**Consequences of X-Time Independence Within a Quantum Wavelength**

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We have argued in previous notes (1) that x and t are independent within a wavelength. This we will argue follows from special relativity applied to uniform motion i.e. -Et+px. In particular, v=x/t on average, not dx/dt, a mathematical abstraction which breaks down within the wavelength, because one cannot follow the particle in time.

We argue that the notion of independent x and t within a wavelength implies a probability distribution of x i.e F(p,x) = the probability to find the particle with momentum p at x. From special relativity we suggest that E is associated with t and so an E(x) suggests also a t(x). (In fact, for KE(x), there is an associated time at each x, but for KE(x)+V(x)=E all x are the same and there is no need for a distinguishing variable time.)

What about the spatial distribution F(p,x)? For a free particle F(p,x) is a function of x, but one which must map into a P(x)=constant. (For a bound state there is a spatial distribution and so the idea of time should exist in that a particle spends "more time" in a high probability region. This is linked to the idea of a p(rms) or velocity(rms)). Even for a bound quantum state we argue that for x and t to be independent, E must be constant at each x. One may argue that a constant E with a p(x) and an introduced V(x) is already present in special relativity i.e. (E-V(x))(E-V(x)) = pp + momo (c=1) which it is, but we argue that x-t independence within a wavelength also follows from special relativity so this seems to be consistent.

In earlier notes we suggested that E constant at each x is an example of equilibrium being linked with a minimum amount of information. Here we show that x-t leads to this minimum amount of information in describing E. (One may note that given a spatial density, there is quite an amount of information present there, but it is linked with the -id/dx operator which is associated with momentum and KE(x) which has time information.)

A second related point which follows from x-t independence is the following. Consider an ensemble of classical bounds states. At an instant of time and a given x, the probability to find the particle moving in the forward and backward directions is the same. If x and t are independent there is no instant of time and F(p,x) and F(-p,x) are not the same. In fact, F(p,x) must show how the particle moves in a forward direction and F(-p,x), motion in a negative direction. As a result, one has constant E at each x in a quantum bound state (i.e. locality), but F(p,x) and F(-p,x) being different suggests that there is nonlocality associated with momentum as it is linked with d/dx and hence x and a wavelength (which is nonlocal i.e. does not exist at a point).

Thus conservation of momentum i.e. an average momentum of zero in a bound state may only be established by integrating over space - it does not hold at a point on average like in a classical system where P(p,x) = P(-p,x). There are in a sense two average momenta in a quantum bound state. The first is linked with F(p,x) not equalling F(-p,x), but E being constant at each x. This is a nonlocal average momentum and includes both p and -p with no time present. We argued above that spatial density varies in x and so one may assign a time to the system. This time is associated with a prms(x) which follows form KE(x) = prms(x)prms(x)/2m used in KE(x)+V(x)= En. Thus there are two "velocities" or momenta. One is a prms (which is classical and associated with time and spatial density) and the other which is nonlocal and linked to x-t being independent meaning that E must be constant at each x.

**X-Time Independence Following from Special Relativity**

The notion of energy E, p (momentum = Ev), x and t exists in special relativity. Consider a rest mass mo at x=0 at time t=to. A frame moving with constant speed -v would see: v=x'/t' E(v,mo) and p=Ev. Thus v=x'/t' is an average speed. One may take d/dt of this to obtain dx/dt, but this is a mathematical abstraction. There is a jump (or finite discontinuity) moving from mo at x=0, t=to to v=x'/t' i.e. x',t' are seen in one frame and x=0, t=to in the other. These ideas lead to the Lorentz transformation with the Lorentz invariant: -Et+px and -EE+pp = momo.

We have argued in previous notes that -Et+px may be thought of in terms of x and t being independent with v=x/t. This is consistent with x'/t' = v with no derivative dx/ddt. Then special length units proportional to 1/p and special time units proportional to 1/E exist i.e. quantum wavelengths and frequencies for a particle moving with a constant speed. -EE+pp=momo (c=1) yields the relationship between energy and momentum for constant speed.

Consider next the case of a V(x) potential and p(x). Then:

$$-(E-V(x))\,(E-V(x)) + p(x)p(x) = mmo \quad (c=1) \quad ((1))$$

Are x and t still independent within a wavelength proportional to 1/p as in the constant motion case? We argue they are, but this means there are two sets of length scales in the picture which is confusing. First each constant p is linked with a length of hbar/p, but p(x) is linked with a point x. We argue that p(x)p(x) is an rms value i.e based on <pp>. If one has an average because x and t are independent, there should be an associated probability distribution F(p,x) for each constant p because each p represents its own wavelength. Different p's must somehow combine to create a <pp> which satisfies ((1)). We next consider the implications of x-t independence within a wavelength on F(p,x).

**Implications of x-t Independence on F(p,x)**

The Lorentz invariant -Et+px for uniform motion associates E with t and p with x. We consider first a free particle. If t and p are independent, one may expect an F(p,x) which does not involve time. This F(p,x) must then distinguish between p and -p (without using time) i.e must be skewed. At the same time it must map into each x having the same probability because constant motion does not distinguish between x points. As argued in (2), this may be done by considering a two dimensional F(p,x) (i.e. exp(ipx)). It differs for p and -p, but has a modulus of 1.

In the case of a free particle, both energy and spatial density must be flat. Otherwise one could introduce the notion of time by stating that the particle spends more time in a particular region than another. If we associate -id/dx (translation generator) with p (which involves motion through space) then exp(ipx) maps to 1 for all x and <pp>/2m = -d/dx d/dx exp(ipx) / exp(ipx) = pp. Thus both spatial density and energy density are flat (in a sense) for constant motion.

What happens in the case of a bound state for a length comparable with the wavelengths of some momenta involved? X and t must be independent within wavelengths, hence within the bound region. If energy depended on x, one could introduce the notion of time into the spatial picture. Note: we focus on energy linked to time because of -Et+px. Thus energy should be constant at each x. This is consistent with ((1)) from special relativity. The notion of frequency and wavelength follow from the relativistic -Et+px with x and t independent, so it is not surprising that ((1)) with no energy source or sink also means constant E at each x.

The independence of t and x, however, means that F(p,x) and F(-p,x) cannot be the same and both are part of a bound state. In fact: W(x)=Sum over p a(p)F(p,x) where a(p) are weights leads to a spatial density of W(x)W(x). From the arguments above, this suggests time is in the picture. At first this may seem like a contradiction. E must be constant at each x because there is no time in the spatial picture, but W(x)W(x) varies with x because there is time.

To resolve this issue, one may note that there is no time in the spatial picture for constant E if one considers <p> = -idW/dx / W. This means that on average positive and negative momenta probabilities do not cancel as they do classically where there is time. This average has x dependence, but it is purely imaginary and so not associated with a real time in the system i.e p and -p appear together so they are not distinguished in time. This is fine because if the length of the system is of the order of a wavelength (or a few) and there is uncertainty in position within a wavelength, it is possible that a p near the turning point has already become a -p (i.e. bounced back).Thus momentum only becomes zero by integrating:

Integral -i W d/dx W dx = Integral spatial density (-idW/dx/W) = 0 ((2))

In other words momentum conservation is nonlocal and linked to wavelength.

What does time associated with W(x)W(x) = spatial density mean? If energy is constant at each x, then:

KE(x) + V(x) = En → -1/2m d/dx dW/dx / W + V(x) = En ((3))

KE(x) = prms(x)prms(x/) 2m ((4))

prms(x) is a function of x and so one may introduce the variable time, but prms(x) is a mathematical average. It describes a classical particle which exists at a point x at t. A quantum object with constant p, however, has a wavelength hbar/p and x and t are independent within this length. On average, however, x/t=v as in the special relativistic case.

**Conclusion**

In conclusion, we argue that both the idea of wavelength and frequency and the independence of x and t follow from special relativity i.e. the Lorentz invariant: -Et+px. We note a nonsmooth jump from x=0, t=to to x’,t’ such that x’/t’=v (as seen from a frame moving with -v constant). Thus v=x’/t’ is an average, but there is an independent spatial distribution (and time one). Taking d/dt to write dx’/dt’ = v is a mathematical abstraction. Thus we argue that t and x are independent for constant motion with one being linked to energy and the other to p. As a result we suggest motion in x should be described by a probability distribution F(p,x) which makes no use of time. We argue that keeping x and t independent leads to two ideas.

First for uniform motion, energy at x must be constant, otherwise one could introduce a time variable and say the particle spends more time in the region with more energy. The same argument holds for spatial density for a free particle. Thus F(p,x) must yield a constant energy at each x and map to a constant for each x for density reasons. exp(ipx) with p→-id/dx satisfies these requirements.

In the case of a bound state, one would like to impose similar arguments. Starting with E being associated with t and an independence of x and t, E should be constant at each x. We note that the length of the bound system is of the order of hbar/p wavelength for characteristic p values suggesting x-t independence. F(p,x) and F(-p,x) cannot be the same as they are classically because it is time which distinguishes the direction of motion in the classical world. This means spatial density is a function of x suggesting time present. How is this possible if x and t are independent? Constant energy implies KE(x) + V(x) = En and KE(x) = prms(x)prms(x)/2m. prms(x), it is associated with time, but it is also associated with a mathematical average -1/2m d/dx dW/dx / W. Thus it is like a classical quantity and can be associated with time. Average momentum (not prms(x)) is also an average -idW/dx / W, but it is imaginary. It integrates to 0 over space showing no overall motion in space associated with a bound state i.e. there is En and Integral WW (-idW/dx / W) dx = 0.

Thus we argue that the notion of time is intimately linked with distributions in space.

References


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