Cross Currency Option Valuation

A cross currency option is a currency translated option of the type foreign equity option struck in domestic currency, which is a call or put on a foreign asset with a strike price set in domestic currency and payoff measured in domestic currency.

The spot underlying price in foreign currency is converted into an amount in domestic currency using the spot exchange rate. This amount is then adjusted by the current value of predicted future discrete dividends, measured in domestic currency.

Let S_t be the stock price measured in a foreign currency. Let X_t be the exchange rate, quoted in domestic currency per one unit of foreign currency. A non-quanto cross currency European vanilla call option has a payoff at the maturity *T*

$$\max(0.0, S_T X_T - K) \tag{1}$$

where *K* is the strike measured in domestic currency. The payoff is also measured in domestic currency. For an Asian call option, the payoff is

$$\max(0.0, \frac{1}{n}\sum_{i=1}^{n} S_{t_i} X_{t_i} - K) \tag{2}$$

where $t_i, i = 1, \cdots, n$ are the average dates and $t_n \le T$.

Since there are two sources of uncertainties involved in the option, one resulting from underlying price changes and the other resulting from changes in the exchange rate, this option is non-quanto. The holder of the option bears the risk caused by the fluctuation of the exchange rate between the underlying currency and the payoff currency.

The domestic risk-neutral processes for S_t and X_t are

$$dX_t = X_t[(r - r_f)dt + \vec{\sigma}_a d\vec{W}] \tag{3}$$

And

$$dS_t = S_t[(r_f - q - \rho\sigma_s\sigma_x)dt + \vec{\sigma}_b d\vec{W}] \tag{4}$$

where *r* is the domestic risk free interest rate, r_f is the foreign risk free interest rate, *q* is the dividend yield, σ_s is the volatility of the underlying stock, σ_x is the volatility of the exchange rate, and ρ is the correlation coefficient between the rate of return of the foreign underlying stock and the exchange rate. Also

$$\vec{\sigma}_a = (\sigma_x, 0) \tag{5}$$

$$\vec{\sigma}_b = (\rho\sigma_s, \sqrt{1-\rho^2}\sigma_s) \tag{6}$$

And

$$d\vec{W} = (dW_1, dW_2)^T. \tag{7}$$

Here $\vec{W}$ is two-dimensional standard Brownian motion under the risk-neutral measure *Q*.

Appling Ito's Lemma, we have

$$d(S_t X_t) = S_t dX_t + X_t dS_t + dS_t dX_t. \tag{8}$$

Substituting (3) to (7) into (8), we obtain

$$d(S_t X_t) = S_t X_t[(r-q)dt + (\vec{\sigma}_a + \vec{\sigma}_b)d\vec{W}]. \quad (9)$$

Thus, $S_t X_t$ is log-normal, with a drift of domestic risk free rate *r* minus dividend yield *q*, and a volatility of

$$\sigma_{sx} = \sqrt{\sigma_s^2 + 2\rho\sigma_s\sigma_x + \sigma_x^2}\ . \quad (10)$$

The values of vanilla European call/put options can be calculated by using the closed form Black-Scholes formula. For Asian options of European style, either Monte Carlo simulation or the Michael Curran's approximation can be employed for pricing. Instead of using σ_s, the composite volatility σ_{sx} must be used in these pricing formulae.

As to the discrete dividends, since they are a riskless component in the stock price dynamic, the spot stock price should be reduced by the present value of all the dividends during the life of the option. Let $t = 0$ be the current value date. Taking the predicted discrete dividends of the underlying stock into account, the translated stock price at time zero is given by

$$S_0^{'} = S_0 X_0 - \sum_{i=1}^{m} F_i d_i e^{-(r_i u_i)} \quad (11)$$

where *m* is the number of dividends paid during the option period, u_i is the dividend payment date, F_i is the forward exchange rate corresponding to the dividend payment date u_i, r_i is the domestic risk free interest rate (ref. https://finpricing.com/lib/IrCurveIntroduction.html) corresponding to u_i.

The aforementioned options are actually currency translated options of the type foreign equity option struck in domestic currency, which is a call or put on a foreign asset with a strike price set in domestic currency and payoff measured in domestic currency.