Capped Accumulated Return Call Option with Two Way Return and Splitting Payoff

We developed and implemented a pricing model for capped-accumulated-return-call (CARC) with two features: two-way-return and splitting payoff.

Let $S(t)$ be a price process of a given underlying asset, $\{t_0 < t_1 < \cdots < t_n\}$ be a set of reset dates and $T \geq t_n$ be a payoff settlement date. The two-way-return CARC with the underlying S is a European type derivative security whose matured payoff at the settlement date is given by

$$N \times \max\{0, R_c - R_f\} \tag{1}$$

where R_f is the global floor (strike) of the return rate, N is the notional principal, and R_c is capped-accumulated-return and defined as

$$R_c = \prod_{i=1}^{n}(1 + R_{cap}^{(i)}) - 1 \tag{2}$$

where $R_{cap}^{(i)}$ is the two-way-capped return-rate for each period explained as follows. Define the actual period return-rate as

$$R_i = \frac{S(t_i) - S(t_{i-1})}{S(t_{i-1})}, i = 1, \cdots, n. \quad (3)$$

If the price of the underlying asset goes down, $R_{cap}^{(i)}$ would be the absolute value of the return up to a local floor c_-; otherwise, $R_{cap}^{(i)}$ would be asset return up to a local cap c_+. Mathematically, $R_{cap}^{(i)}$ can be expressed as

$$R_{cap}^{(i)} = \begin{cases} \min\{c_-, -R_i\}, \text{if } R_i < 0 \\ \min\{c_+, R_i\}, \text{if } R_i \geq 0. \end{cases} \quad (4)$$

Let t be the current value date, then the current value of this CARC can be written as

$$df(t,T) \times N \times E_t\left[\max\left\{0, \prod_{i=1}^{n}(1 + R_{cap}^{(i)}) - (1 + R_f)\right\}\right] \quad (5)$$

where $df(t,T)$ is the discounting factor at the value date. The above formula is in a world that is forward risk-neutral with respect to a specific currency C_p.

As a result, the notional principal N is measured in the currency C_p, and the discounting factor should be calculated by a C_p zero curve (ref. https://finpricing.com/lib/IrInflationCurve.html) given at the value date. If the underlying asset is measured in another currency C_U, assuming the option is a non-Quanto type transaction, the governing price dynamics of the underlying asset in the risk-neutral world of C_p should be written as

$$dS_t = (r^U - q)S_t dt + \sigma_s S_t dW_t \quad (6)$$

where r^U is the short rate of C_U, q is the dividend yield of the asset, σ_s is the volatility of the asset price, and W_t is the Wiener process. All these parameters are assumed deterministic.

For the CARC model, we have made three different payoffs available, denoted P_1, P_2 and P_3. Equation (1) is the definition for P_3. P_1 and P_2 are respectively expressed as

$$P\_1 = N + N \times \max\{R_f, R_c\} \quad (7)$$

$$P\_2 = N \times \max\{R_f, R_c\} \quad (8)$$

An enhanced Quasi-Monte Carlo method is employed to evaluation this feature of CARC. In this method, Sobol sequence in conjunction with Brownian Bridge path generation approach is applied. In this transaction, c_+ is 0.10, c_- is 0.12, R_f is 0.147, and the notional principal is USD 100. Only the term structure of at-the-money volatility is used in calculation, i.e., volatility skew is NOT applied.