom robot (DoFs), in which only the first elastic joint is actuated.

II. Problem Definition

We use both active and passive elastic joints to represent the underactuated compliant arm with m actuators [9], i.e.,

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)+H\dot{q}+Kq=Su\,, \qquad (1)$$

where $q,\dot{q},\ddot{q}\in\mathbb{R}^n$, $M(q)\in\mathbb{R}^{n\times n}$, $C(q,\dot{q})\in\mathbb{R}^{n\times n}$, and $G(q)\in\mathbb{R}^n$ with the usual meaning. $K,H\in\mathbb{R}^{n\times n}$ are the diagonal stiffness and damping matrix. u is the torque input, $S:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^n$ is the underactuation matrix.

Defining the iteration index $j\in\mathbb{N}$ and the state vector $x_j=\left[q_j^\top,\dot{q}_j^\top\right]^\top\in\mathbb{R}^{2n}$; (1) can be written as

$$\begin{cases} \dot{x}_j(t)=f(x_j(t))+g(x_j(t))u_j(t) & (2)\\ y_j(t)=h(x_j(t))\,, & (3)\end{cases}$$

where $t\in[0,t_\mathrm{f}]$. $f(\cdot):\mathbb{R}^n\times[0,t_\mathrm{f}]\to\mathbb{R}^n$, and $g(\cdot):\mathbb{R}^n\times[0,t_\mathrm{f}]\to\mathbb{R}^{n\times m}$ are the drift and control vector field, i.e., $f(x_j)=\left[\dot{q}_j,-M^{-1}(q_j)(C(q_j,\dot{q}_j)\dot{q}_j+G(q_j)+Kq_j+H\dot{q}_j)\right]$, and $g(x_j)=\left[0_{n\times m},M^{-1}(q_j)S\right]$.

Let us assume what follows for the system (2)-(3).

Assumption 1. *The system* (2)-(3) *has a fixed relative degree* r_vect, $\forall x\in\mathbb{R}^{2n}$ *(see, e.g.,[10]), and* $r=r_1=\cdots=r_m$.

Assumption 2. *The function* $f,L_f^o h,o=0,\cdots,r,h,g,$ *and* D *are Lipschitz with constants* $f_0,\theta_o,h_0,g_0,$ *and* $D_0\in\mathbb{R}$.

Finally, given a desired and feasible[1] trajectory $y_\mathrm{d}(t):[0,t_\mathrm{f}]\to\mathbb{R}^m$, the article objective is to design an iterative controller capable for the system (1)-(2) of precise tracking of $y_\mathrm{d}(t)$ and stabilize the trajectory final point without a substantial alteration of the system's stiffness.

[1]Feasible means that for any y_d there exists x_d and u_d that verify (2)-(3).

III. Problem Solution

We here design an iterative control law which relies on both feedback and feedforward actions. We propose a suitable choice of the learning gains, and we ensure a not-drastic modification of the robot elasticity. Finally, we study the equilibrium stability.

A. Iterative Learning Controller

We introduce an ILC controller [2]. Thanks to the repetition of the desired task, the iterative framework boosts tracking efficiency while balancing out model uncertainty.

Recalling (2)-(3), we present the control law, which combines feedback and feedforward terms [2], i.e.,

$$u_{j+1}(t) = u_j(t) + \Lambda_{\mathrm{ff}_j}(t)\,{}^r e_j(t) + \Lambda_{\mathrm{fb}_{j+1}}(t)\,{}^r e_{j+1}(t)\,, \tag{4}$$

where $\Lambda_{\mathrm{fb}_{j+1}}, \Lambda_{\mathrm{ff}_j} \in \mathbb{R}^{m\times m}$ are the learning gains, while the tracking error ${}^r e_j(t) \in \mathbb{R}^m$ is

$${}^r e_j(t) \triangleq \sum_{i=0}^{r} {}^i\Upsilon \left(y_{\mathrm{d}}^{(i)}(t) - y_j^{(i)}(t) \right) \tag{5}$$

where ${}^i\Upsilon$, $i = 0, \cdots, r$ are the control gains [9][2]. Note that $y^{(r)}(t) = \sum_{i=0}^{r} {}^i\Upsilon L_f^i h(x) + {}^r\Upsilon D(x) u(t)$.

In the following, we remove the time-dependency to save space. The following Theorem presents the paper main result.

Theorem 1. *Let us consider the system* (2)-(3) *with assumptions A1-A2, and let* (4) *be the iterative controller. If the gains* $\Lambda_{\mathrm{fb}_{j+1}}, \Lambda_{\mathrm{ff}_j} \in \mathbb{R}^{m\times m}$ *are chosen equal to*

$$\Lambda_{\mathrm{fb}_{j+1}} = {}^r\Upsilon D^{-1}(x_{j+1})\,{}^r\Upsilon^{-1} \quad \Lambda_{\mathrm{ff}_j} = \varepsilon D^{-1}(x_j)\,{}^r\Upsilon^{-1} \tag{6}$$

where $D(x_j) = \partial h(x_j)/\partial q M^{-1}(q_j) S$, *and* $\varepsilon \in (0,1)\ \forall j \in \mathbb{N}_0$ *then* $\lim_{j\to+\infty} \left\| {}^r e_j(t) \right\|_\lambda = 0$.

Furthermore, the feedback component does not modify the robot elasticity more than $\kappa \geq 0$, *meaning that the distance open-closed loop stiffness is such as* $||K - K_{\mathrm{fb}}|| \leq \kappa$.

Sketch Proof. Recalling (4), (5), defining $\Delta u \triangleq u_{\mathrm{d}} - u$, $\Delta x \triangleq x_{\mathrm{d}} - x$, and $\Theta(x, x_{\mathrm{d}}) \triangleq \sum_{i=0}^{r} {}^i\Upsilon \left(L_f^i h(x_{\mathrm{d}}) - L_f^i h(x) \right)$ lead to

$$\begin{aligned} &\Delta u_{j+1} + \Lambda_{\mathrm{fb}_{j+1}} {}^r\Upsilon D(x_{j+1}) \Delta u_{j+1} = \left(I_m - \Lambda_{\mathrm{ff}_j} {}^r\Upsilon D(x_j) \right) \Delta u_j \\ &- \Lambda_{\mathrm{fb}_{j+1}} \left(\Theta(x_{j+1}, x_{\mathrm{d}}) - {}^r\Upsilon \left(D(x_{\mathrm{d}}) - D(x_{j+1}) \right) u_{\mathrm{d}} \right) \\ &- \Lambda_{\mathrm{ff}_j} \left(\Theta(x_j, x_{\mathrm{d}}) - {}^r\Upsilon \left(D(x_{\mathrm{d}}) - D(x_j) \right) u_{\mathrm{d}} \right) . \end{aligned} \tag{7}$$

Applying the triangular inequality to (7), and recalling (6); one can write

$$\left|\left|\Delta u_{j+1}\right|\right| \leq \zeta_{\mathrm{fb}} \left(\mu \left(\left|\left|\Delta x_{j+1}\right|\right| + \left|\left|\Delta x_j\right|\right| \right) + (1-\varepsilon) \left|\left|\Delta u_j\right|\right| \right) , \tag{8}$$

where $\zeta_{\mathrm{fb}} = 1/||I_m + {}^r\Upsilon||$ and $\mu \in \mathbb{R}$. Given assumptions A1 and A2, applying the Gronwall's Lemma, and computing the $\lambda-$norm lead to $\left|\left|\Delta u_{j+1}\right|\right|_\lambda \leq \zeta_{\mathrm{fb}} \nu \left|\left|\Delta u_{j+1}\right|\right|_\lambda + \zeta_{\mathrm{fb}} \left((\varepsilon + \nu) \left|\left|\Delta u_j\right|\right|_\lambda \right)$, where $\nu \triangleq \mu b_1 (1 - \mathrm{e}^{(b_2-\lambda)t_{\mathrm{f}}})/(\lambda - b_2)$, $b_1, b_2 \in \mathbb{R}$, $b_2 = \sup_t f_0 + g_0 ||u_{\mathrm{d}}||$. Defining $\overline{\zeta}_{\mathrm{fb}} \triangleq \zeta_{\mathrm{fb}}/(1 - \zeta_{\mathrm{fb}}\nu)$ and $\overline{\varepsilon} \triangleq (\varepsilon + \nu)$ yield to

$$\left|\left|\Delta u_{j+1}\right|\right|_\lambda \leq \overline{\zeta}_{\mathrm{fb}} \overline{\varepsilon} \left|\left|\Delta u_j\right|\right|_\lambda . \tag{9}$$

[2]The initial guess $u_0(\cdot)$ is arbitrarily selected.

Note that $\forall j, \forall b_2 \geq 0$, $\exists \lambda \geq 0$ such that $\overline{\zeta}_{\mathrm{fb}} < 1$ and $\overline{\varepsilon} < 1$. Eq. (9) is a control contraction, which leads to $\lim_{j\to+\infty} \left|\left|\Delta u_{j+1}\right|\right|_\lambda = 0 \implies \lim_{j\to+\infty} \left|\left|{}^r e_j\right|\right|_\lambda = 0$.

More details of the proof first part can be found [2] as well as the proof of the second statement, Theorem 2-C3. □

Remark 1. *The feedback controller leads to the presence of a Zero Dynamics (see, e.g., [10]). Note that Theorem 1 states* $\lim_{j\to+\infty} ||\Delta x_j||_\lambda = 0$.

B. Equilibrium Stability

Leveraging the Lyapunov indirect theorem, one can study the stability. Let us consider an equilibrium point such as $\tilde{x} = \left[\tilde{q}^\top, 0_{n\times 1}^\top\right]^\top$, $\tilde{u}$, and its linear approximation, i.e., $\dot{z} = \mathscr{A} z + \mathscr{B} w$, $\varsigma = \mathscr{C} z$, where $z \triangleq x - \tilde{x}$, $w \triangleq u - \tilde{u}$, $\varsigma \triangleq y - \mathscr{C}\tilde{x}$

$$\dot{z} = \begin{bmatrix} 0_n & I_n \\ -\mathscr{A}_{21} & -\mathscr{A}_{22} \end{bmatrix} z + \begin{bmatrix} 0_{n\times m} \\ \mathscr{B}_2 \end{bmatrix} \omega\,, \varsigma = \begin{bmatrix} \mathscr{C}_1 & 0_{m\times n} \end{bmatrix} z\,, \tag{10}$$

where $\mathscr{A}_{21} = M^{-1}(q) \left(\partial G(q)/\partial q + K \right)\big|_{\tilde{q}}$, $\mathscr{A}_{22} = M^{-1}(\tilde{q}) H$, $\mathscr{B}_2 = M^{-1}(\tilde{q}) S$, and $\mathscr{C}_1 = \partial h(q)/\partial q\big|_{\tilde{q}}$. Inspecting (10), if $\mathscr{A}_{21} \succ 0$ the eigenvalues of the overall matrix $\mathscr{A}$ have a negative real part, therefore the equilibrium is stable, i.e., Lyapunov indirect Theorem.

The sign of $\mathscr{A}_{21}$ depends on the stiffness and the gravity force. Thus, both design solutions [2] or feedback controller [11] may stabilize any system feasible equilibrium.

Proposition 1. *Let us consider the linear system* (10). *If the number of unstable eigenvalues is less or equal to rm, then* (4) *and its linear counterpart* $u_{\mathrm{fb}} = -(I_m + {}^r\Upsilon)^{-1} \Gamma_{\mathrm{fb}} \sum_{i=0}^{r} {}^i\Upsilon \mathscr{C} \mathscr{A}^i z$, *can stabilize the equilibrium.*

Proof. See [2]. □

Note that, after applying (4), *a-posteriori* linear analysis needs to be carried out to ensure the equilibrium stability.

IV. Validation

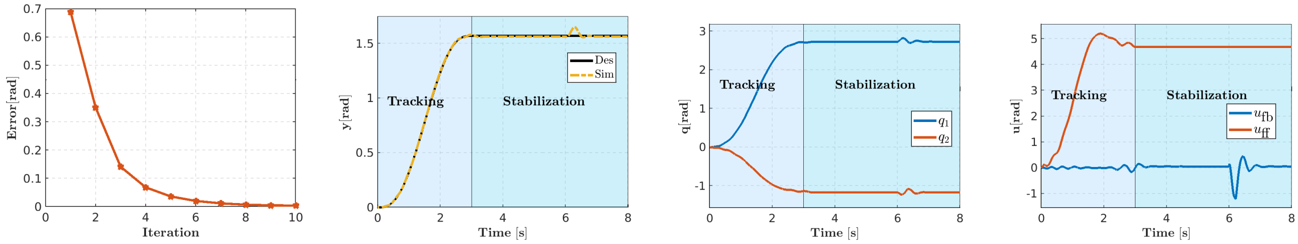
We test the iterative law simulating a two DoFs underactuated compliant arm. This robot is a single-input single-output system ($m = 1$), where only the first elastic joint is actuated.

The selected output function (3) is the robot tip orientation, namely, $y = [1, 1, \cdots, 1]q$, and the decoupling matrix is a scalar function equal to $D(x) = L_g L_f h(x) = [1, 1, \cdots, 1] M^{-1}(q) S$, which is $L_g L_f h(x) \neq 0, \forall x \in \mathbb{R}^{2n}$ thanks to assumption A1. Note that the relative degree is 2.

The desired trajectory to track is a minimum jerk [2], which starts from the origin,i.e.,$y(0) = 0$ up to y_{f} in t_{f}s.In each simulations, the robot is set in the origin $x_j(0) = 0_{2n\times 1}$ and $u_0 = 0$. This is an asymptotically stable point, thus the initial guess is $u_0(t) \equiv 0$. The performance are evaluated using the Root Mean Square (RMS) of the error.

The learning gains are reported in (6) with $\varepsilon = 0.9$; while the control gains ${}^i\Upsilon$ are selected depending on the task, Tab. I. It is worth nothing that the gains choice affects both the convergence velocity and the stability of any equilibrium.

We select $\kappa = 2$ to ensure that the difference between the open and closed-loop elasticity is limited from a factor of 2.



(a) Error evolution over iteration. (b) Final output tracking. (c) Joints evolution. (d) Final control u_{10}.

Fig. 1. Results for the minimum jerk trajectory lasting $t_f = 3$s reaching $y_f = \pi/2$. An external disturbance occurs at $\bar{t} = 6$s.

TABLE I
CONTROL GAINS $[{}^0\Upsilon, {}^1\Upsilon, {}^2\Upsilon]$

Feedback$[20, 10, 1]$	Feedforward $[10, 1, 1]$

A. Simulation Results

The physical parameters of the two DoFs robot: mass $m_i = 0.55$kg, inertia $J_{i,i} = 0.01\text{kgm}^2$, length $l_i = 0.089$m, center of mass distance $a_i = 0.085$m, damping $H_{i,i} = 0.1$Nms/rad, and stiffness $K_{i,i} = 0.5$Nm/rad for $i = 1$ and 2, [2].

We perform a minimum jerk with $t_f = 3$s and $y_f = \pi/2$. Then at $\bar{t} = 6$s, we perturb the system with an external impulse of 0.8rad. The final equilibrium of the trajectory is $\tilde{x} = [2.72, -1.14, 0, 0]^\top$, $\tilde{u} = 4.76$, and recalling (10), whose open-loop eigenvalues are equal to $-\{2.33 \pm 2.88\text{i} \quad 12.02 \quad -2.5\}$. Leveraging the Lyapunov indirect Theorem, the equilibrium is unstable, since one eigenvalue belongs to the real and positive plane. However $1 < rm = 2$ then the control law and its linear counterpart can stabilize the equilibrium point. Applying Theorem 1, choosing the control gains as in Tab. I, the closed-loop eigenvalues are $-\{1.98 \quad 72.19 \quad 5.75 \pm 2.67\text{i}\}$, i.e., asymptotically stable equilibrium.

Fig.1 displays the results. Fig. 1(a) depicts the RMS error evolution w.r.t. iteration domain, Fig.1(b) shows tracking performance at the last iteration, Fig. 1(c) displays the joints evolution, and Fig. 1(d) reports the learned control actions at $j = 10$.

Discussion: The presented algorithm can track the desired trajectory with acceptable performances without the need of a precise model of the underactuated compliant arm. The feedback component stabilizes the system in the final configuration, Fig. 1(b). Since only one eigenvalue is unstable, the feedback controller can stabilize it. Additionally, the closed-loop stiffness variation is minimal.

Finally, final RMS error is ≈ 0.002rad Fig. 1(a).

V. CONCLUSION

In this work, an introduced an Iterative Learning Controller for trajectory tracking dealing with compliant underactuated arms is presented. Without requiring a precise robot model, thanks to the feedforward action, the robot successfully tracks the desired trajectory; while the feedback term ensures equilibrium stability without a drastic modification of the overall compliance. Through simulations, the effectiveness of the suggested technique is evaluated.

The focus of next research will be data-driven learning controllers, e.g., reinforcement learning.

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