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Design and simulation of a mechatronic system with an adaptive digital controller

Methods for designing the new technology of continuous control systems the movement, which are built on the use of concepts reversed dynamics problems in conjunction with optimization on energy criteria, distribute on the mechatronic system. In this case analytical minimization of the local quadratic criterion is replaced by a numerical procedure which allows to obtain the finite-difference equation for the desired control that gives adaptive properties to mechatronic system. The process of adaptation in a mechatronic system with a digital controller has been studied and the conditions for its stability have been obtained. The main ratios for calculating the settings of an adaptive digital controller as part of a mechatronic DC electric drive system are given. The simulation method was used to compare the dynamic characteristics of two mechatronic systems: with adaptive and conventional (P and PI) digital controllers. The simulation results are presented in the form of transient responses obtained under the action of coordinate and parametric disturbances.

Keywords*: mechatronic system, the energy criterion, adaptation, local optimization, control law, digital controller, synthesis, analysis, simulation.*

Dedicated to **Peter Dmitrievich Krutko** – a wonderful person, an excellent engineer and a prominent scientist

Introduction

Mechatronics as "the science of computer control in technical systems" [1] develops successfully due to refinement of computer technology and algorithmic support of digital controllers, that play a central role in modern automation and control systems. As control processes in these systems are associated with the energy change, it seems natural to look for new approaches to the development of control algorithms for dynamic objects on the basis of energy criteria. In this context a series of works of P.D. Krutko should be noted. The main results of these works are stated in the form of "new technologies of analytical design of algorithmic assurance of traffic control systems" [2]. These technologies, based on the concept of dynamics inverse problems, combined with the local quadratic criteria optimization, having the physical meaning of mechanical energy, allow to design control laws analytically for continuous regulators based on the minimization of kinetic or acceleration energy. Control systems with these controllers have little sensitivity to parameter and coordinate perturbations.

Mechatronic system are based on computer technology, so instead of an analytical optimization method that is used to design an analog adaptive controller, it is advisable to use numerical optimization methods. This article covers solving this problem.

Problem definition

Suppose that an object is described by the differential equation

$$\ddot{n} + a_1\dot{n} + a_0 n = b_0 u;\ t = 0,\ n(0) = n_0,\ \dot{n}(0) = \dot{n}_0, \quad (1)$$

where n - controlled variable; u – control function; a_1, a_0 and b_0 – parameters.

The task is to find the discrete control law for the digital controller that processes the data feedback with sampling period T, and forming a control signal in the form of a step function

$$u(n,\dot{n},t) = u(kT) \text{ at } kT \le t < (k+1)T,\ k = 0,1,2,\ ..., \quad (2)$$

which will provide transfer of the control object from the initial state (1) to a given equilibrium steady state $n(t) = \bar{n} = const,\ \dot{n}(t) \to 0$. This requires that the process $n(t) \to \bar{n};\ \dot{n}(t) \to 0$, which characterizes the output of the object in a closed-loop control, was held in a small neighborhood of the reference process $n^*(t) \to \bar{n};\ \dot{n}^*(t) \to 0$, formed by the reference model, described by the differential equation

$$\ddot{n}^* + \alpha_1\dot{n}^* + \alpha_0 n^* = \alpha_0\bar{n},\ \ \alpha_0, \alpha_1 = \text{const} > 0 \quad (3)$$

with appropriate initial conditions. Parameters α_0 and α_1 should be chosen based on the desired dynamic properties of the designed system. The degree of approximation of processes in a controlled system and the reference model is convenient to estimate by the value of the quality function [2], which plays the role of local optimality criterion

$$G(u) = \frac{1}{2}[\ddot{n}^*(t) - \ddot{n}(t,u)]^2,\ t \ge 0, \quad (4)$$

where *G(u)* is the mass normalized value of the acceleration energy [3], calculated in the neighborhood of the trajectory of the reference model. The smaller is the value of *G(u)*, the more the processes in the system are closer to the processes in the standard model. The function of quality (3) in such statement is both a criterion of control and the target condition for adaptation. At the same time as a feasible target function argument stands the current value of the control function.

Design of the discrete control law

Problem posed above has no analytical solution, so we use a numerical procedure to minimize the criterion (4). The most common numerical method of optimization criteria of the form (4) is a simple gradient method which algorithm while keeping symbols adopted in [4], has the following view:

$$x^{i+1} = x^i - \alpha \frac{\partial F(x^i)}{\partial x},\ i = 0,1,2,...,\ \alpha = \text{const}. \tag{5}$$

The choice of a simple gradient method is due to the simplicity of its implementation on computing devices. Suppose that the algorithm (5) is realized on the basis of a microcontroller running with the discrete period *T*. In this case, the control signal at the output of the controller will be presented in the form of a step function (2), which preserves its value constant during the whole period *T*. Subject to this ratio (5) can be rewritten in the form of a finite-difference equation

$$u[(k+1)T] = u(kT) - \lambda \frac{\partial G(u)}{\partial u},\ k = 0,1,2,\ ...,\ \text{at } u = u(kT) \text{ and } \lambda = \text{const}, \tag{6}$$

defining an iterative procedure to calculate the optimal control u_{opt} by the criterion (4). The parameter λ in (6) defines the step size, which remains constant throughout the iterative procedure. It can be seen as a factor determining the stability and speed of convergence of the iterative process for calculating the optimal control u_{opt}.

Since the solution of equation (6) defines the desired equation *u(kT)* for k = 0,1,2, ..., then it will be considered as an equation of the controller. It should be noted that the successful solution of this equation depends on the possibility of calculating of the derivative $\frac{\partial G(u)}{\partial u}$ and the correct choice of the parameter λ.

To calculate the derivative $\frac{\partial G(u)}{\partial u}$ we will do the following. Considering (3), we rewrite the expression for the quality function (4) as follows:

$$G(u) = \frac{1}{2}[-\alpha_1 \dot{n}^* - \alpha_0 n^* + \alpha_0 \bar{n} - \ddot{n}(u)]^2, \tag{7}$$

where $u = u(kT)$ for $kT \le t < (k+1)T,\ k = 0,1,2,...$.

Differentiating both sides of (7) considering (1) with respect to *u*, we find the expression for the derivative

$$\frac{\partial G(u)}{\partial u} = \{\alpha_0[\bar{n} - n^*] - \alpha_1 \dot{n}^* - \ddot{n}(u)\}(-b_0), \tag{8}$$

where $u(t) = u(kT)$ for $kT \le t < (k+1)T,\ k = 0,1,2,...$.

From (8) it follows that to calculate the required derivative information is needed about the current state of the reference model and the second derivative of a controlled coordinate system when $kT \le t < (k+1)T,\ k = 0,1,2,...$, as well as parameters of the reference model α_1, α_2 and the only one parameter b_0 referring to the control object. In such a way, the calculation of the derivative (8) is straightforward at each step of the iterative process.

Substituting (8) into the right side of (6) and using (3), we obtain the recurrence relation in the form of a finite-difference equation

$$u[(k+1)T]=u(kT)+\lambda b_0[\alpha_0[\bar{n}-n^*]-\alpha_0\dot{n}^*-\ddot{n}],\ k=0,1,2,\ \dots\ .$$

Replacing in the resulting equation the reference model variables n^* and $\dot{n}^*$ by the relevant variables n and $\dot{n}$, characterizing the control object output, we can rewrite it as follows:

$$u[(k+1)T]=u(kT)+\lambda b_0[\alpha_0[\bar{n}-n]-\alpha_1\dot{n}-\ddot{n}],\ k=0,1,2,\ \dots\ . \quad (9)$$

The adopted change of variables corresponds to the feedback looping of the system on the output coordinate and its derivative. The presence in (9) of the second derivative $\ddot{n}$ with a minus sign indicates the need to introduce another negative feedback loop to accelerate the controlled coordinate. Thus, the step control signal is calculated based on the information about the current state of continuous control object, which is characterized by the exit coordinate, its speed and acceleration.

Then we introduce the notation that determines the information function

$$\theta(t)=\alpha_0[\bar{n}-n(t)]-\alpha_1\dot{n}(t)-\ddot{n}(t). \quad (10)$$

In this case, equation (9) is transformed as follows:

$$u[(k+1)T]=u(kT)+K\theta(t),\ \ k=0,1,2,\ \dots\ .$$

Considering that the information from the measuring devices goes to the microcontroller discretely through time and in digital form, expression (10) can be transformed to

$$\theta(kT)=\alpha_0[\bar{n}-n(kT)]-\alpha_1\dot{n}(kT)-\ddot{n}(kT), kT\le t<(k+1)T,\ k=0,1,2,\ \dots\ , \quad (11)$$

where $\bar{n}=const$ plays the role of master control.

Finally, a discrete adaptive control law of the continuous object (1) has the form

$$u[(k+1)T]=u(kT)+K\theta(kT),\ \ k=0,1,2,\ \dots\ , \quad (12)$$

where $K=\lambda b_0$ is a gain constant of the adaptive digital controller.

The initial conditions for the equation (12) are defined for k = 0 by the initial value of the information function (11) and an initial value of the control function $u(0)=u_0$.

Analysis of equation (1) shows that a change in any of the parameters of the object a_0, a_1, b_0 (or all together), as well as the influence of external perturbations leads to a deviation of the current acceleration $\ddot{n}$ from the acceleration $\ddot{n}^*$ defined by the reference model. In this case, in accordance with the local optimization criteria (4) with the recursive procedure (12) a digital controller provides control adaptation for new conditions of the system. In this case the adaptation is a result of the current numerical minimization performed in real time. Since the target functionality of this optimization is chosen both as a criterion for object management, and as a target adaptation condition, so the control law (12) can be called direct adaptive control law.

The block diagram of a digital controller that implements the designed discrete adaptive control law in the form of (12), is shown in Figure 1.

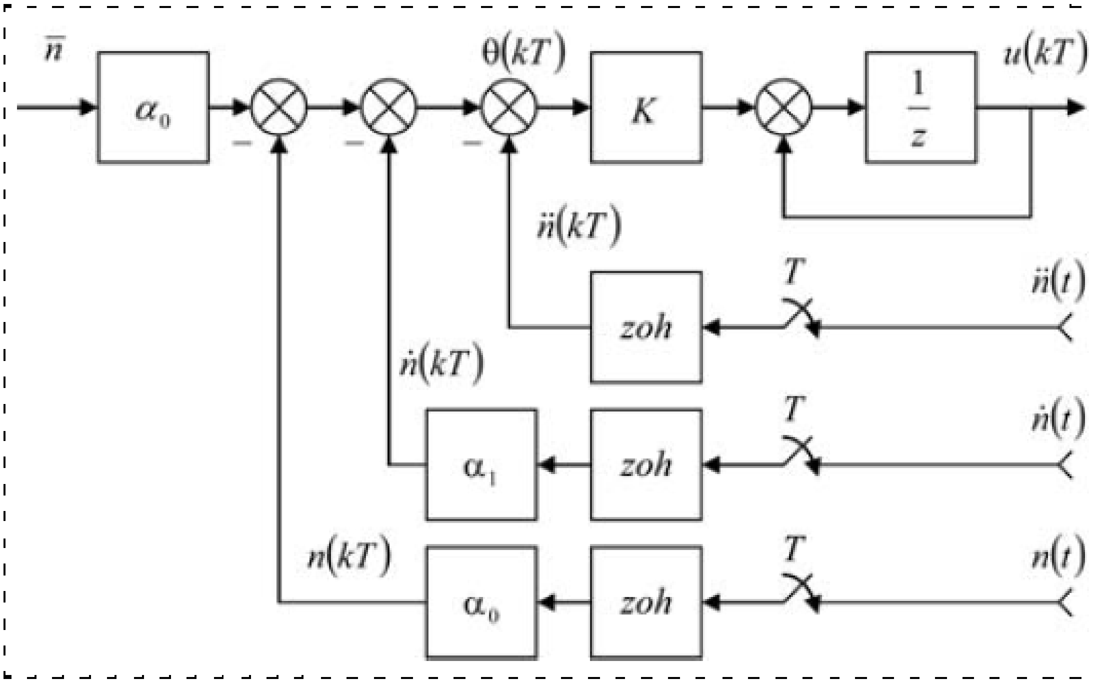

Figure 1. The block diagram of a digital adaptive controller

Here blocks *zoh*, combined with the keys, closing with a period *T*, perform the function of zero-order holder, and the unit $\frac{1}{z}$ performs a control signal delay on the time *T*. It should be noted that the structure of the designed controller is not defined in advance and is found according to the equation for the control law once it is received.

The study of the adaptation process

We research the parameter *K* effect, included in the equation (12), on an iterative process of the current control approximation *u(kT)* to its optimum value u_{opt} at runtime. For this purpose, we introduce the function

$$\tilde{u}[kT] = u(kT) - u_{opt}, \tag{13}$$

characterizing the deviation of the control function current value from its desired value, which is optimal by the criterion (4). Seeking optimal value u_{opt} we find from the equation obtained by equating (8) to zero in view of (1) and performed replacement n^* and $\dot{n}^*$ on n и $\dot{n}$ respectively. We have

$$\frac{\partial G(u)}{\partial u} = \{\alpha_0[\bar{n} - n] - \alpha_1\dot{n} + a_1\dot{n} + a_0 n - b_0 u\}(-b_0) = 0,$$

which implies

$$u_{opt} = \frac{1}{b_0}\phi(t), \tag{14}$$

where

$$\phi(t) = \alpha_0[\bar{n} - n] - \alpha_1\dot{n} + a_1\dot{n} + a_0 n. \tag{15}$$

Then the information function (10) can be represented, considering (15), as

$$\theta(t) = \phi(t) - b_0 u(kT)$$

and we substitute the obtained result in (11). We have

$$u[(k+1)T] = u(kT) + K\phi(t) - Kb_0 u(kT),\, k = 0,1,2,\, \ldots\, .$$

We rewrite the last equation, taking into account (15), in the form

$$u[(k+1)T] = u(kT) + Kb_0 u_{opt} - Kb_0 u(kT),\, k = 0,1,2,\, \ldots\, ,$$

and by subtracting from both sides of the last result u_{opt}, considering (13) we get

$$\tilde{u}[(k+1)T] = (1 - Kb_0)\tilde{u}(kT),\ k = 0,1,2,\, \ldots\, . \tag{16}$$

The homogeneous finite-difference equation (16) characterizes the dynamics of the adaptation process $u(kT) \rightarrow u_{opt}$ of the control function current value to its new optimum which value is determined in accordance with the criterion (4) by new conditions of the operation caused by parametric or coordinate perturbations that affected the change of a controlled coordinate acceleration $\ddot{n}$ in accordance with equation (1). Under $\tilde{u}(kT) = 0$ the control function reaches its optimal value $u(kT) = u_{opt}$. From the equation (12) it follows that the adaptation process quality is determined by the choice of parameter $K = \lambda b_0$ – an adaptive digital controller gain.

We'll find the parameter K in terms of stability of solution of a homogeneous finite-difference equation (16). The expression for the desired solution is determined by the discrete analogue of the Cauchy-Lagrange equations, which in the case of the homogeneous equation has the form of

$$\tilde{u}(NT) = (1 - Kb_0)^N \tilde{u}_0, \text{where } \tilde{u}(0) = \tilde{u}_0,\ \ N = 0,1,2,\, \ldots\, . \tag{17}$$

A necessary and sufficient condition for asymptotic stability of the solution of equation (16) follows from (17) and has the form of

$$\left|1 - Kb_0\right| < 1 \text{ or } 0 < K < 2/b_0. \tag{18}$$

Thus, the choice of K from (18) provides the asymptotic stability of the control function adaptation process, which is due to the finite-difference equation solution (12).

The adaptive controller parameters synthesis

Calculation of the parameters a_0, a_1 of digital controller that implements the control law (12) and providing the desired quality of step response can be performed using the reference model by various methods discussed in [2] and [5]. It is necessary to specify the time t_p (setting time) and the type of step response of designed mechatronic systems. The choice of the gain K should be carried out under the conditions of the adaptation process asymptotic stability according to the formula (18). Sampling period T should be chosen considering the time required for the formation of the information function digital values and calculations on this basis of the control function numerical values. You can use the Smith's inequality [6] $T \le \tau_{\min}/15$, where $\tau_{\min}$ – minimal time constant. As an example, we will carry out the synthesis of a digital controller for DC motor speed in a mechatronic system considered in [7], which is a dual-circuit high-speed DC system with controllers in each circuit. We assume that there are sensors for control object to measure its output coordinate – the angular velocity of the rotor shaft, together with its derivative. To measure the second derivative in the expression for the information function (11), we apply a real differentiator with a time constant T_d. In this case, the adaptive digital controller, shown in Figure 1, forms, together with the control object, a three circuits high-speed direct current system.

To solve this problem, first of all, we write the equation for the DC motor with independent excitation in the form of (1), which while retaining the notation used in [7], takes the form of

$$\ddot{\omega} + a_1\dot{\omega} + a_0\omega = b_0 u_a, \tag{19}$$

where ω – angular velocity; u_a – armature circuit voltage;

$$a_0 = \frac{k_1 k_2 k_E}{T_a};\; a_1 = \frac{1}{T_a};\; b_0 = \frac{k_1 k_2}{T_a}.$$

Parameters of the engine with a power of 0.45 kW correspond to those in [7]:

$$k_1 = 1.72\ 1/ohm,\; k_2 = 0.72\ 1/\mathrm{Ams}^2, k_E = 0.34\, Vs,\;\; T_a = 0.043\, s. \tag{20}$$

In this case, the model (19) parameters will have the following meanings:

$$a_0 = 9.73;\; a_1 = 23.25;\; b_0 = 28.63. \tag{21}$$

Let us choose a reference model in the form of (3) and set the step response setting time of the designed mechatronic system t_p = 0,04 seconds with a value of the overshoot $\sigma \le 5\%$. In this case, if we define a time constant τ_p approximately characterizing the dynamics of the designed mechatronic system with a second order, and the damping rate ζ, the required parameters of the adaptive controller (which are also the reference model parameters) can be calculated using formulas

$$\alpha_0 = \frac{1}{\tau_p^2},\ \alpha_1 = 2\zeta\frac{1}{\tau_p},\ \text{where}\ \tau_\text{p} \approx t_p/\gamma, \tag{22}$$

γ – a certain number in the interval from 3 up to 7.

Choice of $\zeta = 0.707$ provides a process of reference model step response with overshoot $\sigma \le 5\%$, while $\zeta \ge 1$ the overshoot is absent.

Thus, for the synthesis of a digital controller, which imparts adaptive properties to a system with a continuous object (1) it is sufficient to define two measures of quality - the time and form of step response (with or without overshoot). At the same time calculations use only one parameter characterizing the control object - transfer coefficient b_0, which is determined after reducing the object model to the form (1).

For the technical implementation of the adaptive controller it is important to ensure the information function primary calculation before calculating the control function. The mechatronic control system setting is put into effect by choosing only one adjusting parameter - the gain *K*, whose value must satisfy the inequality (18).

Simulation

Figure 2 shows the DC motor speed control mechatronic system simulation block diagram with parameters (20). At the same time controller parameters, calculated by the formulas (22), obtained the following values: $\alpha_0 = 30625$; $\alpha_1 = 247$ at $\xi = 0.707$.

Selected during modeling adaptive controller gain $K = 2.4 \cdot 10^{-4}$ satisfies (18), which in view of (21) takes the form 0 <*K* <0.07.

The differentiation circuit time constant T_D is selected from the relation $T_D = \tau_p / 5$ and the discrete period is assumed to be *T* = 100 microseconds. The model also includes a pulse-width converter (PWM), shown in [7], and a non-linear element, limiting the maximum voltage in the armature circuit within the limits +110 ... – 110 V. At modeling it is provided a connection at *t*= 0.06 seconds of a perturbing action – a load torque.

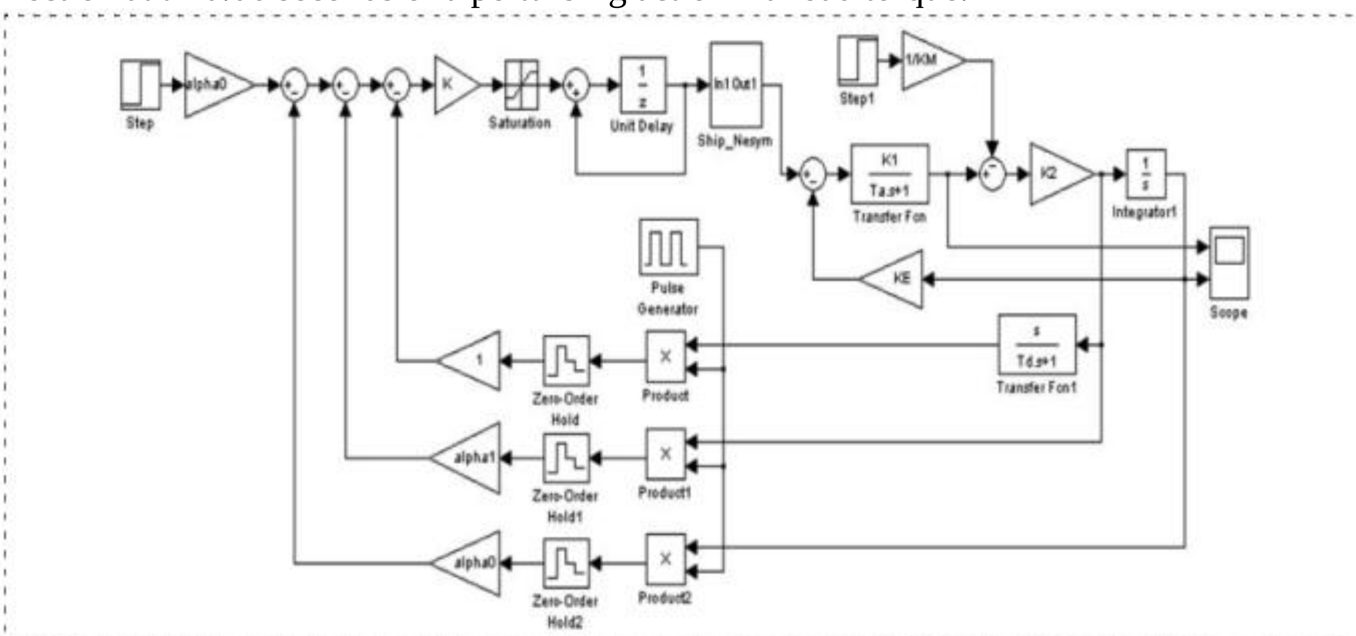

Figure 2. Simulation scheme of the DC electric drive with a digital adaptive controller

The simulation results are illustrated by graphs of the step response. Figure 3 compares the dynamic characteristics of two high-speed digital DC systems, designed for the same work conditions with the same type of DC motor with PWM: adaptive and a two-circuits with two conventional regulators of P-and PI-type. Here, the graphs show the electrical processes in the armature circuit and mechanical processes associated with the controlled engines rotors movement. Figure 4 shows the step response of the two compared systems at a doubling of the time constant of the DC motor armature circuit T_a .

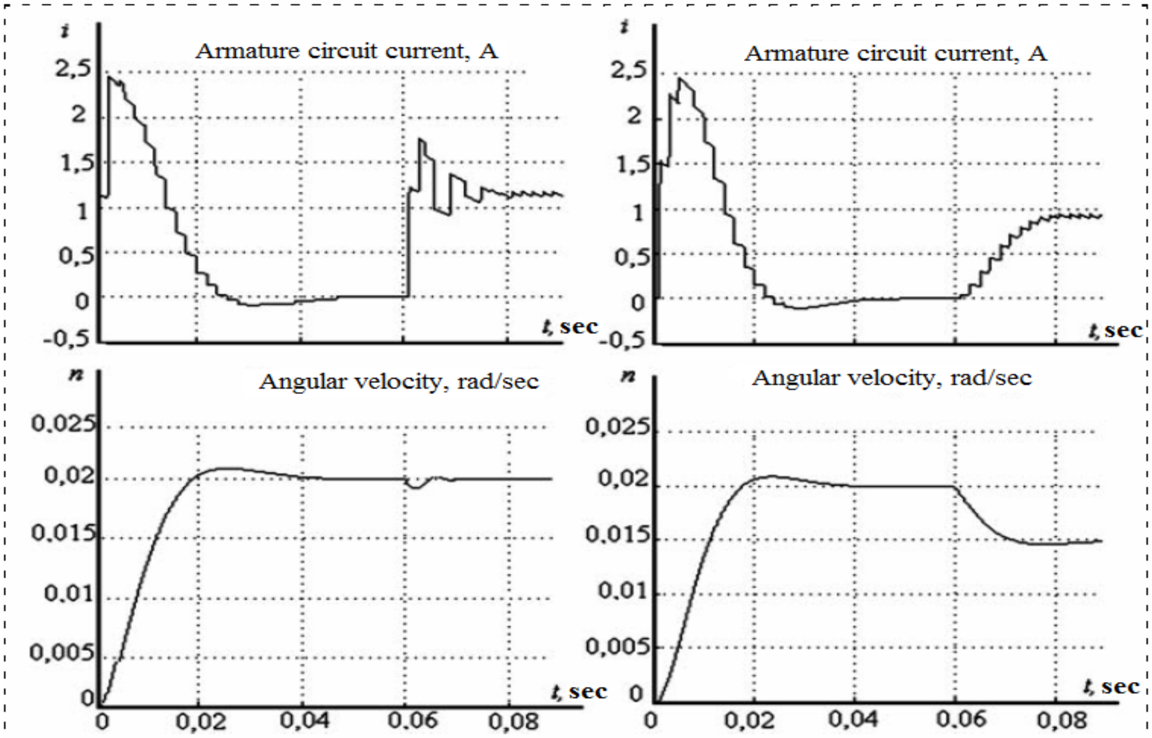

Figure 3. Graphs of step responses in the adaptive system (left) and the two-circuits high-speed DC system (right)

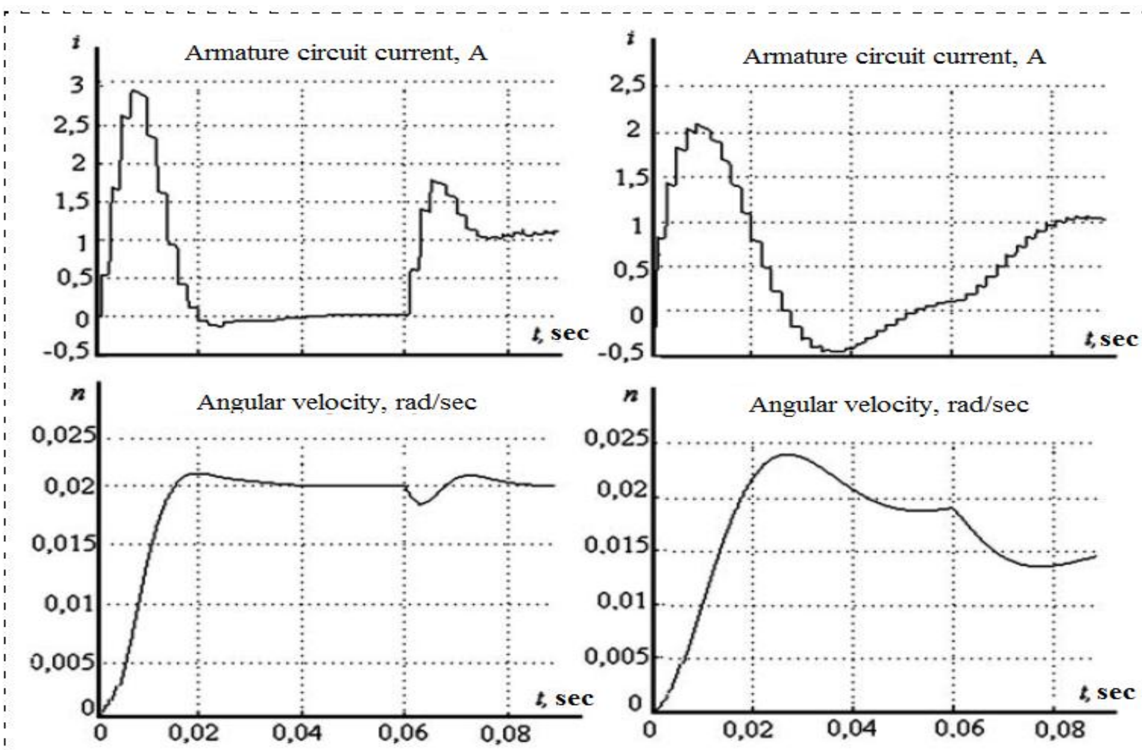

Figure 4. Graphs of step responses in the adaptive system (left) and the two-circuits high-speed DC system (right) at a doubling of the motor time constant T_a

As it follows from the graphs, the mechatronic system with adaptive controller has no static error on the control and disturbing effect and we can see robust properties at parametric and coordinate perturbations.

Conclusion

Development of designing methods for movement control systems, based on the application of the concepts of dynamics inverse problems and local optimization, formulated and detailed by P.D. Krutko for continuous systems, in the direction of their application for the mechatronic systems design allows to create a fundamentally new type of digital control devices using a common methodology. Simple in structure and in setting up such devices are able to adapt to the dynamic objects of different robotic systems, giving them an extremely important property of robustness.

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