

# Perception-aware trajectory planning for a pair of vehicles avoiding indistinguishability

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## I. INTRODUCTION

Autonomous mobile robots are being used in a large variety of environments, with aerial, underwater and ground applications. Planning a trajectory to reach a desired location requires the robot to be able to localise itself in the environment, relying on the on-board sensors and on sensors the environment is equipped with. To this aim, robots are often provided with *ranging sensors*, i.e. sensors measuring the distance between two devices, one of them mounted on the vehicle itself, while the other may be fixed to the frame, or mounted on another vehicle. Since the orientation of the vehicle cannot be reconstructed from range measurements, the robots are *statically unobservable*, and thus they have to actively seek for a sufficiently informative trajectory to estimate their state.

**Related works:** The technical literature presents this problem mainly from a local perspective, investigating whether (and possibly how) small perturbations on the initial state of the vehicle modify the sensor readings along the trajectory. This analysis usually relies on the Observability Matrix and on the Observability and Constructibility Gramians, built on the linearised system. Observability analyses of some given trajectories have been proposed by several authors, with different kinematic models (e.g. [1]), whose common outcome is that straight trajectories are less informative than curved ones. On the other hand, researchers have proposed trajectory planning algorithms aiming at maximising the observability of the robot. In particular, Salaris et al. [2] propose a planning technique maximising some norm of the Constructibility Gramian, showing that this minimises the uncertainty on the final state of the trajectory of the vehicle. Such planning problems have been extended to multi-robot systems (see [3]), where a leader robot moves in the environment, while a follower vehicle plans its trajectory to maximise the observability of the leader.

**Paper contributions:** We analyse the observability/constructibility problem from a “global” perspective, formally associated with the concept of indistinguishability. We consider a two-agents system, relying on absolute and relative range measurements, i.e., collected from a fixed-frame point on the plane or from the other vehicle, and propose a trajectory planning algorithm avoiding *indistinguishability*.

## II. PROBLEM STATEMENT

Let us consider the discrete-time unicycle dynamic model, with state vector  $q_k = [x_k, y_k, \theta_k]$ , where  $x, y$  denote the

Cartesian coordinate of the barycentre of the vehicle, while  $\theta$  denotes its heading with respect to a reference axis. Its discrete-time dynamics can be written as ([4])

$$x_{k+1} = x_k + A_k C_k, y_{k+1} = y_k + A_k S_k, \theta_{k+1} = \theta_k + \omega_k T_s, \quad (1)$$

where

$$A_k = 2 \frac{v_k}{\omega_k} \sin\left(\frac{T_s}{2} \omega_k\right) \text{ and } \lim_{\omega_k \rightarrow 0} A_k = v_k T_s, \\ C_k = \cos\left(\theta_k + \frac{T_s}{2} \omega_k\right), \quad S_k = \sin\left(\theta_k + \frac{T_s}{2} \omega_k\right),$$

where  $T_s$  is the sensors-based sampling period. Moreover, the environment is equipped with  $N_M$  range sensors, which we will refer to as *anchors* or *markers*. The markers will be denoted by  $M_i = [X_i, Y_i]^T$ ,  $i = 1, \dots, N_M$ , and have a finite sensing range  $r$ . Therefore, the measurement  $\rho_k$  is considered unavailable when the distance is larger than  $r$ , hence

$$\rho_{k,i}^2 = (x_k - X_i)^2 + (y_k - Y_i)^2, \quad \forall i \text{ s.t. } \rho_{k,i} < r. \quad (2)$$

By setting an arbitrary initial condition  $q_0$ , we can reconstruct all the sequence of states  $q_k$  where the measurement occur, neglecting the time instants where no measurements are collected since they are not informative. We now consider this sequence of points  $\mathcal{P}_k$  (without the orientation  $\theta$ ), each one labelled with the anchor collecting the measurement at time  $k$ , as a virtual trajectory  $\mathcal{T}$  followed by the vehicle, and, with a slight abuse of notation, refer to it as *indistinguishable* if there is more than one roto-translations  $[R, T]$  of  $\mathcal{T}$  such that

$$\|(R\mathcal{P}_k + T) - M_{i_k}\| = \rho_{k,i}, \quad (3)$$

where  $R = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$  is the rotation matrix and  $T = [\Delta x, \Delta y]^T$  is the translation vector. Given two anchors  $M_A = [0, 0]^T$  and  $M_B = [d, 0]^T$  (two generic anchors at distance  $d$ ), and two vehicles  $a$  and  $b$ , we want to plan the trajectories  $\mathcal{T}_a, \mathcal{T}_b$  of the two vehicles, i.e. plan the points  $\mathcal{P}_{k,a}$  and  $\mathcal{P}_{k,b}$  compliant with the dynamics (1), such that  $\mathcal{T}_a, \mathcal{T}_b$  are *not indistinguishable*, by relying on absolute measurements, i.e. from  $M_A$  and  $M_B$ , and relative measurements.

## III. OVERVIEW OF THE PLANNING ALGORITHM

The algorithm is divided into 3 phases: (1) each vehicle collects the most informative measurements from an anchor, (2) the vehicles navigate to meet each other, and (3) collect the last two mutual measurements.

**Phase 1:** Initially, we consider vehicle  $a$  to be in the sensing range of anchor  $M_A$ , while  $b$  in the sensing range of  $M_B$ .

Since  $d \gg r$ , the robots cannot communicate, and thus they have to plan their trajectories independently. To describe the most informative manoeuvres in the sensing range of one anchor, we report here the following result:

**Proposition 1** ([4]). *Given an anchor collecting  $N$  measurements  $\rho_k$ ,  $k = 0, \dots, N-1$ , (1) for any  $N$ , two trajectories are indistinguishable as far as they are a rotation of the same trajectory about the anchor itself; (2) besides rotations about the anchor, for  $N = 2$  (or  $N > 2$  with collinear points), two trajectories are indistinguishable as long as they are symmetric with respect to an axis passing through the anchor.*

At time  $k = 0$ , the robot builds its local reference frame with the anchor in the origin and the vehicle itself in  $[\rho_0, 0]^\top$  with unknown heading. At the first time step, the vehicle moves to collect the second measurement  $\rho_1$ . After collecting the measurement  $\rho_1$ , the vehicle knows that its position coincides with one of the two intersections between the circle centred in the anchor with radius  $\rho_1$  and the circle centred in the previous point with radius  $A_0$ . Hence, with two measurements, the vehicle has travelled along one of two possible trajectories that are completely known in its local reference frame. Hence, the robot plans the next manoeuvre to have three non-collinear measurement points that, by virtue of Proposition 1, disambiguates the trajectory followed when  $\rho_2$  is collected. The same outcome may be reached when the vehicle falls outside the sensing range of the anchor after the first manoeuvre. Indeed, the vehicle can travel over a circle centred in  $q_0$  (i.e. rotate on the spot by  $\pi/2$  and then move with  $v/\omega = A_0$ ) until it collects a measurement, build the two trajectories with the same procedure as before, and compute the distance of all the intermediate points from the anchor. Since no measurements have been collected, we can discard the trajectory where at least one of the distances is smaller than  $r$ .

**Phase 2:** At the end of **Phase 1**, each vehicle is aware of its distance and orientation with respect to the line connecting its reference anchor to the vehicle itself. In order to meet, i.e., getting closer to each other than  $r$ , the two vehicles reach the orbiting distance  $\frac{d+r}{2}$  from their anchor and start moving on a circle. Vehicle  $a$  moves with maximum velocity, while the velocity of  $b$  is reduced such that when  $a$  travels over an entire circle,  $b$  travels over an arc of amplitude  $2\Omega$ , which is the amplitude of the arc included between the two intersections of the two circles, i.e.,  $\Omega = \arccos(\frac{d}{d+r})$ . This way, the vehicle encounter is guaranteed in finite time.

**Phase 3:** As soon as the two vehicles come sufficiently close to each other (closer than  $r$ ), they collect the first mutual measurement. With the same *rationale* as before, each vehicle considers a local reference frame with the reference anchor in the origin and the current position on the  $x$ -axis, i.e. with state  $q_k = [\frac{d+r}{2}, 0, \pm\pi/2]^\top$ . Vehicle  $b$  stops,  $a$  treats it a fixed-frame anchor  $a$  repeats the same procedure as in **Phase 1**, until it collects the second mutual measurement. With two mutual measurements,  $a$  can reconstruct two possible distances  $d_+, d_-$  from the anchor  $M_B$ , i.e. the anchor associated with vehicle  $b$ , and thus, in a local reference frame,  $M_B$  lies on the

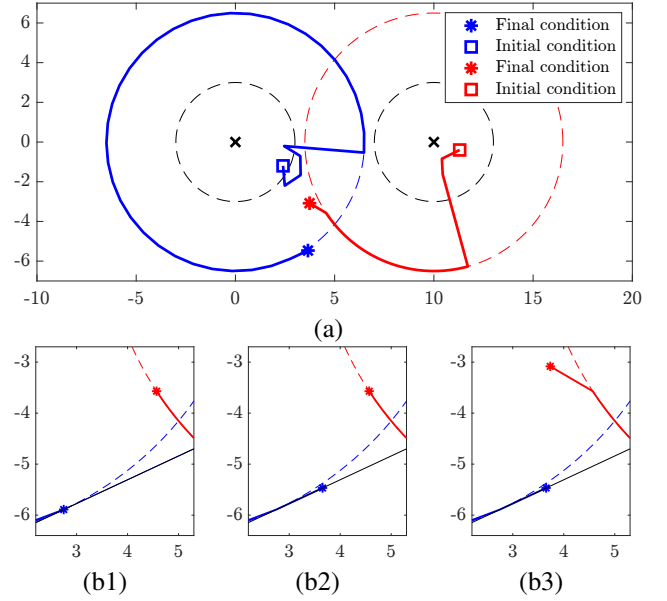


Fig. 1. Simulation example. (a) Paths followed by the vehicles, the blue line represent the vehicle  $a$ , the red line the vehicle  $b$ , the black crosses the position of the two markers  $M_A, M_B$ , with sensing range  $r$  (black dashed line). The blue and red dashed lines represent the orbiting distances. (b) Phase 3, zoom from (a): the two vehicles collect their first mutual measurement (b1),  $a$  moves on the tangent (black solid line) of the dashed circle (b2), and eventually  $b$  moves to resolve the possible ambiguities.

intersection between a circle centred in  $M_A$  with radius  $d$  and a circle centred in the current (known) position of  $a$  with radius  $d_+$  or  $d_-$ , hence yielding a maximum of 4 intersections, i.e., 4 roto-translations of the trajectory of  $b$  compliant with (3). The last manoeuvre executed by  $b$ , while  $a$  stops, is planned to resolve the residual ambiguities with the same *rationale* as in **Phase 1**,  $k = 2$ , hence avoid indistinguishability and concludes the algorithm. Figure 1 shows a simulation with the proposed trajectory planning algorithm.

#### IV. FUTURE WORK

We plan to extend the results obtained in this analysis and take into account more general settings where some information on the manoeuvres are not available, due to failing odometric sensors or to malicious agents sharing wrong information. Furthermore, we plan to investigate the same indistinguishability properties of a unicycle vehicle subject to bearing measurements.

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