Speculation on Temperature in Pure Quantum Bound States

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In (1) it is suggested that Newton's second law for constant acceleration, i.e. F=ma, may be derived using the first law of thermodynamics (with dE=0) and the special relativistic idea of a constant acceleration being linked to a temperature as shown by Unruh i.e. T=a C1 (where C1 is a constant given in terms of hbar, c etc).

In this note, we consider the two Lorentz invariants EE = pp + momo (c=1) (and its generalization (E-V(x))(E-V(x)) = pp + momo)and -Et+px. The former becomes Newton's energy conservation law in the nonrelativistic limit i.e. E= pp/2m +V(x) which is equivalent to Newton's second law (upon taking d/dx), but contains a variety of accelerations.

The Lorentz invariant -Et+px suggests t and x are independent and that t is linked with an eternal clock with frequency hbar/E and x with an internal ruler with spacing hbar/p. Thus even though a particle with constant speed moves as x= p/m t (as measured using an external clock and ruler) there is an internal wavelength hbar/p in which there is a probability for the particle to be at various x points (as seen with an external ruler) and no internal time.

Given that one may consider externally x = p/m t, one may also consider externally the situation of acceleration. Each constant p, however, is associated with exp(ipx (an eigenfuction of the translation operator -id/dx) so at a given x, one has a superposition of exp(ipx)'s. This suggests a different "probability" distribution for p at each x or in other words a temperature linked with {Sum over p a(p) pp/2m exp(ipx)} / {Sum over p a(p)exp(ipx)}.. The temperature changes from one x point to another. If one considers that E is constant at each x. then applying the first law of thermodynamics yields Fdx = dx d/dx (-1/2m d/dx dW/dx) which should be equivalent to TdS. Using Uruh's results, this leads to the expression for entropy given in (1).

This, however, seems to be an entropy which is different from one constructed using Shannon's entropy equation. We consider some of these ideas in this note.

Newton's Law Using Unruh Temperature

In (1) it is noted that using Unruh's temperature and the first law of thermodynamics with dE=0 i.e. Fdx=TdS ((1)) and dS assumed to be proportional to dx, one may obtain Newton's second law F=ma for a constant acceleration.

Unruh's temperature/constant acceleration relationship follows from special relativity applied to a constant acceleration (2):

kT = hbar a / (2*3.14*c) ((2))

Thus a constant acceleration is associated with the notion of a temperature.

Using ((1)) with Fdx = TdS and assuming

dS= 2*3.14*k* m hbar/c dx ((3)) yields F=ma ((4)) i.e. Newton's second law for a constant acceleration.

We try to argue that the idea of a "temperature" associated with acceleration appears in quantum mechanics which we argue also follows from special relativity.

Quantum Mechanics from Special Relativity

We have argued in previous notes that quantum mechanics follows from Lorentz invariants in special relativity.

First we use: EE = pp + momo (c=1) ((5))

Here p=mo v / sqrt(1-vv/cc) ((6a)) where x=vt for a constant velocity. Thus there is an external clock and ruler which may be used to observe the particle move according to x=vt.

Next, one introduces the notion of v, as measured from an external clock and ruler, changing in each "dx" interval, where dx is chosen to be small i.e. dx→0. Using a function V(x) (called a potential) one may write:

(E-V(x))(E-V(x)) = pp+momo (c=1) ((7))

Taking the nonrelativistic limit i.e. pp and V(x) << mo (c=1) yields the Newtonian conservation of energy equation:

E= pp/2m + V(x) ((8)) with p=mov and F=-dV/dx ((9))

((8)) is associated with a different acceleration dv/dt at each x if V(x) is not a constant.

An important issue, however, arises if one considers the Lorentz invariant A= -Et+px.
This invariant suggests that t and x are independent and represent an internal time and internal ruler with:

Frequency = hbar/E ((10a)) and internal ruler spacing = hbar/p ((10b))

How can this be consistent with x=vt which suggests infinite resolution? We suggest that x=vt only holds on average and is measured using an internal clock and ruler with infinite resolution. There are, however, physical repeated half wavelength units of length .5 hbar/p which contain no internal time, but rather a probability for the particle to be at x given by cos(px). exp(ipx) is a sum of cos(px) and i sin(px) i.e. shifted distributions to indicate the direction of motion because internal time is removed. It may also be noted that exp(ipx) is an eigenfunction of -id/dx and so has repeated wavelength units to demonstrate a kind of spatial invariance.
Thus there is no notion of dx→0, but one may still achieve and average acceleration using ((8)) i.e.

E = {Sum over p a(p) pp/2m exp(ipx) } / {Sum over p a(p)exp(ipx)} + V(x) ((11))

The first term on the RHS is equivalent to -1/2m d/dx dW/dx / W where W= Sum over p a(p)exp(ipx)}.

Thus many constant p values create the average kinetic energy at x using interference. This KE(x) is equivalent to the classical value E-V(x) and:

d/dx KE(x) = m a(x) ((12)) where a(x) is acceleration at x.

In classical statistical mechanics kT is equivalent to average kinetic energy. In ((11)) one has an average kinetic energy consisting of a distribution of p values at x, each with weight a(p)cos(px) if one considers the symmetry a(p)=a(-p).

Thus this approach also leads to the notion that a given acceleration is equivalent to a temperature i.e.a pp/2m distribution and also follows directly from special relativity (which we argue is the source of the exp(ipx) formalism of quantum mechanics).

Entropy Considerations

If one uses the first law of thermodynamics at each x point with E=constant so 0 = TdS-Fdx then:

TdS= Fdx = (dx) d/dx KE(x) ((13))

Thus dS is proportional to dx as in (1). Then according to ((2)) T = C1 a so ((13)) implies that:

C1 a dS = dx m a(x) or dS= m/C1 dx where C1= hbar/ (2*3.14*c*k) ((14))

((14)), however, is equivalent to ((3)) presented in (1).

One may note that this S(x) is a constant and not the same entropy density one would obtain using Shannon's entropy formula

Conclusion

In conclusion, we argue that using Lorentz invariants from special relativity i.e. EE = pp + momo (c=1) generalized to (E-V(x))(E-V(x)) = pp + momo and A= -Et+px and taking the nonrelativistic limit one obtains E= pp/2m + V(x) and exp(ipx) representing a quantum particle. Thus there is no dx→0 limit and one must use a superposition of exp(ipx)s to create an average vrms(x) and the notion of acceleration. Given that a distribution of p's is used with weights a(p)cos(px) at each x for symmetry a(p)=a(-p), one has a temperature T(x) associated with a(x). Using the first law of thermodynamics at x with dE=0 one has Fdx = TdS and may show that TdS = dx d/dx KE(x). Using Unruh's relationship for T and a constant acceleration, one finds that dS= 2*3.14*k* m hbar/c dx which matches the result in (1) obtained using different arguments. We note that this entropy is not the same as one obtained using Shannon's entropy formula.

References

1. Verlinde, E. On the Origins of Gravity and the Laws of Newton (2010) https://arxiv.org/pdf/1001.0785.pdf

2. https://en.wikipedia.org/wiki/Unruh_effect