Capped Accumulated Return Call Valuation with Lock-In Feature

A Monte Carlo (Gaussian MC and Quasi MC) pricing model is presented for the product named capped-accumulated-return-call (CARC) with lock in feature. Let $L_1 < L_2 < \ldots < L_p$ be an increasing series of lock in returns. Let L_k be the greatest lock in return such that the maximum of the partial accumulated returns is greater or equal than L_k. If L_k exists, then the final accumulated return will not be smaller than L_k. By its definition L_k is path-dependent. It acts as a path-dependent floor, as opposed to the global floor which is fixed.

Let $S_1, \cdots, S_M$ be M stocks in a given basket, $\mathrm{S_j(t)}$ be the price process of the jth stock and $1 \le j \le M$, and $\{\mathrm{t_0 < t_1 < \cdots < t_n}\}$ be a set of reset dates and $\mathrm{T \ge t_n}$ be a payoff settlement date. The CARC with the multiple underlyings $\mathrm{S_1, \cdots, S_N}$ is a European type derivative security whose matured payoff at the settlement date is given by

$$\mathrm{N + N \times max\left\{R_c, R_f\right\}} \tag{1}$$

where $\mathrm{R_f}$ is the global floor of the return rate, N is the notional principal, and $\mathrm{R_c}$ is capped-accumulated-return and defined as

$$\mathrm{R_c = \prod_{i=1}^{n}(1 + R_{cap}^{(i)}) - 1} \tag{2}$$

where $R_{cap}^{(i)}$ is the capped return-rate for each period described as follows: Define the actual period return-rate as

$$R_i = \frac{\overline{S}(t_i) - \overline{S}(t_{i-1})}{\overline{S}(t_{i-1})}, \, i = 1, \cdots, n, \tag{3}$$

where

$$\overline{S}(t_i) = \sum_{j=1}^{M} w_j S_j(t_i). \tag{4}$$

Here w_j, $j = 1, \cdots, M$ are the weights and

$$\sum_{j=1}^{M} w_j = 1. \tag{5}$$

Then we define

$$R_{cap}^{(i)} = \min\{c, R_i\}, \tag{6}$$

where *c* is the cap.

The new feature is represented by an increasing series of attributes LOCK_IN_RETURN.

Let $L_1 < L_2 < \ldots < L_p$ be this series of increasing lock in returns. Define the *j*th partial accumulated return as follows: $R_c(j) = \prod_{i=1}^{j} (1 + R_{cap}^{(i)}) - 1$. In particular, the final accumulated return, R_c, is equal to the *n*th partial accumulated return: $R_c = R_c(n)$.

Let L_k be the greatest lock in return such that the maximum of the partial accumulated returns is greater or equal than L_k, that is $L_k := \max\{L_s \mid s = 1, \ldots, p, \max_{j=1,\ldots,n} R_c(j) \geq L_s\}$. In our simulation, L_k is path-dependent (just like R_c's), as opposed to R_f which is fixed. If there is no such L_k, that is the set above is empty, we set $L_k = -\infty$.

We impose that the final accumulated return will not be less than L_k (see the payoff below).

Let t be the current value date, then the current value of CARC with lock in feature can be written

$$df(t,T) \times N \times \left[1 + E_t\left[\max\left\{\max\{R_c, L_k\}, R_f\right\}\right]\right] \qquad (7),$$

where $\mathrm{df}(t,T)$ is the discounting factor at the value date (note that the inner term is equal to $\max\{R_c, L_k, R_f\}$). The above formula is in a world that is forward risk-neutral with respect to a specific currency C_p. As a result, the notional principal N is measured in the currency C_p, and the discounting factor should be calculated by a C_p zero curve (ref. https://finpricing.com/lib/IrCurveIntroduction.html) given at the value date. If the underlying asset is measured in another currency C_U, assuming the option is a Quanto type transaction, the governing price dynamics of the underlying asset in the risk-neutral world of C_p should be written as

$$dS_t = (r^U - q - \rho\sigma_x\sigma_s)S_t dt + \sigma_s S_t dW_t \qquad (8)$$

where r^U is the short rate of C_U, q is the dividend yield of the asset, σ_s is the volatility of the asset price, σ_x is the volatility of the exchange rate between C_p and C_U, ρ is correlation coefficient between the asset price and the exchange rate, and W_t is the Wiener process. All these parameters are assumed deterministic.

Quasi-Monte Carlo (GED QMC) simulation method with Sobol sequence associated with Brownian Bridge path generation was used to calculate the option price. GED used 8,000 simulations and 2,000 stratified samples. Also results obtained using crude Monte Carlo method (GED MC) at 100,000 simulations are reported.