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Cascade Theory for Turbulence, Dark Matter and SMBH Evolution

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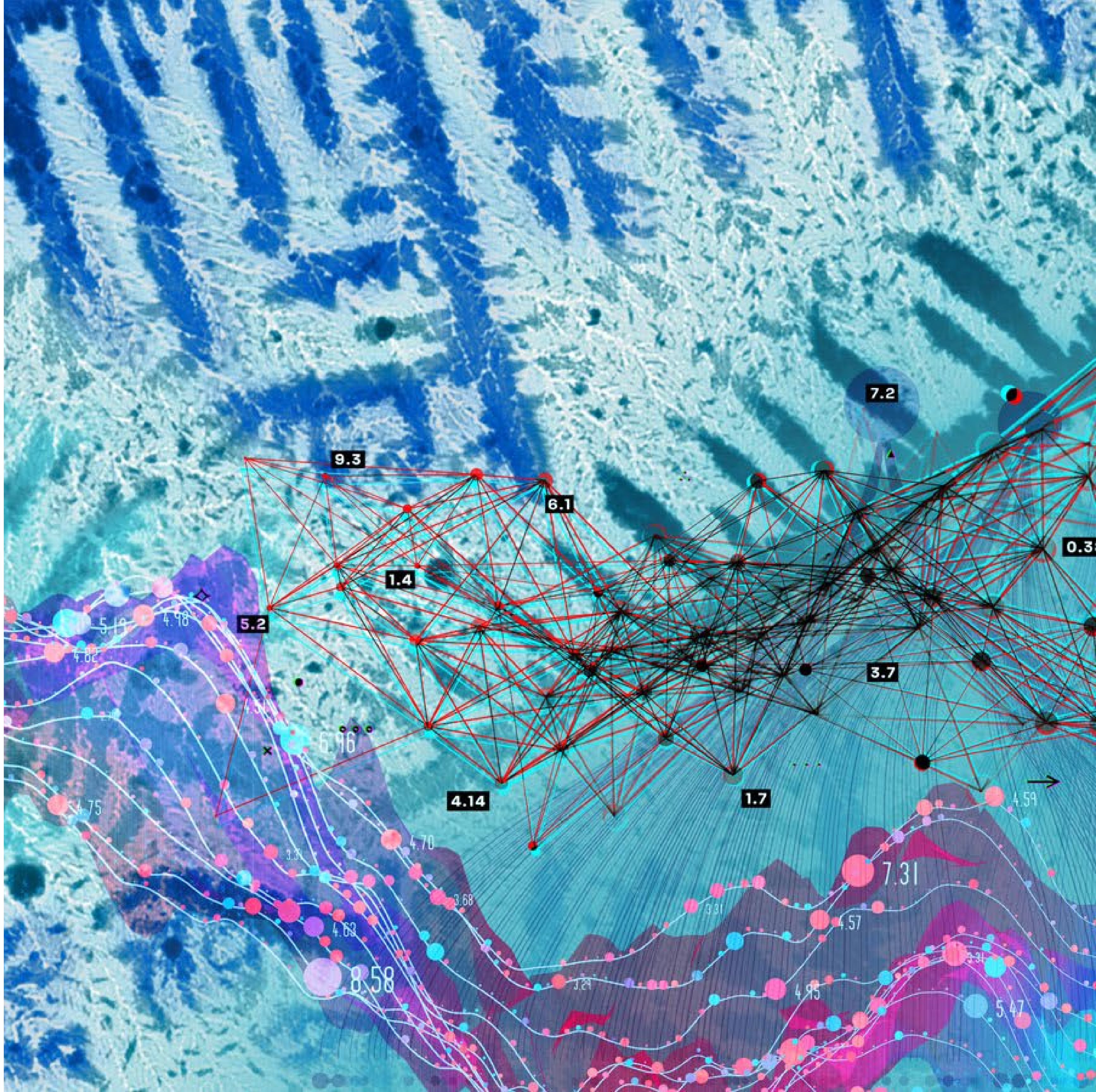


Outline

- Introduction
- Turbulence vs. the flow of dark matter: similarities and differences?
- Inverse mass cascade in dark matter flow
  - Random walk of halos in mass space and halo mass function
  - Random walk of dark matter in real space and halo density profile
- Energy cascade in dark matter flow
  - Universal scaling laws from N-body simulations and rotation curves
  - Dark matter properties from energy cascade
  - Uncertainty principle for energy cascade?
  - Extending to self-interacting dark matter
- Velocity/density correlation/moment functions
- Maximum entropy distributions for dark matter
- Energy cascade for the origin of MOND acceleration
- Energy cascade for the baryonic-to-halo mass relation
- Energy cascade for SMBH-bulge coevolution

Relevant datasets are available at:

"A comparative study of dark matter flow & hydrodynamic turbulence and its applications“ http://dx.doi.org/10.5281/zenodo.6569901

Energy cascade for baryonic-to-halo mass relation

- Total galaxy baryonic mass = stellar mass + cold gas.
- Stellar-to-halo mass relation (SHMR)
    - halo abundance matching

Goals:

- Baryonic-to-halo mass ration (BHMR>SHMR)
- The average mass fraction of baryons in all halos?
- The fraction of total baryons residing in all galaxies?

- Baryonic Tully-Fisher (BTFR) for flat rotation speed:

$$v_f^4 = G m_b a_0 \Leftarrow$$

observed baryonic mass

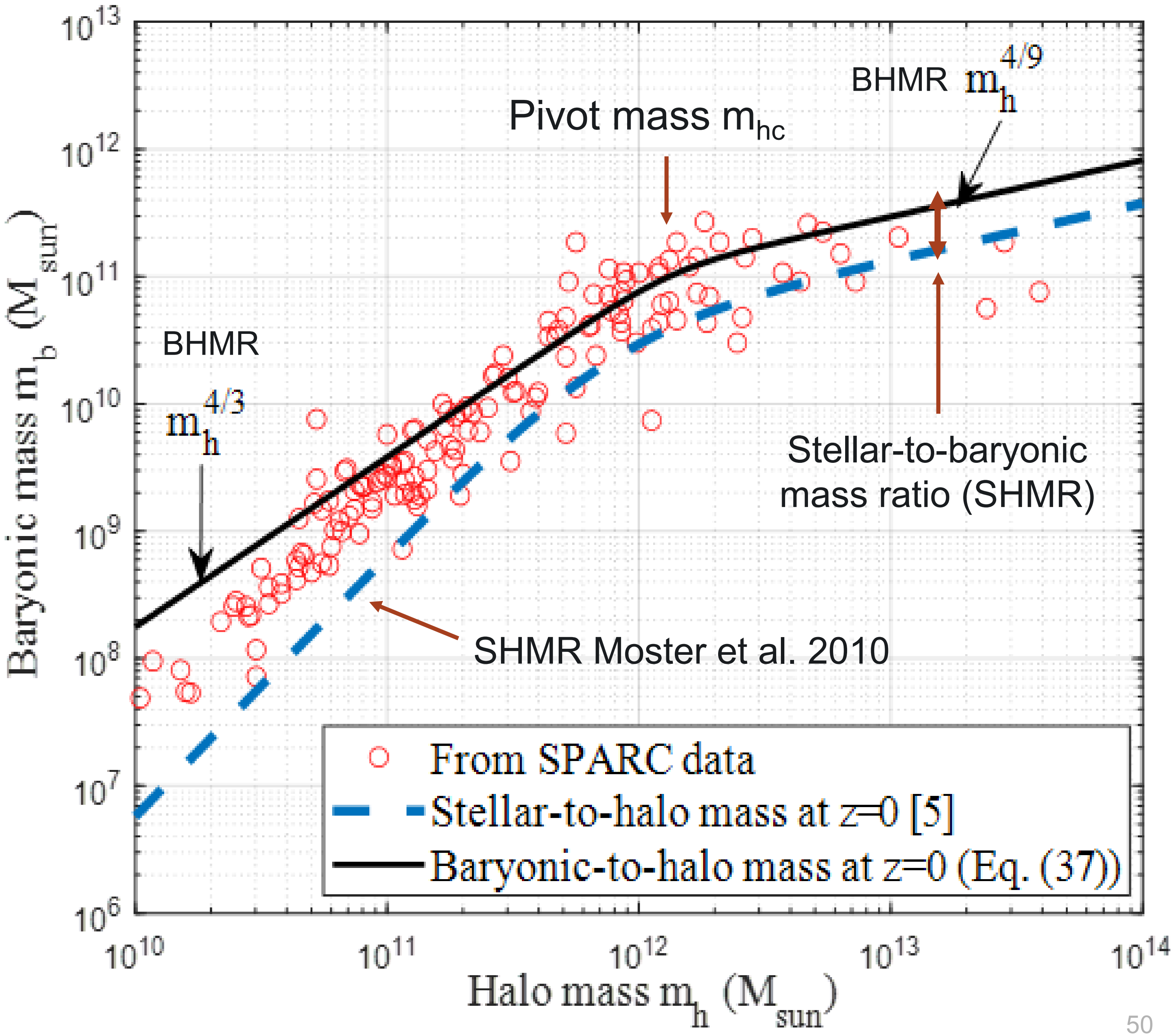
- Halo mass m_h can be related to the halo virial radius r_h through constant density ratio Δ_c

$$\boxed{m_h} = \frac{4}{3}\pi\left(r_h\right)^3 \Delta_c \bar{\rho}_0\left(a\right)$$

- The BHMR (between m_b and m_h) can be obtained only if the relation between v_f and r_h is known.

Relate to energy cascade in baryonic flow? see 2/3 law

$$\varepsilon_u \propto v_f^3 / r_h$$



Energy cascade for the flow of baryonic mass

Baryonic Tully-Fisher relation (BTFR):

$$v_f^4 = G m_b a_0$$

Halo mass and halo size relation:

$$m_h = \frac{4}{3}\pi r_h^3 \Delta_c \bar{\rho}_0 a^{-3}$$

<u>Rate of energy cascade</u>

$$\varepsilon_u = -\beta_f \frac{u^2}{r_h / v_f} a^q$$

Small halos <m$_L$:
Baryonic mass in equilibrium with DM, i.e. same kinetic energy as DM particles u^2

DM Circular velocity

$$v_{cir} = \frac{4}{9}\sqrt{\frac{\Delta_c}{2}}\beta_f v_f a^q \propto v_f$$

DM halo size

$$r_h = \frac{4}{9}\beta_f v_f H^{-1} a^q \propto v_f$$

Flat rotation speed

$$v_f = \frac{9}{4\beta_f}\left(\frac{2}{\Delta_c}\right)^{\frac{1}{3}} \left(G m_h H\right)^{1/3} a^{-q} \propto \left(m_h\right)^{1/3}$$

Baryonic Tully-Fisher relation (BTFR):

$$v_f^4 = G m_b a_0$$

Halo mass and halo size relation:

$$m_h = \frac{4}{3}\pi r_h^3 \Delta_c \bar{\rho}_0 a^{-3}$$

$$\varepsilon_u = -\alpha_f \frac{v_f^2}{r_h / v_f} a^p$$

Turnaround time

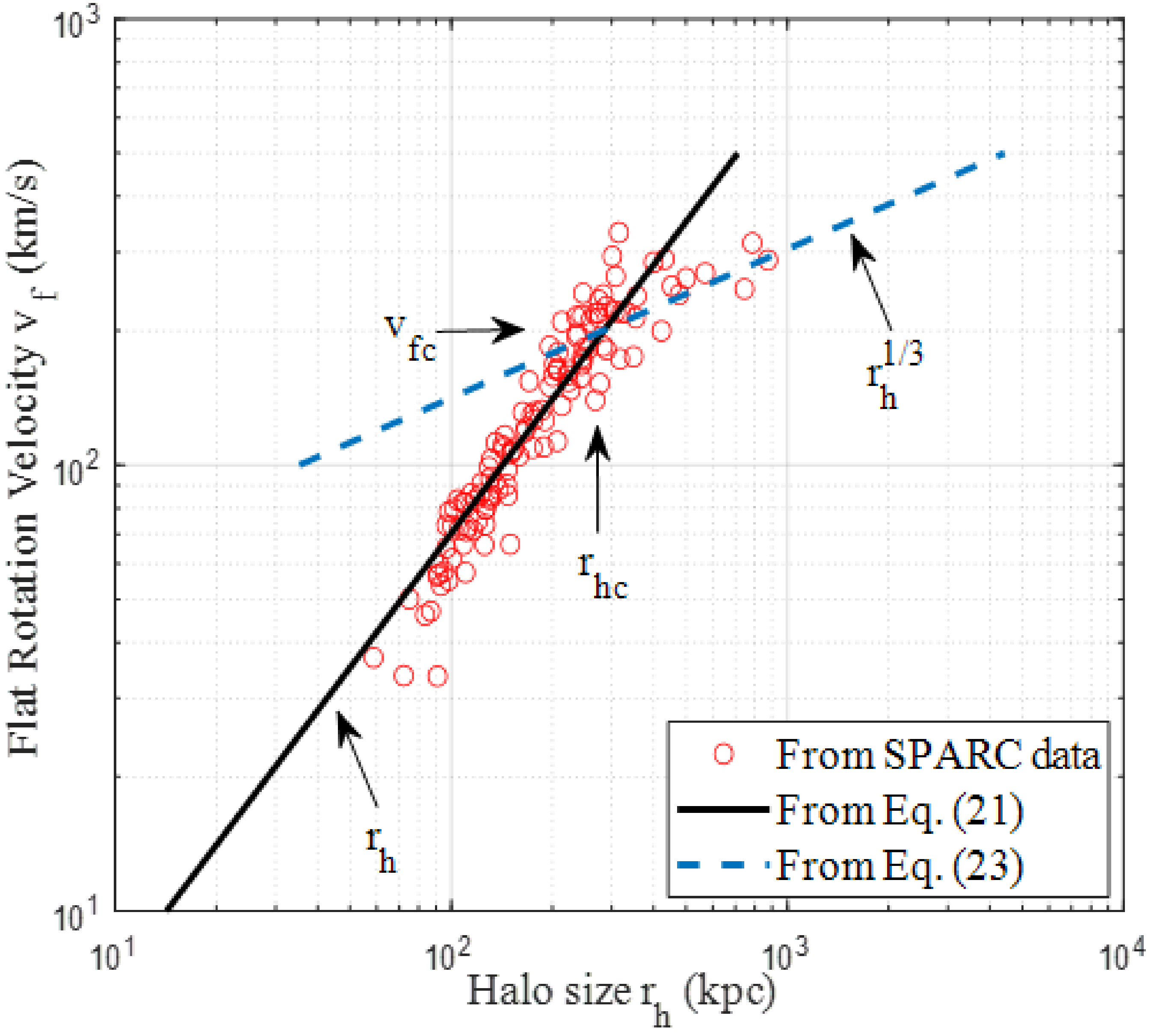
Large halos >m$_L$:
Baryonic mass and DM are two miscible phases sharing the same rate of cascade.

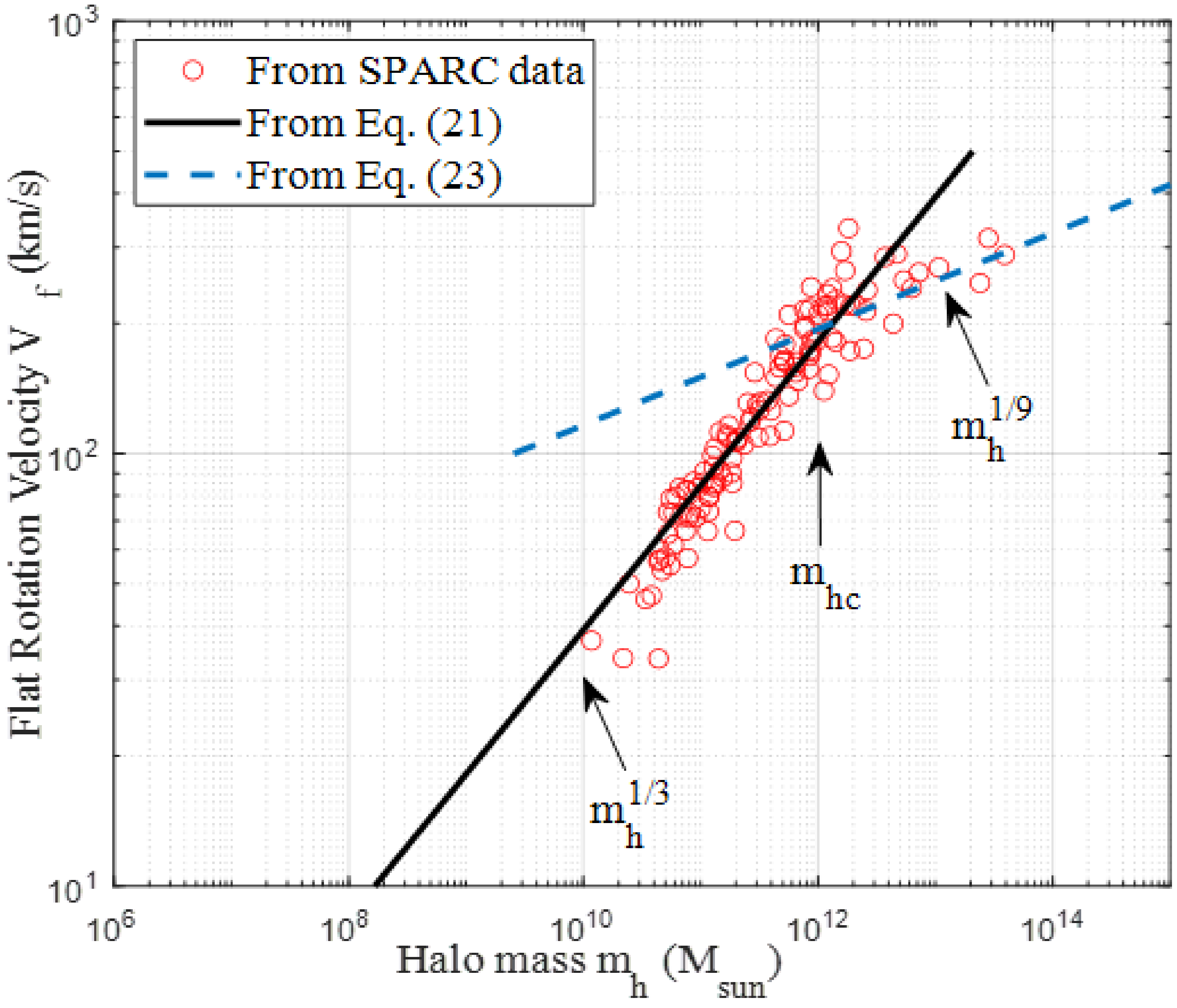
$$v_{cir} = \frac{4}{9}\sqrt{\frac{\Delta_c}{2}}\alpha_f \frac{v_f^3}{u^2} a^p \propto v_f^3$$

$$r_h = \frac{4}{9}\alpha_f \frac{v_f^3}{H u^2} a^p \propto v_f^3$$

$$v_f = \left(\frac{3}{2\sqrt{\alpha_f}}\right)^{\frac{2}{3}} \left(\frac{2}{\Delta_c}\right)^{\frac{1}{9}} \left(G m_h H\right)^{1/9} u^{2/3} a^{-p/3} \propto \left(m_h\right)^{1/9}$$

Model prediction and validation by SPARC data I





Model prediction and validation by SPARC data II

Baryonic mass in small halos < pivot mass m_{hc}:

$$m_b = \left(M_{c1}\right)^{-1/3}\left(m_h\right)^{4/3}$$

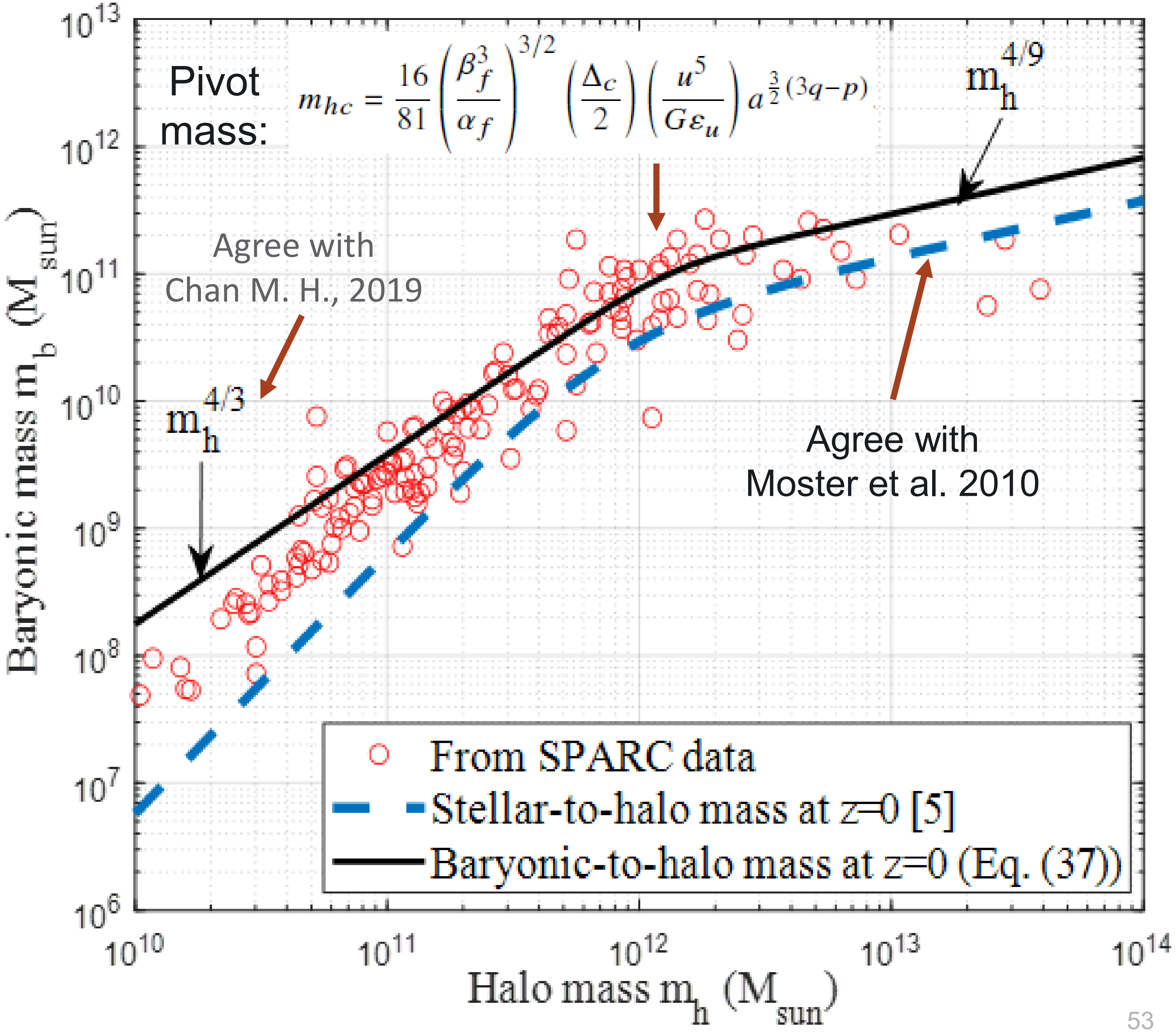
Baryonic mass in large halos < pivot mass m_{hc}:

$$m_b = \left(M_{c2}\right)^{5/9}\left(m_h\right)^{4/9}$$

Model incorporate two limits:

$$\frac{m_b}{m_h} = 2^{\frac{1}{m}} A(z)\left[\left(\frac{m_h}{m_{hc}(z)}\right)^{-\frac{m}{3}} + \left(\frac{m_h}{m_{hc}(z)}\right)^{\frac{5m}{9}}\right]^{-\frac{1}{m}}$$

- Dash line: the stellar-to-halo mass ratio (SHMR) obtained from halo abundance matching (required to match the stellar mass function)
- The 4/9 scaling law for both SHMR and BHMR



Pivot mass: $m_{hc} = \frac{16}{81}\left(\frac{\beta_f^3}{\alpha_f}\right)^{3/2}\left(\frac{\Delta_c}{2}\right)\left(\frac{u^5}{G\varepsilon_u}\right)a^{\frac{3}{2}(3q-p)}$

Agree with Chan M. H., 2019

Agree with Moster et al. 2010

Redshift evolution of baryonic-halo-mass ratio

Overall cosmic baryonic-to-DM mass ratio (including both halos and out-of-halo) is ~18.8% in ΛCDM model:

Baryonic-to-DM mass ratio in out-of-halos

Average Baryonic-to-halo mass ratio in all halos

$$A_{boh}(z) = \frac{0.188 - A_{dh}(z)\,A_{bh}(z)}{1 - A_{dh}(z)}$$

Cosmic ratio

Fraction of DM mass in halos

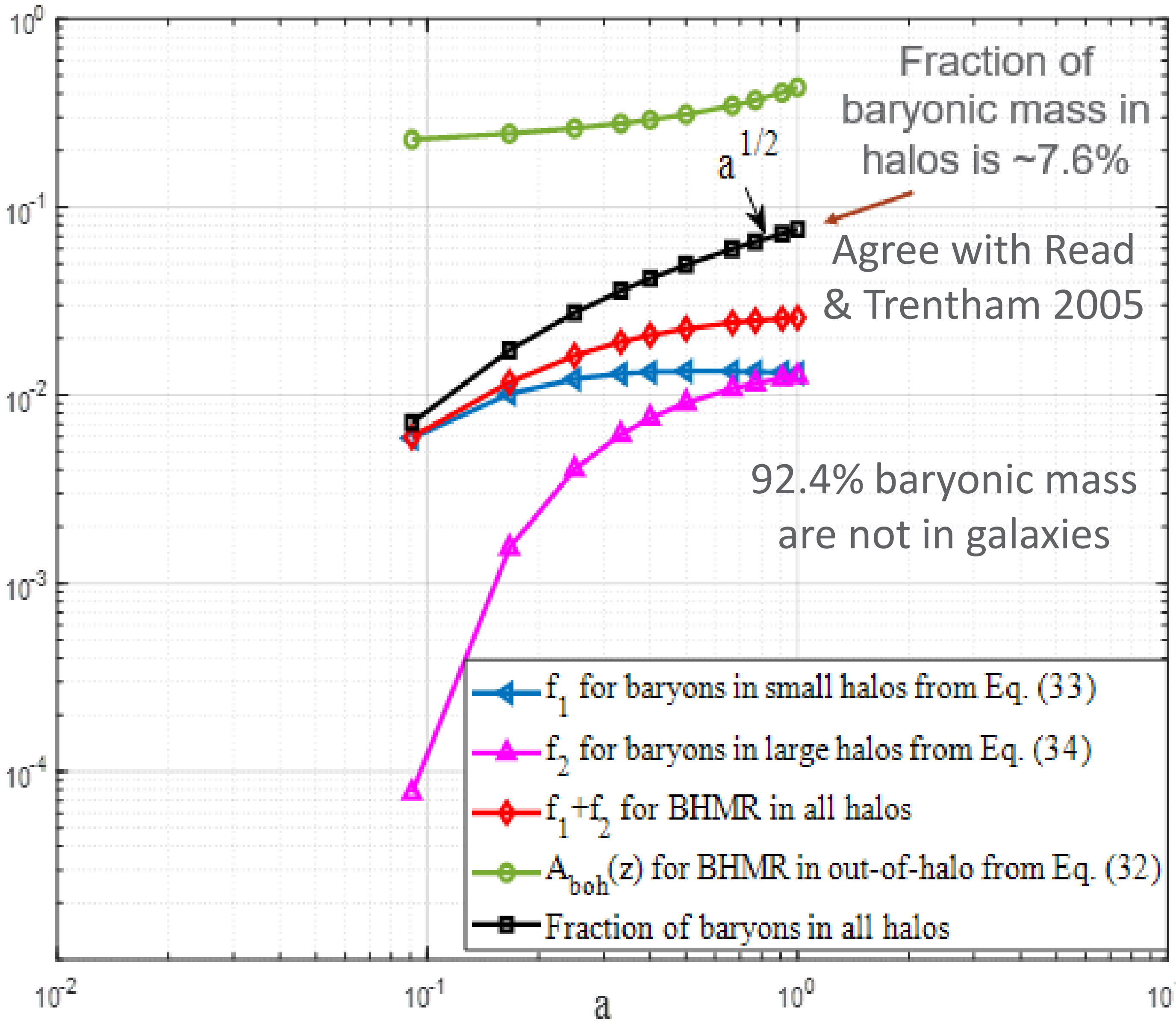
Use double-λ mass function to compute:

$$f_1 = \int_0^{\nu_c} f_{D\lambda}(\nu)\left(M_{c1}\right)^{-1/3}\left(\nu^{3/2} m_h^*\right)^{1/3} d\nu$$

The baryonic-to-halo mass ratio in small halos

$$f_2 = \int_{\nu_c}^{\infty} f_{D\lambda}(\nu)\left(M_{c2}\right)^{5/9}\left(\nu^{3/2} m_h^*\right)^{-5/9} d\nu$$

The baryonic-to-halo mass ratio in large halos



Fraction of baryonic mass in halos is ~7.6%

Agree with Read & Trentham 2005

92.4% baryonic mass are not in galaxies

Redshift evolution of BHMR

Energy cascade for SMBH-galaxy evolution

- Strong correlations between supermassive black holes (SMBHs) and host galaxies suggest a co-evolution.
  - M_B–σ_b relation (BH mass vs. velocity dispersion)
  - M_B–M_b relation (BH mass vs. bulge mass)
  - M_B–L_b relation (BH mass vs. bulge luminosity)
- Proposed mechanisms for BH-galaxy co-evolution
  - AGN Feedback
  - Statistical origin
  - Effect of energy cascade?

$M_B \propto \sigma_b^5$	$M_B \propto M_b$	$M_b \propto r_b \sigma_b^2$
M_B–σ_b correlation	M_B–M_b correlation	Virial theorem

Bulge dispersion

$$\frac{\sigma_b^3}{r_b} = Const$$

Bulge size

why?

- Two-thirds law: $\sigma_b^2 \propto (\varepsilon_b r_b)^{2/3}$ ➡ $\varepsilon_b = \sigma_b^3 / r_b$ ➡ The rate of energy cascade in bulge
- Does energy cascade exist in SMBH-bulge system?
- How energy cascade impacts SMBH-galaxy coevolution?
- Can cascade induced accretion exceed Eddington limit?

(Ferrarese et al. 2005) $$\frac{M_B}{10^8 M_\odot} = 1.66\left(\frac{\sigma}{200km/s}\right)^{4.86}$$

(Marconi et al. 2003) $$M_B \approx 0.002 M_b$$

Virial theorem $$M_b \approx 3 r_b \sigma_b^2 / G$$

$$\varepsilon_b = \sigma_b^3 / r_b \approx 10^{-4} m^2/s^3$$

For comparison:

M31 bulge: $6\times10^{-5}\ m^2/s^3$
Average local galaxies: $10^{-4}\ m^2/s^3$
Sun: mass-to-light ratio 5122 kg/W or $2\times10^{-4}\ m^2/s^3$
Cascade in dark matter: $4.6\times10^{-7}\ m^2/s^3$

Energy cascade in galaxy bulge

Dynamics on large scale does not feel the dissipation of baryons. Flow is self-gravitating collisionless with the same scaling laws on scale r:

Mass:	$m_r = \alpha_r \varepsilon_b^{2/3} G^{-1} r^{5/3}$	5/3 law
Density:	$\rho_r = \beta_r \varepsilon_b^{2/3} G^{-1} r^{-4/3}$	-4/3 law
Kinetic energy:	$v_r^2 = \left(\varepsilon_b r\right)^{2/3}$	2/3 law
Cascade pressure:	$P_r = \rho_r v_r^2 \propto \varepsilon_b^{4/3} G^{-1} r^{-2/3}$	Due to random motion
Cascade Force:	$F_r = 4\pi r^2 P_r \propto \varepsilon_b^{4/3} G^{-1} r^{4/3} \propto v_r^4 / G$	

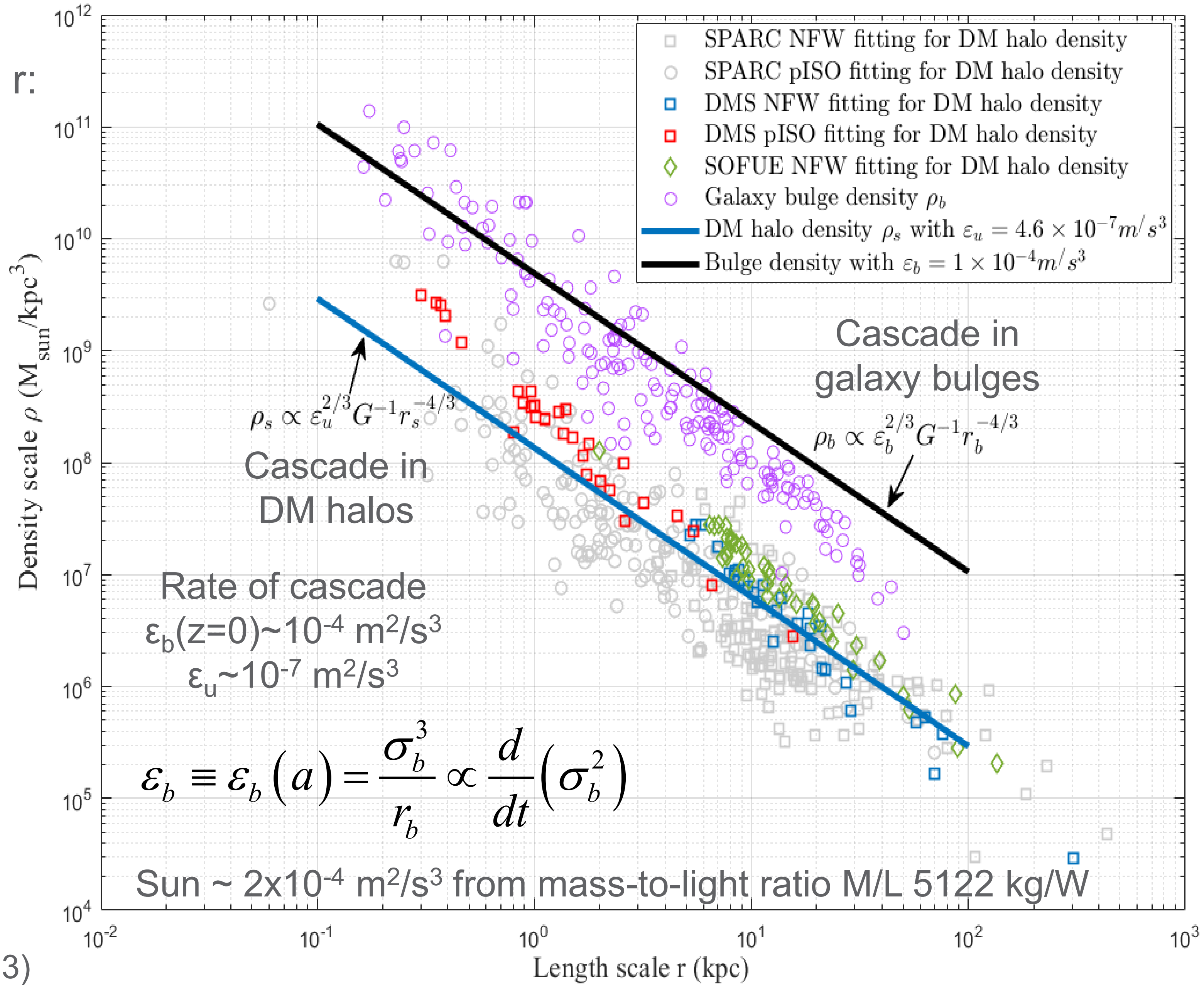
Predicted galaxy mass-size relation: $r \propto m_r^{3/5}$

Observed mass-size relation (ETG only, why?):

$r \propto m_r^{[0.5\ \ 0.6]}$ (Huertas-Company et al. 2013)

$r \propto m_r^{0.6}$ (Mowla et al. 2019a)

$r \propto m_r^{0.55}$ (Shen et al. 2003)

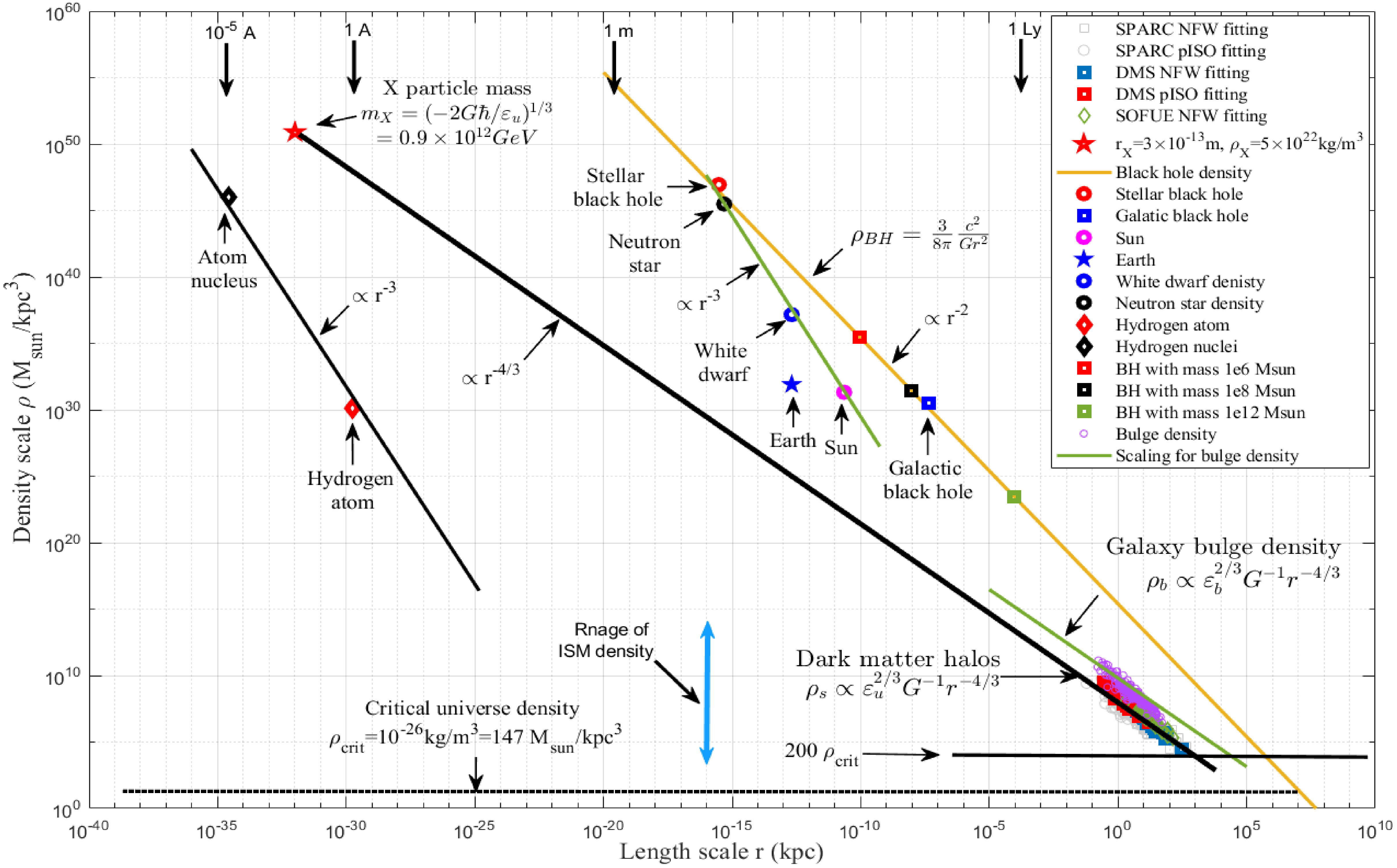


Rate of cascade
$\varepsilon_b(z=0) \sim 10^{-4}\ m^2/s^3$
$\varepsilon_u \sim 10^{-7}\ m^2/s^3$

$$\varepsilon_b \equiv \varepsilon_b(a) = \frac{\sigma_b^3}{r_b} \propto \frac{d}{dt}\left(\sigma_b^2\right)$$

Sun ~ $2\times10^{-4}\ m^2/s^3$ from mass-to-light ratio M/L 5122 kg/W

Astronomical density variation on length scales



Dynamics on the bulge scale and time-variation of ε_b

$$\varepsilon_b = \frac{\sigma_b^3}{r_b} \propto \frac{d}{dt}\left(\sigma_b^2\right) \qquad \sigma_b^2 r_b^n = Const \qquad \sigma_b^2 \propto GM_b/r_b$$

$$r_b \propto a^{\frac{3}{2+n}}, \quad \sigma_b \propto a^{-\frac{3n}{4+2n}}, \quad M_b \propto a^{\frac{3-3n}{2+n}}$$

$$\rho_b \propto a^{-3}, \quad \varepsilon_b \propto a^{-\frac{6+9n}{4+2n}}, \quad r_M \propto a^{\frac{6+9n}{10+5n}}$$

From the observed evolution of galaxy mass-size relation

r_M: size with fixed bulge mass at different z

r_M: the size of bulge with a fixed mass M_b at different z

$r_M \propto a^{1.01}$ (Huertas-Company et al. 2013)

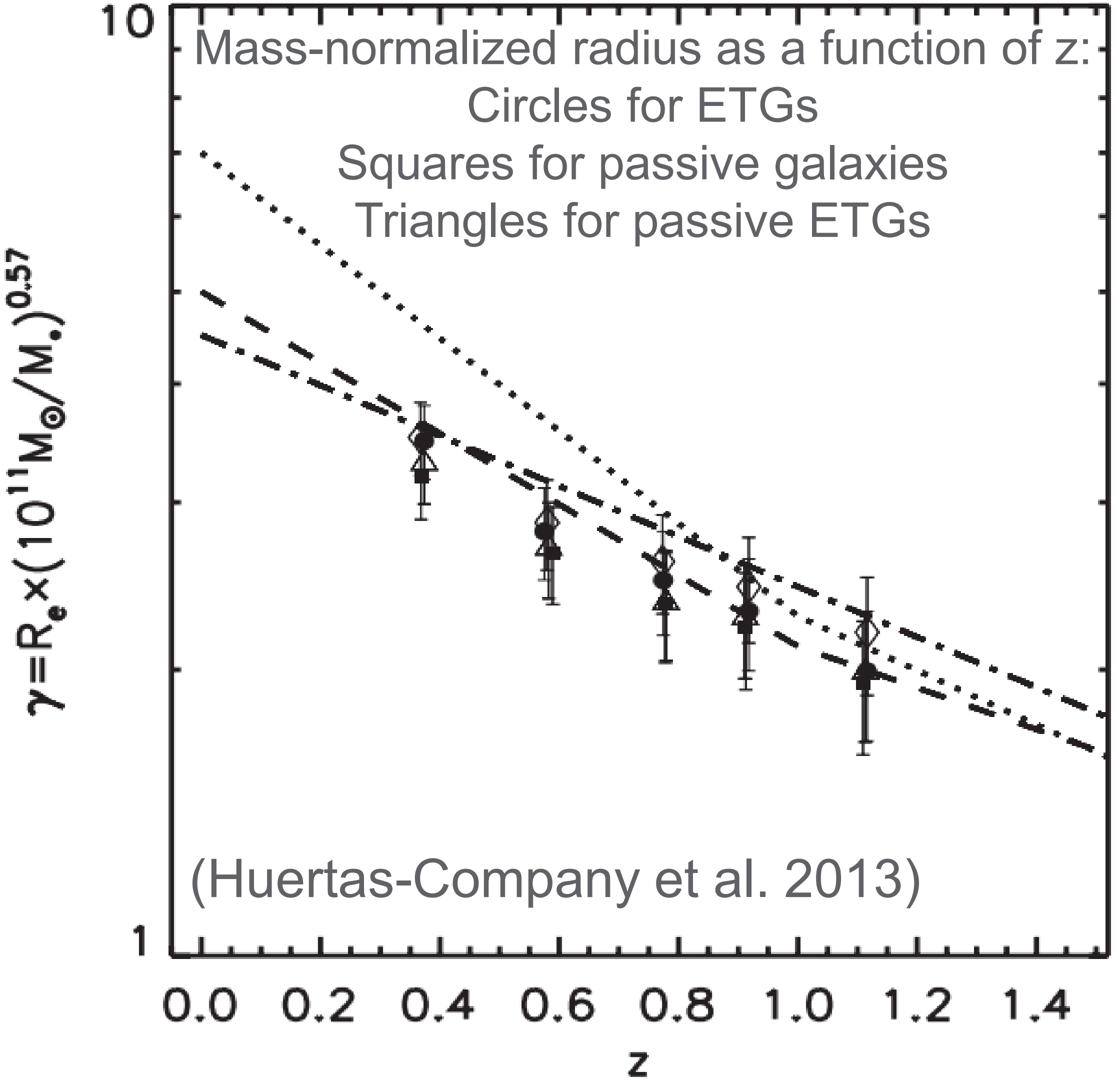
$r_M \propto a^{1.05}$ (Yang et al. 2020)

$r_M \propto a^{0.95}$ (Mowla et al. 2019b)

$n = 1$

$$r_b \propto a \quad \sigma_b \propto a^{-1/2} \quad M_b \propto a^0$$

$$\rho_b \propto a^{-3} \quad \varepsilon_b \propto a^{-5/2} \quad r_M \propto a$$



Mass-normalized radius as a function of z:
Circles for ETGs
Squares for passive galaxies
Triangles for passive ETGs

(Huertas-Company et al. 2013)

Key quantities and length scales for SMBH-Bulge

Six physical quantities:

Bulge mass M_b
BH mass M_B
BH luminosity L_B

Rate of energy cascade ε_b
Gravitational constant G
Light speed c

Five length scales:

Bulge scale: $r_b = (1/\alpha_r)^{3/5} M_b^{3/5} G^{3/5} \varepsilon_b^{-2/5}$

BH sphere of influence: $r_B = (1/\alpha_r)^{3/5} M_B^{3/5} G^{3/5} \varepsilon_b^{-2/5}$

Schwarzschild Radius: $r_s = 2GM_B/c^2$

Radiation scale: $r_p = \left(\frac{GL_B}{3\alpha_r^2 \gamma_r c}\right)^{\frac{3}{4}} \varepsilon_b^{-1}$

Dissipation scale: $r_x = \left(\frac{\nu_B^3}{\varepsilon_b}\right)^{\frac{1}{4}} = \left(\frac{8z_r^3 G^3 M_B^3}{c^3 \varepsilon_b}\right)^{\frac{1}{4}}$

Equivalent BH kinematic viscosity: $\nu_B = z_r c r_s = 2 z_r G M_B / c$

Scaling laws:

$$m_r = \alpha_r \varepsilon_b^{2/3} G^{-1} r^{5/3}$$

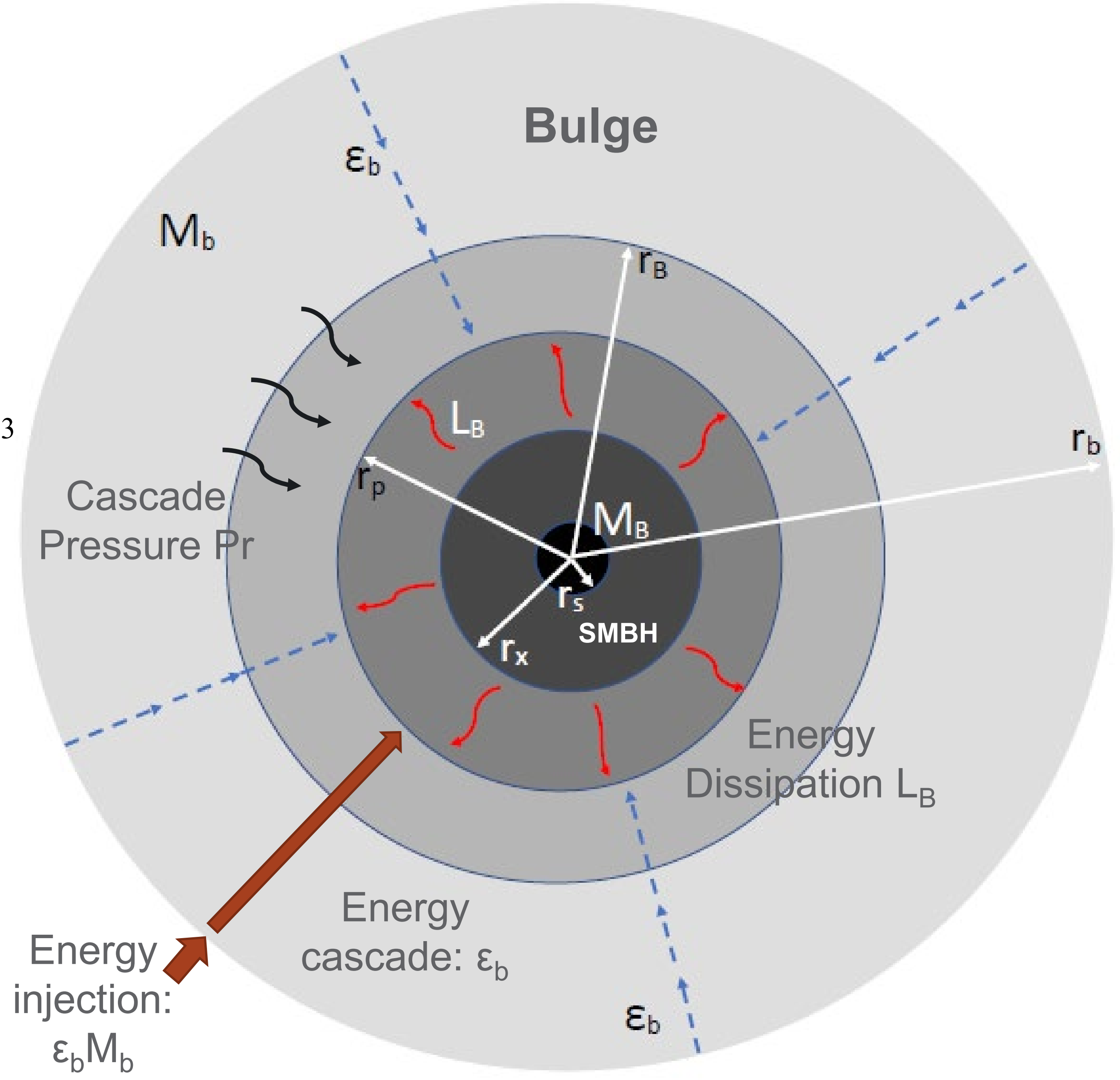
Radiation pressure

$$p_{rad}(r) = \frac{L_B}{4\pi r^2}$$

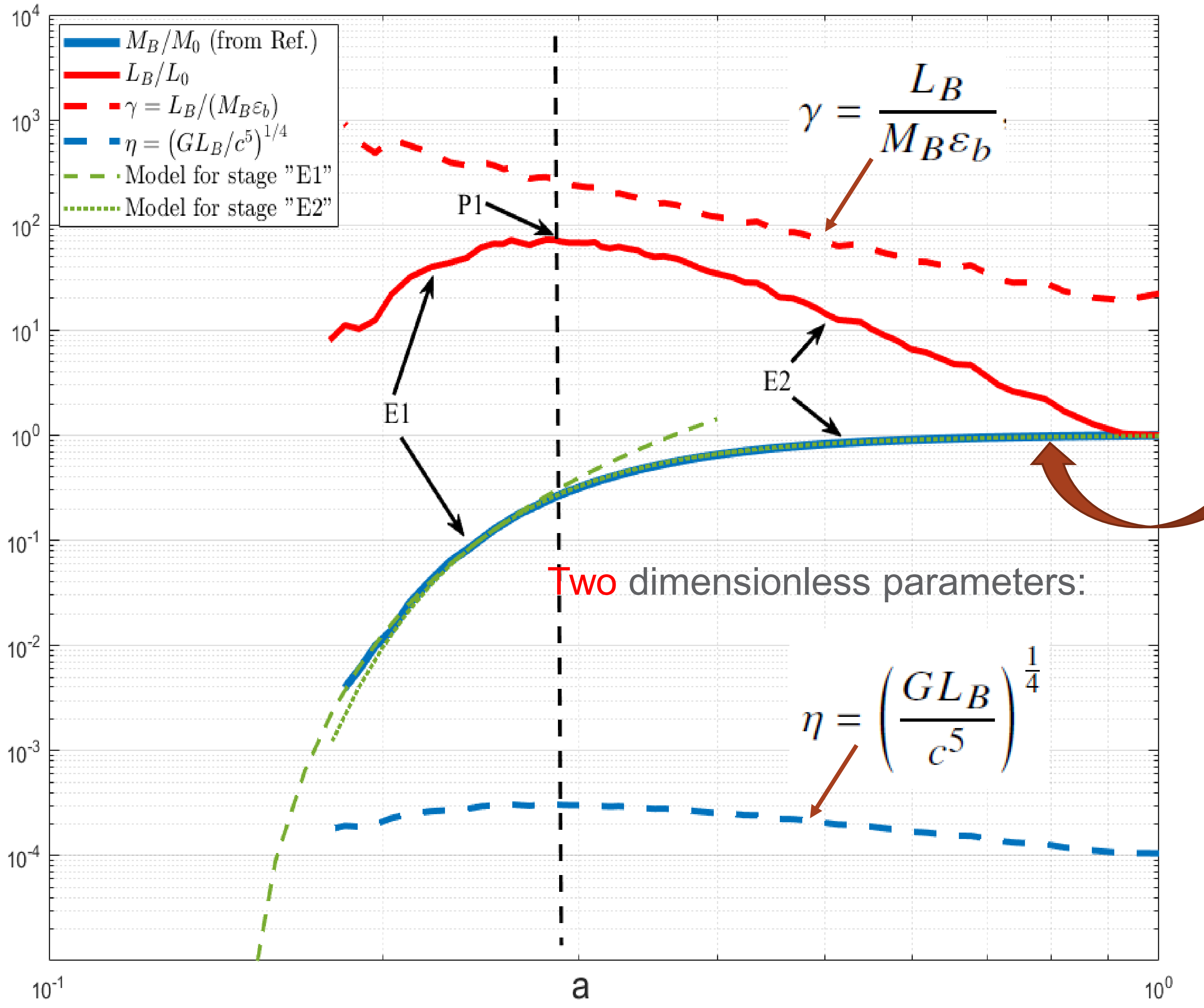
$$\updownarrow$$

$$p_r(r) = \frac{\varepsilon_b^{4/3}}{G r^{2/3}}$$

Cascade pressure



SMBH evolution from quasar luminosity function

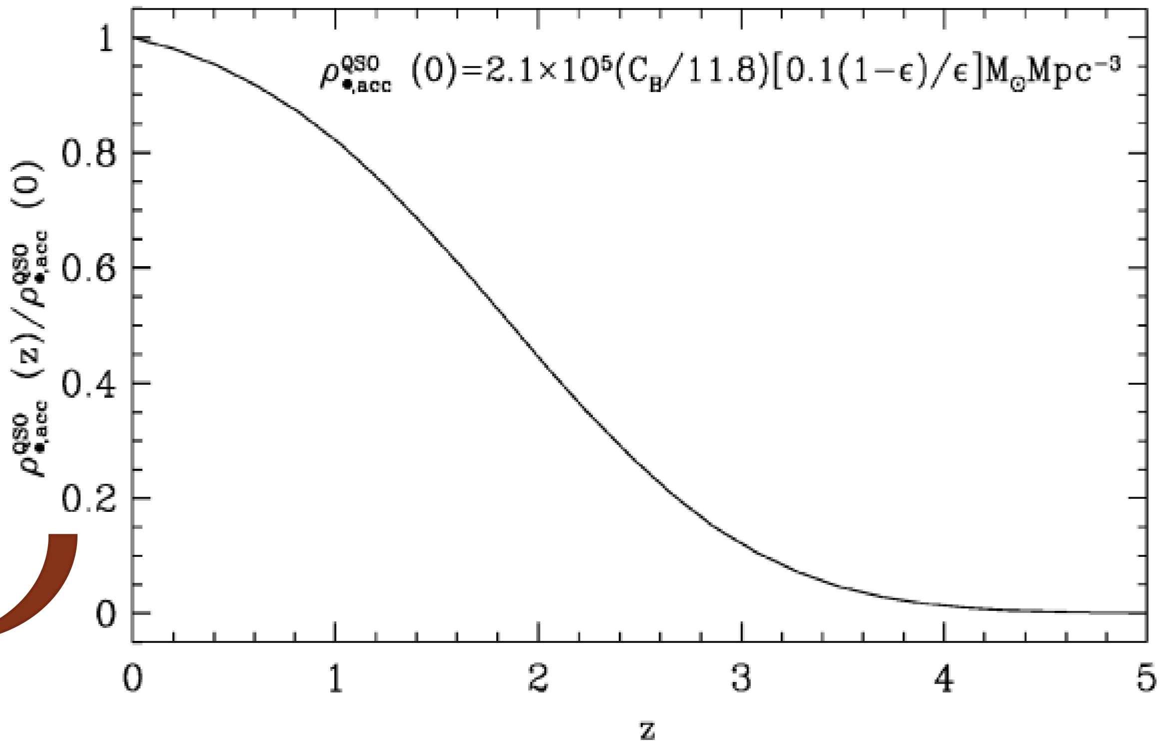


$$\gamma = \frac{L_B}{M_B \varepsilon_b}$$

Two dimensionless parameters:

$$\eta = \left(\frac{GL_B}{c^5}\right)^{\frac{1}{4}}$$

Time evolution of BH mass M_B, Luminosity L_B, dimensionless γ and η

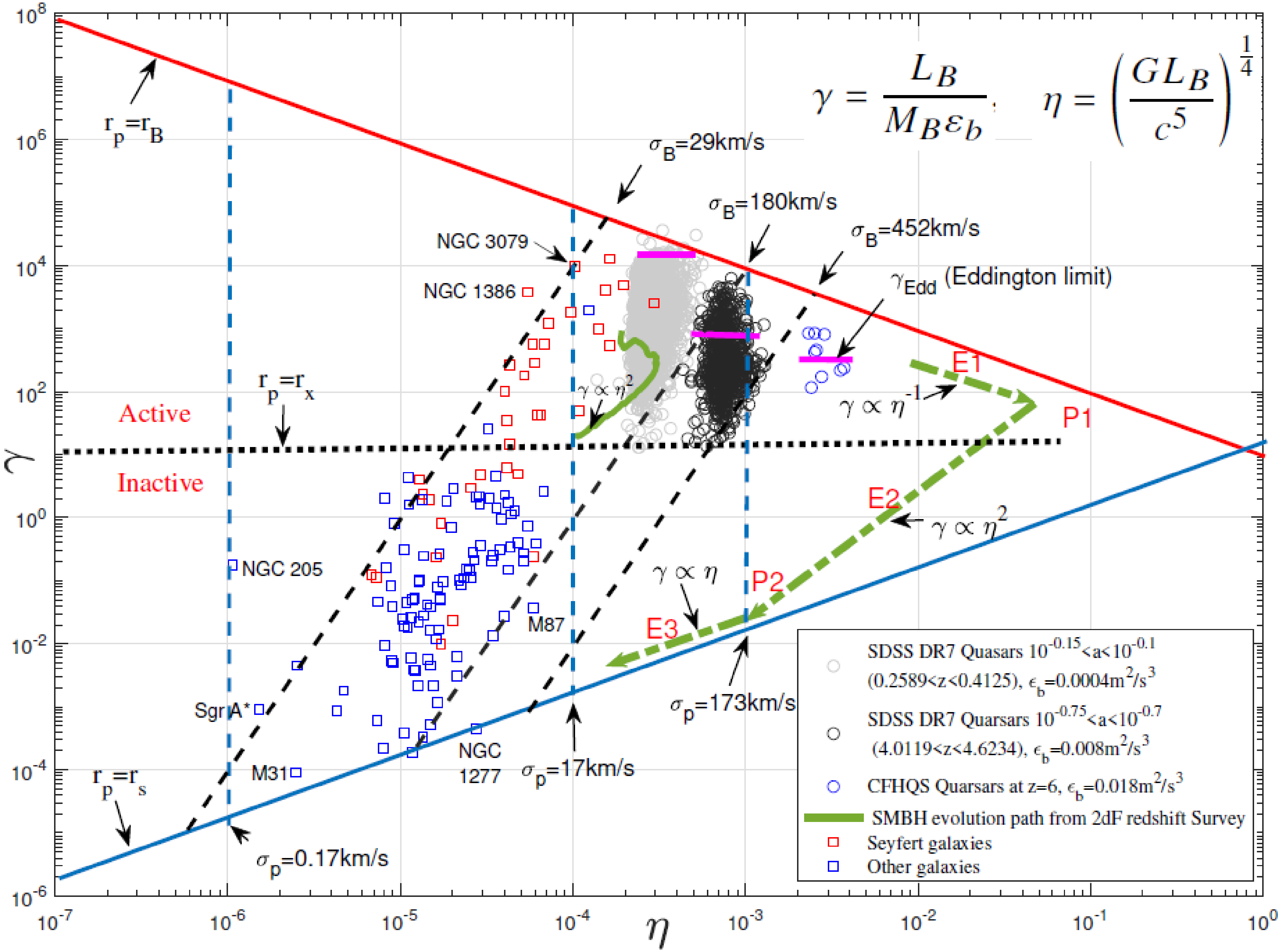


Evolution of co-moving BH mass density from Quasar luminosity function from 2dF Redshift Survey (Yu & Tremaine 2002)

Luminosity is converted from mass evolution :

$$\frac{L_B}{M_0} = \frac{\dot{M}_B}{M_0}\frac{\epsilon c^2}{1-\epsilon} = \frac{\partial(M_B/M_0)}{\partial a}H_0 a^{-1/2}\frac{\epsilon c^2}{1-\epsilon}$$

The SMBH distribution and evolution in γ -- η plane



Data sources:

1) Survey of local galaxies from literature (squares) Multiple sources

2) Quasars from Sloan Digital Sky Survey DR7 (gray and black circles) Schneider et.al 2010, Shen et al. 2011.

3) High redshift quasars from Canada–France High-z Quasar Survey (blue circles) Willott et.al 2010

4) BH evolution from the luminosity function from 2dF Redshift Survey (solid green) Yu & Tremaine et.al 2002

Any other potential sources?

Galaxy bulge and SMBH data

Velocity scales

Length scales

Rate of cascade

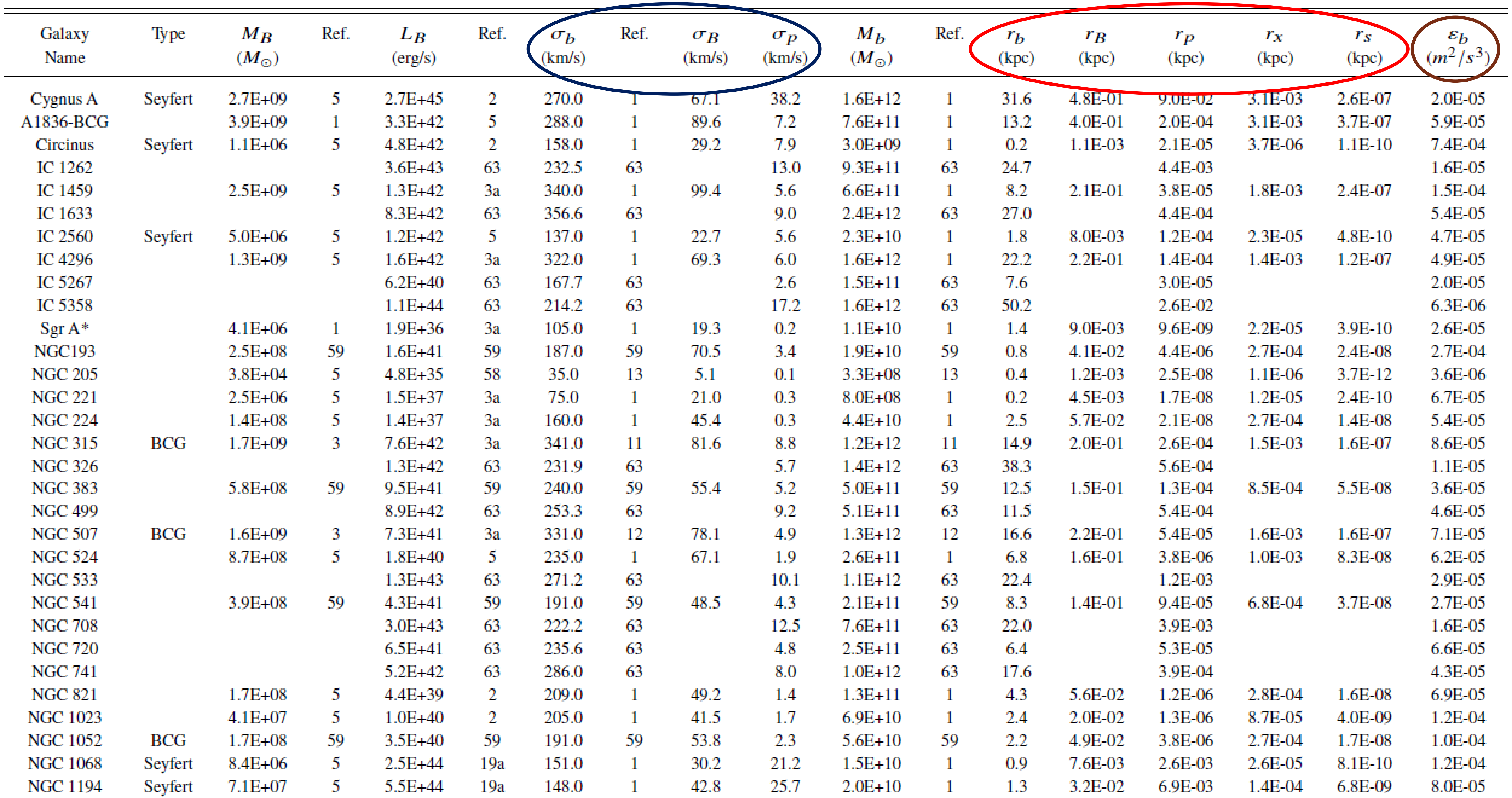
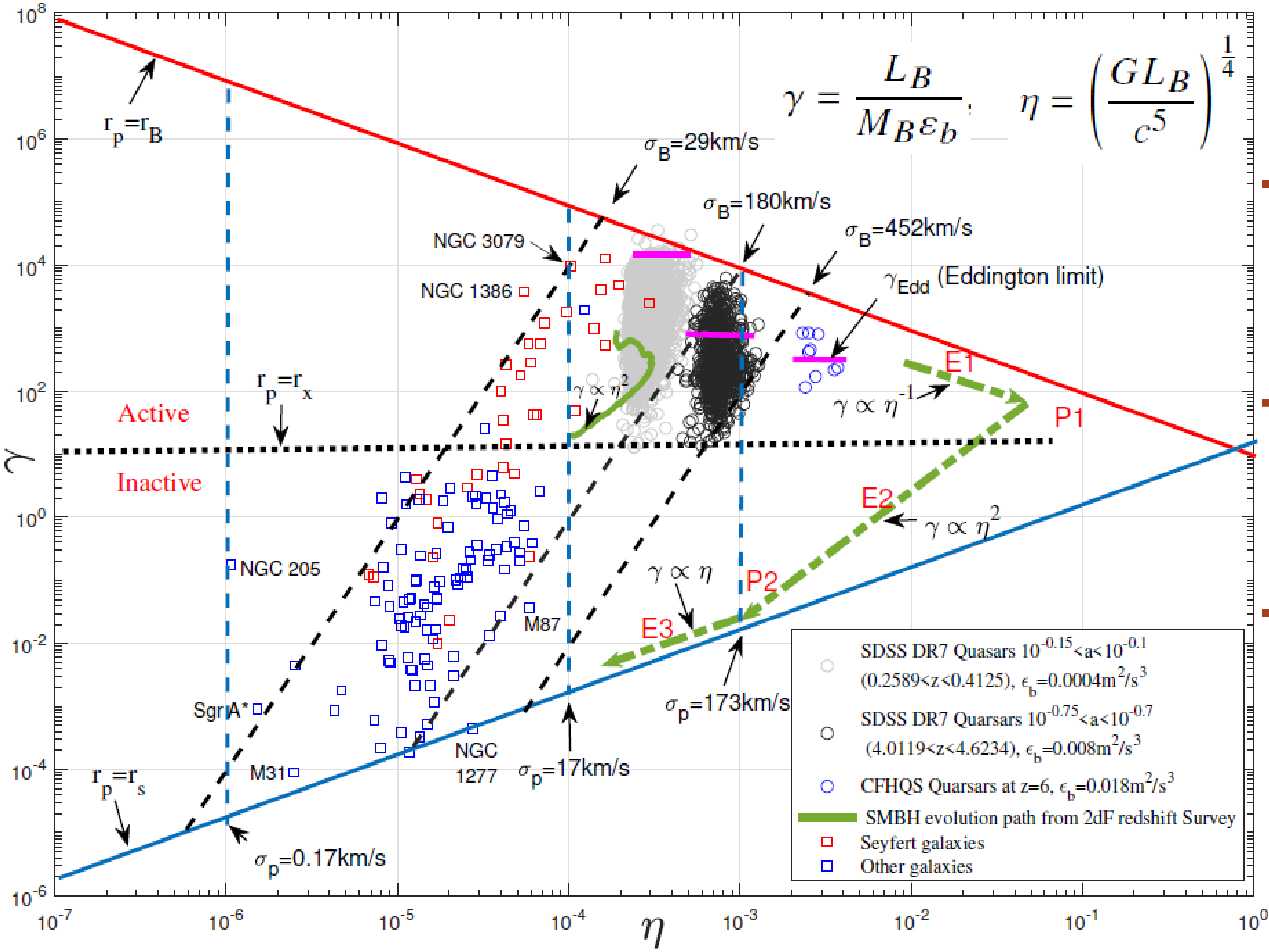


Table A1. Samples of SMBHs and their host galaxies

Galaxy Name	Type	M_B ($M_\odot$)	Ref.	L_B (erg/s)	Ref.	σ_b (km/s)	Ref.	σ_B (km/s)	σ_P (km/s)	M_b ($M_\odot$)	Ref.	r_b (kpc)	r_B (kpc)	r_P (kpc)	r_X (kpc)	r_S (kpc)	ε_b (m^2/s^3)
Cygnus A	Seyfert	2.7E+09	5	2.7E+45	2	270.0	1	67.1	38.2	1.6E+12	1	31.6	4.8E-01	9.0E-02	3.1E-03	2.6E-07	2.0E-05
A1836-BCG		3.9E+09	1	3.3E+42	5	288.0	1	89.6	7.2	7.6E+11	1	13.2	4.0E-01	2.0E-04	3.1E-03	3.7E-07	5.9E-05
Circinus	Seyfert	1.1E+06	5	4.8E+42	2	158.0	1	29.2	7.9	3.0E+09	1	0.2	1.1E-03	2.1E-05	3.7E-06	1.1E-10	7.4E-04
IC 1262				3.6E+43	63	232.5	63		13.0	9.3E+11	63	24.7		4.4E-03			1.6E-05
IC 1459		2.5E+09	5	1.3E+42	3a	340.0	1	99.4	5.6	6.6E+11	1	8.2	2.1E-01	3.8E-05	1.8E-03	2.4E-07	1.5E-04
IC 1633				8.3E+42	63	356.6	63		9.0	2.4E+12	63	27.0		4.4E-04			5.4E-05
IC 2560	Seyfert	5.0E+06	5	1.2E+42	5	137.0	1	22.7	5.6	2.3E+10	1	1.8	8.0E-03	1.2E-04	2.3E-05	4.8E-10	4.7E-05
IC 4296		1.3E+09	5	1.6E+42	3a	322.0	1	69.3	6.0	1.6E+12	1	22.2	2.2E-01	1.4E-04	1.4E-03	1.2E-07	4.9E-05
IC 5267				6.2E+40	63	167.7	63		2.6	1.5E+11	63	7.6		3.0E-05			2.0E-05
IC 5358				1.1E+44	63	214.2	63		17.2	1.6E+12	63	50.2		2.6E-02			6.3E-06
Sgr A*		4.1E+06	1	1.9E+36	3a	105.0	1	19.3	0.2	1.1E+10	1	1.4	9.0E-03	9.6E-09	2.2E-05	3.9E-10	2.6E-05
NGC193		2.5E+08	59	1.6E+41	59	187.0	59	70.5	3.4	1.9E+10	59	0.8	4.1E-02	4.4E-06	2.7E-04	2.4E-08	2.7E-04
NGC 205		3.8E+04	5	4.8E+35	58	35.0	13	5.1	0.1	3.3E+08	13	0.4	1.2E-03	2.5E-08	1.1E-06	3.7E-12	3.6E-06
NGC 221		2.5E+06	5	1.5E+37	3a	75.0	1	21.0	0.3	8.0E+08	1	0.2	4.5E-03	1.7E-08	1.2E-05	2.4E-10	6.7E-05
NGC 224		1.4E+08	5	1.4E+37	3a	160.0	1	45.4	0.3	4.4E+10	1	2.5	5.7E-02	2.1E-08	2.7E-04	1.4E-08	5.4E-05
NGC 315	BCG	1.7E+09	3	7.6E+42	3a	341.0	11	81.6	8.8	1.2E+12	11	14.9	2.0E-01	2.6E-04	1.5E-03	1.6E-07	8.6E-05
NGC 326				1.3E+42	63	231.9	63		5.7	1.4E+12	63	38.3		5.6E-04			1.1E-05
NGC 383		5.8E+08	59	9.5E+41	59	240.0	59	55.4	5.2	5.0E+11	59	12.5	1.5E-01	1.3E-04	8.5E-04	5.5E-08	3.6E-05
NGC 499				8.9E+42	63	253.3	63		9.2	5.1E+11	63	11.5		5.4E-04			4.6E-05
NGC 507	BCG	1.6E+09	3	7.3E+41	3a	331.0	12	78.1	4.9	1.3E+12	12	16.6	2.2E-01	5.4E-05	1.6E-03	1.6E-07	7.1E-05
NGC 524		8.7E+08	5	1.8E+40	5	235.0	1	67.1	1.9	2.6E+11	1	6.8	1.6E-01	3.8E-06	1.0E-03	8.3E-08	6.2E-05
NGC 533				1.3E+43	63	271.2	63		10.1	1.1E+12	63	22.4		1.2E-03			2.9E-05
NGC 541		3.9E+08	59	4.3E+41	59	191.0	59	48.5	4.3	2.1E+11	59	8.3	1.4E-01	9.4E-05	6.8E-04	3.7E-08	2.7E-05
NGC 708				3.0E+43	63	222.2	63		12.5	7.6E+11	63	22.0		3.9E-03			1.6E-05
NGC 720				6.5E+41	63	235.6	63		4.8	2.5E+11	63	6.4		5.3E-05			6.6E-05
NGC 741				5.2E+42	63	286.0	63		8.0	1.0E+12	63	17.6		3.9E-04			4.3E-05
NGC 821		1.7E+08	5	4.4E+39	2	209.0	1	49.2	1.4	1.3E+11	1	4.3	5.6E-02	1.2E-06	2.8E-04	1.6E-08	6.9E-05
NGC 1023		4.1E+07	5	1.0E+40	2	205.0	1	41.5	1.7	6.9E+10	1	2.4	2.0E-02	1.3E-06	8.7E-05	4.0E-09	1.2E-04
NGC 1052	BCG	1.7E+08	59	3.5E+40	59	191.0	59	53.8	2.3	5.6E+10	59	2.2	4.9E-02	3.8E-06	2.7E-04	1.7E-08	1.0E-04
NGC 1068	Seyfert	8.4E+06	5	2.5E+44	19a	151.0	1	30.2	21.2	1.5E+10	1	0.9	7.6E-03	2.6E-03	2.6E-05	8.1E-10	1.2E-04
NGC 1194	Seyfert	7.1E+07	5	5.5E+44	19a	148.0	1	42.8	25.7	2.0E+10	1	1.3	3.2E-02	6.9E-03	1.4E-04	6.8E-09	8.0E-05

The SMBH distribution in γ -- η plane



The upper limit (red): $r_p = r_B \Rightarrow \gamma\eta = 12$

$$\Downarrow$$

$$L_B \propto \varepsilon_b^{4/5} M_B^{4/5} G^{-1/5} c$$

The lower limit (blue): $r_p = r_s \Rightarrow \gamma = 24\eta$

$$\Downarrow$$

$$L_B \propto \varepsilon_b^{4/3} M_B^{4/3} G^{1/3} c^{-5/3}$$

The boundary of active and inactive (black): $r_p = r_x \Rightarrow \gamma = 18.6$

$$\Downarrow$$

$$L_B \sim \varepsilon_b M_B$$

$$\frac{\sigma_p}{c} = \left(\frac{\gamma_r}{3}\right)^{\frac{1}{4}} \eta \Rightarrow L_B/c \propto \sigma_p^4/G$$

Radiation force Cascade force

$$\frac{\sigma_B}{c} = \frac{(\alpha_r \gamma_r)^{1/2}}{\alpha_r^{1/5}} \left(\frac{\eta^4}{\gamma}\right)^{\frac{1}{5}} \Rightarrow \boxed{\frac{M_B}{10^8 M_\odot} = 0.99 \left(\frac{\sigma_b}{200 km/s}\right)^5}$$

M_B – σ relation

The three-stage SMBH evolution in γ -- η plane

Co-evolution stage (E1): parallel to the upper limit

$$r_p \propto r_B \Rightarrow \gamma\eta = Const \Rightarrow L_B \propto \varepsilon_b^{4/5} M_B^{4/5} G^{-1/5} c$$

$$\boxed{L_B \sim \varepsilon_b M_b \gg \varepsilon_b M_B} \quad \frac{dM_B}{dt} = L_B \frac{1-\epsilon}{\epsilon c^2} \quad \Downarrow \quad \boxed{\varepsilon_b = a^{-m} \quad \text{m=5/2}}$$

$$M_B = \left[\alpha_r^{-3/2} \gamma_r^{-5/2} \xi_r^{-5/3}\right] \frac{\sigma_p^5}{\varepsilon_b G} \Leftarrow M_B = M_{\infty 1}\left[1-\left(\frac{a}{a_1}\right)^{-\frac{4}{5}m+\frac{3}{2}}\right]^5$$

Transitional stage (E2):

$$L_B \propto M_B \varepsilon_b^2 / \varepsilon_b^* \text{ or } \gamma \propto \eta^2$$

$$\boxed{L_B \sim \varepsilon_b M_B} \quad \Downarrow$$

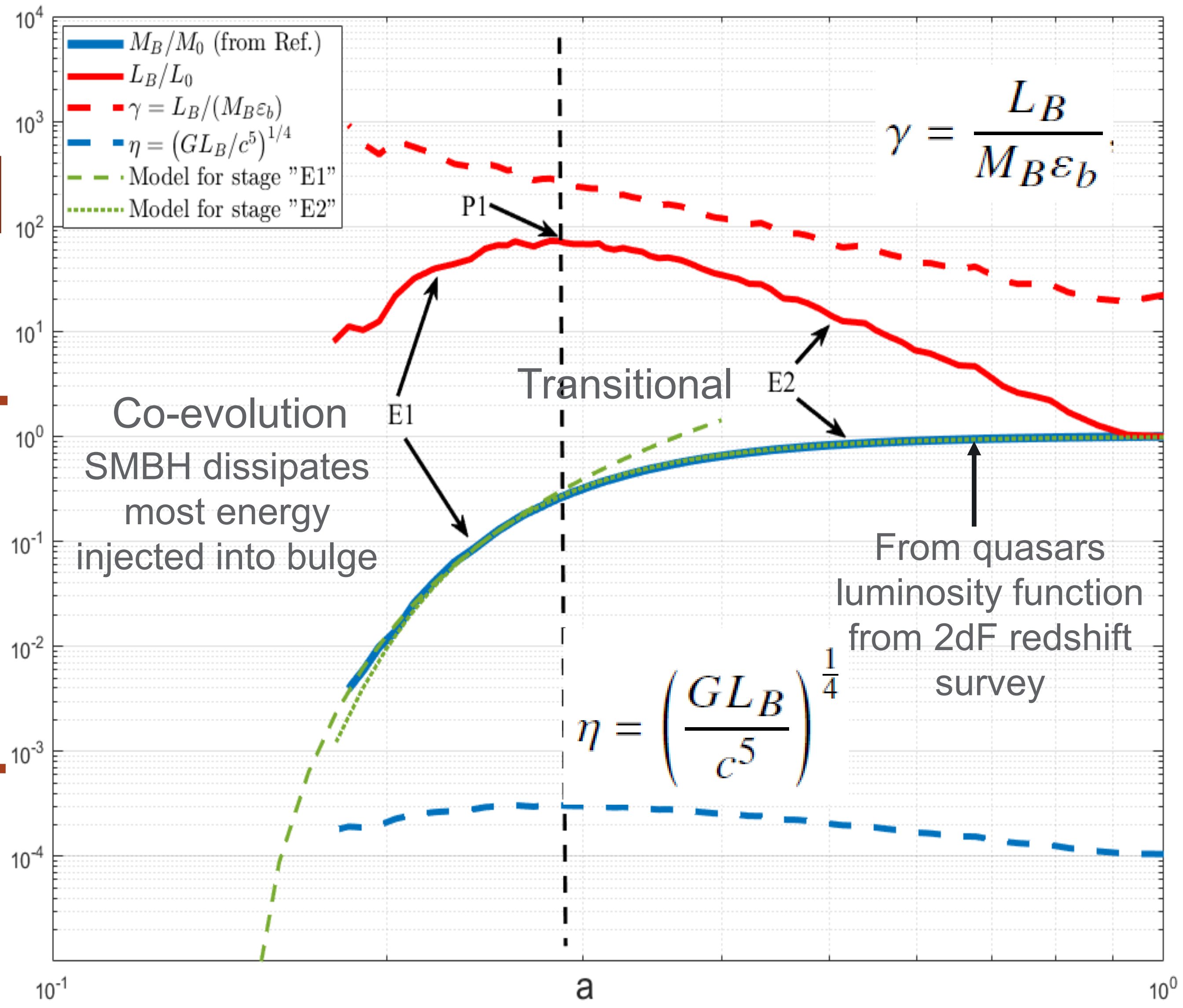
$$M_B = \left(\frac{3}{\gamma_r \gamma^*}\right)\left(\frac{\varepsilon_b}{\varepsilon_b^*}\right)^{1-p} \frac{\sigma_p^4 c}{\varepsilon_b G} \Leftarrow M_B = M_{\infty 2} \exp\left[-\left(\frac{a}{a_2}\right)^{-mp+\frac{3}{2}}\right]$$

Dormant stage (E3): parallel to the lower limit

$$r_p = r_s \Rightarrow \gamma \propto \eta \Rightarrow L_B \propto \varepsilon_b^{4/3} M_B^{4/3} G^{1/3} c^{-5/3}$$

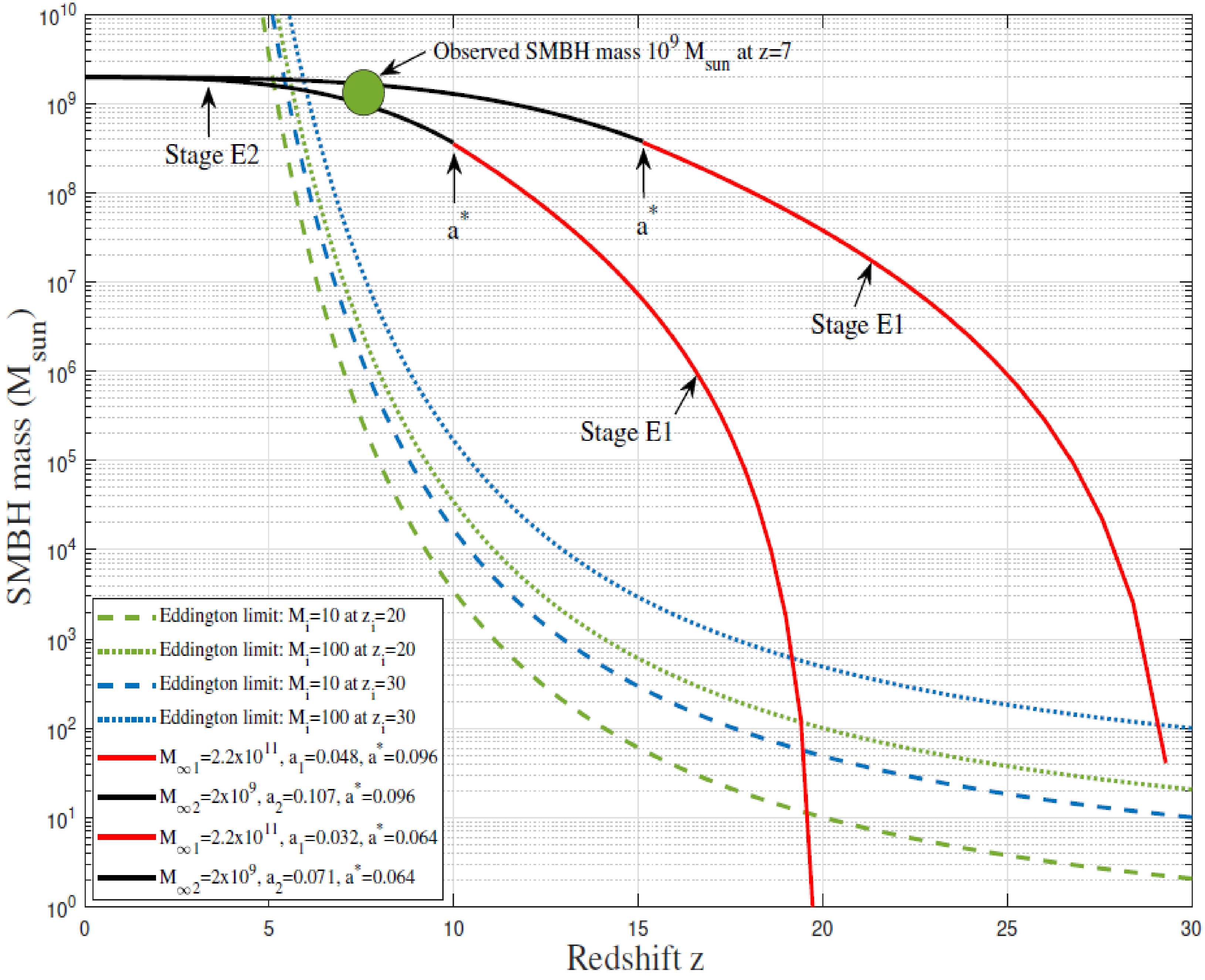
$$\boxed{L_B \ll \varepsilon_b M_B} \quad \Downarrow$$

$$M_B = \left(\frac{\alpha_r^{-3/2} \gamma_r^{-3/2}}{2}\right) \frac{\sigma_p^3 c^2}{\varepsilon_b G} \Leftarrow M_B = M_{\infty 3}\left[1+\left(\frac{a}{a_3}\right)^{-\frac{4}{3}m+\frac{3}{2}}\right]^{-3}$$



Three-stage SMBH evolution

Cascade induced accretion vs. Eddington accretion



Eddington accretion:

$$M_B = M_i \exp\left(\frac{t-t_i}{t_{sal}}\right)$$

Radiation force balances the weight of static gas:

$$\frac{L_{Edd}}{4\pi c r^2} = \frac{G M_B m_p}{r^2 \sigma_T} \quad \text{or} \quad \frac{L_{Edd}}{c} \approx M_B \times \left(2.1\times 10^{-8}\frac{m}{s^2}\right)$$

Alternatively, radiation force must balance the cascade force:

$$M_B \propto \sigma_p^5 \big/ \varepsilon_b G \quad \text{(in stage E1)}$$

$$\frac{L_B}{c} \propto \frac{\sigma_p^4}{G} \propto M_B \times \left(\frac{\varepsilon_b}{\sigma_p}\right) \gg \frac{L_{Edd}}{c} \Leftarrow \varepsilon_b \propto a^{-5/2}$$

Cascade induced accretion (first stage E1):

$$M_B = M_{\infty 1}\left[1-\left(\frac{a}{a_1}\right)^{-\frac{4}{5}m+\frac{3}{2}}\right]^5 \qquad a_1 = 1\big/\left(1+z_i\right)$$

$$M_B = M_{\infty 2}\exp\left[-\left(\frac{a}{a_2}\right)^{-mp+\frac{3}{2}}\right] \qquad \text{(second stage E2)}$$

In early universe, cascade accretion >> Eddington?

Potential flaws in this argument?

Conclusions, keywords, and hyperlinks

- Cascade is ubiquitous in our universe
- Inverse mass cascade with a scale-independent rate ε_m (kg/s)
    - Random walk of halos in mass space (diffusion) ➡ Double-λ halo mass function
    - Random walk of DM particles ➡ Double-γ halo density profile
    - Halo mass function and density profile share the same origin and similar functional form.
    - No critical density ratio δc or spherical/ellipsoidal collapse model required
- Energy cascade with a constant rate ε_u (m^2/s^3)
    - 2/3 law for kinetic energy $v_r^2 \propto (\varepsilon_u r)^{2/3}$
    - 5/3 law for enclosed mass, $m_r \propto \varepsilon_u^{2/3} G^{-1} r^{5/3}$
    - -4/3 law for halo density, $\rho_r \propto \varepsilon_u^{2/3} G^{-1} r^{-4/3}$
    - The fundamental origin of cascade on the smallest scale (uncertainty principle)?

⬅ In propagation range, all quantity by ε_u, G, and r

- The smallest scale dependent on the nature of dark matter:
    - Collisionless dark matter: $r_\eta \propto (\varepsilon_u G h)^{1/3}$ ➡ DM particle mass & properties ⬅ All quantity by ε_u, G, and *h*
    - Self-interacting dark matter: $r_\eta \propto \varepsilon_u^2 G^{-3} (\sigma/m)^3$ ➡ the smallest structure ⬅ All quantity by ε_u, G, and σ/m
- The largest scale determined by u_0, ε_u, and G ➡ the largest halo & its properties ⬅ All quantity by ε_u, G, u_0, a

- Velocity/density correlation/moment functions
- The maximum entropy distributions in dark matter
- Energy cascade for the origin or MOND acceleration
- Energy cascade for the baryonic-to-halo mass relation
- Energy cascade for SMBH-galaxy co-evolution

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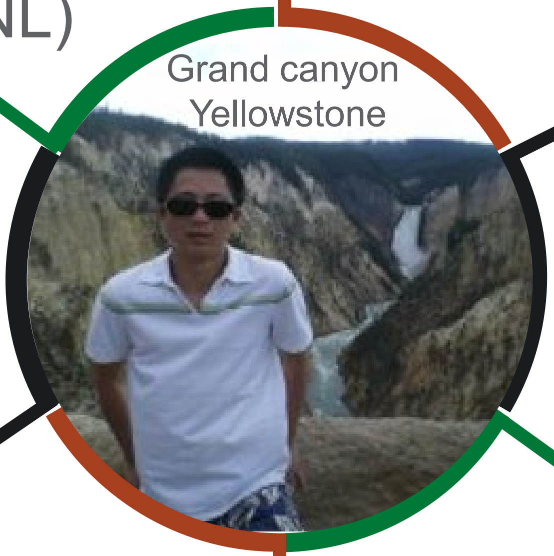

Grand canyon
Yellowstone

INTERESTS:

- Fluid dynamics
- Cosmological flow
- Multiscale Modeling

HOBBIES:

- Travel
- Hiking, biking

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