ЧИСЛЕННОЕ ИНТЕГРИРОВАНИЕ ОБЫКНОВЕННЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

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NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS

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Аннотация

Для многих прикладных задач получение точного решения дифференциальных уравнений практически невозможно. В этих случаях применяют методы приближенного решения дифференциальных уравнений. В статье рассматривается решение дифференциального уравнения различными численными методами.

Abstract

For many applied problems it is practically impossible to obtain the exact solution of differential equations. In these cases, methods of approximate solution of differential equations are used. This article considers the solution of a differential equation by various numerical methods.

Ключевые слова: математика, дифференциальное уравнение, метод Эйлера, метод Адамса, численные методы.

Keywords: mathematics, differential equation, Euler method, Adams method, numerical methods.

Methods for obtaining the exact solution of differential equations are possible only for a relatively small part of the equations encountered in practice.

Therefore, methods of approximate solution of differential equations are of great importance, which, depending on the form of representation of the solution, can be separated into two groups:

1) analytical methods, which give an approximate solution of a differential equation in the form of an analytical expression (application of the Taylor formula);

2) numerical methods giving an approximate solution in the form of a table. These methods are widely developed with great application of the computers and its capabilities (methods of Euler, Adams, etc.)[1], [3].

Nowadays, the amount of work involved in training to solve differential equations on a computers is not so large and does not exceed the amount of work involved in writing the solution. With the help of the computers it is possible to obtain a graph or its image on the screen. As a result, there is no urgent need to study theoretical ways of integrating differential equations [2].

Recently, a large number of different software products (MathCAD, MathLAB, etc.) with which, specifying only the input data, it is possible to solve many different tasks of differential equations, algebra and other sections of mathematics have appeared [5].

The use of such programs significantly reduces the time to solve important problems. Nevertheless, using these programs without analyzing the method by which the problem is solved cannot guarantee that the problem is solved correctly. Therefore, to fully understand how various kinds of differential equations and their systems are calculated, it is necessary to study and analyze numerical methods of solution [4].

One of the developed methods for approximate solution of differential equations is the method of decomposition of required solution in power series

$$y(x) = y_0 + xy'_0 + \frac{x^2}{2!}y''_0 + \frac{x^3}{3!}y'''_0 + \cdots \quad (1)$$

where $y_0, y'_0, y''_0, \ldots$ are determined from the initial condition of the problem.

For example, solve the differential equation $y' = y - x$ under initial conditions $x_0 = 0, y_0 = 1{,}5$.

To determine the values of $y_0, y'_0, y''_0, \ldots$ we use this equation. So $y'' = y' - 1, y''' = y'', y^{(IV)} = y''' = y'', \ldots, y^{(n)} = y^{(n-1)} = \cdots = y''$

When $x_0 = 0,\ y_0 = 1{,}5, y' = 1{,}5 - 0 = 1{,}5, y'' = 1{,}5 - 1 = 0{,}5,$

$y''' = 0{,}5, y^{(IV)} = 0{,}5, \ldots, y^{(n)} = 0{,}5.$

Substituting these values into formula (1), we obtain

$$y(x) = 1{,}5 + 1{,}5x + \frac{x^2}{2\cdot 2!} + \frac{x^3}{2\cdot 3!} + \frac{x^4}{2\cdot 4!} + \cdots$$

By setting values of $x = 0;\ 0{,}25;\ 0{,}5;\ 0{,}75;\ 1;\ 1{,}25;\ 1{,}5$, we find

$$y(0) = 1{,}5;\ y(0{,}25) = 1{,}8919;\ y(0{,}5) = 2{,}3138;\ y(0{,}75) = 2{,}8025;$$
$$y(1) = 3{,}3541;\ y(1{,}25) = 3{,}8165;\ y(1{,}5) = 4{,}6693.$$

Let us solve the same problem under the same initial conditions by numerical Euler method, dividing the interval [0; 1,5] into six parts with step length $h = 0{,}25$.

In the equation replace the derivative by the finite difference ratio

$$\frac{\Delta y}{\Delta x} = f(x; y)$$

Hence

$$y_k - y_{k-1} = f(x_{k-1}; y_{k-1})\Delta x$$

or

$$y_k = y_{k-1} + f(x_{k-1}; y_{k-1})\Delta x$$

When $x_0 = 0,\ y_0 = 1{,}5$;

if $x_1 = 0{,}25$, then

$y_1 = y_0 + (y_0 - x_0)h = 1{,}5 + (1{,}5 - 0)\cdot 0{,}25 = 1{,}875;$

if $x_2 = 0{,}5$, then

$y_2 = y_1 + (y_1 - x_1)h = 1{,}875 + (1{,}875 - 0{,}25)\cdot 0{,}25 = 2{,}2313;$

if $x_3 = 0{,}75$, then

$y_3 = y_2 + (y_2 - x_2)h = 2{,}2313 + (2{,}2313 - 0{,}5)\cdot 0{,}25 = 2{,}6641;$

if $x_4 = 1$, then

$y_4 = y_3 + (y_3 - x_3)h = 2{,}6641 + (2{,}6641 - 0{,}75)\cdot 0{,}25 = 3{,}1426;$

if $x_5 = 1{,}25$, then

$y_5 = y_4 + (y_4 - x_4)h = 3{,}1426 + (3{,}1426 - 1)\cdot 0{,}25 = 3{,}6782;$

if $x_6 = 1{,}5$, then

$y_6 = y_5 + (y_5 - x_5)h = 3{,}6782 + (3{,}6782 - 1{,}25)\cdot 0{,}25 = 4{,}2853.$

Now find the solution of the equation $y' = y - x$ with initial condition $x_0 = 0,\ y_0 = 1{,}5$ on the interval [0; 1,5] by Adams method with step $h = 0{,}25$.

Using the Adams method, find the first two values of the solution y_1 and y_2 by the below formulas

$$y_1 = y_0 + h\,y'_0 + \frac{h^2}{2!}y''_0 + \frac{h^3}{3!}y'''_0 + \cdots$$

$$y_2 = y_1 + 2h\,y'_0 + \frac{(2h)^2}{2!}y''_0 + \frac{(2h)^3}{3!}y'''_0 + \cdots$$

where

$$y'_0 = f(x_0;\ y_0) = (y - x)|_{x_0=0} = 1{,}5;$$

$$y''_0 = f'(x_0;\ y_0) = (y' - x)|_{x_0=0} = 0{,}5;$$
$$y'''_0 = f''(x_0;\ y_0) = (y'' - x)|_{x_0=0} = 0{,}5.$$

Next, determine the values of y_3 and y_4 by the formula

$$y_{k+1} = y_k + h\,y'_k + h\Delta y_{k-1} + \frac{5h}{12}\Delta^2 y_{k-2} + \cdots$$

So, for example, when $k = 2$

$$y_3 = y_2 + h(y_1 - x_1) + \frac{h^2}{2}(y_2 - y_1) + \frac{5h}{12}(y_2 - 2y_1 + y_0),$$
$$y_4 = y_3 + h(y_2 - x_2) + \frac{h^2}{2}(y_3 - y_2) + \frac{5h}{12}(y_3 - 2y_2 + y_1),$$
$$y_5 = y_4 + h(y_3 - x_3) + \frac{h^2}{2}(y_4 - y_3) + \frac{5h}{12}(y_4 - 2y_3 + y_2),$$
$$y_6 = y_5 + h(y_4 - x_4) + \frac{h^2}{2}(y_5 - y_4) + \frac{5h}{12}(y_5 - 2y_4 + y_3).$$

When $x_0 = 0,\ y_0 = 1{,}5$; when $x_1 = 0{,}25,\ y_1 = 1{,}8920$; when $x_2 = 0{,}5$, $y_2 = 2{,}3243$; when $x_3 = 0{,}75,\ y_3 = 2{,}8084$; when $x_4 = 1,\ y_4 = 3{,}3585$; when $x_5 = 1{,}25,\ y_5 = 3{,}9944$; when $x_6 = 1{,}5,\ y_6 = 4{,}7344$.

Now let us put the obtained results of solving the equation $y' = y - x$ by different methods into the table

	y_i			
x_i	according to Taylor	Euler's method	Adams method	analytical solution $y = x + 1 + \frac{1}{2}e^x$
$x_0 = 0$	1,5	1,5	1,5	1,5
$x_1 = 0{,}25$	1,8919	1,875	1,8920	1,892
$x_2 = 0{,}5$	2,3138	2,2313	2,3243	2,3243
$x_3 = 0{,}75$	2,8075	2,6641	2,8084	2,8782
$x_4 = 1$	3,3541	3,1426	3,3585	3,359
$x_5 = 1{,}25$	3,8165	3,6782	3,9944	3,995
$x_6 = 1{,}5$	4,6693	4,2853	4,7344	4,7404

The results most similar to the analytical solution were obtained when solving the equation by the Adams method.

Numerical methods allow us to solve equations of higher orders and systems of differential equations and obtain solutions close to analytical ones.

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