Local quantum theory with fluids in space-time: Supplemental Information

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1 Introduction and Notation

This document contains detailed examples of the local quantum fluid treatment of various significant quantum experiments and thought experiments. For each case, you will find a space-time diagram of the experiment which shows the trajectory of the systems involved, as well as a detailed table which shows the detailed evolution of the internal and external memory of each system. The tables should be read from the bottom up, since this is the direction of increasing time in the corresponding space-time diagrams.

In the tables, system labels are superscripts, and indexes which distinguish different subfluids of a system are subscripts. When fluid particles from two different systems meet at an event (and thus share the same past light cone), their internal memories are instantaneously synchronized, and then any interaction unitaries between those systems are applied. After the interaction, the different external memories of each system correspond to the orthogonal terms in the internal memory state, written in the macroscopic preferred basis. If there is no macroscopic preferred basis, then the external memories of the systems are microscopic and unobserved, so the choice of representational basis is arbitrary.

In the space-time diagrams, systems which are isolated from the macroscopic environment are shown using dashed lines, while systems with a clear macroscopic preferred basis in the global environment are shown using solid lines. Microscopic systems in coherent quantum superposition are isolated from the environment in this way, but in general, any system that is isolated from another system is in quantum superposition relative to that system. Importantly, special relativity implies that space-like separated system states are isolated from one another, and may thus may be in quantum superposition relative to one another - even for macroscopic systems. Isolation where a system is kept as physically shielded from the environment as also possible, despite constant interactions with fields which carry information at c, because the physical shielding keeps the isolated system and the environment from becoming strongly entangled (i.e., orthogonal states of the environment are correlated to nearly identical states of the isolated system).

The experimenter E and detectors D in many of these diagrams are assumed to belong to the macroscopic laboratory environment, in which systems are not isolated at all, and interact frequently, so that any relative superpositions are extremely short-lived, and something approximating fuzzy global worlds emerge.

2 Example Experiments

2.1 Wigner's Friend

In the Wigner's Friend [1] diagram of Fig. 1, lines with dashes of different sizes are used to demonstrate that even macroscopic systems can theoretically remain in superposition relative to an external environment provided they are kept in nearly perfect isolation. In this version of the *gedanken* experiment, we begin with a microscopic radioisotope in a quantum superposition $|\psi\rangle$, which the Cat then effectively measures locally (M^{IC}), resulting in a local superposition of the Cat and Isotope, where $|0\rangle$ corresponds to the Cat being alive, and $|1\rangle$ to it being dead. The Cat is macroscopic, and must experience being either alive or killed, but due to the isolation of the box, it remains in superposition relative to Schrödinger and the Friend. Thus, there are two local macroscopic worlds for the Cat, even though Schrödinger and the Friend are each still experiencing just one world relative to the start of the experiment. Next, Schrödinger peeks into the box to examine the cat (M^{CS}). This puts Schrödinger into a superposition of having seen the cat alive and dead, so now there are two local worlds for Schrödinger, but due to the isolation of the sealed laboratory, he remains in superposition relative to the Friend (and note also that the memory of the Isotope was not updated by this remote interaction). Then the Friend peeks into the laboratory and checks with Schrödinger (M^{SF}), resulting in two local worlds for the Friend. Lastly, this macroscopic state spreads rapidly through the environment as systems interact locally and synchronize memories, which ultimately divides each of the systems in the macroscopic environment into two worlds with the same orthogonal indexes for the isotope, Cat, Schrödinger, and Friend.

2.1.1 Extended Wigner's Friend

As an aside, we can consider the question, when do we say that something has collapsed permanently and will always be seen to have collapsed by any observer? Specifically, even if a system collapses relative to a local observer, and then becomes space-like separated from that observer, under what circumstances would we still be certain that the system was *not* in quantum superposition relative to the observer?

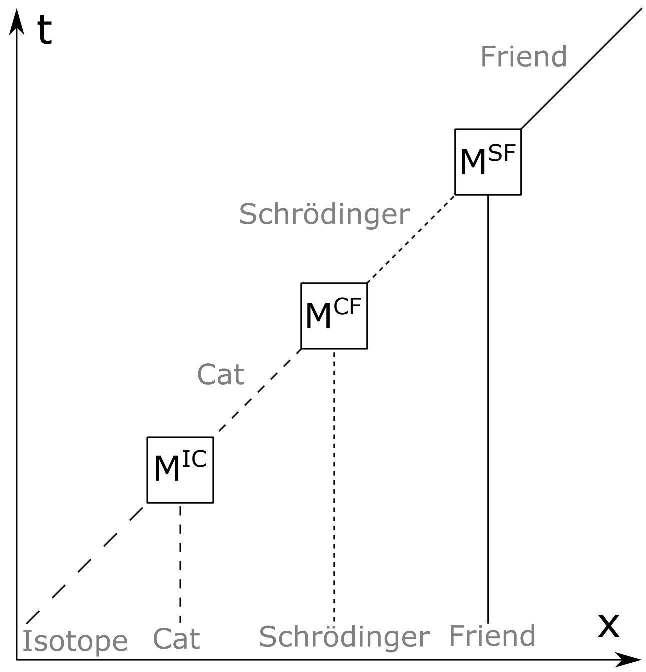

Figure 1: A version of the Wigner's friend *gedanken* experiment, beginning with Schrödinger's cat, but with Schrödinger himself isolated from his friend within a sealed laboratory. The experiment emphasizes how local interactions result in division into multiple local worlds, without affecting remote systems at all. Before the Friend opens the lab, this naturally means the Friend and the Schrödinger who saw a living cat would assign different quantum states to the cat, but this all follows from the one consistent narrative that plays out in space-time, wherein all superpositions are relative.

The general answer seems to be that macroscopic systems of sufficient complexity must collapse, since the unitary operations to reverse a collapse for such a system is so unlikely to occur as to be negligible, even though it is still possible in theory. When such a complex system enters a superposition relative to an observer in a particular macroscopic preferred basis, the system has collapsed into a different orthogonal state relative to the observer in each local world, and that operation will never be reversed.

Now, if we consider Wigner's argument, he assumed that the experimenter in the sealed laboratory was macroscopic, and roughly speaking, had "experiences and impressions like ours." This seems to imply the stronger form of complexity-based collapse. So, even though Schrödinger is in superposition relative to the Friend outside the lab, Schrödinger and the Cat have entered a macroscopic superposition that will never be reversed, and when the Friend opens the lab, they will also enter a superposition with the same alive/dead macroscopic preferred basis.

Now, in several Extended Wigner's Friend *gedanken* experiments [2, 3, 4], the idea of applying unitary operations on an entire sealed laboratory from the outside has been under significant recent consideration. In particular, the Local Friendliness inequalities that have been recently derived assume that a macroscopic conscious observer can be put into a quantum superposition, while still experiencing a collapse into just one outcome. Furthermore, a proposed experiment [5] would use a human-level artificial intelligence (AI) within a macro-scale quantum computer that purports to meet Wigner's criterion of having "experiences and impressions like ours" as the conscious observer. Then, because the AI lives inside quantum computer, those experiences and impression can be erased by applying the inverse of the measurement unitary that created the experience of a collapse, returning the quantum computer to its prior state.

But now there seems to be a logical problem: If collapse of the type Wigner was considering is the type that is *defined* as never being reversed, and this type of collapse is a prerequisite for having "experiences and impressions like ours," then the AI in the quantum computer can never satisfy this definition *because* some of its experiences will be reversed. This means that the assumption that the AI experiences a single outcome like our own may be fundamentally flawed.

Of course, this also means if we had the technological means to put a living person into a coherent superposition wherein they must experience a collapse and a single outcome, and then reverse the entire operation to return the person to their prior state, then the person would likewise fail to meet the standard of never being reversed, even though it is fair to say they should have experiences like any other person. This ultimately suggests that Wigner's standard never really made sense to begin with: There is no complexity-based condition under which a system can be considered to have permanently collapsed, even if we have observed that collapsed result ourselves, and thus "impressions and experienced like our own" are still insufficient to draw the conclusion that a single definite outcome exists relative to all observers.

2.2 Wheeler's Delayed Choice

Wheeler's Delayed Choice experiment [6] involves putting a single quantum particle into a spatial superposition, and only then deciding whether to make different path interfere, demonstrating wave behavior and erasing the information about a unique trajectory, or to directly measure its position demonstrating particle behavior with a unique trajectory and no wave interference. The idea of the delay is to show the quantum particle cannot simply decide to be a particle or a wave at the moment it is put into spatial superposition, since either behavior can be observed due to a later choice.

We give a 1D treatment where two adjacent cavities are each treated as a quantum system with Fock space occupation number levels. A beam splitter is represented by an H between the cavities. The case where the two paths interfere after the particle is superposed being superposed and the path information is erased is shown in Fig. 2 (upper left), while the case where the path is measured is shown in Fig. 2 (upper right). In the case with interference, a phase shifter in one cavity can change the probabilities for the two detectors. This case in shown Fig. 2 (lower right) and the example table is included to demonstrate how the quantum fluid model gives a local treatment of interference effects.

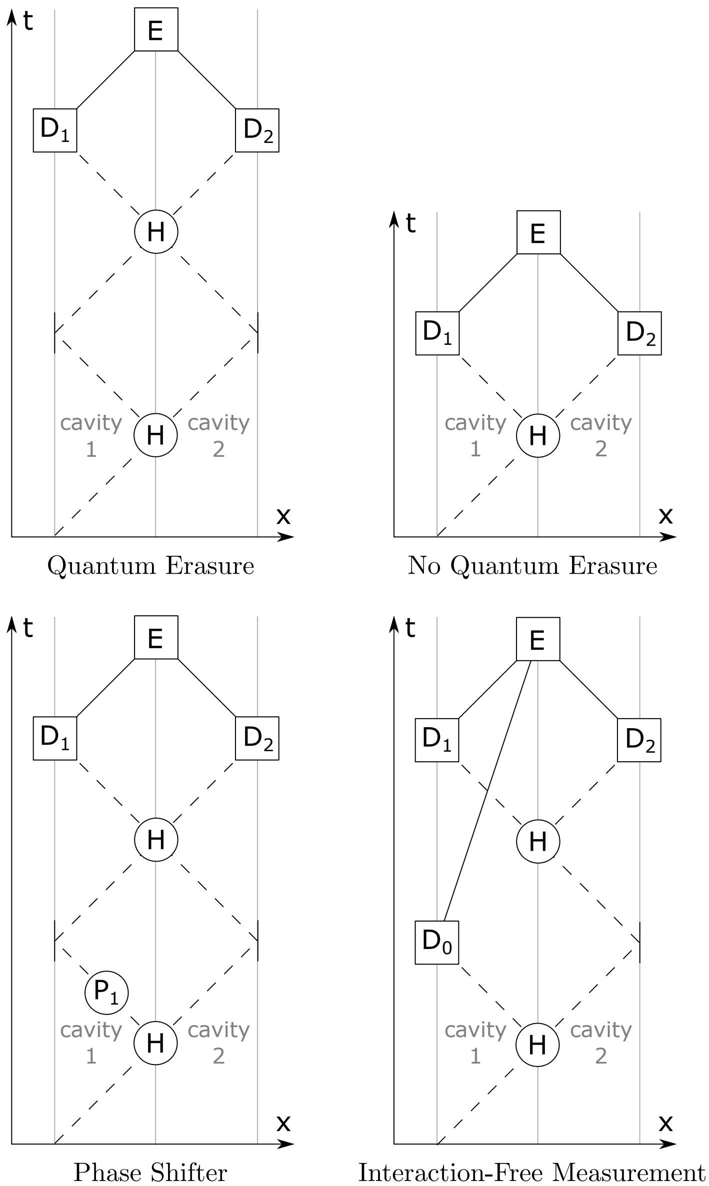

Figure 2: Experiments with a single particle and two cavities separated by a beam splitter.

The full treatment of these examples with multiple space-like separated detectors also emphasizes that they just are another case of entanglement correlations that are realized by delayed local matching, no different than the treatment of the Bell experiment in the main text.

2.3 Interaction-Free Measurement

If a new detector D_0 (bomb) is placed in the first cavity, it prevents quantum fluid from passing, so there is no interference with fluid from the other cavity, as shown in Fig. 2 (lower left). The setup is tuned so that if D_0 were absent, all of the fluid would go to D_1. When D_0 is present, a quarter of the fluid now goes to D_2, and that fluid must have traveled through the second cavity because D_0 obstructs the first. This situation is called *interaction-free measurement* because when D_2 is observed to fire, this constitutes a measurement of the presence of D_0 in the first cavity, while the fluid particle that made it to D_2 could not have visited the first cavity en route.

A well-known version of this experiment is the Elitzur-Vaidman Bomb Tester [6], where the role of D_0 is played by an ultra-sensitive bomb that will explode if the particle hits it, and the goal is to detect the presence or absence of this bomb without triggering it.

2.3.1 Nondemolition Measurements and Enviromental Decoherence

To round out the discussion of the Mach-Zehnder interferometer (or our 1D cavity simulacrum), we should also consider some other effects which reduce or destroy the visibility of quantum interference. First, consider a quantum nondemolition detector (QND) that is placed in cavity 1, which detects the presence of the particle without absorbing or deflecting it. This particle and QND become entangled, and then the signal from the QND becomes entangled with the experimenter. This results in four local worlds for the experimenter. Of the two local worlds where the QND detected the particle, one saw the particle at D_1 and the other at D_2, and likewise for the two where the QND did not detect the particle. In either case, the ensemble statistics show that each detector fires half the time, meaning the visibility of interference is zero.

Although it is somewhat less obvious, environmental decoherence works the same way. To see this, imagine that the QND is now just a tiny molecule that is placed in cavity 1. The molecule becomes entangled with the particle, just as in the other case, but now there is no macroscopic signal from the QND to the experimenter - the experimenter never explicitly observes the state of the molecule. However, if we presume the molecule is in contact with the laboratory environment, the local interactions in the environment, and the memory synchronizations that accompany them still carry this information to the experimenter, resulting in an entangled state with the same structure as before, but without any macroscopic observation of the QND result. Either way, we get the same ensemble statistics for the other two detectors, with no visible interference.

The Delayed-Choice Quantum Eraser is a generalization of the QND experiment, where the particle on paths 1 and/or 2 is the primary system which is detected at an earlier time, and the particle on paths 3 and/or 4 in Fig. 3 is the isolated microscopic QND signal, which may or may not be erased before that particle is detected at a later time.

2.4 The Delayed-Choice Quantum Eraser

The Delayed-Choice Quantum Eraser experiment [7, 8] (Fig. 3) is an extension of Wheeler's Delayed Choice, where entangled particle pairs are used to delay the choice of whether or not to let paths 1 and 2 interfere (erasure of the path information) is delayed until even after the fluid on those paths has reached detectors D_1 and D_2. After those detectors have fired, a choice is made whether to make path 3 and 4 interfere (erasure, Fig. 3, left) or not (Fig. 3, right). Once the ensemble data from D_1 and D_2 are separated into bins based on whether D_3 or D_4 fired, the cases where the paths 3 and 4 were later made to interfere

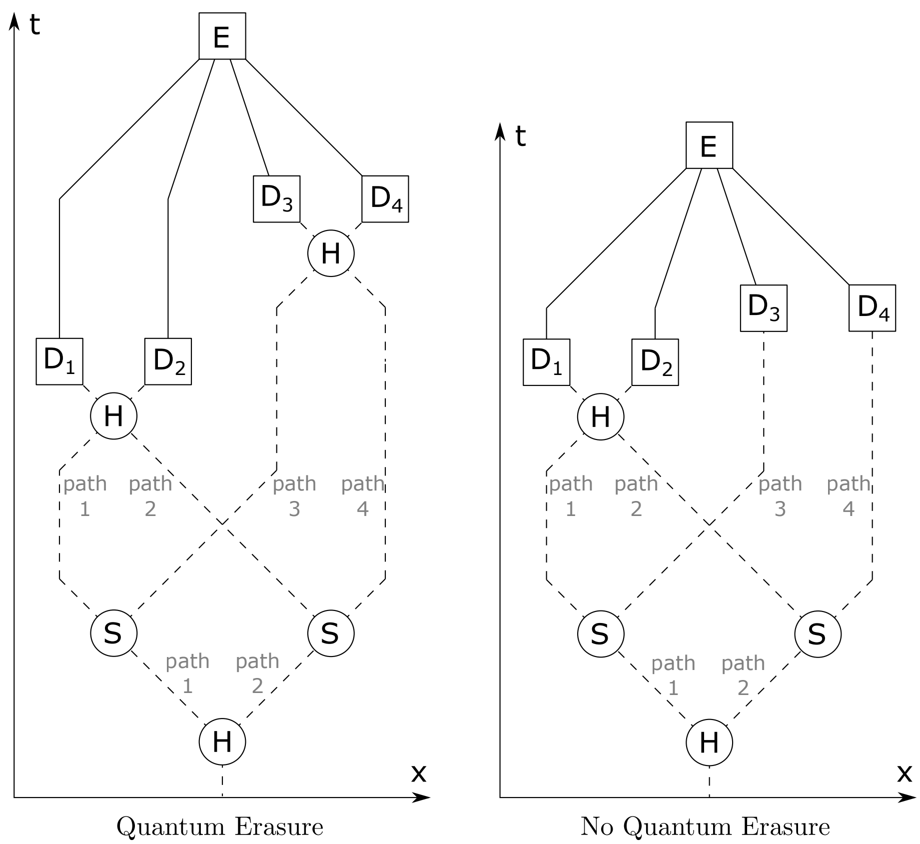

Figure 3: The Delayed Choice Quantum Eraser

(erasure) reveal perfect constructive and destructive interference on paths 1 and 2, and the cases where paths 3 and 4 were later kept apart reveal no interference on paths 1 and 2.

This counterintuitive experiment is an important example for the local quantum treatment because it involves a mixture of spatial interference and entanglement effects.

2.5 Quantum Teleportation

The local quantum fluid treatment of quantum teleportation [9] is an important example because it shows clearly how the quantum information is transported locally through space-time.

The goal of this experiment is to transmit the information in an isolated quantum state $|\psi\rangle$ of system C into another system isolated quantum system B without system C ever crossing the space between them, effectively teleporting the coherent quantum state $|\psi\rangle$ from system C to system B.

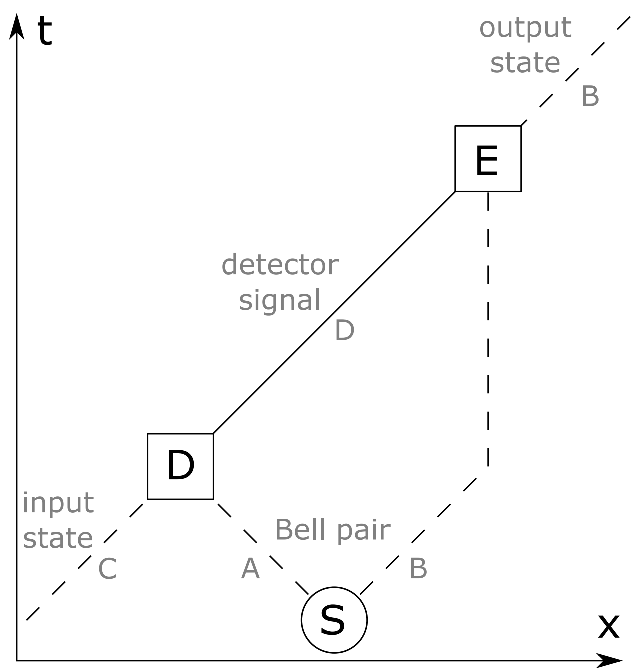

Figure 4: A quantum teleportation setup. Presuming the Experimenter (E) performs the indicated unitary operation, the output state $|\psi\rangle$ of the system B is identical to the input state of system C. After the measurement, there are four different local worlds for the detector and signal and the quantum information is carried by each of those signals. The indicated unitary puts system B into the state $|\psi\rangle$ relative to all four local worlds of the Experimenter.

This is done beginning with an entangled state of systems A and B. Systems A and C are then brought together and measured in an entangled joint eigenbasis, and the macroscopic result is sent by a classical channel to system B, as shown in Fig. 4 This measurement has four outcomes, and the detector is macroscopic, so the detector has been divided into four local worlds, each sending its result to B.

The quantum state of the detector signal contains the quantum information comprising $|\psi\rangle$, and when the macroscopic signal D arrives at system B, their internal memories synchronize, which adds the information $|\psi\rangle$ into system B, and correlates each of the four worlds with a particular state of system B. In each world, the signal D indicates which unitary needs to be applied to B in order to put it into the quantum state $|\psi\rangle$, which completes the protocol. We can characterize all of those operations as a single controlled unitary coupling U^{BD} where D is the control system, and B the target system.

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Beam Splitter and Two Detectors (No Erasure)

Local Information Type	Cavity 1	Cavity 2
External Memories (Experimenter)	$\frac{1}{\sqrt{2}}\lvert 10\rangle^{E}_{\lvert 1010\rangle^{12D_1D_2}},\ \frac{1}{\sqrt{2}}\lvert 01\rangle^{E}_{\lvert 0101\rangle^{12D_1D_2}}$	
Internal Memory Evolution	$M_E M_1 M_2 H\lvert 10\rangle^{12}\lvert 0\rangle^{D_1}\lvert 0\rangle^{D_2}\lvert 00\rangle^{E}$	
	$= \frac{1}{\sqrt{2}}\big(\lvert 1010, 10\rangle + \lvert 0101, 01\rangle\big)^{12D_1D_2E}$	
Internal Memory Synch	$M_1 M_2 H\lvert 10\rangle^{12}\lvert 0\rangle^{D_1}\lvert 0\rangle^{D_2}\lvert 00\rangle^{E}$	
	$= \frac{1}{\sqrt{2}}\big(\lvert 1010\rangle + \lvert 0101\rangle\big)^{12D_1D_2}\lvert 00\rangle^{E}$	
External Memories (Detect.)	$\frac{1}{\sqrt{2}}\lvert 1\rangle^{D_1}_{\lvert 10\rangle^{12}},\ \frac{1}{\sqrt{2}}\lvert 0\rangle^{D_1}_{\lvert 01\rangle^{12}}$	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{D_2}_{\lvert 01\rangle^{12}},\ \frac{1}{\sqrt{2}}\lvert 1\rangle^{D_2}_{\lvert 10\rangle^{12}}$
Internal Memory Evolution	$M_1 H\lvert 10\rangle^{12}\lvert 0\rangle^{D_1} = \frac{1}{\sqrt{2}}\big(\lvert 101\rangle + \lvert 010\rangle\big)^{12D_1}$	$M_2 H\lvert 10\rangle^{12}\lvert 0\rangle^{D_2} = \frac{1}{\sqrt{2}}\big(\lvert 100\rangle + \lvert 011\rangle\big)^{12D_2}$
Internal Memory Synch	$H\lvert 10\rangle^{12}\lvert 0\rangle^{D_1} = \frac{1}{\sqrt{2}}\big(\lvert 10\rangle + \lvert 01\rangle\big)^{12}\lvert 0\rangle^{D_1}$	$H\lvert 10\rangle^{12}\lvert 0\rangle^{D_2} = \frac{1}{\sqrt{2}}\big(\lvert 10\rangle + \lvert 01\rangle\big)^{12}\lvert 0\rangle^{D_2}$
External Memories (Cavities)	$\frac{1}{\sqrt{2}}\lvert 1\rangle^{1}_{\lvert 0\rangle^{2}},\ \frac{1}{\sqrt{2}}\lvert 0\rangle^{1}_{\lvert 1\rangle^{2}}$	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{2}_{\lvert 1\rangle^{1}},\ \frac{1}{\sqrt{2}}\lvert 1\rangle^{2}_{\lvert 0\rangle^{1}}$
Internal Memory Evolution	$H\lvert 10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\lvert 10\rangle + \lvert 01\rangle\big)^{12}$	$H\lvert 10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\lvert 10\rangle + \lvert 01\rangle\big)^{12}$
Internal Memory Synch	$\lvert 10\rangle^{12}$	$\lvert 10\rangle^{12}$
External Memories (Cavities)	$\lvert 1\rangle^{1}$	$\lvert 0\rangle^{2}$
Internal Memory	$\lvert 1\rangle^{1}$	$\lvert 0\rangle^{2}$

Two Beam Splitters and Two Detectors (Erasure)

Local Information Type	Cavity 1	Cavity 2
External Memories (Experimenter)	$\|10\rangle^{E}_{\|1010\rangle^{12D_1D_2}}$	
Internal Memory Evolution	$M_E M_E M_1 M_2 HH\|10\rangle^{12}\|0\rangle^{D_1}\|0\rangle^{D_2}\|00\rangle^{E}$	
	$= \|1010, 10\rangle^{12D_1D_2E}$	
Internal Memory Synch	$M_1 M_2 HH\|10\rangle^{12}\|0\rangle^{D_1}\|0\rangle^{D_2}\|00\rangle^{E}$	
	$= \|1010\rangle^{12D_1D_2}\|00\rangle^{E}$	
External Memories (Detect.)	$\|1\rangle^{D_1}_{\|10\rangle^{12}}$,	$\|0\rangle^{D_2}_{\|01\rangle^{12}}$
Internal Memory Evolution	$M_1 HH\|10\rangle^{12}\|0\rangle^{D_1} = \|101\rangle^{12D_1}$	$M_2 HH\|10\rangle^{12}\|0\rangle^{D_2} = \|100\rangle^{12D_2}$
Internal Memory Synch	$HH\|10\rangle^{12}\|0\rangle^{D_1} = \|10\rangle^{12}\|0\rangle^{D_1}$	$HH\|10\rangle^{12}\|0\rangle^{D_2} = \|10\rangle^{12}\|0\rangle^{D_2}$
External Memories (Cavities)	$\|1\rangle^{1}_{\|0\rangle^{2}}$	$\|0\rangle^{2}_{\|1\rangle^{1}}$
Internal Memory Evolution	$HH\|10\rangle^{12} = \|10\rangle^{12}$	$HH\|10\rangle^{12} = \|10\rangle^{12}$
Internal Memory Synch	$H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\|10\rangle + \|01\rangle\big)^{12}$	$H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\|10\rangle + \|01\rangle\big)^{12}$
External Memories (Cavities)	$\frac{1}{\sqrt{2}}\|1\rangle^{1}_{\|0\rangle^{2}}$, $\frac{1}{\sqrt{2}}\|0\rangle^{1}_{\|1\rangle^{2}}$	$\frac{1}{\sqrt{2}}\|0\rangle^{2}_{\|1\rangle^{1}}$, $\frac{1}{\sqrt{2}}\|1\rangle^{2}_{\|0\rangle^{1}}$
Internal Memory Evolution	$H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\|10\rangle + \|01\rangle\big)^{12}$	$H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\|10\rangle + \|01\rangle\big)^{12}$
Internal Memory Synch	$\|10\rangle^{12}$	$\|10\rangle^{12}$
External Memories (Cavities)	$\|1\rangle^{1}$	$\|0\rangle^{2}$
Internal Memory	$\|1\rangle^{1}$	$\|0\rangle^{2}$

Local Information Type	Cavity 1	Cavity 2
External Memories (Experimenter)	$\cos(\frac{\phi}{2})\|10\rangle^{E}_{\|1010\rangle^{12D_1D_2}}, i\sin(\frac{\phi}{2})\|01\rangle^{E}_{\|0101\rangle^{12D_1D_2}}$	
Internal Memory Evolution	$M_E M_1 M_2 H P_1 H\|10\rangle^{12}\|0\rangle^{D_1}\|0\rangle^{D_2}\|00\rangle^{E}$	
	$= \big(\cos(\frac{\phi}{2})\|1010,10\rangle + i\sin(\frac{\phi}{2})\|0101,01\rangle\big)^{12D_1D_2E}$	
Internal Memory Synch	$M_1 M_2 H P_1 H\|10\rangle^{12}\|0\rangle^{D_1}\|0\rangle^{D_2}\|00\rangle^{E}$	
	$= \big(\cos(\frac{\phi}{2})\|1010\rangle + i\sin(\frac{\phi}{2})\|0101\rangle\big)^{12D_1D_2}\|00\rangle^{E}$	
External Memories (Detect.)	$\cos(\frac{\phi}{2})\|1\rangle^{D_1}_{\|10\rangle^{12}}, i\sin(\frac{\phi}{2})\|0\rangle^{D_1}_{\|01\rangle^{12}}$	$\cos(\frac{\phi}{2})\|0\rangle^{D_2}_{\|10\rangle^{12}}, i\sin(\frac{\phi}{2})\|1\rangle^{D_2}_{\|01\rangle^{12}}$
Internal Memory Evolution	$M_1 H P_1 H\|10\rangle^{12}\|0\rangle^{D_1}$	$M_2 H P_1 H\|10\rangle^{12}\|0\rangle^{D_2}$
	$= \big(\cos(\frac{\phi}{2})\|101\rangle + i\sin(\frac{\phi}{2})\|010\rangle\big)^{12D_1}$	$= \big(\cos(\frac{\phi}{2})\|100\rangle + i\sin(\frac{\phi}{2})\|011\rangle\big)^{12D_2}$
Internal Memory Synch	$H P_1 H\|10\rangle^{12}\|0\rangle^{D_1}$	$H P_1 H\|10\rangle^{12}\|0\rangle^{D_2}$
	$= \big(\cos(\frac{\phi}{2})\|10\rangle + i\sin(\frac{\phi}{2})\|01\rangle\big)^{12}\|0\rangle^{D_1}$	$= \big(\cos(\frac{\phi}{2})\|10\rangle + i\sin(\frac{\phi}{2})\|01\rangle\big)^{12}\|0\rangle^{D_2}$
External Memories (Cavities)	$\cos(\frac{\phi}{2})\|1\rangle^{1}_{\|0\rangle^{2}}, i\sin(\frac{\phi}{2})\|0\rangle^{1}_{\|1\rangle^{2}}$	$\cos(\frac{\phi}{2})\|0\rangle^{2}_{\|1\rangle^{1}}, i\sin(\frac{\phi}{2})\|1\rangle^{2}_{\|0\rangle^{1}}$
Internal Memory Evolution	$H P_1 H\|10\rangle^{12}$	$H P_1 H\|10\rangle^{12}$
	$= \big(\cos(\frac{\phi}{2})\|10\rangle + i\sin(\frac{\phi}{2})\|01\rangle\big)^{12}$	$= \big(\cos(\frac{\phi}{2})\|10\rangle + i\sin(\frac{\phi}{2})\|01\rangle\big)^{12}$
Internal Memory Synch	$P_1 H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(e^{i\phi}\|10\rangle + \|01\rangle\big)^{12}$	$P_1 H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(e^{i\phi}\|10\rangle + \|01\rangle\big)^{12}$
External Memories (Cavities)	$\frac{1}{\sqrt{2}}e^{i\phi}\|1\rangle^{1}_{\|0\rangle^{2}}, \frac{1}{\sqrt{2}}\|0\rangle^{1}_{\|1\rangle^{2}}$	$\frac{1}{\sqrt{2}}\|0\rangle^{2}_{\|1\rangle^{1}}, \frac{1}{\sqrt{2}}\|1\rangle^{2}_{\|0\rangle^{1}}$
Internal Memory Evolution	$P_1 H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(e^{i\phi}\|10\rangle + \|01\rangle\big)^{12}$	$H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\|10\rangle + \|01\rangle\big)^{12}$
External Memories (Cavities)	$\frac{1}{\sqrt{2}}\|1\rangle^{1}_{\|0\rangle^{2}}, \frac{1}{\sqrt{2}}\|0\rangle^{1}_{\|1\rangle^{2}}$	$\frac{1}{\sqrt{2}}\|0\rangle^{2}_{\|1\rangle^{1}}, \frac{1}{\sqrt{2}}\|1\rangle^{2}_{\|0\rangle^{1}}$
Internal Memory Evolution	$H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\|10\rangle + \|01\rangle\big)^{12}$	$H\|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\|10\rangle + \|01\rangle\big)^{12}$
Internal Memory Synch	$\|10\rangle^{12}$	$\|10\rangle^{12}$
External Memories (Cavities)	$\|1\rangle^{1}$	$\|0\rangle^{2}$
Internal Memory	$\|1\rangle^{1}$	$\|0\rangle^{2}$

Local Information Type	Cavity 1	Cavity 2
External Memories (Exper.)	$\frac{1}{\sqrt{2}}\lvert 100\rangle^{E}_{\lvert 00100\rangle^{12D_0D_1D_2}},\ \frac{1}{2}\lvert 010\rangle^{E}_{\lvert 10010\rangle^{12D_0D_1D_2}},$	
	$-\frac{1}{2}\lvert 001\rangle^{E}_{\lvert 01001\rangle^{12D_0D_1D_2}}$	
Internal Memory Evolution	$M_EM_1M_2HM_0H\lvert 10\rangle^{12}\lvert 0\rangle^{D_1}\lvert 0\rangle^{D_2}\lvert 000\rangle^{E}$	
	$= \frac{1}{\sqrt{2}}\Big(\lvert 00100, 100\rangle$	
	$+\frac{1}{\sqrt{2}}\big(\lvert 10010, 010\rangle - \lvert 01001, 001\rangle\big)\Big)^{12D_0D_1D_2,E}$	
Internal Memory Synch	$M_1M_2HM_0H\lvert 10\rangle^{12}\lvert 0\rangle^{D_1}\lvert 0\rangle^{D_2}\lvert 000\rangle^{E}$	
	$= \frac{1}{\sqrt{2}}\Big(\lvert 00100\rangle + \frac{1}{\sqrt{2}}\big(\lvert 10010\rangle - \lvert 01001\rangle\big)\Big)^{12D_0D_1D_2}$	
	$\otimes\lvert 000\rangle^{E}$	
External Memories (Detect.)	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{D_1}_{\lvert 01\rangle^{12D_0}},\ \frac{1}{2}\lvert 1\rangle^{D_1}_{\lvert 00\rangle^{12D_0}},\ -\frac{1}{2}\lvert 0\rangle^{D_1}_{\lvert 10\rangle^{12D_0}}$	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{D_2}_{\lvert 01\rangle^{12D_0}},\ \frac{1}{2}\lvert 0\rangle^{D_2}_{\lvert 10\rangle^{12D_0}},\ -\frac{1}{2}\lvert 1\rangle^{D_2}_{\lvert 00\rangle^{12D_0}}$
Internal Memory Evolution	$M_1HM_0H\lvert 10\rangle^{12}\lvert 0\rangle^{D_1}$	$M_2HM_0H\lvert 10\rangle^{12}\lvert 0\rangle^{D_2}$
	$= \frac{1}{\sqrt{2}}\Big(\lvert 0010\rangle + \frac{1}{\sqrt{2}}\big(\lvert 1001\rangle - \lvert 0100\rangle\big)\Big)^{12D_0D_1}$	$= \frac{1}{\sqrt{2}}\Big(\lvert 0010\rangle + \frac{1}{\sqrt{2}}\big(\lvert 1000\rangle - \lvert 0101\rangle\big)\Big)^{12D_0D_2}$
Internal Memory Synch	$HM_0H\lvert 10\rangle^{12}\lvert 0\rangle^{D_1}$	$HM_0H\lvert 10\rangle^{12}\lvert 0\rangle^{D_2}$
	$= \frac{1}{\sqrt{2}}\Big(\lvert 001\rangle + \frac{1}{\sqrt{2}}\big(\lvert 100\rangle - \lvert 010\rangle\big)\Big)^{12D_0}\lvert 0\rangle^{D_1}$	$= \frac{1}{\sqrt{2}}\Big(\lvert 001\rangle + \frac{1}{\sqrt{2}}\big(\lvert 100\rangle - \lvert 010\rangle\big)\Big)^{12D_0}\lvert 0\rangle^{D_2}$
External Memories (Cavities)	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{1}_{\lvert 01\rangle^{2D_0}},\ \frac{1}{2}\lvert 1\rangle^{1}_{\lvert 00\rangle^{2D_0}},\ -\frac{1}{2}\lvert 0\rangle^{1}_{\lvert 10\rangle^{2D_0}}$	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{2}_{\lvert 01\rangle^{1D_0}},\ \frac{1}{2}\lvert 0\rangle^{2}_{\lvert 10\rangle^{1D_0}},\ -\frac{1}{2}\lvert 1\rangle^{2}_{\lvert 00\rangle^{1D_0}}$
Internal Memory Evolution	$HM_0H\lvert 10\rangle^{12}$	$HM_0H\lvert 10\rangle^{12}$
	$= \frac{1}{\sqrt{2}}\Big(\lvert 001\rangle + \frac{1}{\sqrt{2}}\big(\lvert 100\rangle - \lvert 010\rangle\big)\Big)^{12D_0}$	$= \frac{1}{\sqrt{2}}\Big(\lvert 001\rangle + \frac{1}{\sqrt{2}}\big(\lvert 100\rangle - \lvert 010\rangle\big)\Big)^{12D_0}$
Internal Memory Synch	$M_0H\lvert 10\rangle^{12}\lvert 0\rangle^{D_0}$	$M_0H\lvert 10\rangle^{12}\lvert 0\rangle^{D_0}$
	$= \frac{1}{\sqrt{2}}\big(\lvert 001\rangle + \lvert 010\rangle\big)^{12D_0}$	$= \frac{1}{\sqrt{2}}\big(\lvert 001\rangle + \lvert 010\rangle\big)^{12D_0}$
External Memories (Cavities)	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{1}_{\lvert 01\rangle^{2D_0}},\ \frac{1}{\sqrt{2}}\lvert 0\rangle^{1}_{\lvert 10\rangle^{2D_0}}$	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{2}_{\lvert 1\rangle^{1}},\ \frac{1}{\sqrt{2}}\lvert 1\rangle^{2}_{\lvert 0\rangle^{1}}$
Internal Memory Evolution	$M_0H\lvert 10\rangle^{12}\lvert 0\rangle^{D_0} = \frac{1}{\sqrt{2}}\big(\lvert 001\rangle + \lvert 010\rangle\big)^{12D_0}$	$H\lvert 10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\lvert 10\rangle + \lvert 01\rangle\big)^{12}$
Internal Memory Synch	$H\lvert 10\rangle^{12}\lvert 0\rangle^{D_0} = \frac{1}{\sqrt{2}}\big(\lvert 10\rangle + \lvert 01\rangle\big)^{12}\lvert 0\rangle^{D_0}$	$H\lvert 10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\lvert 10\rangle + \lvert 01\rangle\big)^{12}$
External Memories (Cavities)	$\frac{1}{\sqrt{2}}\lvert 1\rangle^{1}_{\lvert 0\rangle^{2}},\ \frac{1}{\sqrt{2}}\lvert 0\rangle^{1}_{\lvert 1\rangle^{2}}$	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{2}_{\lvert 1\rangle^{1}},\ \frac{1}{\sqrt{2}}\lvert 1\rangle^{2}_{\lvert 0\rangle^{1}}$
Internal Memory Evolution	$H\lvert 10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\lvert 10\rangle + \lvert 01\rangle\big)^{12}$	$H\lvert 10\rangle^{12} = \frac{1}{\sqrt{2}}\big(\lvert 10\rangle + \lvert 01\rangle\big)^{12}$
Internal Memory Synch	$\lvert 10\rangle^{12}$	$\lvert 10\rangle^{12}$
External Memories (Cavities)	$\lvert 1\rangle^{1}$	$\lvert 0\rangle^{2}$
Internal Memory	$\lvert 1\rangle^{1}$	$\lvert 0\rangle^{2}$

Delayed Choice Quantum Eraser (No Erasure)

External Memories (Exper.)	$\frac{1}{2}\lvert 1010\rangle^{E}_{\lvert 10101010\rangle^{1234D_{1234}}},\ \frac{1}{2}\lvert 0110\rangle^{E}_{\lvert 01100110\rangle^{1234D_{1234}}},$ $\frac{1}{2}\lvert 0110\rangle^{E}_{\lvert 01100110\rangle^{1234D_{1234}}},\ -\frac{1}{2}\lvert 1001\rangle^{E}_{\lvert 10011001\rangle^{1234D_{1234}}}$	
Internal Memory Evolution	$M_E M_1 M_2 M_3 M_4 H^{12} S_1 S_2 H^{12}$ $\times\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{4}\lvert 0\rangle^{D_1}\lvert 0\rangle^{D_2}\lvert 0\rangle^{D_3}\lvert 0\rangle^{D_4}\lvert 0000\rangle^{E}$ $=\frac{1}{2}\big(\lvert 10101010,1010\rangle+\lvert 01100110,0110\rangle$ $+\lvert 01010101,0101\rangle-\lvert 10011001,1001\rangle\big)^{1234D_{1234}E}$	
Internal Memory Synch	$M_1 M_2 M_3 M_4 H^{12} S_1 S_2 H^{12}$ $\times\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{4}\lvert 0\rangle^{D_1}\lvert 0\rangle^{D_2}\lvert 0\rangle^{D_3}\lvert 0\rangle^{D_4}\lvert 0000\rangle^{E}$ $=\frac{1}{2}\big(\lvert 10101010\rangle+\lvert 01100110\rangle$ $+\lvert 01010101\rangle-\lvert 10011001\rangle\big)^{1234D_1D_2D_3D_4}\otimes\lvert 0000\rangle^{E}$	
External Memories (Detect.)	$\frac{1}{2}\lvert 1\rangle^{D_1}_{\lvert 1010\rangle^{1234}},\ \frac{1}{2}\lvert 0\rangle^{D_1}_{\lvert 0110\rangle^{1234}},$ $\frac{1}{2}\lvert 0\rangle^{D_1}_{\lvert 0101\rangle^{1234}},\ -\frac{1}{2}\lvert 1\rangle^{D_1}_{\lvert 1001\rangle^{1234}}$ $\frac{1}{2}\lvert 0\rangle^{D_2}_{\lvert 1010\rangle^{1234}},\ \frac{1}{2}\lvert 1\rangle^{D_2}_{\lvert 0110\rangle^{1234}},$ $\frac{1}{2}\lvert 1\rangle^{D_2}_{\lvert 0101\rangle^{1234}},\ -\frac{1}{2}\lvert 0\rangle^{D_2}_{\lvert 1001\rangle^{1234}}$	$\frac{1}{\sqrt{2}}\lvert 1\rangle^{D_3}_{\lvert 101\rangle^{123}},\ \frac{1}{\sqrt{2}}\lvert 0\rangle^{D_3}_{\lvert 010\rangle^{123}}$ $\frac{1}{\sqrt{2}}\lvert 0\rangle^{D_4}_{\lvert 100\rangle^{124}},\ \frac{1}{\sqrt{2}}\lvert 1\rangle^{D_4}_{\lvert 011\rangle^{124}}$
Internal Memory Evolution	$M_1 H^{12} S_1 S_2 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{4}\lvert 0\rangle^{D_1}$ $=\frac{1}{2}\big(\lvert 10101\rangle+\lvert 01100\rangle+\lvert 01010\rangle-\lvert 10011\rangle\big)^{1234D_1},$ $M_2 H^{12} S_1 S_2 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{4}\lvert 0\rangle^{D_2}$ $=\frac{1}{2}\big(\lvert 10100\rangle+\lvert 01101\rangle+\lvert 01011\rangle-\lvert 10010\rangle\big)^{1234D_2}$	$M_3 S_1 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{D_3}$ $=\frac{1}{\sqrt{2}}\big(\lvert 1011\rangle+\lvert 0100\rangle\big)^{123D_3},$ $M_4 S_2 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{4}\lvert 0\rangle^{D_4}$ $=\frac{1}{\sqrt{2}}\big(\lvert 1000\rangle+\lvert 0111\rangle\big)^{124D_4}$
Internal Memory Synch	$H^{12} S_1 S_2 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{4}\lvert 0\rangle^{D_1}$ $=\frac{1}{2}\big(\lvert 1010\rangle+\lvert 0110\rangle+\lvert 0101\rangle-\lvert 1001\rangle\big)^{1234}\lvert 0\rangle^{D_1},$ $H^{12} S_1 S_2 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{4}\lvert 0\rangle^{D_2}$ $=\frac{1}{2}\big(\lvert 1010\rangle+\lvert 0110\rangle+\lvert 0101\rangle-\lvert 1001\rangle\big)^{1234}\lvert 0\rangle^{D_2}$	$S_1 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{D_3}$ $=\frac{1}{\sqrt{2}}\big(\lvert 101\rangle+\lvert 010\rangle\big)^{123}\lvert 0\rangle^{D_3},$ $S_2 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{4}\lvert 0\rangle^{D_4}$ $=\frac{1}{\sqrt{2}}\big(\lvert 100\rangle+\lvert 011\rangle\big)^{124}\lvert 0\rangle^{D_4}$
External Memories (Cavities)	$\frac{1}{2}\lvert 1\rangle^{1}_{\lvert 010\rangle^{234}},\ \frac{1}{2}\lvert 0\rangle^{1}_{\lvert 110\rangle^{234}},\ \frac{1}{2}\lvert 0\rangle^{1}_{\lvert 101\rangle^{234}},\ -\frac{1}{2}\lvert 1\rangle^{1}_{\lvert 001\rangle^{234}}$ $\frac{1}{2}\lvert 0\rangle^{2}_{\lvert 110\rangle^{134}},\ \frac{1}{2}\lvert 1\rangle^{2}_{\lvert 010\rangle^{134}},\ \frac{1}{2}\lvert 1\rangle^{2}_{\lvert 001\rangle^{134}},\ -\frac{1}{2}\lvert 0\rangle^{2}_{\lvert 101\rangle^{134}}$	$\frac{1}{\sqrt{2}}\lvert 1\rangle^{3}_{\lvert 10\rangle^{12}},\ \frac{1}{\sqrt{2}}\lvert 0\rangle^{3}_{\lvert 01\rangle^{12}}$ $\frac{1}{\sqrt{2}}\lvert 0\rangle^{4}_{\lvert 10\rangle^{12}},\ \frac{1}{\sqrt{2}}\lvert 1\rangle^{4}_{\lvert 01\rangle^{12}}$
Internal Memory Evolution	$H^{12} S_1 S_2 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{4}$ $=\frac{1}{2}\big(\lvert 1010\rangle+\lvert 0110\rangle+\lvert 0101\rangle-\lvert 1001\rangle\big)^{1234}$	
Internal Memory Synch	$S_1 S_2 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}\lvert 0\rangle^{4}$ $=\frac{1}{\sqrt{2}}\big(\lvert 1010\rangle+\lvert 0101\rangle\big)^{1234}$	
External Memories (Cavities)	$\frac{1}{\sqrt{2}}\lvert 1\rangle^{1}_{\lvert 01\rangle^{23}},\ \frac{1}{\sqrt{2}}\lvert 0\rangle^{1}_{\lvert 10\rangle^{23}}$ $\frac{1}{\sqrt{2}}\lvert 1\rangle^{3}_{\lvert 10\rangle^{12}},\ \frac{1}{\sqrt{2}}\lvert 0\rangle^{3}_{\lvert 01\rangle^{12}}$	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{2}_{\lvert 10\rangle^{14}},\ \frac{1}{\sqrt{2}}\lvert 1\rangle^{2}_{\lvert 01\rangle^{14}}$ $\frac{1}{\sqrt{2}}\lvert 0\rangle^{4}_{\lvert 10\rangle^{12}},\ \frac{1}{\sqrt{2}}\lvert 1\rangle^{4}_{\lvert 01\rangle^{12}}$
Internal Memory Evolution	$S_1 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}=\frac{1}{\sqrt{2}}\big(\lvert 101\rangle+\lvert 010\rangle\big)^{123}$	$S_2 H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{4}=\frac{1}{\sqrt{2}}\big(\lvert 100\rangle+\lvert 011\rangle\big)^{124}$
Internal Memory Synch	$H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{3}=\frac{1}{\sqrt{2}}\big(\lvert 100\rangle+\lvert 010\rangle\big)^{12}$	$H^{12}\lvert 10\rangle^{12}\lvert 0\rangle^{4}=\frac{1}{\sqrt{2}}\big(\lvert 100\rangle+\lvert 010\rangle\big)^{124}$
External Memories (Cavities)	$\frac{1}{\sqrt{2}}\lvert 1\rangle^{1}_{\lvert 0\rangle^{2}},\ \frac{1}{\sqrt{2}}\lvert 0\rangle^{1}_{\lvert 1\rangle^{2}}$	$\frac{1}{\sqrt{2}}\lvert 0\rangle^{2}_{\lvert 1\rangle^{1}},\ \frac{1}{\sqrt{2}}\lvert 1\rangle^{2}_{\lvert 0\rangle^{1}}$
Internal Memory Evolution	$H\lvert 10\rangle^{12}=\frac{1}{\sqrt{2}}\big(\lvert 10\rangle+\lvert 01\rangle\big)^{12}$	$H\lvert 10\rangle^{12}=\frac{1}{\sqrt{2}}\big(\lvert 10\rangle+\lvert 01\rangle\big)^{12}$
Internal Memory Synch	$\lvert 10\rangle^{12}$	$\lvert 10\rangle^{12}$
External Memories (Cavities)	$\lvert 1\rangle^{1}$	$\lvert 0\rangle^{2}$
Internal Memory	$\lvert 1\rangle^{1}$	$\lvert 0\rangle^{2}$

<table>
<tr><td>External Memories (Exper.)</td><td>$\frac{1}{\sqrt{2}}|1010\rangle^{E}_{|10101010\rangle^{1234D_{1234}}},\ \frac{1}{\sqrt{2}}|0101\rangle^{E}_{|01010101\rangle^{1234D_{1234}}},$</td><td></td></tr>
<tr><td>Internal Memory Evolution</td><td>$M_E M_1 M_2 M_3 M_4 H^{12} H^{34} S_1 S_2 H^{12}$
$\otimes|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_1}|0\rangle^{D_2}|0\rangle^{D_3}|0\rangle^{D_4}|0000\rangle^{E}$
$= \frac{1}{\sqrt{2}}\big(|10101010, 1010\rangle$
$+|01010101, 0101\rangle\big)^{1234D_{1234}E}$</td><td></td></tr>
<tr><td>Internal Memory Synch</td><td>$M_1 M_2 M_3 M_4 H^{12} H^{34} S_1 S_2 H^{12}$
$\otimes|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_1}|0\rangle^{D_2}|0\rangle^{D_3}|0\rangle^{D_4}|0000\rangle^{E}$
$= \frac{1}{\sqrt{2}}\big(|10101010\rangle+|01010101\rangle\big)^{1234D_{1234}} \otimes |0000\rangle^{E}$</td><td></td></tr>
<tr><td>External Memories (Detect.)</td><td>$\frac{1}{2}|1\rangle^{D_1}_{|1010\rangle^{1234}},\ \frac{1}{2}|0\rangle^{D_1}_{|0110\rangle^{1234}},$
$\frac{1}{2}|0\rangle^{D_1}_{|0101\rangle^{1234}},\ -\frac{1}{2}|1\rangle^{D_1}_{|1001\rangle^{1234}}$

$\frac{1}{2}|0\rangle^{D_2}_{|1010\rangle^{1234}},\ \frac{1}{2}|1\rangle^{D_2}_{|0110\rangle^{1234}},$
$\frac{1}{2}|1\rangle^{D_2}_{|0101\rangle^{1234}},\ -\frac{1}{2}|0\rangle^{D_2}_{|1001\rangle^{1234}}$</td><td>$\frac{1}{2}|1\rangle^{D_3}_{|1010\rangle^{1234}},\ \frac{1}{2}|0\rangle^{D_3}_{|1001\rangle^{1234}}$
$\frac{1}{2}|0\rangle^{D_3}_{|0101\rangle^{1234}},\ -\frac{1}{2}|1\rangle^{D_3}_{|0110\rangle^{1234}}$

$\frac{1}{2}|0\rangle^{D_4}_{|1010\rangle^{1234}},\ \frac{1}{2}|1\rangle^{D_4}_{|1001\rangle^{1234}},$
$\frac{1}{2}|1\rangle^{D_4}_{|0101\rangle^{1234}},\ -\frac{1}{2}|0\rangle^{D_4}_{|0110\rangle^{1234}}$</td></tr>
<tr><td>Internal Memory Evolution</td><td>$M_1 H^{12} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_1}$
$= \frac{1}{2}\big(|10101\rangle + |01100\rangle + |01010\rangle - |10011\rangle\big)^{1234D_1},$

$M_2 H^{34} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_2}$
$= \frac{1}{2}\big(|10100\rangle + |01101\rangle + |01011\rangle - |10010\rangle\big)^{1234D_2}$</td><td>$M_3 H^{34} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_3}$
$= \frac{1}{2}\big(|10101\rangle + |10010\rangle + |01010\rangle - |01101\rangle\big)^{1234D_3},$

$M_4 H^{12} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_4}$
$= \frac{1}{2}\big(|10100\rangle + |10011\rangle + |01011\rangle - |01100\rangle\big)^{1234D_4}$</td></tr>
<tr><td>Internal Memory Synch</td><td>$H^{12} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_1}$
$= \frac{1}{2}\big(|1010\rangle + |0110\rangle + |0101\rangle - |1001\rangle\big)^{1234}|0\rangle^{D_1},$

$H^{12} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_2}$
$= \frac{1}{2}\big(|1010\rangle + |0110\rangle + |0101\rangle - |1001\rangle\big)^{1234}|0\rangle^{D_2}$</td><td>$H^{34} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_3}$
$= \frac{1}{2}\big(|1010\rangle + |1001\rangle + |0101\rangle - |0110\rangle\big)^{1234}|0\rangle^{D_3},$

$H^{34} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}|0\rangle^{D_4}$
$= \frac{1}{2}\big(|1010\rangle + |1001\rangle + |0101\rangle - |0110\rangle\big)^{1234}|0\rangle^{D_4}$</td></tr>
<tr><td>External Memories (Cavities)</td><td>$\frac{1}{2}|1\rangle^{1}_{|010\rangle^{234}},\ \frac{1}{2}|0\rangle^{1}_{|110\rangle^{234}},\ \frac{1}{2}|0\rangle^{1}_{|101\rangle^{234}},\ -\frac{1}{2}|1\rangle^{1}_{|001\rangle^{234}}$

$\frac{1}{2}|0\rangle^{2}_{|110\rangle^{134}},\ \frac{1}{2}|1\rangle^{2}_{|010\rangle^{134}},\ \frac{1}{2}|1\rangle^{2}_{|001\rangle^{134}},\ -\frac{1}{2}|0\rangle^{2}_{|101\rangle^{134}}$</td><td>$\frac{1}{2}|1\rangle^{3}_{|100\rangle^{124}},\ \frac{1}{2}|0\rangle^{3}_{|101\rangle^{124}},\ \frac{1}{2}|0\rangle^{3}_{|011\rangle^{124}},\ -\frac{1}{2}|1\rangle^{3}_{|010\rangle^{124}}$

$\frac{1}{2}|0\rangle^{4}_{|101\rangle^{123}},\ \frac{1}{2}|1\rangle^{4}_{|100\rangle^{123}},\ \frac{1}{2}|1\rangle^{4}_{|010\rangle^{123}},\ -\frac{1}{2}|0\rangle^{4}_{|011\rangle^{123}}$</td></tr>
<tr><td>Internal Memory Evolution</td><td>$H^{12} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}$
$= \frac{1}{2}\big(|1010\rangle + |0110\rangle + |0101\rangle - |1001\rangle\big)^{1234}$</td><td>$H^{34} S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}$
$= \frac{1}{2}\big(|1010\rangle + |1001\rangle + |0101\rangle - |0110\rangle\big)^{1234}$</td></tr>
<tr><td>Internal Memory Synch</td><td>$S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}$
$= \frac{1}{\sqrt{2}}\big(|1010\rangle + |0101\rangle\big)^{1234}$</td><td>$S_1 S_2 H^{12}|10\rangle^{12}|0\rangle^{3}|0\rangle^{4}$
$= \frac{1}{\sqrt{2}}\big(|1010\rangle + |0101\rangle\big)^{1234}$</td></tr>
<tr><td>External Memories (Cavities)</td><td>$\frac{1}{\sqrt{2}}|1\rangle^{1}_{|01\rangle^{23}},\ \frac{1}{\sqrt{2}}|0\rangle^{1}_{|10\rangle^{23}}$

$\frac{1}{\sqrt{2}}|1\rangle^{3}_{|10\rangle^{12}},\ \frac{1}{\sqrt{2}}|0\rangle^{3}_{|01\rangle^{12}}$</td><td>$\frac{1}{\sqrt{2}}|0\rangle^{2}_{|10\rangle^{14}},\ \frac{1}{\sqrt{2}}|1\rangle^{2}_{|01\rangle^{14}}$

$\frac{1}{\sqrt{2}}|0\rangle^{4}_{|10\rangle^{12}},\ \frac{1}{\sqrt{2}}|1\rangle^{4}_{|01\rangle^{12}}$</td></tr>
<tr><td>Internal Memory Evolution</td><td>$S_1 H^{12}|10\rangle^{12}|0\rangle^{3} = \frac{1}{\sqrt{2}}\big(|101\rangle + |010\rangle\big)^{123}$</td><td>$S_2 H^{12}|10\rangle^{12}|0\rangle^{4} = \frac{1}{\sqrt{2}}\big(|100\rangle + |011\rangle\big)^{124}$</td></tr>
<tr><td>Internal Memory Synch</td><td>$H^{12}|10\rangle^{12}|0\rangle^{3} = \frac{1}{\sqrt{2}}\big(|100\rangle + |010\rangle\big)^{12}$</td><td>$H^{12}|10\rangle^{12}|0\rangle^{4} = \frac{1}{\sqrt{2}}\big(|100\rangle + |010\rangle\big)^{124}$</td></tr>
<tr><td>External Memories (Cavities)</td><td>$\frac{1}{\sqrt{2}}|1\rangle^{1}_{|0\rangle^{2}},\ \frac{1}{\sqrt{2}}|0\rangle^{1}_{|1\rangle^{2}}$</td><td>$\frac{1}{\sqrt{2}}|0\rangle^{2}_{|1\rangle^{1}},\ \frac{1}{\sqrt{2}}|1\rangle^{2}_{|0\rangle^{1}}$</td></tr>
<tr><td>Internal Memory Evolution</td><td>$H|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(|10\rangle + |01\rangle\big)^{12}$</td><td>$H|10\rangle^{12} = \frac{1}{\sqrt{2}}\big(|10\rangle + |01\rangle\big)^{12}$</td></tr>
<tr><td>Internal Memory Synch</td><td>$|10\rangle^{12}$</td><td>$|10\rangle^{12}$</td></tr>
<tr><td>External Memories (Cavities)</td><td>$|1\rangle^{1}$</td><td>$|0\rangle^{2}$</td></tr>
<tr><td>Internal Memory</td><td>$|1\rangle^{1}$</td><td>$|0\rangle^{2}$</td></tr>
</table>

<table>
<tr><td>Local Information Type</td><td></td><td></td></tr>
<tr><td>External Memories</td><td>$\frac{1}{2}|0\rangle^D_{|\Phi^+\rangle^{AC}\left(a|0\rangle+b|1\rangle\right)^B}$, $\frac{1}{2}|1\rangle^D_{|\Phi^-\rangle^{AC}\left(a|0\rangle+b|1\rangle\right)^B}$,
$\frac{1}{2}|2\rangle^D_{|\Psi^+\rangle^{AC}\left(a|0\rangle+b|1\rangle\right)^B}$, $\frac{1}{2}|3\rangle^D_{|\Psi^-\rangle^{AC}\left(a|0\rangle+b|1\rangle\right)^B}$</td><td>$\frac{1}{2}\left(a|0\rangle+b|1\rangle\right)^B_{|\Phi^+\rangle^{AC}|0\rangle^D}$,
$\frac{1}{2}\left(a|0\rangle+b|1\rangle\right)^B_{|\Phi^-\rangle^{AC}|1\rangle^D}$,
$\frac{1}{2}\left(a|0\rangle+b|1\rangle\right)^B_{|\Psi^+\rangle^{AC}|2\rangle^D}$,
$\frac{1}{2}\left(a|0\rangle+b|1\rangle\right)^B_{|\Psi^-\rangle^{AC}|3\rangle^D}$,</td></tr>
<tr><td>Internal Memory Evolution
(Teleportation Complete)</td><td>$U^{BD}M^{ACD}U^{AB}|00\rangle^{AB}\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$
$=\frac{1}{2}\Big(|\Phi^+\rangle^{AC}|0\rangle^D+|\Phi^-\rangle^{AC}|1\rangle^D+|\Psi^+\rangle^{AC}|2\rangle^D+|\Psi^-\rangle^{AC}|3\rangle^D\Big)$
$\otimes\left(a|0\rangle+b|1\rangle\right)^B$</td><td></td></tr>
<tr><td>Internal Memory Synch
(D and B meet)</td><td>$M^{ACD}U^{AB}|00\rangle^{AB}\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$
$=\frac{1}{2}\Big(|\Phi^+\rangle^{AC}|0\rangle^D\left(a|0\rangle+b|1\rangle\right)^B+|\Phi^-\rangle^{AC}|1\rangle^D\left(a|0\rangle-b|1\rangle\right)^B$
$+|\Psi^+\rangle^{AC}|2\rangle^D\left(a|1\rangle+b|0\rangle\right)^B+|\Psi^-\rangle^{AC}|3\rangle^D\left(a|1\rangle-b|0\rangle\right)^B\Big)$</td><td></td></tr>
<tr><td>External Memories</td><td>$\frac{1}{2}|0\rangle^D_{|\Phi^+\rangle^{AC}\left(a|0\rangle+b|1\rangle\right)^B}$, $\frac{1}{2}|1\rangle^D_{|\Phi^-\rangle^{AC}\left(a|0\rangle-b|1\rangle\right)^B}$,
$\frac{1}{2}|2\rangle^D_{|\Psi^+\rangle^{AC}\left(a|1\rangle+b|0\rangle\right)^B}$, $\frac{1}{2}|3\rangle^D_{|\Psi^-\rangle^{AC}\left(a|1\rangle-b|0\rangle\right)^B}$</td><td>$\frac{1}{\sqrt{2}}|0\rangle^B_{|00\rangle^{AB}}$, $\frac{1}{\sqrt{2}}|1\rangle^B_{|11\rangle^{AB}}$</td></tr>
<tr><td>Internal Memory Evolution</td><td>$M^{ACD}U^{AB}|00\rangle^{AB}\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$
$=\frac{1}{2}\Big(|\Phi^+\rangle^{AC}|0\rangle^D\left(a|0\rangle+b|1\rangle\right)^B+|\Phi^-\rangle^{AC}|1\rangle^D\left(a|0\rangle-b|1\rangle\right)^B$
$+|\Psi^+\rangle^{AC}|2\rangle^D\left(a|1\rangle+b|0\rangle\right)^B+|\Psi^-\rangle^{AC}|3\rangle^D\left(a|1\rangle-b|0\rangle\right)^B\Big)$</td><td>$U^{AB}|00\rangle^{AB}=\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right)^{AB}$</td></tr>
<tr><td>Internal Memory Synch
(A meets C and D)</td><td>$U^{AB}|00\rangle^{AB}\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$
$=\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right)^{AB}\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$</td><td>$U^{AB}|00\rangle^{AB}=\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right)^{AB}$</td></tr>
<tr><td>External Memories
(arbitrary basis)</td><td>$\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$</td><td>$\frac{1}{\sqrt{2}}|0\rangle^A_{|00\rangle^{AB}}$, $\frac{1}{\sqrt{2}}|1\rangle^A_{|11\rangle^{AB}}$,
$\frac{1}{\sqrt{2}}|0\rangle^B_{|00\rangle^{AB}}$, $\frac{1}{\sqrt{2}}|1\rangle^B_{|11\rangle^{AB}}$</td></tr>
<tr><td>Internal Memory Evolution</td><td>$\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$</td><td>$U^{AB}|00\rangle^{AB}=\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right)^{AB}$</td></tr>
<tr><td>Internal Memory Synch</td><td>$\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$</td><td>$|00\rangle^{AB}$</td></tr>
<tr><td>External Memories</td><td>$\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$</td><td>$|0\rangle^A$, $|0\rangle^B$</td></tr>
<tr><td>Internal Memory</td><td>$\left(a|0\rangle+b|1\rangle\right)^C|0\rangle^D$</td><td>$|0\rangle^A$, $|0\rangle^B$</td></tr>
</table>

Local Info Type	Isotope	Cat	Schrödinger	Friend
Ext. Mem.	$\frac{1}{\sqrt{2}}\vert 0\rangle^I_{\vert 00\rangle^{IC}},\ \frac{1}{\sqrt{2}}\vert 1\rangle^I_{\vert 11\rangle^{IC}}$	$\frac{1}{\sqrt{2}}\vert 0\rangle^C_{\vert 000\rangle^{ICS}},\ \frac{1}{\sqrt{2}}\vert 1\rangle^C_{\vert 111\rangle^{ICS}}$	$\frac{1}{\sqrt{2}}\vert 0\rangle^S_{\vert 0000\rangle^{ICSF}},\ \frac{1}{\sqrt{2}}\vert 1\rangle^S_{\vert 1111\rangle^{ICSF}}$	$\frac{1}{\sqrt{2}}\vert 0\rangle^F_{\vert 0000\rangle^{ICSF}},\ \frac{1}{\sqrt{2}}\vert 1\rangle^F_{\vert 1111\rangle^{ICSF}}$
Int. Mem. Evol.	$M^{IC}\vert\psi\rangle^I\vert 0\rangle^C$	$M^{CS}M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S$	$M^{SF}M^{CS}M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S\vert 0\rangle^F$	$M^{SF}M^{CS}M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S\vert 0\rangle^F$
	$= \frac{1}{\sqrt{2}}\big(\vert 00\rangle + \vert 11\rangle\big)^{IC}$	$= \frac{1}{\sqrt{2}}\big(\vert 000\rangle + \vert 111\rangle\big)^{ICS}$	$= \frac{1}{\sqrt{2}}\big(\vert 0000\rangle + \vert 1111\rangle\big)^{ICSF}$	$= \frac{1}{\sqrt{2}}\big(\vert 0000\rangle + \vert 1111\rangle\big)^{ICSF}$
Int. Mem. Synch	$M^{IC}\vert\psi\rangle^I\vert 0\rangle^C$	$M^{CS}M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S$	$M^{CS}M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S\vert 0\rangle^F$	$M^{CS}M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S\vert 0\rangle^F$
	$= \frac{1}{\sqrt{2}}\big(\vert 00\rangle + \vert 11\rangle\big)^{IC}$	$= \frac{1}{\sqrt{2}}\big(\vert 000\rangle + \vert 111\rangle\big)^{ICS}$	$= \frac{1}{\sqrt{2}}\big(\vert 000\rangle + \vert 111\rangle\big)^{ICS}\vert 0\rangle^F$	$= \frac{1}{\sqrt{2}}\big(\vert 000\rangle + \vert 111\rangle\big)^{ICS}\vert 0\rangle^F$
Ext. Mem.	$\frac{1}{\sqrt{2}}\vert 0\rangle^I_{\vert 00\rangle^{IC}},\ \frac{1}{\sqrt{2}}\vert 1\rangle^I_{\vert 11\rangle^{IC}}$	$\frac{1}{\sqrt{2}}\vert 0\rangle^C_{\vert 000\rangle^{ICS}},\ \frac{1}{\sqrt{2}}\vert 1\rangle^C_{\vert 111\rangle^{ICS}}$	$\frac{1}{\sqrt{2}}\vert 0\rangle^S_{\vert 000\rangle^{ICS}},\ \frac{1}{\sqrt{2}}\vert 1\rangle^S_{\vert 111\rangle^{ICS}}$	$\vert 0\rangle^F$
Int. Mem. Evol.	$M^{IC}\vert\psi\rangle^I\vert 0\rangle^C$	$M^{CS}M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S$	$M^{CS}M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S$	$\vert 0\rangle^F$
	$= \frac{1}{\sqrt{2}}\big(\vert 00\rangle + \vert 11\rangle\big)^{IC}$	$= \frac{1}{\sqrt{2}}\big(\vert 000\rangle + \vert 111\rangle\big)^{ICS}$	$= \frac{1}{\sqrt{2}}\big(\vert 000\rangle + \vert 111\rangle\big)^{ICS}$	
Int. Mem. Synch	$M^{IC}\vert\psi\rangle^I\vert 0\rangle^C$	$M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S$	$M^{IC}\vert\psi\rangle^I\vert 0\rangle^C\vert 0\rangle^S$	$\vert 0\rangle^F$
	$= \frac{1}{\sqrt{2}}\big(\vert 00\rangle + \vert 11\rangle\big)^{IC}$	$= \frac{1}{\sqrt{2}}\big(\vert 00\rangle + \vert 11\rangle\big)^{IC}\vert 0\rangle^S$	$= \frac{1}{\sqrt{2}}\big(\vert 00\rangle + \vert 11\rangle\big)^{IC}\vert 0\rangle^S$	
Ext. Mem.	$\frac{1}{\sqrt{2}}\vert 0\rangle^I_{\vert 00\rangle^{IC}},\ \frac{1}{\sqrt{2}}\vert 1\rangle^I_{\vert 11\rangle^{IC}}$	$\frac{1}{\sqrt{2}}\vert 0\rangle^C_{\vert 00\rangle^{IC}},\ \frac{1}{\sqrt{2}}\vert 1\rangle^C_{\vert 11\rangle^{IC}}$	$\vert 0\rangle^S$	$\vert 0\rangle^F$
Int. Mem. Evol.	$M^{IC}\vert\psi\rangle^I\vert 0\rangle^C$	$M^{IC}\vert\psi\rangle^I\vert 0\rangle^C$	$\vert 0\rangle^S$	$\vert 0\rangle^F$
	$= \frac{1}{\sqrt{2}}\big(\vert 00\rangle + \vert 11\rangle\big)^{IC}$	$= \frac{1}{\sqrt{2}}\big(\vert 00\rangle + \vert 11\rangle\big)^{IC}$		
Int. Mem. Synch	$\vert\psi\rangle^I\vert 0\rangle^C$	$\vert\psi\rangle^I\vert 0\rangle^C$	$\vert 0\rangle^S$	$\vert 0\rangle^F$
Ext. Mem.	$\vert\psi\rangle^I$	$\vert 0\rangle^C$	$\vert 0\rangle^S$	$\vert 0\rangle^F$
Int. Mem.	$\vert\psi\rangle^I$	$\vert 0\rangle^C$	$\vert 0\rangle^S$	$\vert 0\rangle^F$