Capped Accumulated Return Call Valuation with Flexible Return

A Monte Carlo (Monte Carlo, MC, and Quasi Monte Carlo, QMC) pricing model is presented for a new variation on the product named capped-accumulated-return-call (CARC): CARC with pick and choose the return period. User specifies the dates where equity returns are to be calculated and then used in the final payoff.

Let S be a stock, $S(t)$ be the price process of the stock, and $\{t_0 < t_1 < \cdots < t_n\}$ be a set of reset dates and $T \geq t_n$ be a payoff settlement date. The simple CARC with underlying S is a European type derivative security whose matured payoff at the settlement date is given by

$$N + N \times \max\left\{R^{acc}, R^{f}\right\} \tag{1}$$

where R^f is the global floor of the return rate, N is the notional principal, and R^{acc} is accumulated return and defined as

$$R^{acc} = \prod_{i=1}^{n}(1 + R_i^{cap}) - 1 \tag{2}$$

where R_i^{cap} is the capped return-rate for each period described as follows: Define the actual period return-rate as

$$R_i = \frac{S(t_i) - S(t_{i-1})}{S(t_{i-1})}, i = 1, \cdots, n, \tag{3}$$

Then we define

$$R_i^{cap} = \min\{c, R_i\}, \tag{4}$$

where c is the cap.

The notional N may or may not be added to the payoff.

The new attribute RETURN_CALCULATION_DATE_FILE specifies the dates where the capped-returns are to be calculated and then used for the computation of the payoff. Let $\theta(1) < ... < \theta(m)$ an increasing subsequence of the sequence $1, \ldots, n$. The payoff (with notional) of CARC with pick and choose return period feature is then:

$$N + N \cdot \max\left(R^{pc}, R^{f}\right),$$

where $R^{pc} = \prod_{i=1}^{m} (1 + R_{\theta(i)}^{cap}) - 1.$

Let t be the current value date, then the current value of CARC with pick and choose return feature (with notional) can be written

$$df(t,T) \times N \times \left[1 + E_t \left[\max\left\{R^{pc}, R^{f}\right\}\right]\right], \qquad (6)$$

where $df(t,T)$ is the discounting factor at the value date. The above formula is in a world that is risk-neutral. The governing price dynamics of the underlying asset in the risk-neutral world is

$$dS_t = (r - q)S_t dt + \sigma S_t dW_t \qquad (7)$$

where r is the short rate, q is the dividend yield of the asset, σ_s is the volatility of the asset price, and W_t is the Wiener process. All these parameters are assumed deterministic. The above dynamics is used to simulate the returns at reset dates.

Let $f(\ )$ be the option value, σ be the volatility of the underlying asset (SPX), and C the USD discount curve (ref. https://finpricing.com/lib/IrInflationCurve.html $C \oplus 0.001$ denotes parallel shift by *10* bps). The formulas used are:

$$Vega = f(\sigma + 0.01) - f(\sigma)$$

and

$$Rho = f(C \oplus 0.001) - f(C).$$

Let $f(S)$ be the option value when the underlying price is S (in our case SPX-IC), and let $\delta = 0.01$. The formula used is:

$$Delta = \frac{f\left(S \cdot (1+\delta)\right) - f\left(\frac{S}{1+\delta}\right)}{S \cdot (1+\delta) - \frac{S}{1+\delta}}$$