Two Kinds of Equilibria: Particle and Wave/Dynamic Based? Part III Combining the Two

Francesco R. Ruggeri Hanwell, N.B. Jan. 10, 2022

In Parts I and II we examined equilibrium situations based on an additive scalar (distributive variable) such as ei= pi dot pi/2m in a Maxwell-Boltzmann gas and a vector dynamic quantity such as the momentum vector in a reflection/refraction of light situation or a single particle quantum bound state. We argued that a loss of information in the form P(a1)P(a2)=P(a3) holds in both cases, but in the additive scalar situation this leads to P(a1) = C exp(-a1/T) while in the vector case to exp(-i a1 dot conjugate variable). For a one dimensional quantum bound state a1=p and the conjugate variable is x. In such a case, exp(ipx) is a periodic eigenstate of the translational generator multiplied by -i. Within the bound state, stochastic hits from V(x) the potential lead to an ensemble i..e W(x) = Sum over p a(p)exp(ipx) where a(p)'s are fixed by considering average energy at each x to be the same i.e. { Sum over p a(p) pp/2m exp(ipx) } / { Sum over p a(p)exp(ipx)} + V(x) = En.

In this note, we consider the a(p)'s as also being probabilities like exp(ipx) and ask whether they could exhibit the same loss of information as in the vector p if one associate p in a(p) with the additive scalare pp/2m. We argue that such considerations lead to the ground state solution of a quantum oscillator.

Loss of Information

In Parts I and II, we argued that there seem to be two different kinds of equilibria in nature, with both existing in the classical world and both in the quantum world as well. In the classical world, one first has the well-known Maxwell-Boltzmann (MB) gas. The stochastic process is represented by elastic two body collisions (either with gas particles with gas particles or gas particles with wall particles). The elastic condition means an additive scalar is conserved, thus this is a distributed variable or additive scalar problem. We argued that the key to equilibrium is the loss of information in the form of P(a1)P(a2)=P(a3). For the MB gas, a1= e=pp/2m i.e. the kinetic energy of a gas molecule.

Then e1+e2=e3 and P(e1)P(e2)=P(e3) —> P(ei) = C exp(-ei/T) ((1))

1/T is a parameter such that: <e> known= Sum over ei C exp(-ei/T) ei ((2))

Matters become a little trickier if a potential V(x) is introduced into the MB gas. Motion in a potential is Newtonian, not stochastic. ei=kinetic energy of a particle is stochastic, yet for a given value of the stochastic value at V(x1)=0, there is a linking equation at another x i.e. x2:

Ei(1) + V(x1) = ei(2) + V(x2) where V(x1)=0 ((3))

Given that ei(1) is represented by a probability (it is a stochastic variable), then ei(2)+V(x2) substituted in P(ei(1)) gives the same value of probability, but ei(2)+V(x2) consists of two pieces.

Thus the deterministic piece $V(x2)$ is now part of a probability expression and gives the appearance of $V(x)$ being stochastic as well i.e.

$$\text{Probability} = C \exp(-ei/T) \exp(-V(x)/T) \quad ((4))$$

Thus a spatial density is created as well which is associated with an entropic type of force, namely pressure. $PV=nRT$ describes the situation for an ideal gas. It is interesting to note that known Newtonian force $-dV(x)/dx$ is balanced by pressure differences to allow the kinetic energies to freely (in a sense) form their T based equilibrium i.e. $\exp(-ei/T)$ holds at any x point. This is consistent with the arguments above because $P(ei(2)+V(x2))$ becomes separable such that every ei at x2 is associated with the same $\exp(-V(x2)/T)$ factor showing that $V(x2)$ is not really involved in the stochastic process of two body collisions. Thus we argue that information or loss of information appears in the ration in the sense that:

$$P(ei(2) + V(x2)) / P(ej(2) + V(x2)) = P(ei(2))/ P(ej(2)) \quad ((5))$$

What happens in the case of a dynamical vector changing in the stochastic process? The reflection/refraction of light from a medium (along the y axis at x=0) is a stochastic process because there is a probability to reflect and another to refract. In this case, however, it is the vector p momentum which changes, not an additive scalar. For instance, if the incident beam moves in the positive x direction with photons of momentum p, then a reflected photon has momentum -p and a refracted one p2=pn2 where n2 is the index of refraction and p has an index of 1. Nevertheless, the same loss of information statement holds.:

$$P(p1)P(p2) = P(p3) \quad ((6))$$

In this case, however, p is a vector and so a scalar is created from the vector and a conjugate variable. The conjugate variable is associated with the motion of the dynamical variable i.e. p moves through x so px or (p dot r) is used. A solution to ((6)) which is associated with the spatial invariance present is:

$$\exp(ipx) \quad ((7))$$

Thus one has again an exp(scalar) result, but now it is periodic because a second variable x had to be introduced and there is spatial invariance associated with it.

The exp(ipx) solution may be used to describe light in a reflection/refraction situation, or an electron in a scattering problem for V(x) confined to a region near x=0. An example is hard sphere scattering.It may also be used in a quantum single particle bound state because the particle again interacts with V(x).

Thus in the classical world, there exists both the additive scalar MB gas example and the dynamical vector equilibrium in the reflection/refraction of light.

In the quantum world, the dynamical vector approach exists in scattering from a V(x) or in a bound state (in which a single particle interacts with V(x)). The additive scalar (distributive variable) approach also exists in the quantum world because a single particle may be stochastically sent to higher energy levels by the absorption of photons or decay to lower energy levels by emitting photons. The energy of the level is then the scalar additive variable.

So far only the situation of either an equilibrium based on a scalar additive or one based on a dynamical vector has been considered. This begs the question: Can one have both in the same problem?

A Quantum Bound State Which Contains Both a Scalar and Vector Equilibrium Together

As noted above, the equilibrium approach for a dynamical vector which changes in a stochastic interaction, such as the momentum vector receiving impulse hits from a potential V(x) ,leads to exp(ipx) as a probability such that exp(ip1x)exp(ip2x) = exp(i (p1+p2)x). Here p1 and p2 are constant vectors. In Newtonian mechanics, however, V(x) accelerates a particle i.e. its momentum is a function of x. In the dynamical vector equilibrium approach one creates an interfering average linked to conservation of energy which creates a p(rms) which represents the Newtonian accelerating situation. exp(ipx)s don't accelerate in the quantum picture. Thus one considers the ensemble:

W(x) = Sum over p a(p)exp(ipx) ((8a)) and the energy conservation requirement at each x yields:

{ Sum over p a(p) pp/2m exp(ipx) } / { Sum over p a(p)exp(ipx)} + V(x) = En. ((8b))

In the above we consider exp(ipx) as the equilibrium probability which is involved in loss of information i.e. exp(ip1x)exp(ip2x) = exp(i (p1+p2)x). What about a(p)? These are weights which may be thought of as unnormalized probabilities. Why don't they also exhibit information loss? ((8b)) shows these weights are governed by the energy conservation equation.

Nevertheless one may propose a situation in which one had double information loss. First, the information loss in exp(ipx) is guaranteed, but given that a(p) may be associated with the scalar pp/2m (nonrelativistic case) one may also consider information loss within a(p) i.e.

a(p1p2/2m) a(p2p2/2m) = a(p3p3/2m) where p1p2/2m + p2p2/2m = p3p3/2m ((9a))

and exp(ip1x)exp(ip2x) = exp(i (p1+p2)x) so p1+p2 vector = p3 vector ((9b))

((9a)) and ((9b)) contradict each other because converting ((9b) into a scalar equation yields:

(p1+p2)/2m = p1p1/2m + p2p2/2m + p1p2/m = p3p3/2m ((10))

Does this mean one cannot have both a scalar and vector equilibrium together? We suggest one can. In particular, because a(p) may be linked to pp/2m one may note that:

a(p) is associated with pp/2m and a(-p) is associated with pp/2m ((11))

This association is often made in quantum mechanics and suggests a fixed parity.

Then a(p1)a(-p2) = a(p1)a(p2) etc so:

P1-p2 vector $\rightarrow$ p1p1/2m+p2p2/2m - p1p2/m, but a(p1)a(-p2) are associated with p1p1/2m + p2p2/2m

P1+p2 vector $\rightarrow$ p1p1/2m+p2p2/2m + p1p2/m but a(p1)a(-p2) are associated with p1p1/2m + p2p2/2m

Thus if p1-p2 and p1+p2 are associated with the same weight a(p1)a(p2)=a(p1)a(-p2) then the average kinetic energy is: (p1p1+p2p2)/2m.

Thus there is a way to reconcile, on AVERAGE, the loss of information associated with the vector p and a loss of information associated with the scalar pp/2m.

For the condition: a(p1p1/2m) a(p2p2/2m) = a(p3p3/2m) such that p1p1p/2m + p2p2/2m = p3p3p/2m one has a(p1) = C exp(-bp1p1). ((12))

This is of the form of a Maxwell-Boltzmann gas probability for kinetic energy. Matters, however, are different because in an MB gas, each particle follows pp/2m + V(x) = E i.e. p(x). In the quantum bound case, p is a constant, only p(rms) changes with x.

To proceed further, we suggest examining the form exp(ipx) which is symmetric in p and x. Given W(x) = Sum over p a(p)exp(ipx), W(x) is the Fourier transform of a(p). If a(p) is an unnormalized probability in p space, then W(x) may be considered an unnormalized probability in x space. One may then impose the same loss of information condition on W(x) that one imposed on a(p) except that p was associated with pp2/m as energy and x is associated with V(x) as energy.

Then W(x1)W(x2) = W(x3) where V(x1)+V(x2) = V(x3) ((13))

This suggests that W(x1) = C2 exp(-b2 V(x)), but this is the spatial form of probability in an MB gas with a potential V(x).

In order to have ((12)) and ((13)) hold, one might expect a symmetry between pp2/m and V(x) namely:

V(x)= dxx where d is a constant ((14))

Thus, the joint additive scalar, dynamical vector equilbria may be associated with a quantum oscillator. A quantum oscillator, however, has many energy levels which are created by absorbing phonons, so one would speculate that the idea of W(x) = C2 exp(-b3 xx) applies to the ground state. This is the case, but the factor also appears in all higher states.

Thus it is possible to have a double set of information loss (one for an additive scalar and one for a dynamical vector) in the same problem.

Conclusion

In Parts I and II we argued there are two types of equilibria existing in both the classical and quantum worlds which follow the same loss of information pattern namely: P(a1)P(a2) = P(a3). In one case, a1 is an additive scalar (distributive variable) as in the MB gas case or a quantum bound single particle jumping from one energy level to another and decaying. In the other case, one has a1=vector as in the momentum vector in the case of light reflecting/refracting in classical physics or a quantum particle with rest mass scattering off a potential in a scattering problem or interacting with V(x) in a bound state problem.

For the additive scalar case, P(ei) =C exp(-ei/T) where ei=a = kinetic energy in a Maxwell-Boltzmann gas with no potential. For the vector case. one has exp(ipx) for the quantum particle with a fixed p vector. In the presence of a potential V(x), an ensemble of exp(ipx)s is used with weights a(p). These a(p) are fixed by imposing a conservation of energy at each x i.e. { Sum over p a(p) pp/2m exp(ipx) } / { Sum over p a(p)exp(ipx)} + V(x) = En.

In this note, we ask whether a(p) may be associated with a loss of information condition on pp/2m i.e. a(p1p1/2m) a(p2p2/2m) = a(p3p3/2m) with p1p1/2m + p2p2/2m + p3p3/2m because this is an additive scalar situation. At the same time, exp(ip1x)exp(ip2x) = exp(i (p1+p2)x) which is the vector loss of information condition. At first there seems to be a contradiction because p1+p2=p3 vectors yields p1p1/2m + p2p2/2m + p1p2/m = p3p3/2m. If one considers an averaging process, however, with a(p1)a(p2)=a(p1)a(-p2), the cross terms p1p2/2m and -p1p2/2m average out. Thus it may be possible to have both an additive scalar and dynamical vector equilibrium together in the same problem. The a(p) scalar loss of information condition implies a(p) = C exp(-bpp). Now W(x) = Sum over a(p)exp(ipx) is the Fourier transform of a(p). If one argues that the loss of information should also apply to energy associated with x i.e. V(x), then: W(x1)W(x2)=W(x3) where V(x1)+V(x2)=V(x3) implies that W(x) = C2 exp(-b2 V(x)). In order for both conditions to hold one might expect to have symmetry between x and p i.e. pp/2m and V(x)=k/2 xx. This is in fact the case and applies to the ground state of the quantum oscillator, although the exp(-b2xx) factor is present in all oscillator energy levels.