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A Ratio-Based approach to identifying the most basic feasible solution to bottleneck transportation problems

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Abstract: The most frequently studied topic in operational research and computer science is the transportation problem. In general, it is considered here how to reduce the overall cost of transporting goods from each source to another source. But in the case of transportation, transportation costs are only one important factor that is considered. Many other elements must be considered in transportation for real-world applications. The challenge of time-saving transportation provides a powerful foundation for identifying better ways to deliver items to consumers on time. This model was built with the goal of decreasing transportable time. The problems based on transportation time are known as "bottleneck transportation problems (BTP)" or time minimizing transportation problems. The literature attests that various algorithms have been presented to obtain a basic, feasible solution as well as an optimal solution to the BTP. Through this research paper, we hope to introduce a new algorithm to obtain a basic, feasible solution to a bottleneck transportation problem. For that, the new algorithm was built based on the ratio of the unit times in each cell to the total time in the transportation table. By using this method, a basic, feasible solution can be easily obtained for balanced and unbalanced BTPs. The initial solution obtained by using this algorithm is an optimal or near-optimal value. This new algorithm is a relatively effective method that can be easily understood.

Keywords: Bottleneck transportation problem, Basic solution, Optimum value, Ratio value, Time.

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Introduction

The transportation problem formulated as a linear programming problem where the constraints have a special structure. In classical form the transportation problem minimizes the cost of transporting some product that is available at some sources (m) and required at some destinations (n). However, in most real world problems the complexity of the social and economic environment requires the explicit consideration of objective functions other than cost. Different approaches are also used to solve these types of transportation problems (TPs) [1]. They are designed to be a basic solution as well as an optimal solution. Accordingly, F.L. Hitchcock [2] presented the first approach in 1941, and then various researchers came forward to find solutions for this through their research reports. However, in real-life transportation problems, it is not reasonable to consider only the cost [3]. Because of that, various transportation models were presented. Related studies have been done by Yadvendra Kacher and Pitam Singh [4], Ekanayake et al. [5]. These research papers illustrate various aspects of transportation problems, such as the proposed TP based on transportation time. It is called the bottleneck transportation problem. The bottleneck transportation problem (BTP) is one of the specific transportation problems mentioned in this research study. In bottleneck transportation problems, the purpose is to minimize the maximum time instead of the transportation of goods from suppliers to destinations. Many researchers have contributed articles to this special problem.

Bindu Kaushal and Shalini Arora presented " A Priority Based Time Minimization Transportation Problem" in 2018[6], Peerayuth Charnsethikul and Saeree Svetasreni presented "The Constrained Bottleneck Transportation Problem" in 2007[7], Bhabani Mallia et al [8], Gaurav Sharma et al puplished the paper in 2016[9] and B. Mallia et al pulished "Fundamentals of Transportation Problem" in 2021[10]. In addition, research solutions have been presented in relation to fuzzy environment problems. V. Vidhya and K. Ganesan " A simple method for the solution of bottleneck-cost transportation problem under fuzzy environment" in 2020[11], V.E. Sobana et al [12], P.Maliniandand M.Ananthanarayanan presented new algorithm in 2016 [13], Amarpreet Kau presented method in 2011 [14] and "Fully Fuzzy Transportation Problem" published in 2015 [15]. Meanwhile, algorithms have been created based on both the factors of time and cost.they are, A.Ahmed et al presented paper in 2014 [16], Sune Lauth Gadegaar et al pupblished paper in 2016 [17],P. Pandian and G. Natarajan in 2018 [18], R. Sophia Porchelvi and M. Anitha published the "On Minimizing the Cost and Time of a Multi Objective Transportation Model “in 2018[19] and Dr. Kirtiwant et al published paper in 2021[20].

All of the above-mentioned research methods are models used to solve bottleneck transportation problems. Here, the objective is to minimize the maximum time for all suppliers to reach the destination by transportation. Thus, the purpose of this study is to introduce an alternative algorithm that can obtain a basic, feasible solution by solving such problems. It is also hoped to study the success of that method through mathematical problems.

Materials and Methods

The bottleneck transportation problem is a problem related to determining the correct distribution of goods from a set of suppliers to a set of clients to minimize the maximum process time under supply capacity, demand requirements, and constrained process (objective function is $min\ z = \max(t_{ij}\ /\ x_{ij})$. The bottleneck transportation problem is a particular example of the linear programming problem.it also uses of liner model to find the solution [8].

Consider following BTP

Objective function; time minimizing

$$minimize\ Z = [Maximize\ t_{ij}\ /\ x_{ij}\ > 0]$$

Subject to;

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i = 1,2, \dots , m$$

$\sum_{i=1}^{m} X_{ij} = b_j,\quad j = 1,2,\dots,n$ and

$X_{ij} \geq 0$ for all $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$

a_i ; supply available i^{th} sources.

b_j; demand required in j^{th} destination

X_{ij}; indicates the number of units transported from the source (i) to destination (j)

t_{ij}; given where t_{ij} is the time of transporting from the source (i) to destination (j)

In this article we using the following algorithm for find the efficient solution of bottleneck transportation problems.

Proposed algorithm

- Step 1. If the transport problem is unbalanced, it should be balanced by adding a dummy column or a dummy row.
- Step 2. Then calculate the sum of the times of each column and row in the transportation table and the total transportation time separately.
- Step 3. After that, calculate the ratio of total time with the "sum of time in each column and row" as well as " unit time in each cell" separately.
- Step 4. Then first select the column or row with the highest ratio value for each column and row. Assign values in order of increasing ratio in the cell that row or column. Cross the respective row or column after all the assigned values are complete.
- Step 5. In this way, select the relevant column or row in descending order of the ratio value of each column and row. Appropriately assign values in increasing order of ratio values to cells in that selected row or column. After all assignment values are complete, remove the corresponding column or row.
- Step 6. Repeat the above step until all the supply and demand values in the transportation table are satisfied.
- Step 7. Finally, each assigned value is assigned to the original time values in the corresponding transportation table, and finally, select the cell with the highest time among the cells with an assignment. Also calculate the total time value.

Using the above algorithm, an efficient initial solution is first found, as in a general transportation problem. It is then observed to be optimal or close to it.

Results and Discussion

In this section, through numerical examples, we hope to explain step-by-step how the final answer was obtained using the above-mentioned algorithm. Also, the basic solutions obtained are compared with the solutions obtained by the existing methods, thereby evaluating the effectiveness of the proposed method.

Ex.1 [8]

Table 01. bottleneck transportation problem

	D_1	D_2	D_3	supply
S_1	5	7	9	20

S_2	3	8	3	30
S_3	4	6	6	50
demand	10	20	40	

Table 02. step 01, The transportation problem is unbalanced and must be balanced first.

	D_1	D_2	D_3	Dummy column	supply
S_1	5	7	9	0	20
S_2	3	8	3	0	30
S_3	4	6	6	0	50
demand	10	20	40	30	

Table 03. step 02, The total time of the transportation problem is 51. Also, the total time of each column and row has been calculated.

	D_1	D_2	D_3	Dummy column	supply	Total time in each row
S_1	5	7	9	0	20	21
S_2	3	8	3	0	30	14
S_3	4	6	6	0	50	16
demand	10	20	40	30		
Total time in each column	12	21	18			

Table 04. step 03, The ratio of the sum total of each row and column to the total time is calculated. The relevant rows and columns are named as 1,2, 3... in the decreasing order of the ratio value, and the column or row to be assigned in that order should be selected.

	D_1 6	D_2 2	D_3 3	Dummy column	supply	Ratio time in each row
S_1 1	0.098	0.137	0.176	0	20	0.412
S_2 5	0.059	0.157	0.059	0	30	0.275
S_3 4	0.078	0.118	0.118	0	50	0.314
demand	10	20	40	30		
ratio time in each column	0.196	0.412	0.353			

Table 05. step 04, step,05 &step 06, After that, the ratio of the unit time of each cell to the total time is calculated. It is represented as $()^1$, $()^2$, $()^3$…in increasing order of the selected row or column. Give the corresponding assignment values to those orders.

	D_1 6	D_2 2	D_3 3	Dummy column	supply	Ratio time in each row
S_1 1	0.098	0.137	0.176	0 $(20)^1$	20 (0)	0.412
S_2 5	0.059	0.157	0.059 $(30)^3$	0	30 (0)	0.275
S_3 4	0.078 $(10)^6$	0.118 $(20)^2$	0.118 $(10)^4$	0 $(10)^5$	50 (30) (20) (10) (0)	0.314

demand	10 (0)	20 (0)	40 (10) (0)	30 (10) (0)		
Ratio time in each column	0.196	0.412	0.353			

Table 06. step 07, Assign the assignment values to the corresponding original transportation table and obtain the corresponding minimum time as well as total transportation time.

	D_1	D_2	D_3	Dummy column	supply	Total time in each row
S_1	5	7	9	0 (20)	20	21
S_2	3	8	3 (30)	0	30	14
S_3	4 (10)	6 (20)	6 (10)	0 (10)	50	16
demand	10	20	40	30		
Total time in each column	12	21	18			

Objective function $MinZ = Max\ \{\ 4,6,3,6,0,0\} = 6\ \ time\ \ units$

The solution of $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}C_{ij}\ is\ \ (4 \times 10) + (6 \times 20) + (3 \times 30) + (6 \times 10) + (0 \times 20) + (0 \times 10) = 310$

Ex.2 [8]

Table 07. bottleneck transportation problem

	D_1	D_2	D_3	D_4	D_5	supply
S_1	25	30	20	40	45	35
S_2	30	25	20	30	40	22
S_3	40	20	40	35	45	32
S_4	25	24	50	27	30	13
demand	12	20	15	25	20	

Table 08. step 01

	D_1	D_2	D_3	D_4	D_5	Dummy column	supply
S_1	25	30	20	40	45	0	35
S_2	30	25	20	30	40	0	22
S_3	40	20	40	35	45	0	32
S_4	25	24	50	27	30	0	13
demand	12	20	15	25	20	10	

Table 09. step 02

	D_1	D_2	D_3	D_4	D_5	Dummy column	supply	Total time in each row
S_1	25	30	20	40	45	0	35	160
S_2	30	25	20	30	40	0	22	145
S_3	40	20	40	35	45	0	32	180
S_4	25	24	50	27	30	0	13	156

demand	12	20	15	25	20	10		
Total time in each column	120	99	130	132	160			

Table 10. step 03

	D_1 8	D_2 9	D_3 7	D_4 6	D_5 2	Dummy column	supply	Ratio time in each row
S_1 3	0.039	0.047	0.031	0.062	0.070	0	35	0.250
S_2 5	0.047	0.039	0.031	0.047	0.062	0	22	0.226
S_3 1	0.062	0.031	0.062	0.055	0.070	0	32	0.281
S_4 4	0.039	0.037	0.078	0.042	0.047	0	13	0.243
demand	12	20	15	25	20	10		
Ratio time in each column	0.187	0.154	0.203	0.206	0.250			

Table 11. step 04, step,05 &step 06

	D_1 8	D_2 9	D_3 7	D_4 6	D_5 2	Dummy column	supply	Ratio time in each row
S_1 3	0.039 $(12)^7$	0.047	0.031 $(15)^6$	0.062 $(8)^8$	0.070	0	35 (20) (8) (0)	0.250
S_2 5	0.047	0.039	0.031	0.047 $(15)^9$	0.062 $(7)^5$	0	22 (15) (0)	0.226
S_3 1	0.062	0.031 $(20)^2$	0.062	0.055 $(2)^3$	0.070	0 $(10)^1$	32 (22) (2) (0)	0.281
S_4 4	0.039	0.037	0.078	0.042	0.047 $(13)^4$	0	13 (0)	0.243
demand	12 (0)	20 (0)	15 (0)	25 (23) (15) (0)	20 (7) (0)	10 (0)		
Ratio time in each column	0.187	0.154	0.203	0.206	0.250			

Table 12. step 07

	D_1	D_2	D_3	D_4	D_5	Dummy column	supply
S_1	25 (12)	30	20 (15)	40 (8)	45	0	35
S_2	30	25	20	30 (15)	40 (7)	0	22
S_3	40	20 (20)	40	35 (2)	45	0 (10)	32
S_4	25	24	50	27	30 (13)	0	13
demand	12	20	15	25	20	10	

Objective function $MinZ = Max\ \{\ 25,20,20,30,35,40,40,30,0\} = 40\ \ time\ \ units$

The solution of $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} C_{ij}\ is\ \ (25 \times 12) + (20 \times 20) + (20 \times 15) + (40 \times 8) + (30 \times 15) + (35 \times 2) + (40 \times 7) + (30 \times 13) + (0 \times 10) = 2510$

Ex.3 [8]

Table 13. bottleneck transportation problem

	D_1	D_2	D_3	D_4	D_5	supply
S_1	8	10	9	8	0	15
S_2	6	3	4	7	0	20
S_3	12	6	8	4	0	30
S_4	9	5	6	7	0	10
S_5	0	0	0	0	0	10
demand	10	15	20	10	30	

Table 14. step 02

	D_1	D_2	D_3	D_4	D_5	supply	Total time in each row
S_1	8	10	9	8	0	15	35
S_2	6	3	4	7	0	20	20
S_3	12	6	8	4	0	30	30
S_4	9	5	6	7	0	10	27
S_5	0	0	0	0	0	10	0
demand	10	15	20	10	30		
Total time in each column	35	24	27	26	0		

Table 15. step 03

	D_1 2	D_2 7	D_3 4	D_4 6	D_5	supply	Ratio time in each row
S_1 1	0.071	0.089	0.080	0.071	0	15	0.313
S_2 8	0.054	0.027	0.036	0.063	0	20	0.179
S_3 3	0.107	0.054	0.071	0.036	0	30	0.268
S_4 5	0.083	0.045	0.054	0.063	0	10	0.241
S_5	0	0	0	0	0	10	0
demand	10	15	20	10	30		
Ratio time in each column	0.313	0.214	0.241	0.232	0		

Table 16. step 04, step,05 &step 06

	D_1 2	D_2 7	D_3 4	D_4 6	D_5	supply	Ratio time in each row
S_1 1	0.071	0.089	0.080	0.071	0 (15)^{1}	15 (0)	0.313
S_2 8	0.054	0.027	0.036 (20)^{6}	0.063	0	20 (0)	0.179
S_3 3	0.107	0.054 (5)^{5}	0.071	0.036 (10)^{4}	0 (15)^{3}	30 (15) (5) (0)	0.268
S_4 5	0.083	0.045 (10)^{7}	0.054	0.063	0	10 (0)	0.241

S_5	0 (10)²	0	0	0	0	10 (0)	0
demand	10 (0)	15 (10) (0)	20 (0)	10 (0)	30 (15) (0)		
Ratio time in each column	0.313	0.214	0.241	0.232	0		

Table 17. step 07

	D_1	D_2	D_3	D_4	D_5	supply
S_1	8	10	9	8	0 (15)	15
S_2	6	3	4 (20)	7	0	20
S_3	12	6 (5)	8	4 (10)	0 (15)	30
S_4	9	5 (10)	6	7	0	10
S_5	0 (10)	0	0	0	0	10
demand	10	15	20	10	30	

Objective function $MinZ = Max \{ 0,6,5,4,4,0,0\} = 6 \ time \ units$

The solution of $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} C_{ij} \ is \ (0 \times 10) + (6 \times 5) + (5 \times 10) + (4 \times 20) + (4 \times 10) + (0 \times 15) + (0 \times 15) = 200$

Table 18. comparison with the existing methods

Methods	Ex.1	Ex.2	Ex.3
Northwest corner method	380	2775	320
Vogel'sapproximation method	310	2725	210
Bhanbani mallia et al [8]	320 & 6-units time	2775 & 40-units time	200 & 6-units time
Proposed algorithm	310 & 6-units time	2510 & 40-units time	200 & 6-units time
Optimal solution	310 & 6-units time	2090 & 40-units time	200 & 6-units time

Ex.4 [18]

Table 19. bottleneck transportation problem

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	supply
S_1	12	13	34	7	8	29	19	15
S_2	7	18	36	40	38	6	10	7
S_3	11	20	30	21	21	29	31	45
S_4	27	12	39	31	5	36	12	30
S_5	15	17	32	36	22	16	14	12
S_6	17	38	16	33	23	30	29	16
demand	20	13	11	27	9	5	40	

Table 20. step 02

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	supply	Total time in each row
S_1	12	13	34	7	8	29	19	15	122
S_2	7	18	36	40	38	6	10	7	155

S_3	11	20	30	21	21	29	31	45	163
S_4	27	12	39	31	5	36	12	30	162
S_5	15	17	32	36	22	16	14	12	152
S_6	17	38	16	33	23	30	29	16	186
demand	20	13	11	27	9	5	40		
Total time in each column	89	118	187	168	117	146	115		

Table 21. step 03

	D_1 13	D_2 10	D_3 1	D_4 3	D_5 11	D_6 8	D_7 12	supply	Ratio time in each row
S_1 9	0.013	0.014	0.036	0.007	0.008	0.031	0.020	15	0.130
S_2 6	0.007	0.019	0.038	0.043	0.040	0.006	0.011	7	0.165
S_3 4	0.012	0.021	0.032	0.022	0.022	0.031	0.033	45	0.173
S_4 5	0.029	0.013	0.041	0.033	0.005	0.038	0.013	30	0.172
S_5 7	0.016	0.018	0.034	0.034	0.023	0.017	0.015	12	0.162
S_6 2	0.018	0.040	0.017	0.035	0.024	0.032	0.031	16	0.198
demand	20	13	11	27	9	5	40		
Ratio time in each column	0.095	0.126	0.199	0.179	0.124	0.155	0.122		

Table 22. step 04, step,05 &step 06

	D_1 13	D_2 10	D_3 1	D_4 3	D_5 11	D_6 8	D_7 12	supply	Ratio time in each row
S_1 9	0.013	0.014	0.036	0.007 (15)^{3}	0.008	0.031	0.020	15 (0)	0.130
S_2 6	0.007	0.019	0.038	0.043	0.040	0.006 (5)^{10}	0.011 (2)^{11}	7(2) (0)	0.165
S_3 4	0.012 (15)^{5}	0.021 (13)^{6}	0.032	0.022 (12)^{4}	0.022 (5)^{7}	0.031	0.033	45 (33) (18) (5) (0)	0.173
S_4 5	0.029	0.013	0.041	0.033	0.005 (4)^{8}	0.038	0.013 (26)^{9}	30 (26) (0)	0.172
S_5 7	0.016	0.018	0.034	0.034	0.023	0.017	0.015 (12)^{12}	12 (0)	0.162
S_6 2	0.018 (5)^{2}	0.040	0.017 (11)^{1}	0.035	0.024	0.032	0.031	16 (5) (0)	0.198
demand	20 (15) (0)	13 (0)	11 (0)	27 (12) (0)	9 (4) (0)	5 (0)	40(14) (12) (0)		

Ratio time in each column	0.095	0.126	0.199	0.179	0.124	0.155	0.122		

Table 23. step 07

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	supply
S_1	12	13	34	7 (15)	8	29	19	15
S_2	7	18	36	40	38	6 (5)	10 (2)	7
S_3	11 (15)	20 (13)	30	21 (12)	21 (5)	29	31	45
S_4	27	12	39	31	5 (4)	36	12 (26)	30
S_5	15	17	32	36	22	16	14 (12)	12
S_6	17 (5)	38	16 (11)	33	23	30	29	16
demand	20	13	11	27	9	5	40	

Objective function $MinZ = Max\ \{\ 11,17,20,16,7,21,21,5,6,10,12,14\} = 21\ \ time\ \ units$

- The minimum time transportation is 21 [18]. answer is same with the p.snanthi pandian el al method.

Ex.5

Table 24. bottleneck transportation problem

	D_1	D_2	D_3	supply
S_1	10	2	20	10
S_2	3	7	9	15
S_3	12	14	16	5
demand	10	8	12	

Table 25. step 02

	D_1	D_2	D_3	supply	Total time in each row
S_1	10	2	20	10	32
S_2	3	7	9	15	19
S_3	12	14	16	5	42
demand	10	8	12		
Total time in each column	25	23	45		

Table 26. step 03

	D_1 4	D_2 5	D_3 1	supply	Ratio time in each row
S_1 3	10	2	20	10	0.344
S_2 6	3	7	9	15	0.204
S_3 2	12	14	16	5	0.452
demand	10	8	12		
Ratio time in each column	0.269	0.247	0.484		

Table 27. step 04, step,05 &step 06

	D_1 4	D_2 5	D_3 1	supply	Ratio time in each row
S_1 3	0.108 (2)[4]	0.022 (8)[3]	0.215	10 (2) (0)	0.344
S_2 6	0.032 (3)[5]	0.075	0.098 (12)[1]	15 (3) (0)	0.204
S_3 2	0.129 (5)[2]	0.151	0.172	5 (0)	0.452
demand	10 (5) (3) (0)	8 (0)	12 (0)		
Ratio time in each column	0.269	0.247	0.484		

Table 28. step 07

	D_1	D_2	D_3	supply
S_1	10 (2)	2 (8)	20	10
S_2	3 (3)	7	9 (12)	15
S_3	12 (5)	14	16	5
demand	10	8	12	

Objective function $MinZ = Max\ \{\ 10{,}3{,}12{,}2{,}9\} = 12\ \ time\ \ units$

The optimum transportation time is 12 [21].

Ex.6 [22]

Table 29. bottleneck transportation problem

	D_1	D_2	D_3	supply
S_1	13	21	14	13
S_2	8	12	21	20
S_3	15	17	19	5
demand	12	15	11	

Table 30. step 02

	D_1	D_2	D_3	supply	Total time in each row
S_1	13	21	14	13	48
S_2	8	12	21	20	41
S_3	15	17	19	5	51
demand	12	15	11		
Total time in each column	36	50	54		

Table 31. step 03

	D_1 6	D_2 3	D_3 1	supply	Ratio time in each row
S_1 4	13	21	14	13	0.343
S_2 5	8	12	21	20	0.293
S_3 2	15	17	19	5	0.364
demand	12	15	11		
Ratio time in each column	0.257	0.357	0.386		

Table 32. step 04, step,05 &step 06

	D_1 6	D_2 3	D_3 1	supply	Ratio time in each row
S_1 4	0.093 (2)^4	0.150	0.100 (11)^1	13 (2) (0)	0.343
S_2 5	0.057 (5)^5	0.086 (15)^3	0.150	20 (5) (0)	0.293
S_3 2	0.107 (5)^2	0.121	0.136	5 (0)	0.364
demand	12 (7) (5) (0)	15 (0)	11 (0)		
Ratio time in each column	0.257	0.357	0.386		

Table 33. step 07

	D_1	D_2	D_3	supply
S_1	13 (2)	21	14 (11)	13
S_2	8 (5)	12 (15)	21	20
S_3	15 (5)	17	19	5
demand	12	15	11	

Objective function $MinZ = Max\ \{\ 13{,}8{,}15{,}12{,}14\} = 15\ \ time\ \ units$

Ex.7 [22]

Table 34. bottleneck transportation problem

	D_1	D_2	D_3	supply
S_1	15	7	25	12
S_2	8	12	14	17
S_3	17	19	21	7
demand	12	10	14	

Table 35. step 02

	D_1	D_2	D_3	supply	Total time in each row
S_1	15	7	25	12	47
S_2	8	12	14	17	34
S_3	17	19	21	7	57
demand	12	10	14		

Total time in each column	40	38	60		

Table 36. step 03

	D_1 4	D_2 5	D_3 1	supply	Ratio time in each row
S_1 3	15	7	25	12	0.341
S_2 6	8	12	14	17	0.246
S_3 2	17	19	21	7	0.364
demand	12	10	14		
Ratio time in each column	0.290	0.275	0.435		

Table 37. step 04, step,05 &step 06

	D_1 4	D_2 5	D_3 1	supply	Ratio time in each row
S_1 3	0.109 (2)^5	0.051 (10)^3	0.181	12 (2) (0)	0.341
S_2 6	0.058 (3)^4	0.087	0.101 (14)^1	17 (3) (0)	0.246
S_3 2	0.123 (7)^2	0.137	0.152	7 (0)	0.364
demand	12 (5) (2) (0)	10 (0)	14 (0)		
Ratio time in each column	0.290	0.275	0.435		

Table 38. step 07

	D_1	D_2	D_3	supply
S_1	15 (2)	7 (10)	25	12
S_2	8 (3)	12	14 (14)	17
S_3	17 (7)	19	21	7
demand	12	10	14	

Objective function $MinZ = Max\ \{\ 15,8,17,7,14\} = 17\ \ time\ \ units$

Table 39. comparison with the existing methods

Method	Ex.06	Ex.07
M.M.Ahmed et al [22]	15	17
Proposed algorithm	15	17
Optimum solution [22]	15	17

Ex.8 [23]

Table 40. bottleneck transportation problem

	D_1	D_2	D_3	D_4	supply
S_1	2	4	6	8	52

S_2	5	7	6	7	59
S_3	16	20	10	12	28
S_4	19	18	17	28	94
demand	40	55	68	70	

Table 41. step 02

	D_1	D_2	D_3	D_4	supply	Total time in each row
S_1	2	4	6	8	52	20
S_2	5	7	6	7	59	25
S_3	16	20	10	12	28	58
S_4	19	18	17	28	94	82
demand	40	55	68	70		
Total time in each column	42	49	39	55		

Table 42. step 03

	D_1 5	D_2 4	D_3 6	D_4 3	supply	Total time in each row
S_1 8	2	4	6	8	52	0.108
S_2 7	5	7	6	7	59	0.135
S_3 2	16	20	10	12	28	0.314
S_4 1	19	18	17	28	94	0.443
demand	40	55	68	70		
Total time in each column	0.227	0.265	0.211	0.297		

Table 43. step 04, step,05 &step 06

	D_1 5	D_2 4	D_3 6	D_4 3	supply	Total time in each row
S_1 8	0.011 (23)^6	0.022 (29)^5	0.032	0.043	52 (23) (0)	0.108
S_2 7	0.027 (17)^7	0.038	0.032	0.038 (42)^4	59 (17) (0)	0.135
S_3 2	0.086	0.108	0.054	0.065 (28)^3	28 (0)	0.314
S_4 1	0.103	0.097 (26)^2	0.092 (68)^1	0.151	94 (26) (0)	0.443
demand	40 (17) (0)	55 (29) (0)	68 (0)	70 (42) (0)		
Total time in each column	0.227	0.265	0.211	0.297		

Table 44. step 07

	D_1	D_2	D_3	D_4	supply
S_1	2 (23)	4 (29)	6	8	52
S_2	5 (17)	7	6	7 (42)	59

S_3	16	20	10	12 (28)	28
S_4	19	18 (26)	17 (68)	28	94
demand	40	55	68	70	

Objective function $MinZ = Max\ \{\ 2,5,4,18,17,7,12\} = 18\ \ time\ \ units$

The optimum transportation time is 18 [23].

Conclusions

Time-minimizing TPs is the most important part of a real-life situation. According to the above result, the proposed method is more efficient for finding an initial solution to the bottleneck transportation problem compared with other existing methods. Existing problems are used to test the suggested approach for optimality and compare the proposed method with the existing method. According to the results obtained by the comparative analysis, it was concluded that the solutions obtained by this algorithm give similar answers to those obtained by the existing methods. Also, the fact that the initial solution is the same as the optimal solution means that the algorithm provides successful solutions. It was observed that this algorithm provides the optimal solution, or one that is close to it, for transportation problems. Accordingly, this algorithm provides a successful and efficient initial solution for bottleneck transportation problems.

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