Figured Magic Squares of Orders 6, 10, 12, 14 and 16 Using Bordered Magic Rectangles: A Systematic Procedure

Inder J. Taneja[1]

Abstract

Recently, author constructed even order magic squares from orders 6 to 20 with different styles and models, for examples the order 20 is with 1616 magic squares, order 18 with 810 magic squares, etc. These can be seen at [31, 32, 33, 33, 34, 35, 36, 37]. The aim is to proceed further orders of magic squares. In this work there are few examples of magic squares given only in figures of orders 6, 8, 10, 12, 14 and 16. A systematic procedure to construct these magic squares is given. It is based on the magic squares and bordered magic rectangles (BMR) of orders 4, 6, 8 etc forming external borders. Then the internal borders are filled with previous known magic squares. For the orders multiples of 4, we can always write magic squares with equal sums blocks of magic squares of order 4. This procedure is very helpful for the orders of type $\boldsymbol{2p}$*, where* $\boldsymbol{p}$ *is a prime number, for examples, 14, 22, 26, 34, 38, etc. For the orders like 18, 30, etc.,we can make good external blocks with order 4, and for orders like 16, 20, 28, 32, etc. we can make good external borders of order 6, and so on. The explanations of constructions are given for the orders 14 and 16..*

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Contents

1 Introduction

The magic sums of order n of consecutive numbers from 1 to n^2 is given by

$$S_{n\times n} := \frac{n\times(1+n^2)}{2}, n\geq 3. \tag{1}$$

Recently, the author [31, 32, 33, 34, 35, 36, 37] constructed magic squares of even orders from 6 to 20 using **bordered magic rectangles**. This construction is based on two aspects:

(i) Using **magic rectangles** or **bordered magic rectangles**.

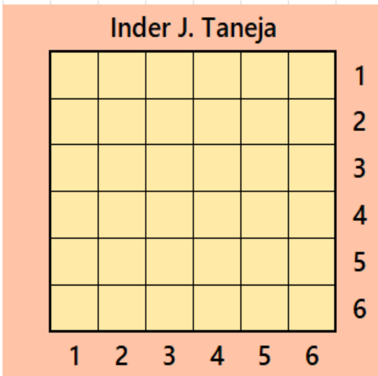
(ii) Using algebraic formula like $(\mathbf{a}+\mathbf{b})^2, \mathbf{a}\neq\mathbf{b}$.

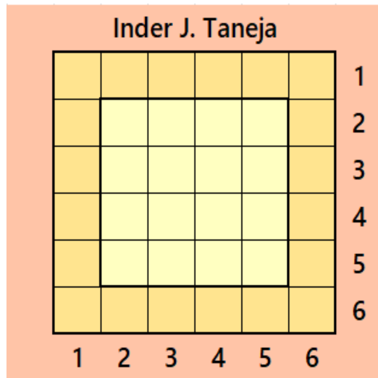
For the above magic squares no construction procedure is explained. The aim is to proceed further orders of magic squares. In this work, a systematic procedure to construct these magic squares is given. It is based on the magic squares and bordered magic rectangles (BMR) of orders 4, 6, 8 etc forming external borders. Then the internal borders are filled with previous known magic squares. For the orders multiples of 4, we can always write magic squares with equal sums blocks of magic squares of order 4. This procedure is very helpful for the orders of type **2p**, where **p** is a prime number, for examples, 14, 22, 26, 34, 38, etc. For the orders like 18, 30, etc.,we can make good external blocks with order 4, and for orders like 16, 20, 28, 32, etc. we can make good external borders of order 6, and so on. There is no explainations for the orders 6, 8, 10 and 12. The real construction starts from the order 14.

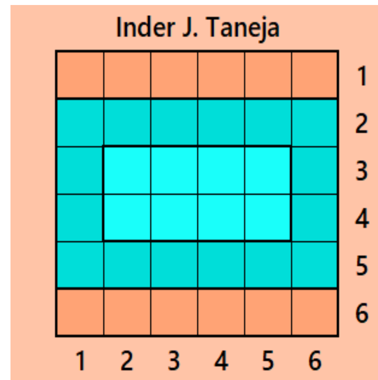
The whole the work is done manually, without use of any programming language, except for the constructions of small blocks of **bordered magic rectangles**. This construction is based on the software due to H. While. Later, these BRM's are readopted according to distribution of each magic square. The distribution of magic squares or bordered magic rectangles is based on **half-sequential** numbers. By **half-sequential** numbers we understand that the total numbers in each case are divided in two equal parts. First part is one sequence and second part is another sequence. Due to **half-sequential** numbers, it is not possible to construct all orders **bordered magic rectangles**. In Appendix 8, there are tables showing the existence of these **bordered magic rectangles** for **half-sequential**. For simplicity, we shall write **BMR** as **bordered magic rectangle**.

2 Magic Squares of Order 6

Below are three magic squares of order 6. These are normal, bordered and based on **bordered magic rectangle**. See below:



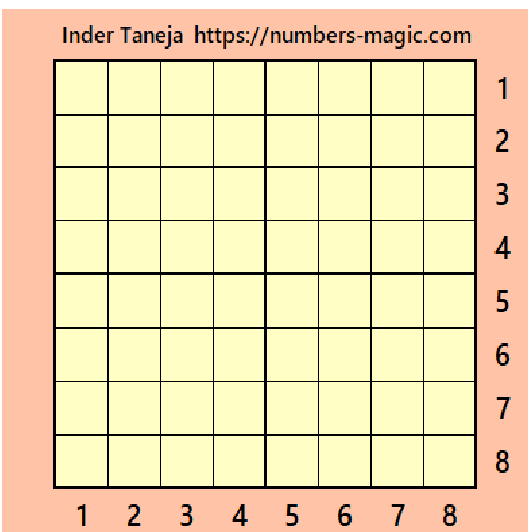


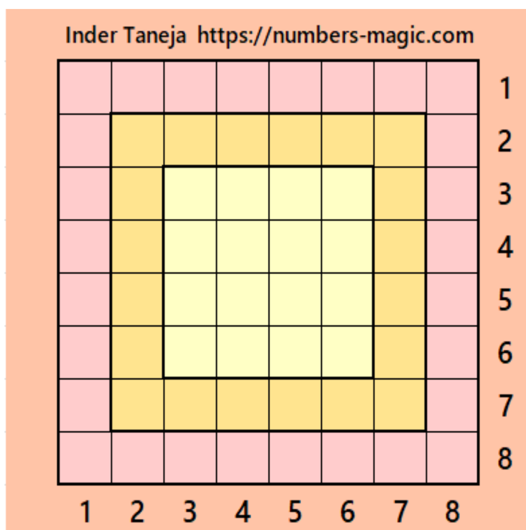


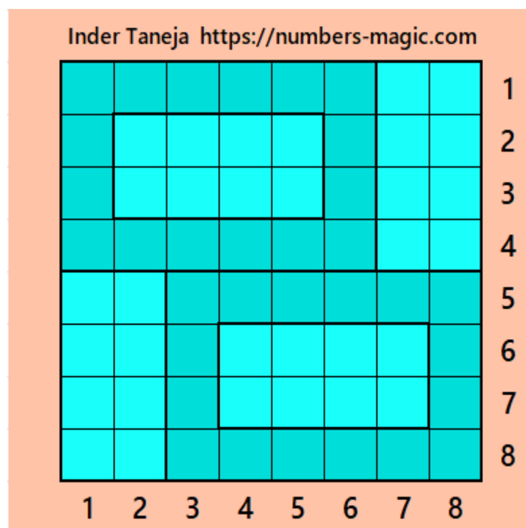
The first two are well known in the literature and the third is new. It is appearing for the first time.

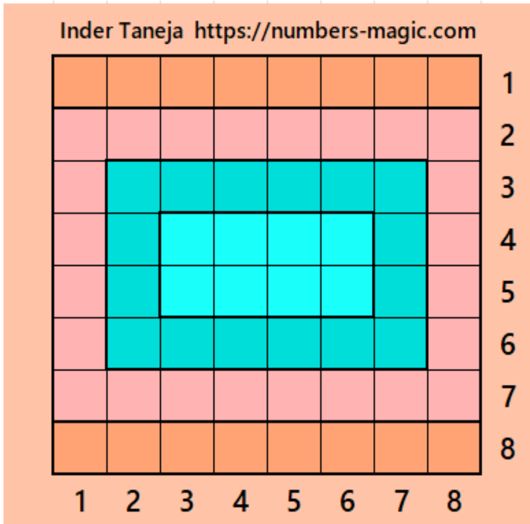
3 Magic Squares of Order 8

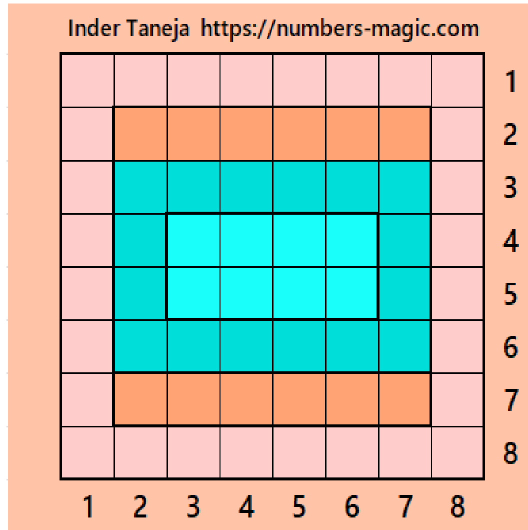
In this section we have written 6 magic squares of order 8. Some of them are by using **bordered magic rectangles**. These includes of type $(a+b)^2, a \neq b$.





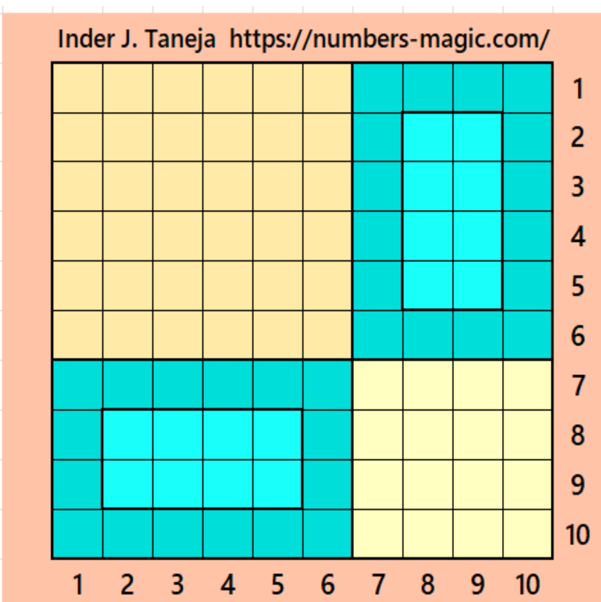


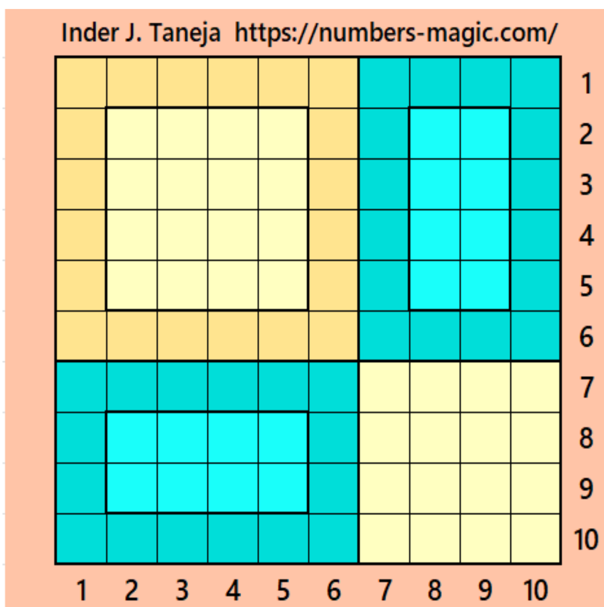


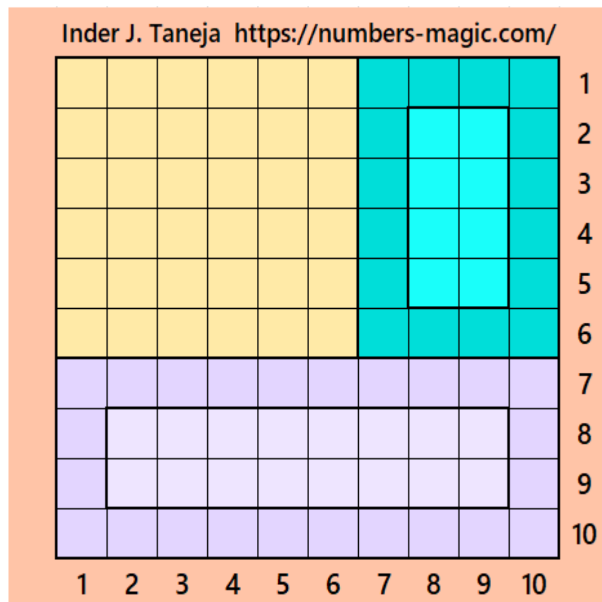


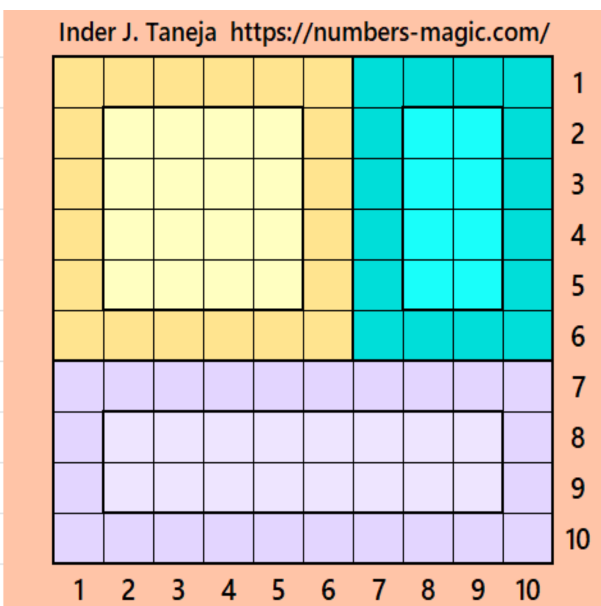
4 Magic Squares of Order 10

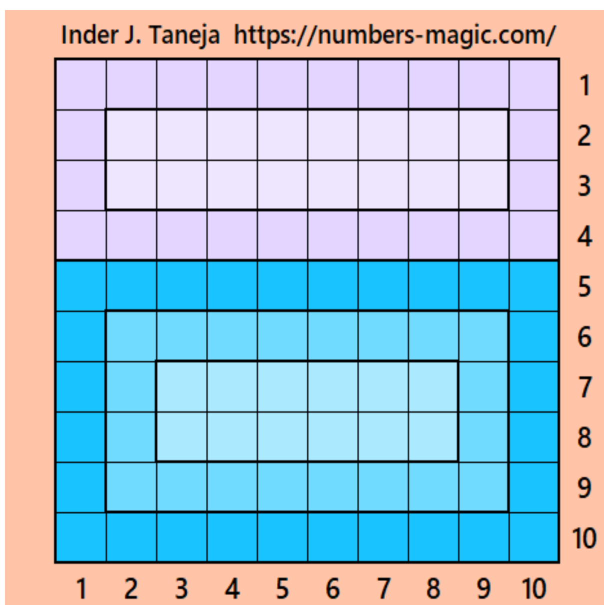
In this section we have written 16 magic squares of order 10. Some of them are by using **bordered magic rectangles**. These includes of type $(a+b)^2, a \neq b$ and bordered with magic squares of order 8.

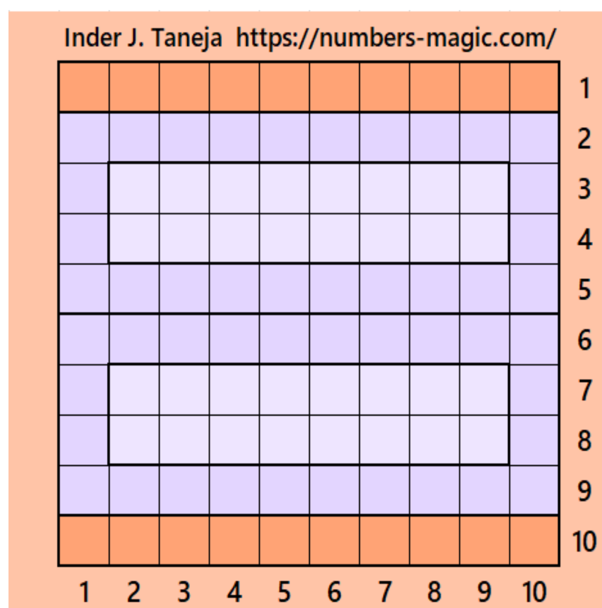


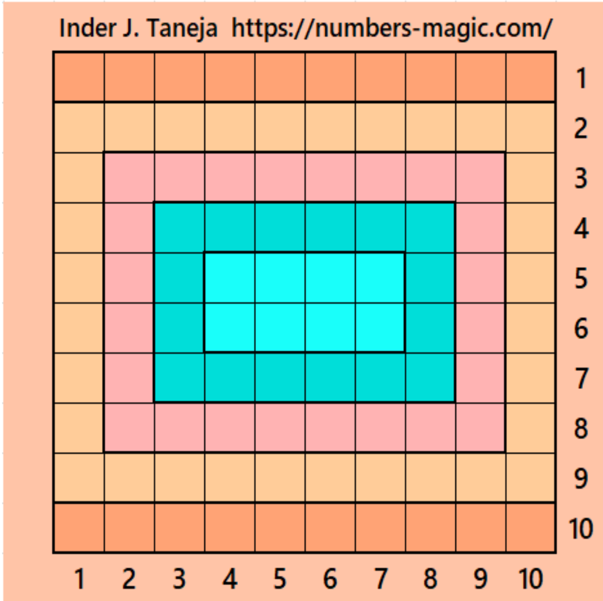


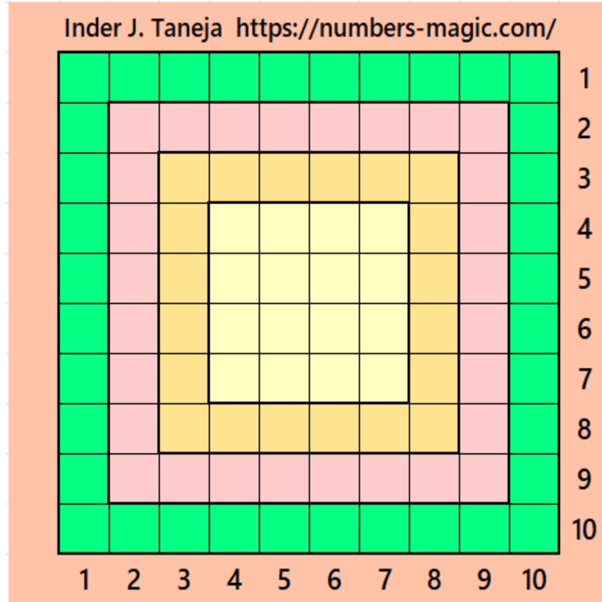


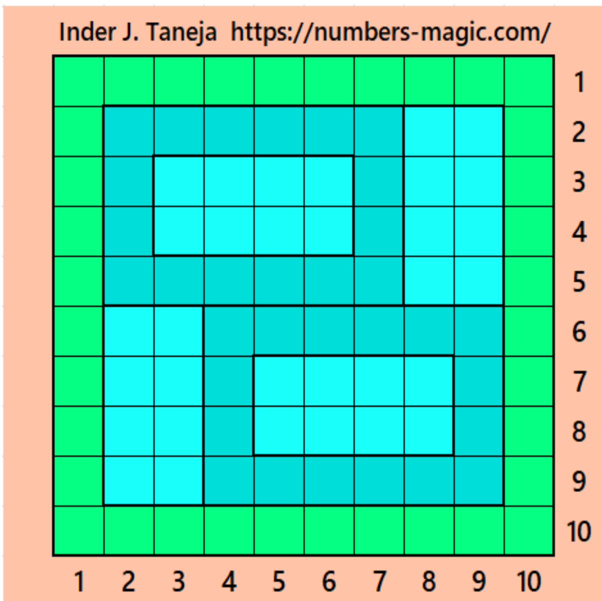


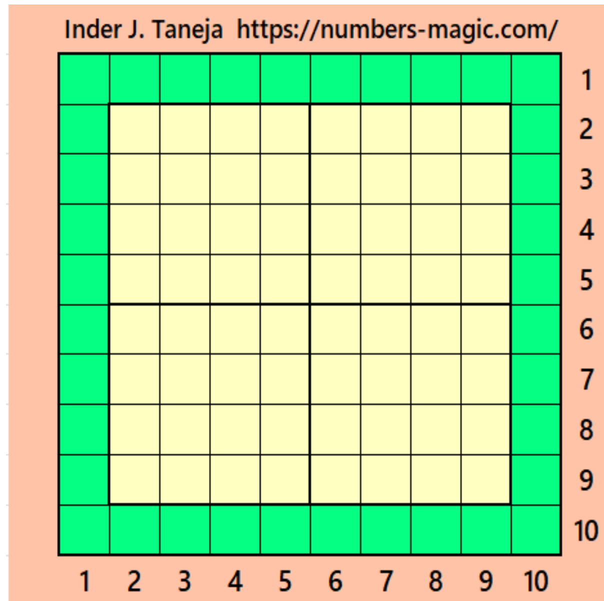








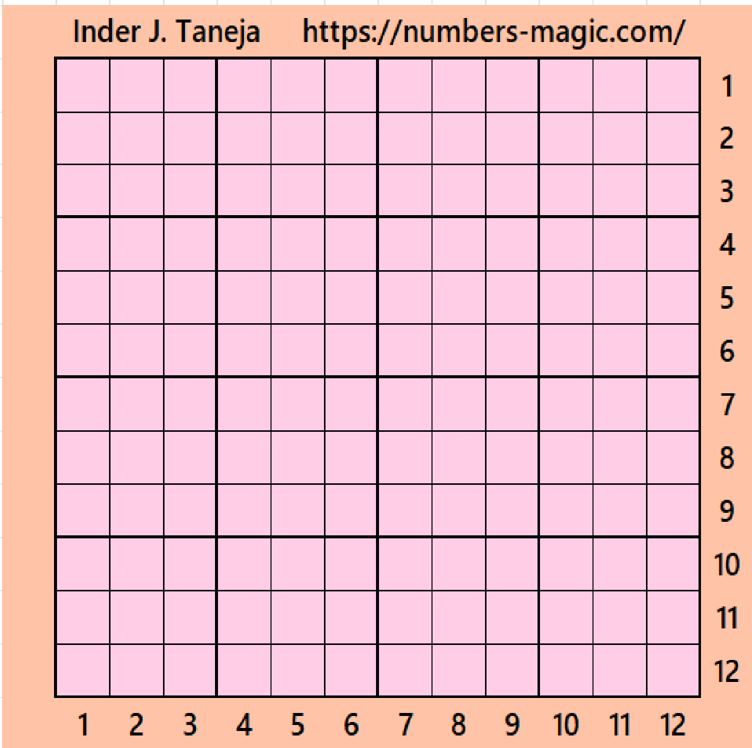




We have written here only ten, but in another work [31], the author wrote 16 magic squares of order 10

5 Magic Squares of Order 12

In this section we have written 45 magic squares of order 12. These are of three types based on **bordered magic rectangles**. These includes of type $(a+b)^2, a \neq b$ and bordered with magic squares of order 10. The first two magic squares are pandiagonal and the others are normal magic squares.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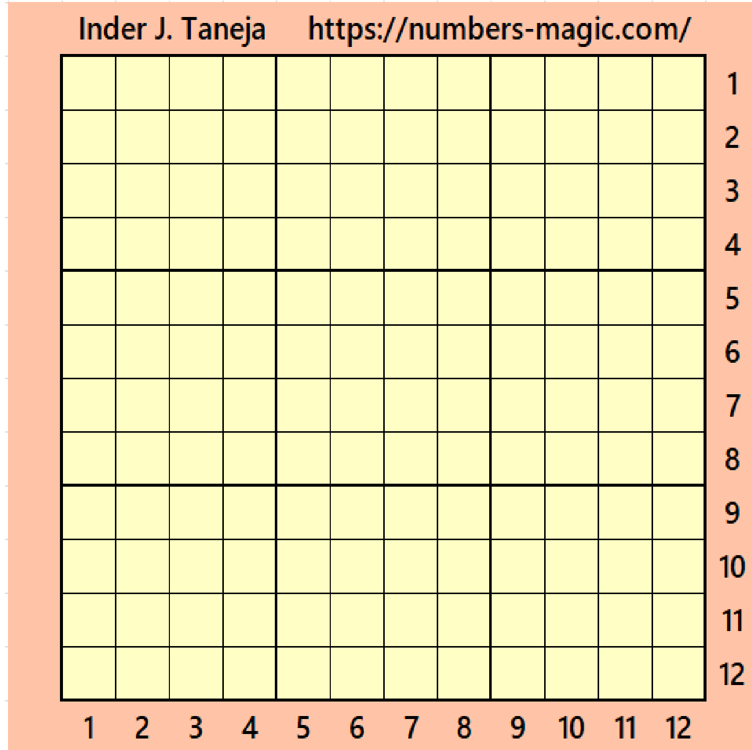
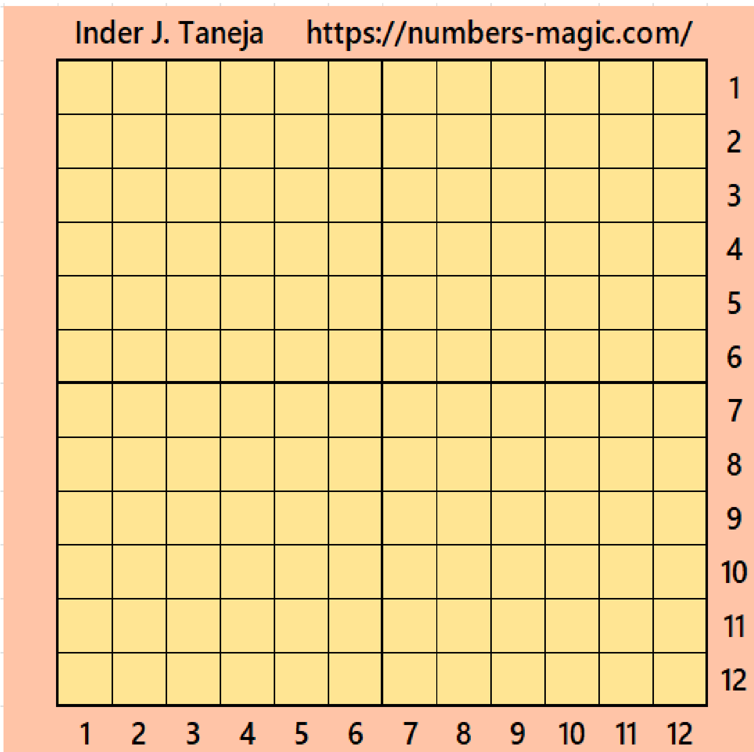

Inder J. Taneja https://numbers-magic.com/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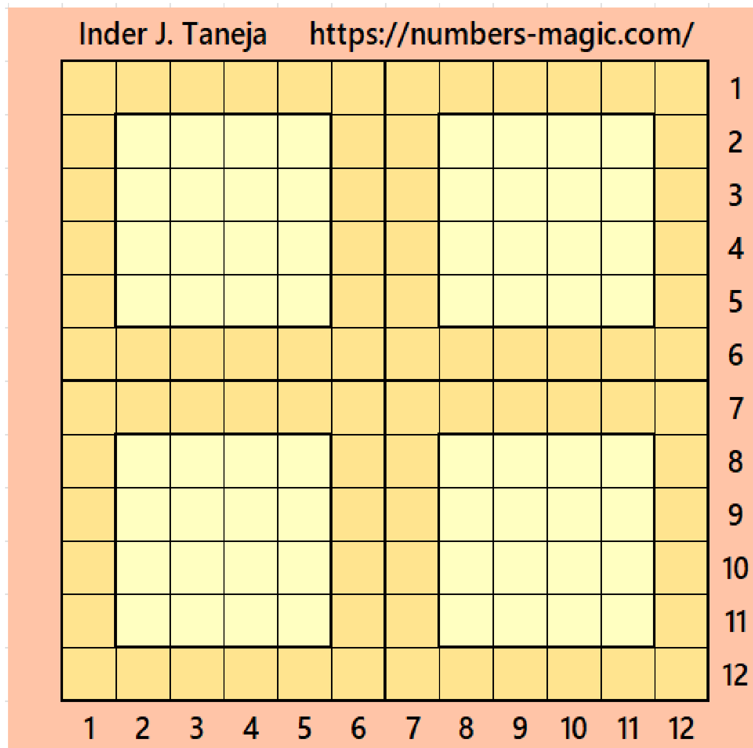
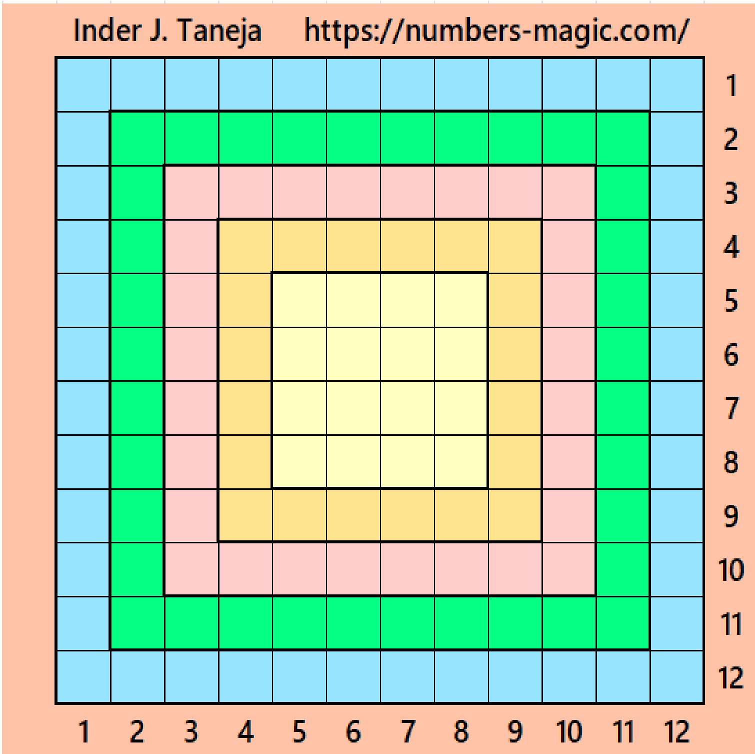
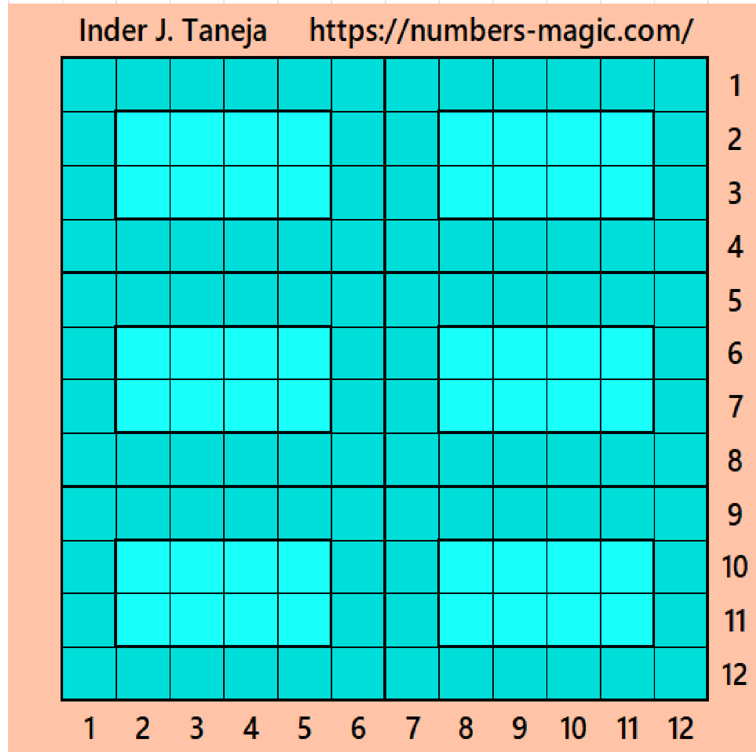

Inder J. Taneja https://numbers-magic.com/


Inder J. Taneja https://numbers-magic.com/
1 2 3 4 5 6 7 8 9 10 11 12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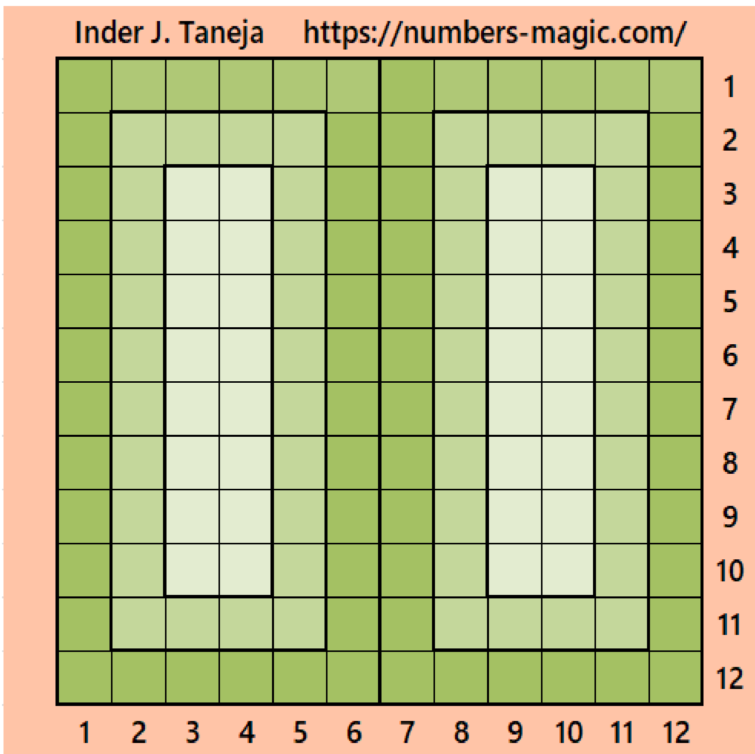
1 2 3 4 5 6 7 8 9 10 11 12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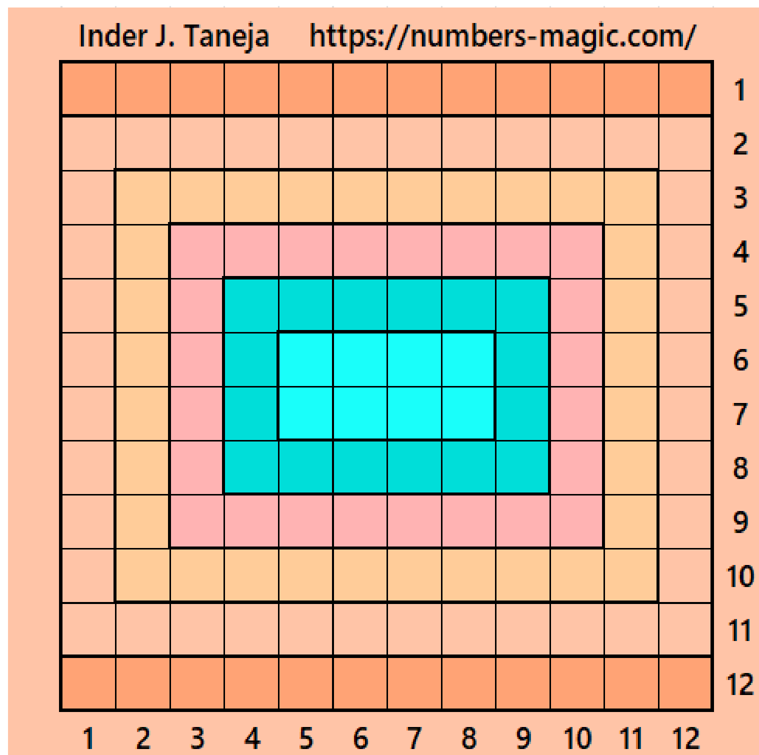
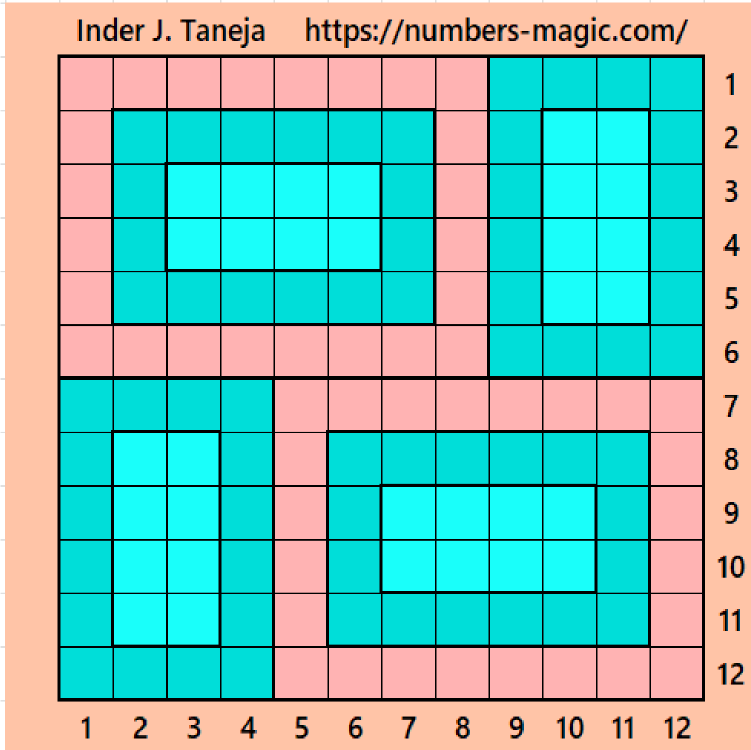
1 2 3 4 5 6 7 8 9 10 11 12


Inder J. Taneja https://numbers-magic.com/
1 2 3 4 5 6 7 8 9 10 11 12



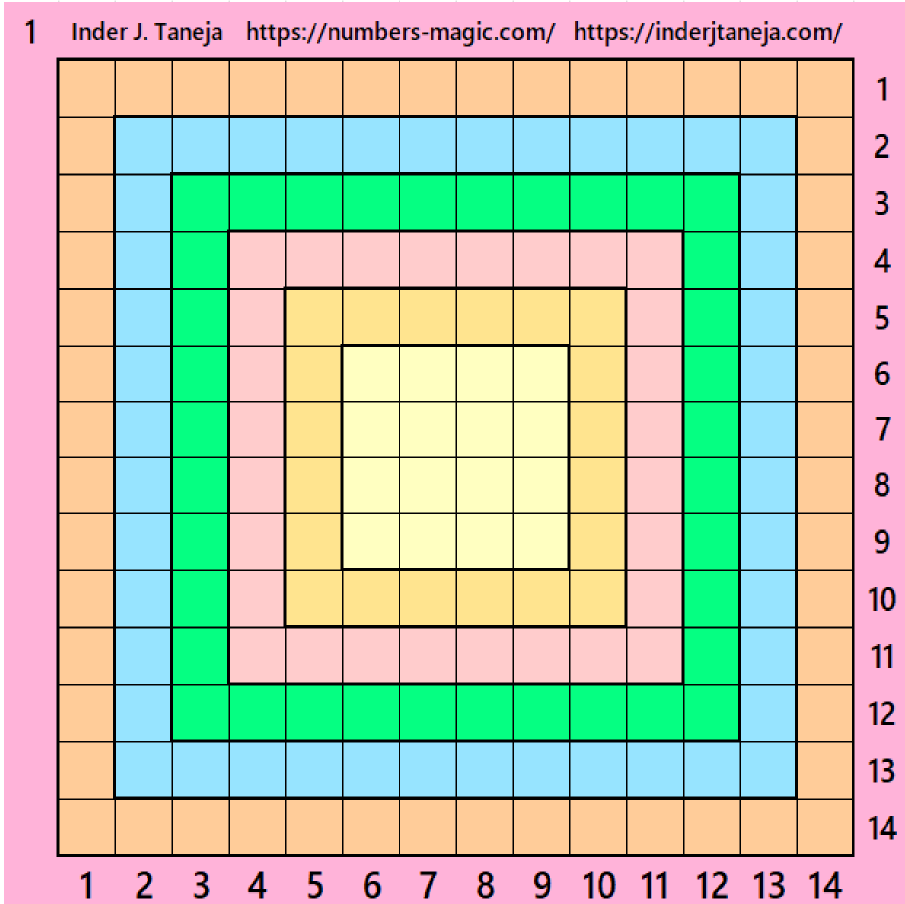
We have written here only nine, but in another work [31], the author wrote 45 magic squares of order 12

6 Magic Squares of Order 14

This section brings magic squares of order 14 only with figures. In some case, the idea of construction is also explained. Below are only few examples, the complete list with numbers can be seen in author's work [32].

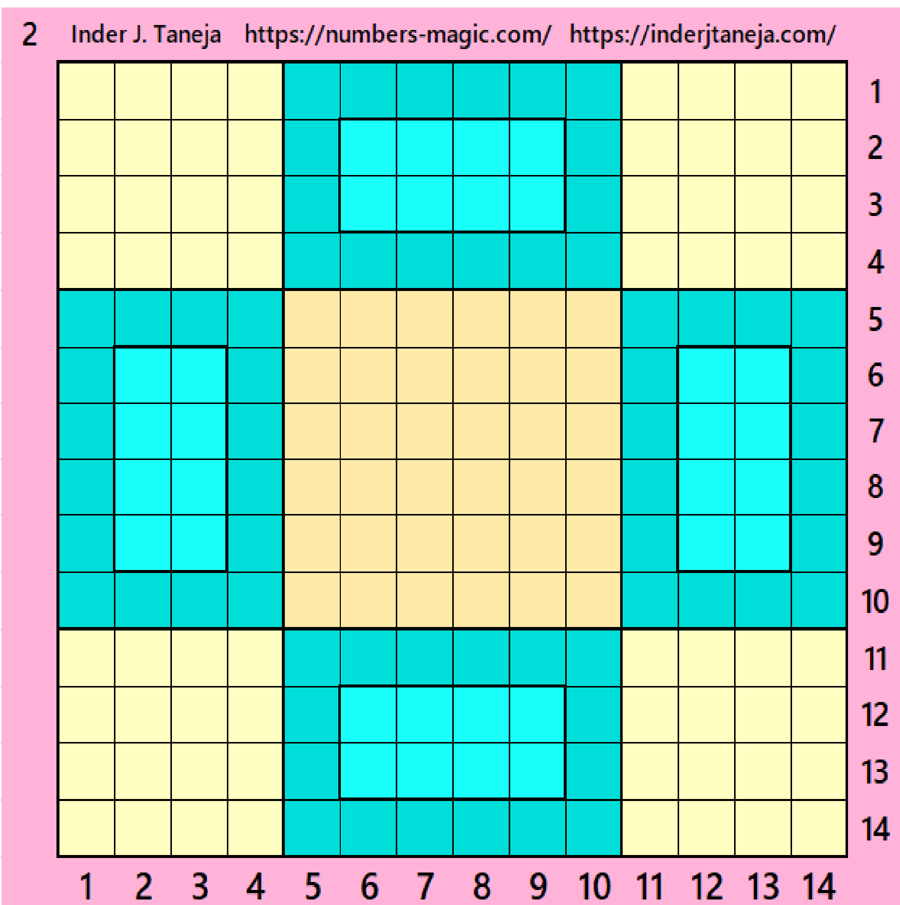
6.1 Bordered Magic Square of Order 14

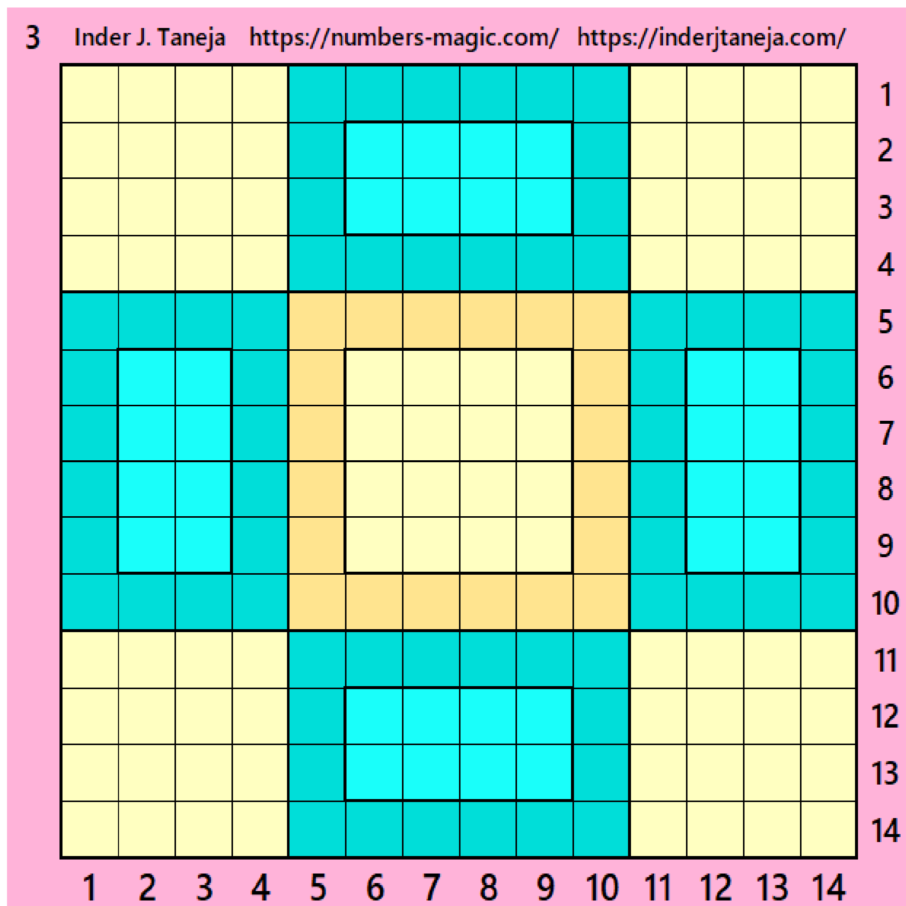
Below is a bordered magic square of order 14. It is well known in literature.

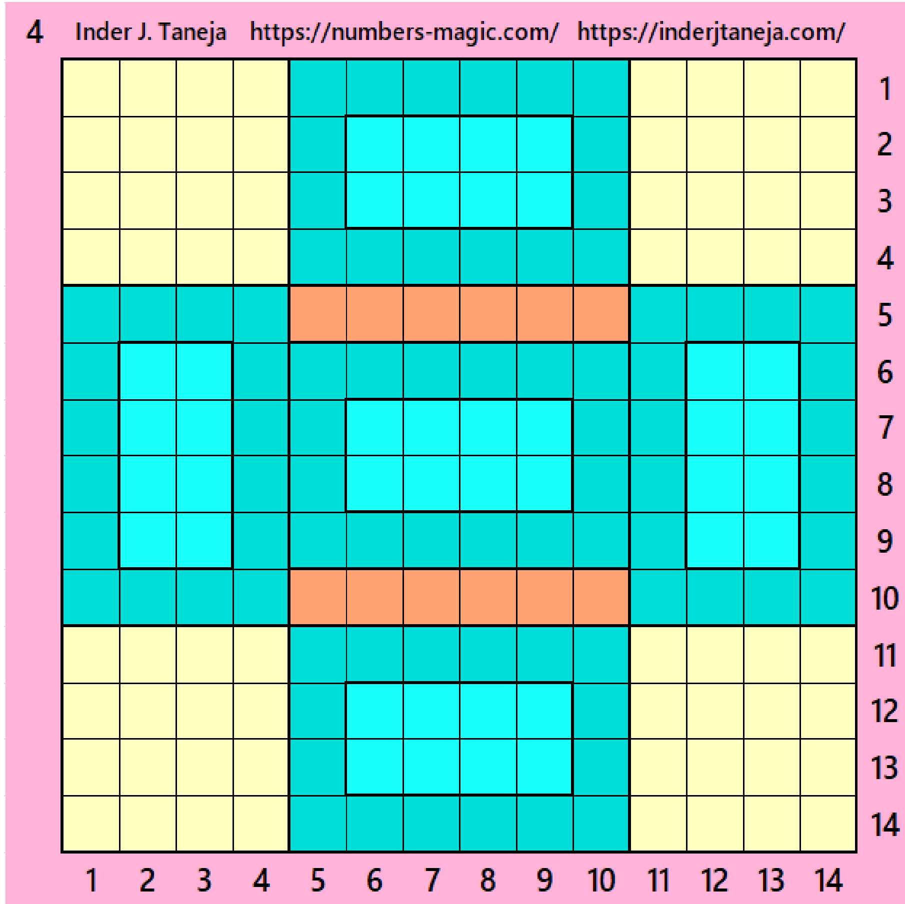


6.2 Cornered Magic Squares of Order 4

From now onwards, we shall apply the procedure of constructions. Let's consider an external border of where there are 4 magic squares of order 4 at the corners. In middle, let's put a bordered magic rectangles of order 4×6 or 6×4. In between we are left with a magic square of order 6. See the three figures below:

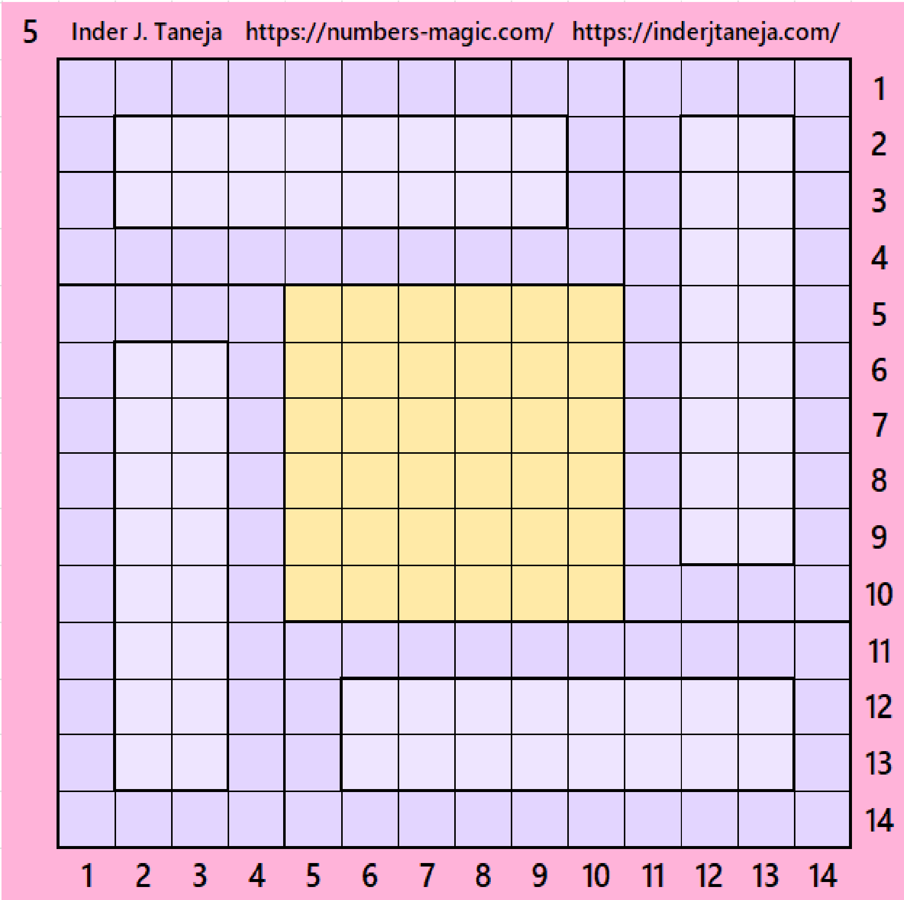


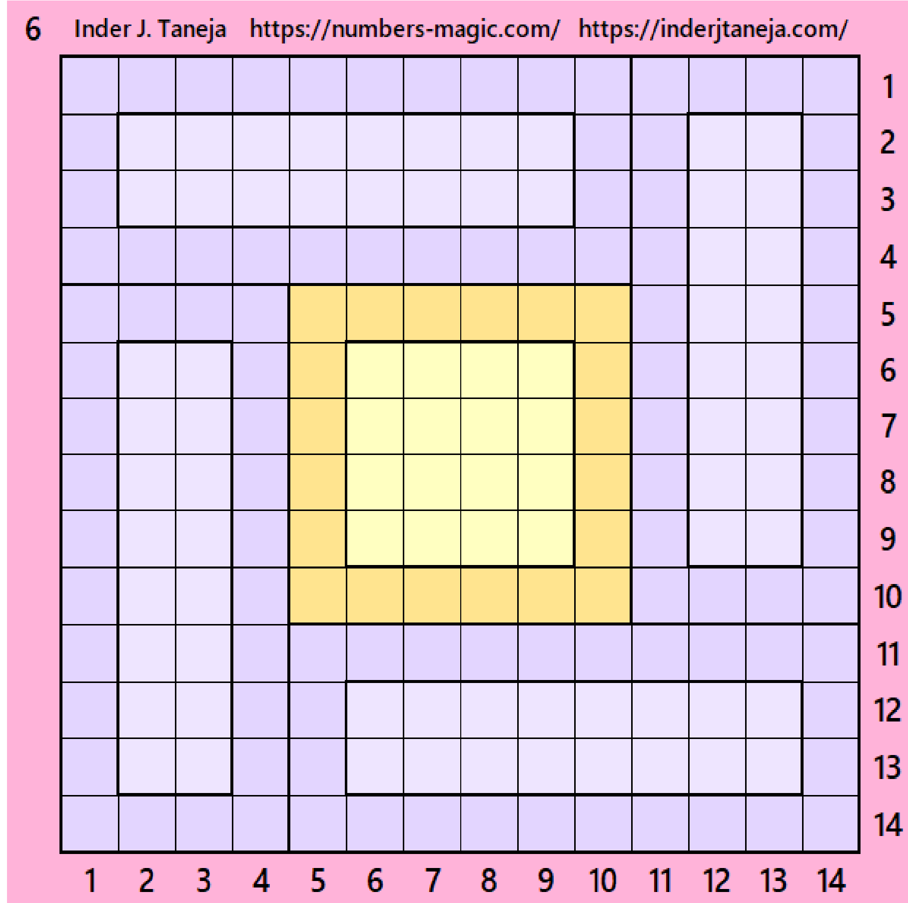


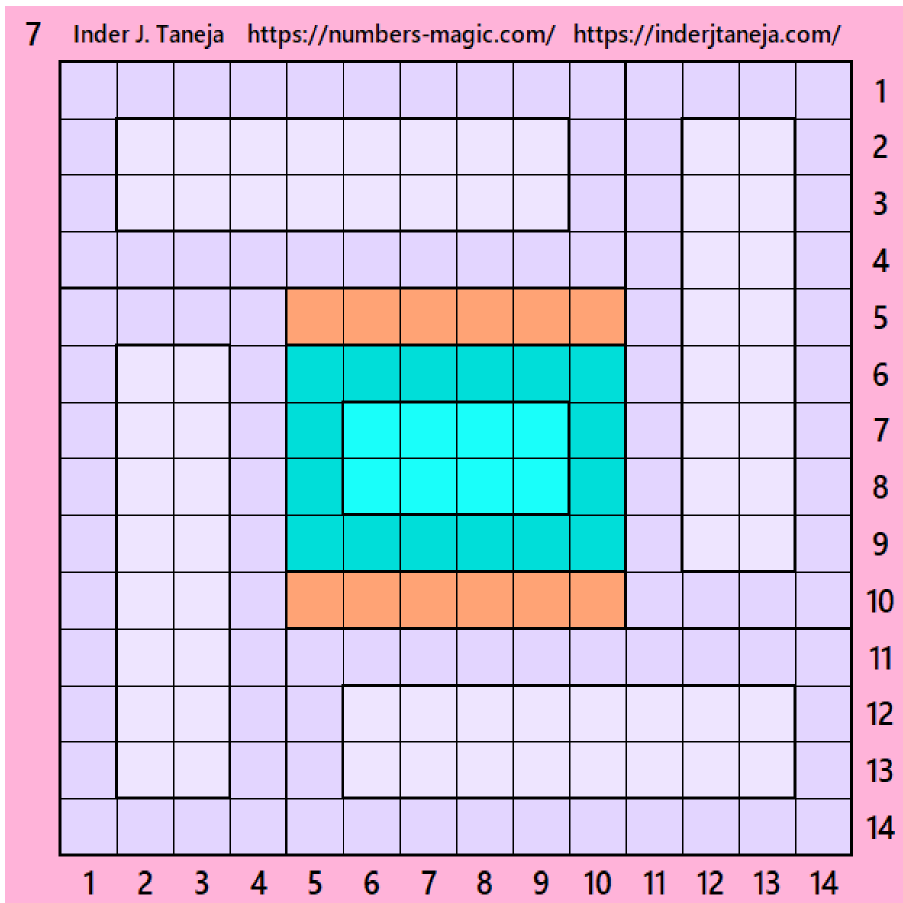


6.3 Closed Border With BMRs of Order 4×10

The above procedure is very simple and don't require any adjustment. Instead of considering 4 magic squares at corners, let's consider a close border by considering 4 equal sums BRMs of order 4×6. In this case, we need some manual adjustment to make it is a magic square. Similar to above below are three magic squares of order 4 with close external border of order 4:

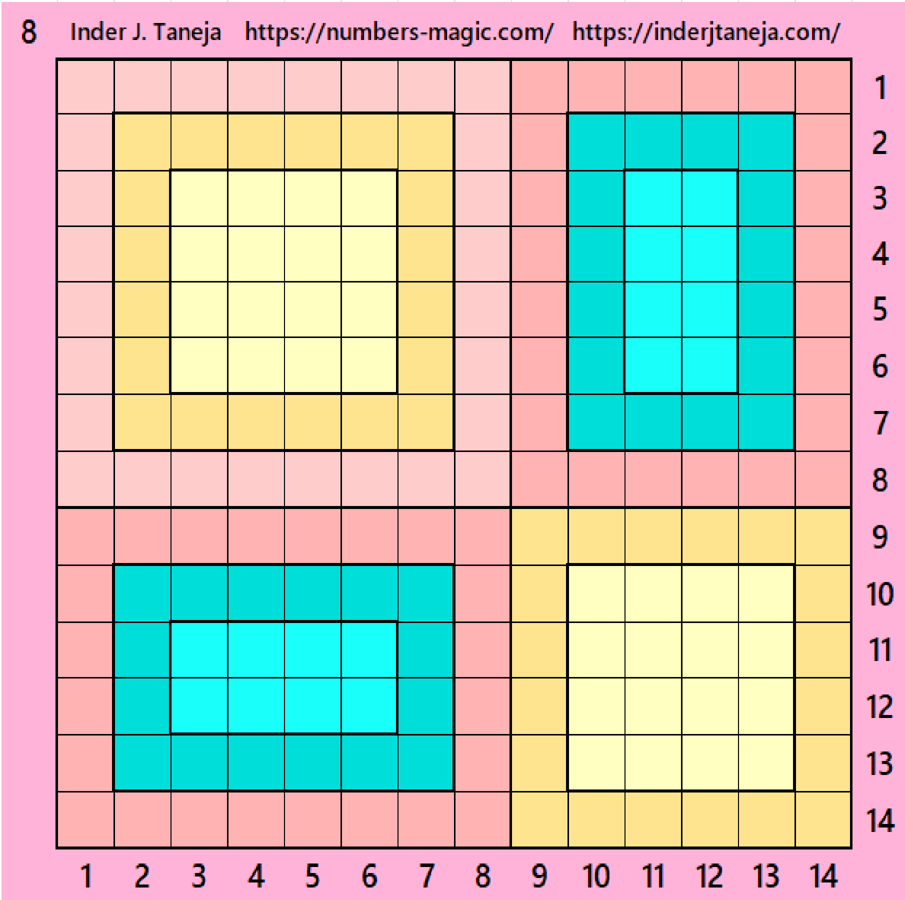


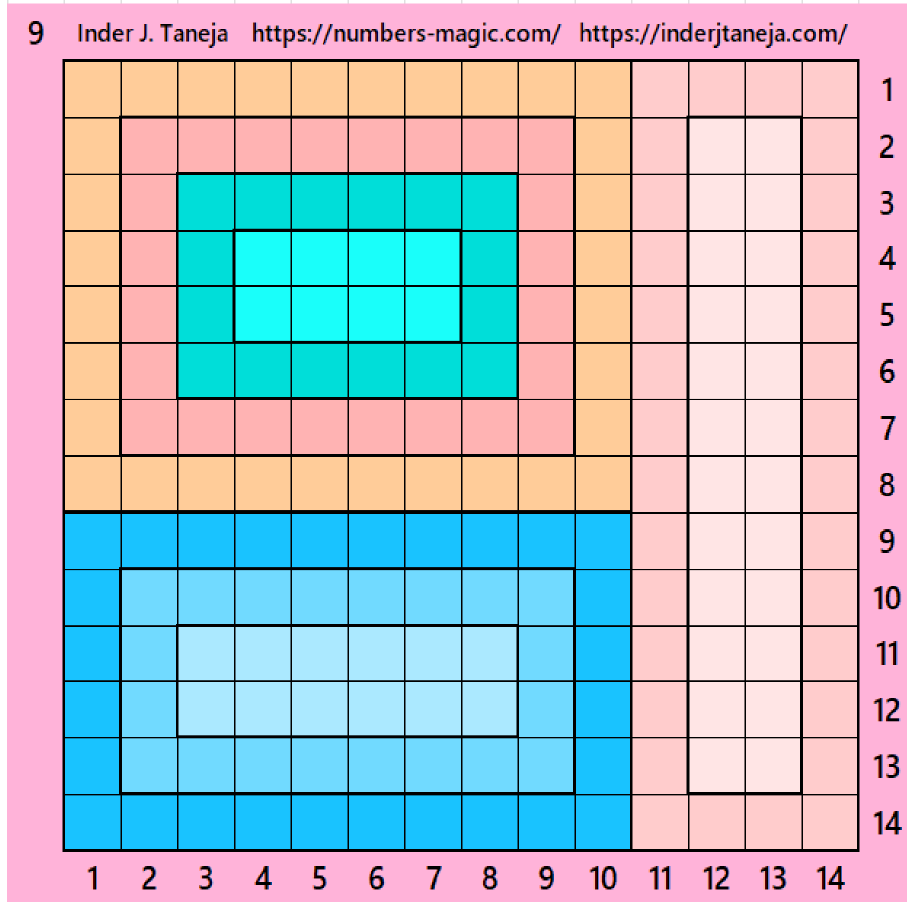




6.4 Extra Examples

The following magic squares are constructed with different combinations of small blocks of magic squares and BMRs:


8
Inder J. Taneja https://numbers-magic.com/ https://inderjtaneja.com/
1 2 3 4 5 6 7 8 9 10 11 12 13 14


9
Inder J. Taneja https://numbers-magic.com/ https://inderjtaneja.com/
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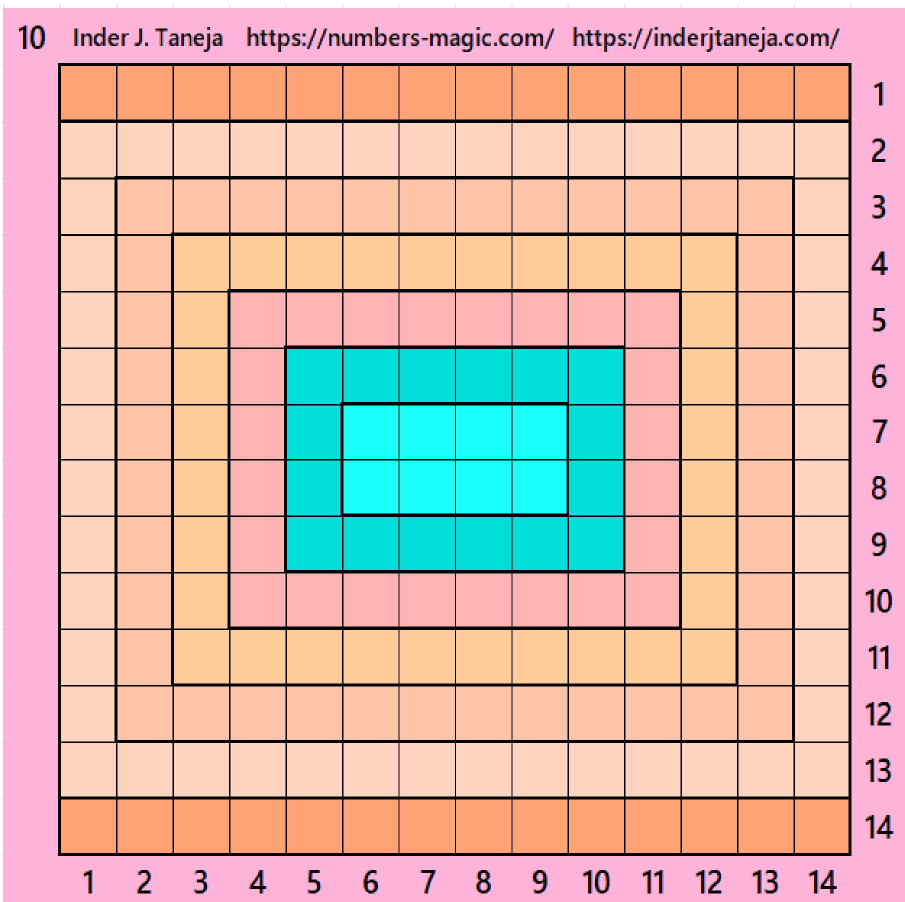
10
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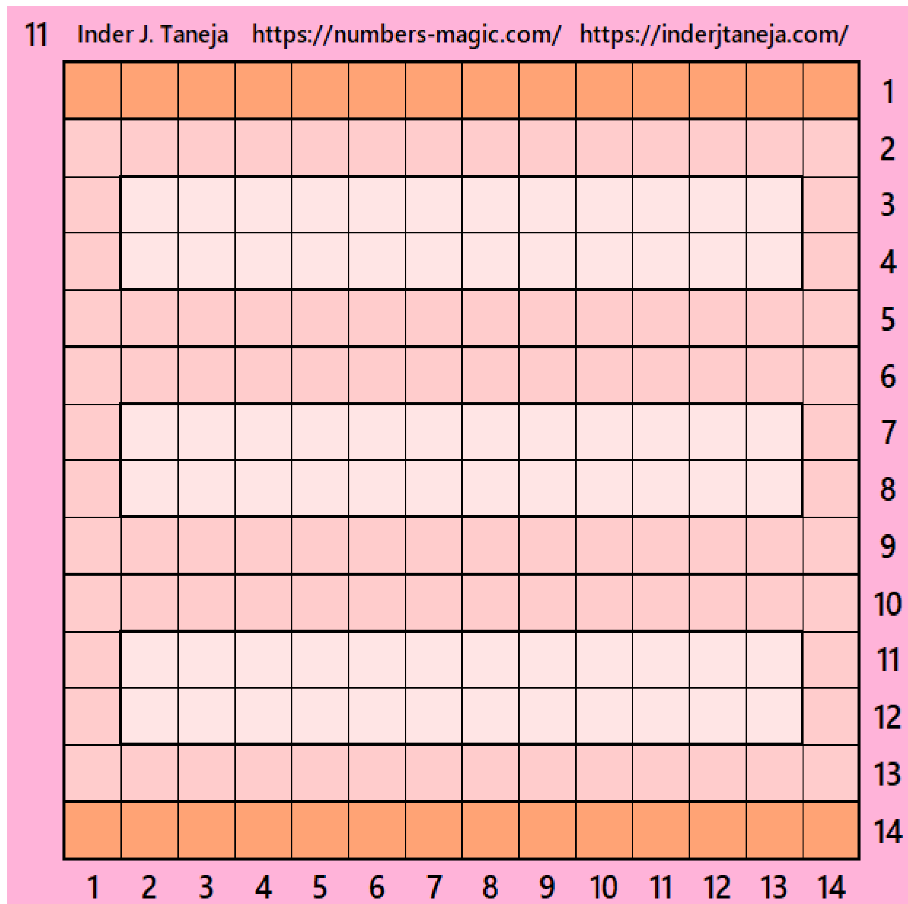
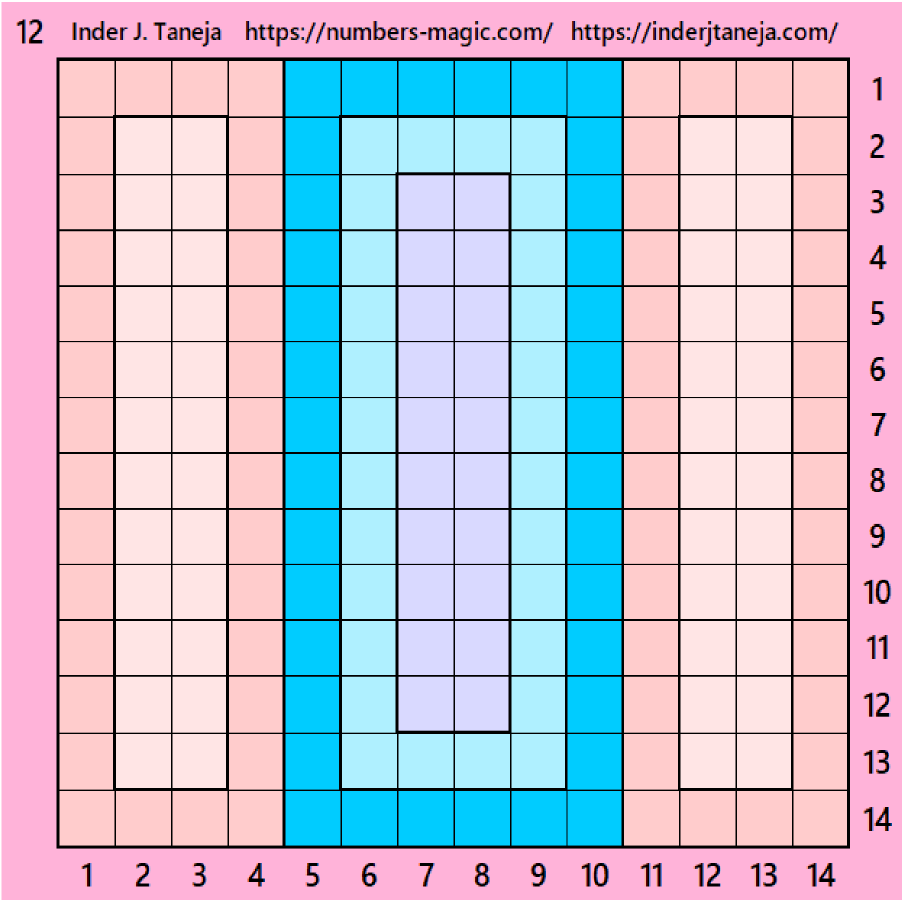
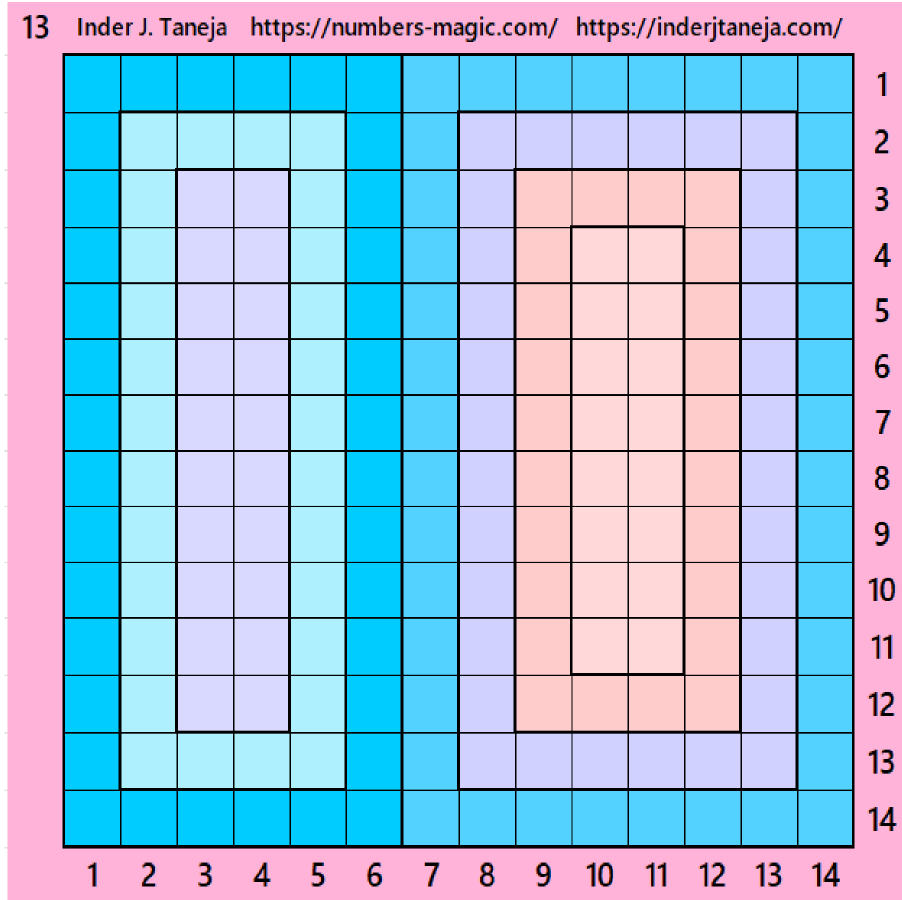
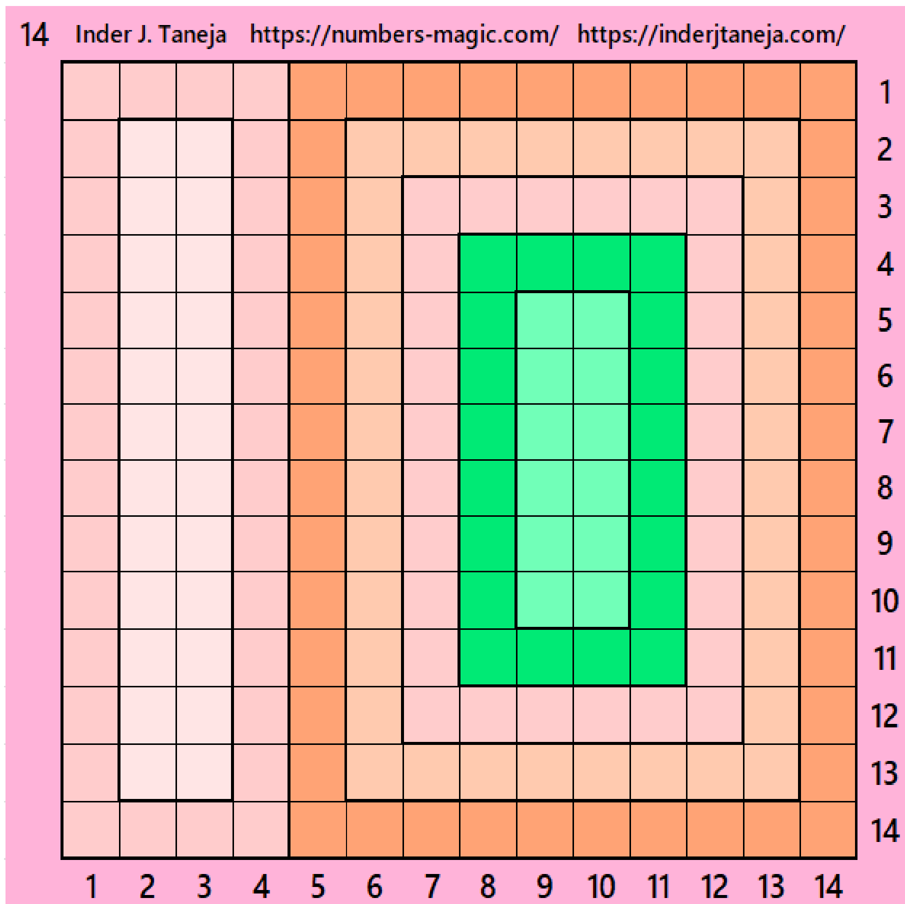
1 2 3 4 5 6 7 8 9 10 11 12 13 14


11
Inder J. Taneja https://numbers-magic.com/ https://inderjtaneja.com/
1 2 3 4 5 6 7 8 9 10 11 12 13 14







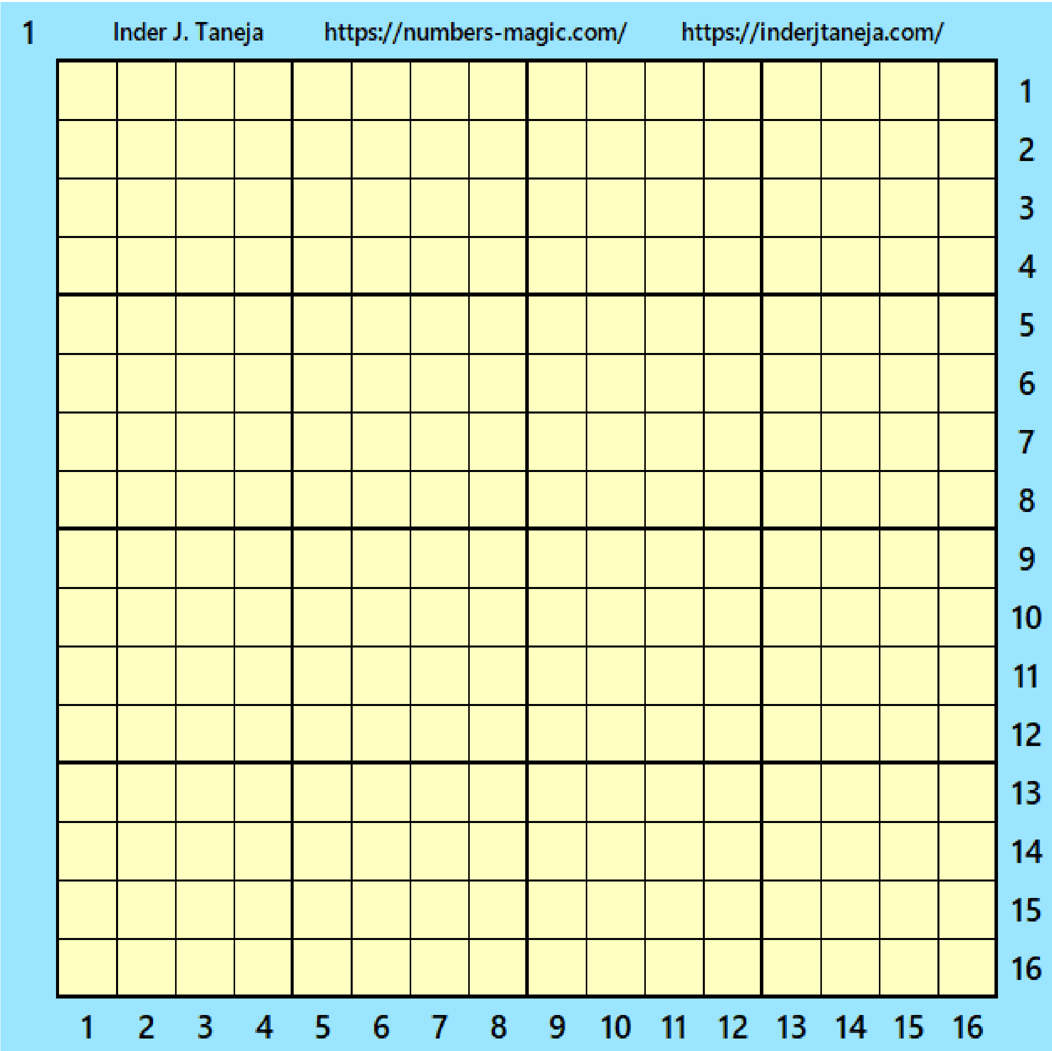
For more details on magic squares of order 14 with BMRs refer author's work [32]

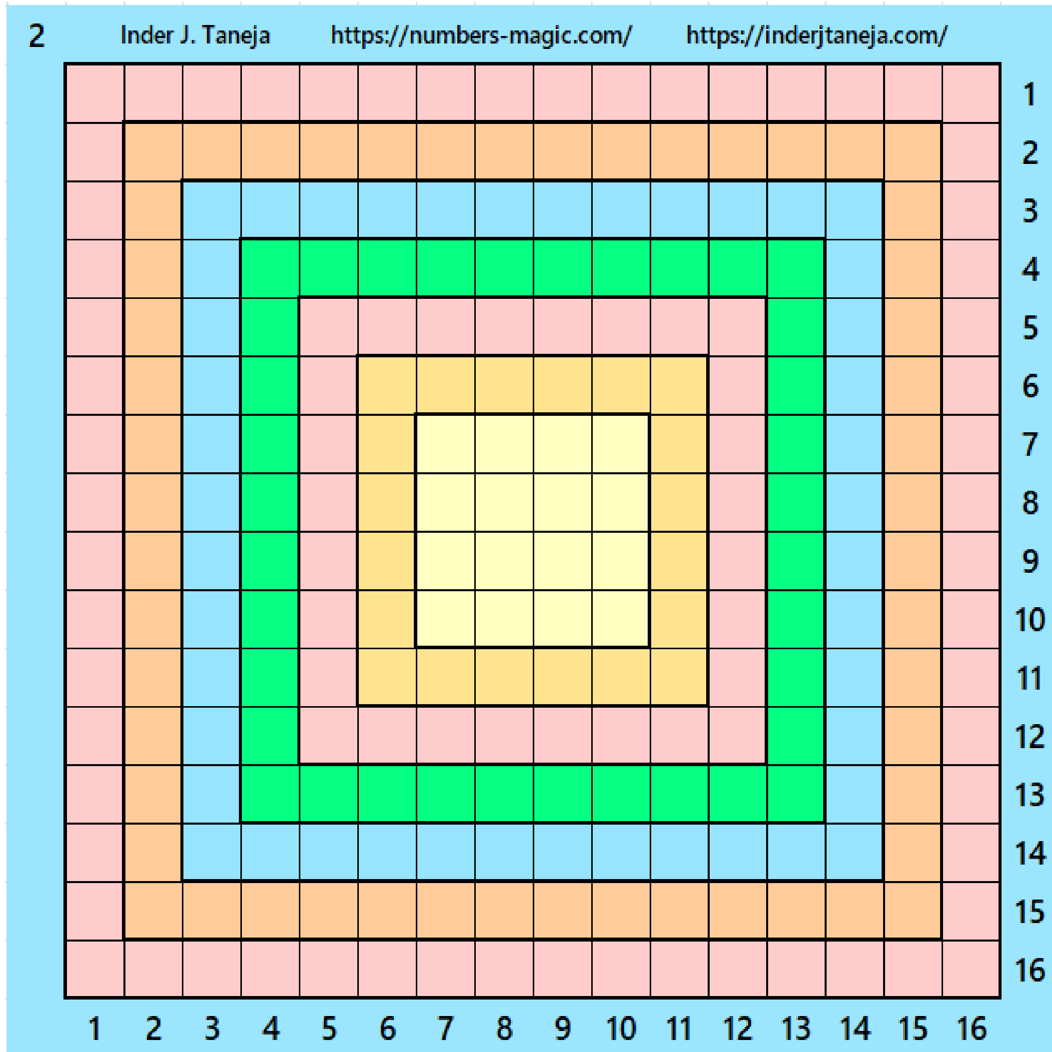
7 Magic Squares of Order 16

This section brings in figures (without numbers) of magic squares of order 16. In some case the idea of construction is explained. More details with numbers can be seen in author's work [33].

7.1 Bordered and Block-Wise Magic Squares of Order 16

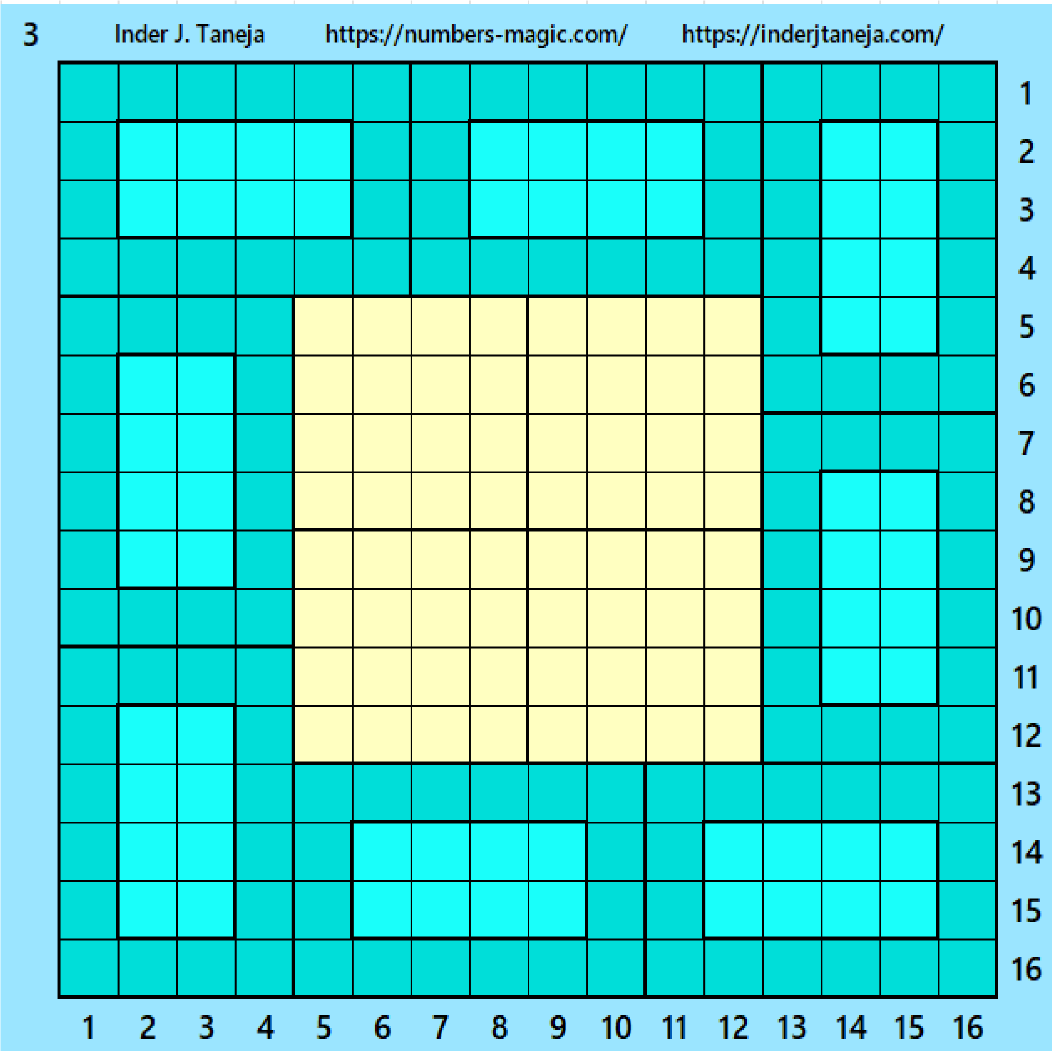
Below are two magic squares of order 16 already known in the literature.

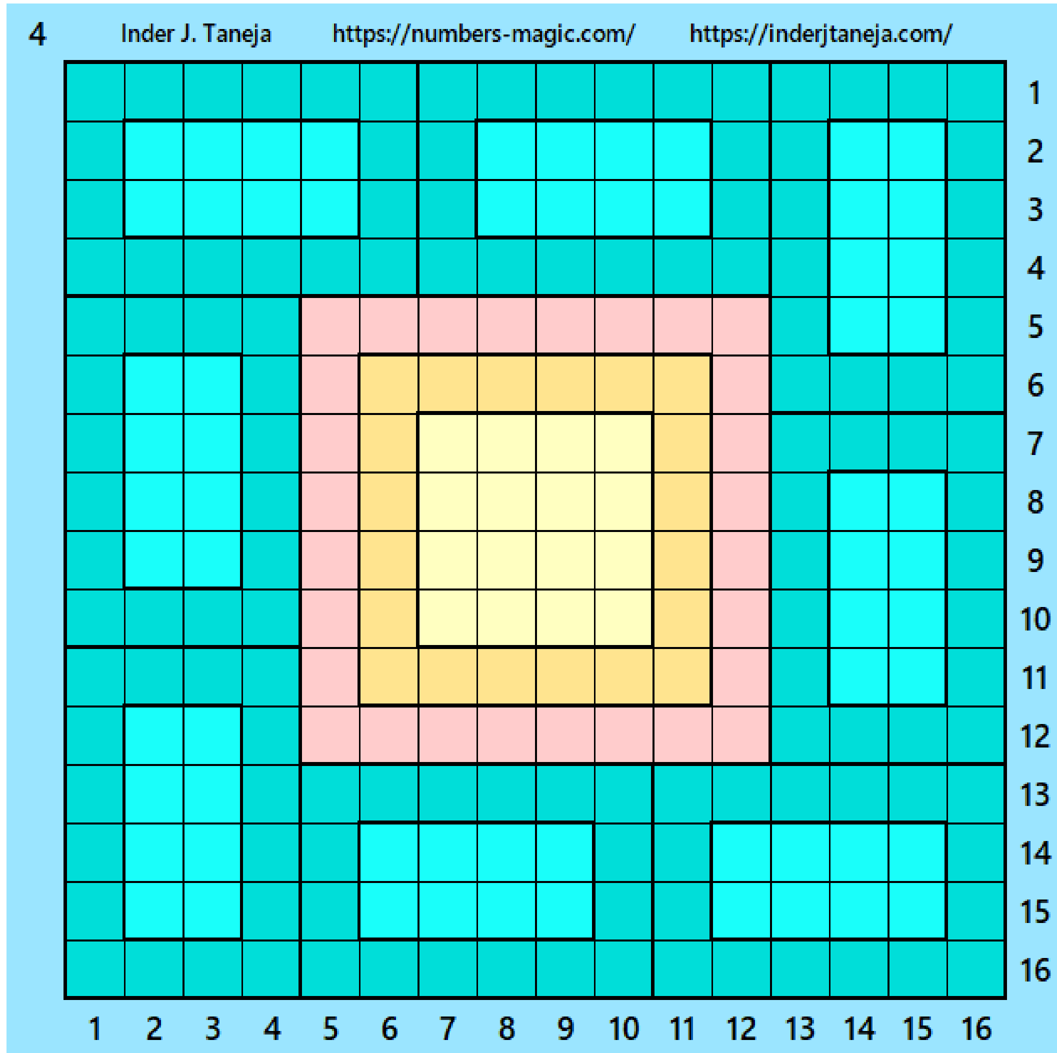




7.2 Closed Border of Order 4

According to Fig 1 subsection 7.1, we don't require open corners with magic squares of order 4. Let's see them with closed border. Let's consider 8 BMRs of order 4×6 and make an external border of order 4. We are left with inner block of order 8. Taking different types of magic squares of order 8, we get magic squares of order 16. See below few examples:


3
Inder J. Taneja
https://numbers-magic.com/
https://inderjtaneja.com/
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16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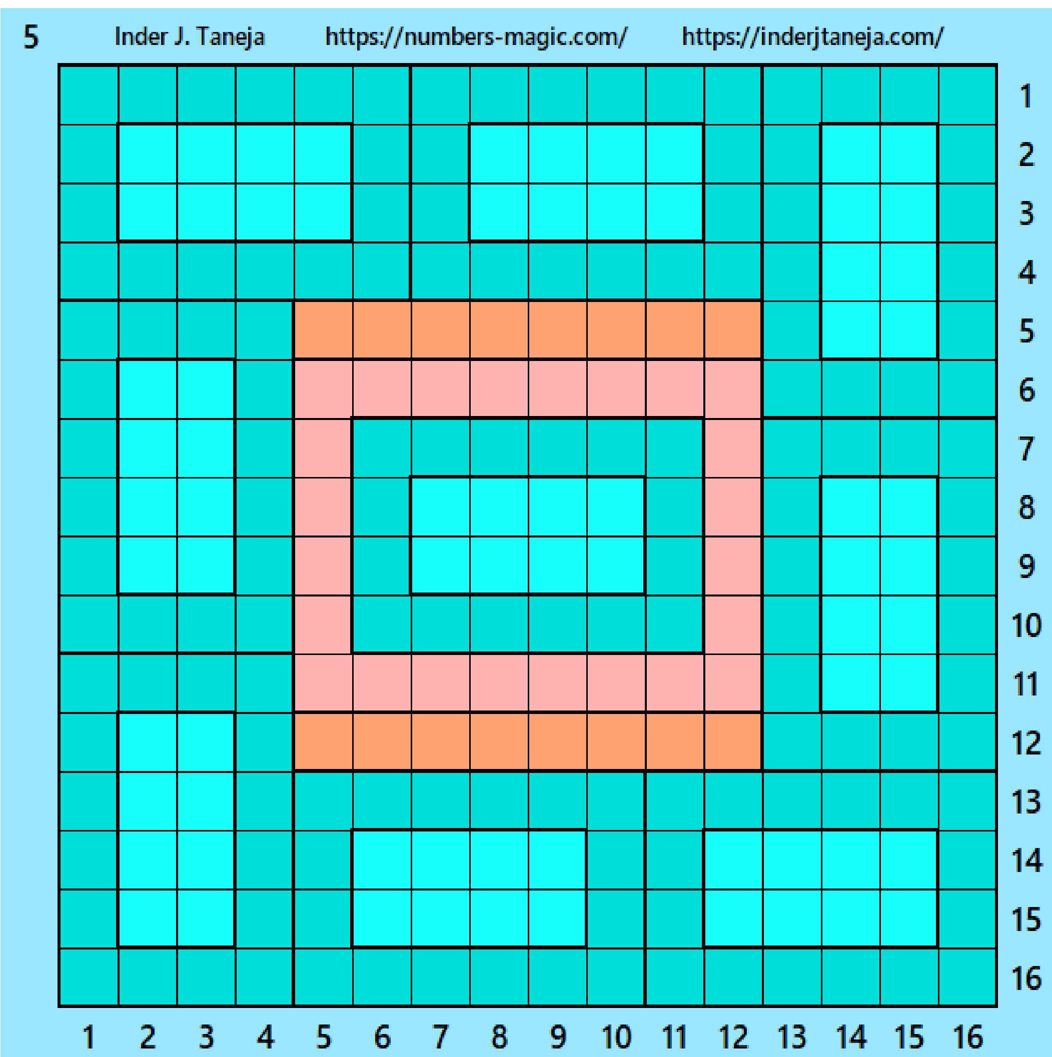
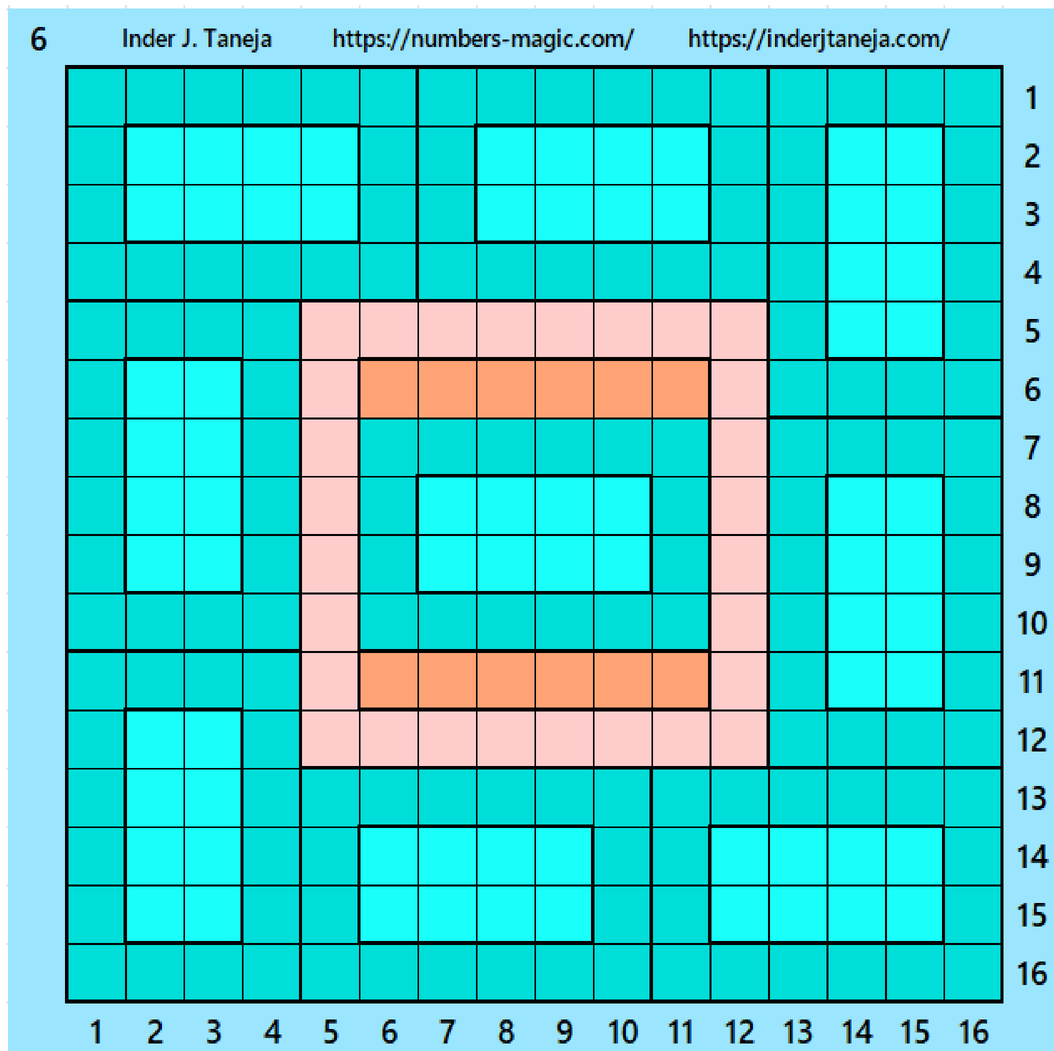
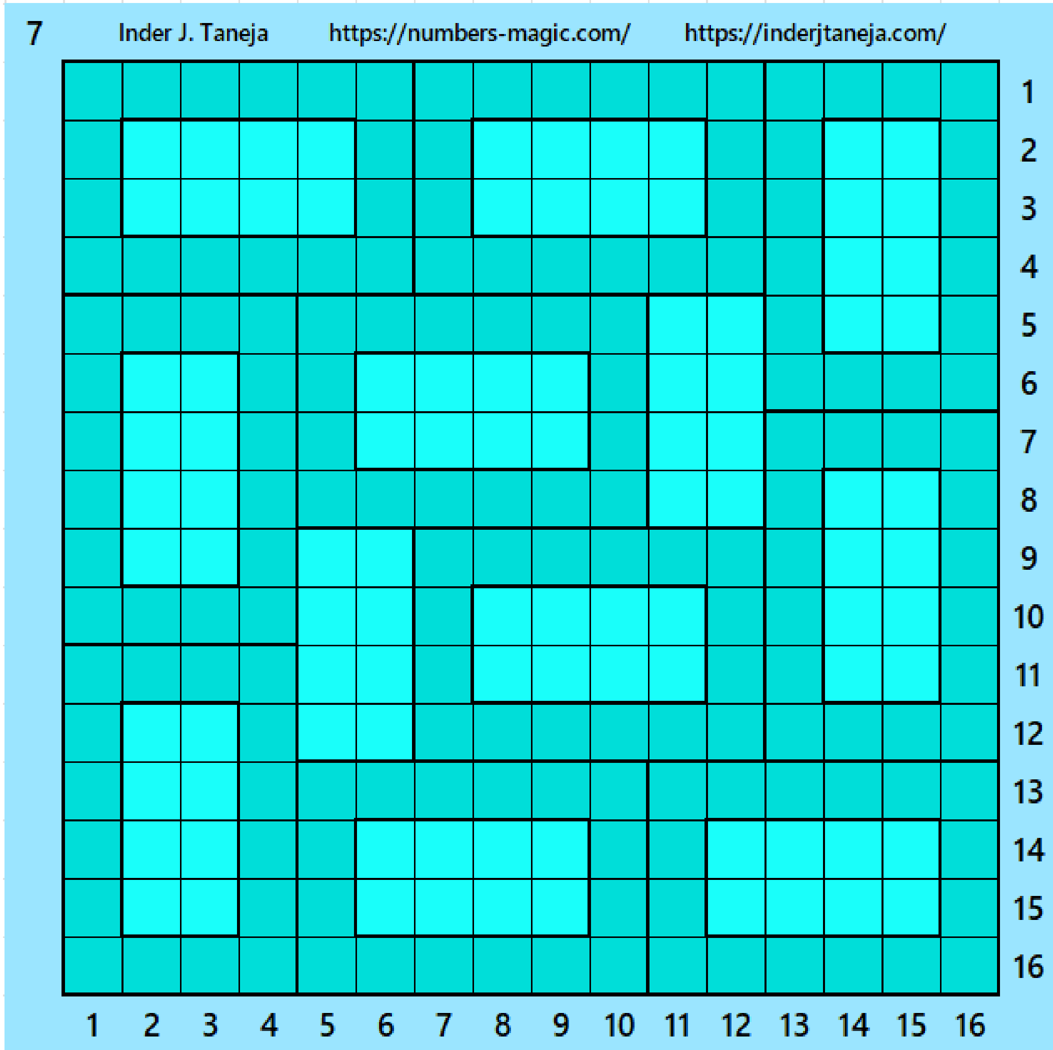
Inder J. Taneja
https://numbers-magic.com/
https://inderjtaneja.com/
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

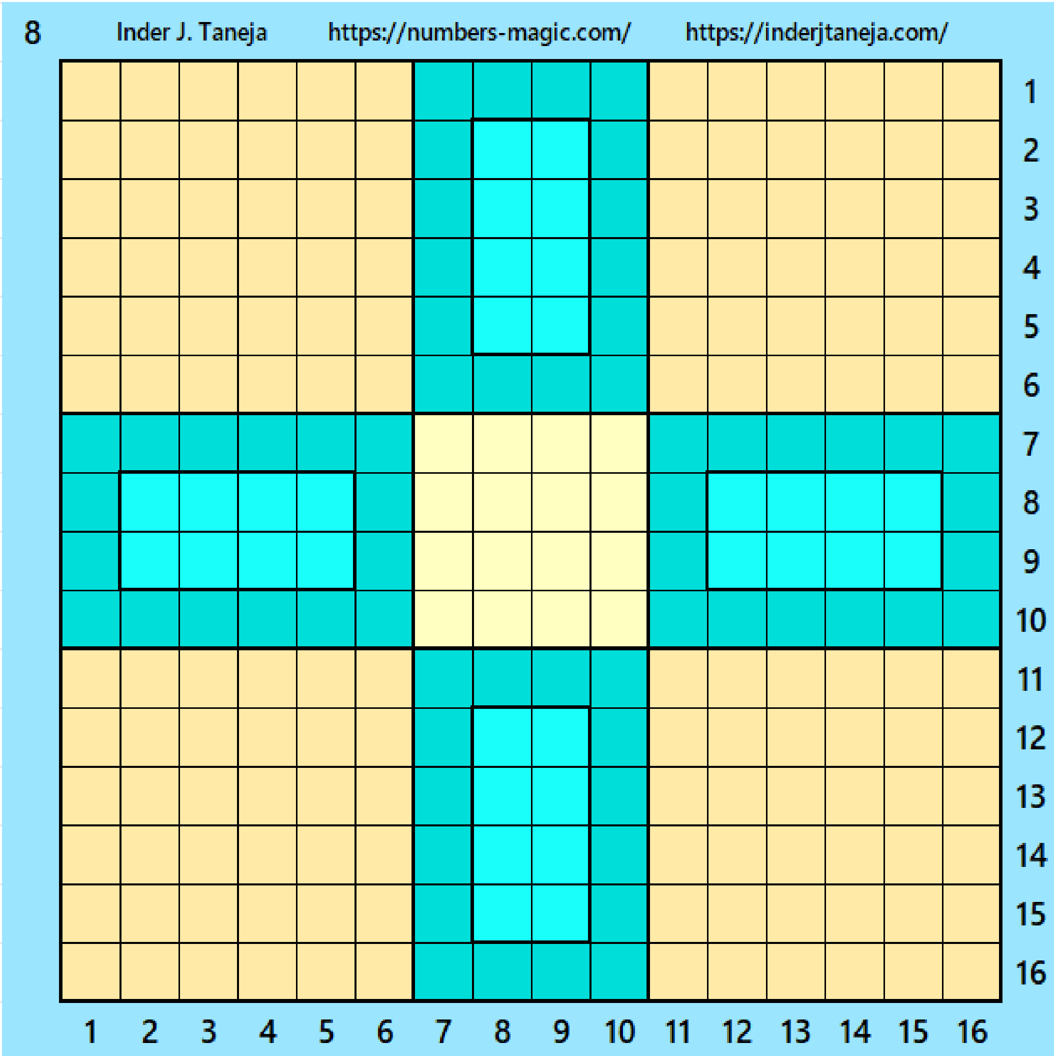

5
Inder J. Taneja
https://numbers-magic.com/
https://inderjtaneja.com/
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

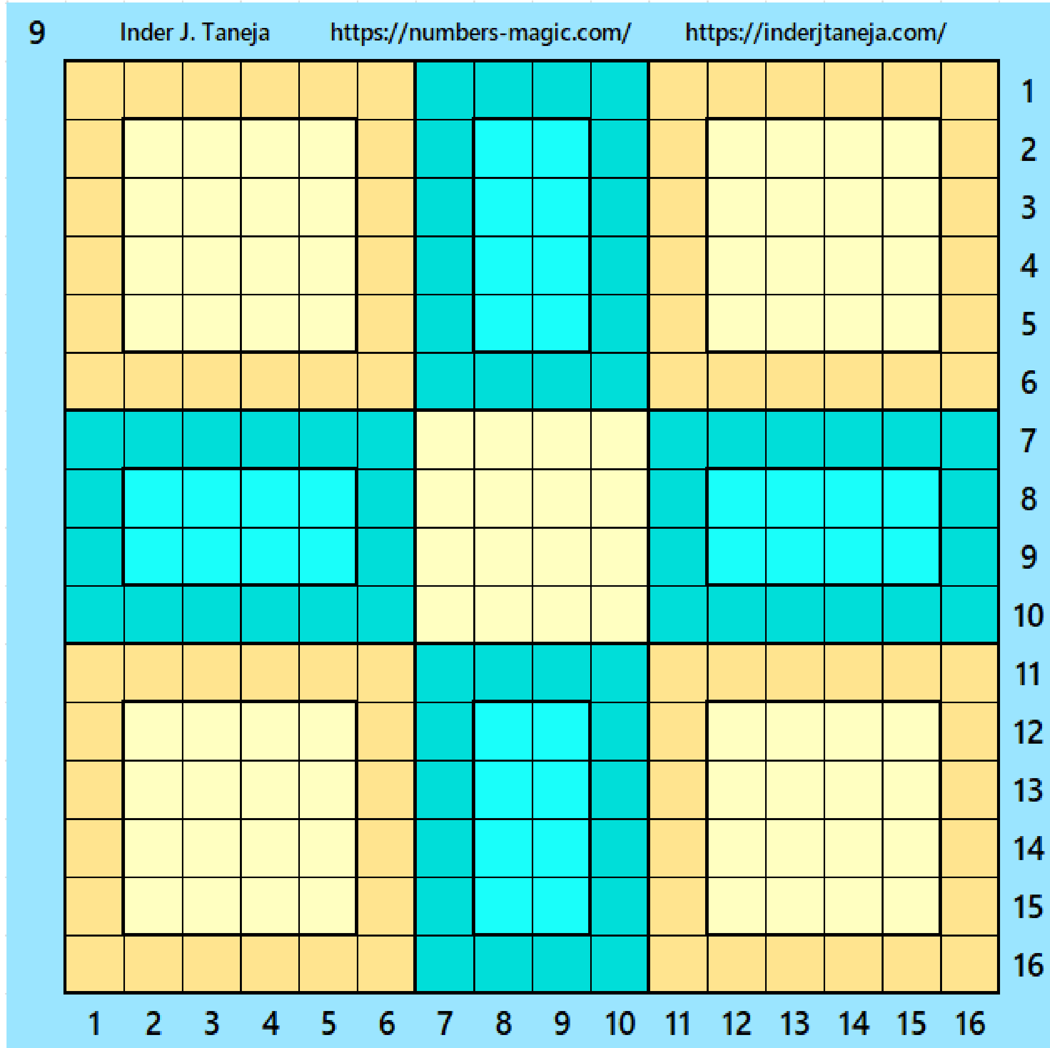

6
Inder J. Taneja
https://numbers-magic.com/
https://inderjtaneja.com/
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16



7.3 Corners With Magic Squares of Order 6

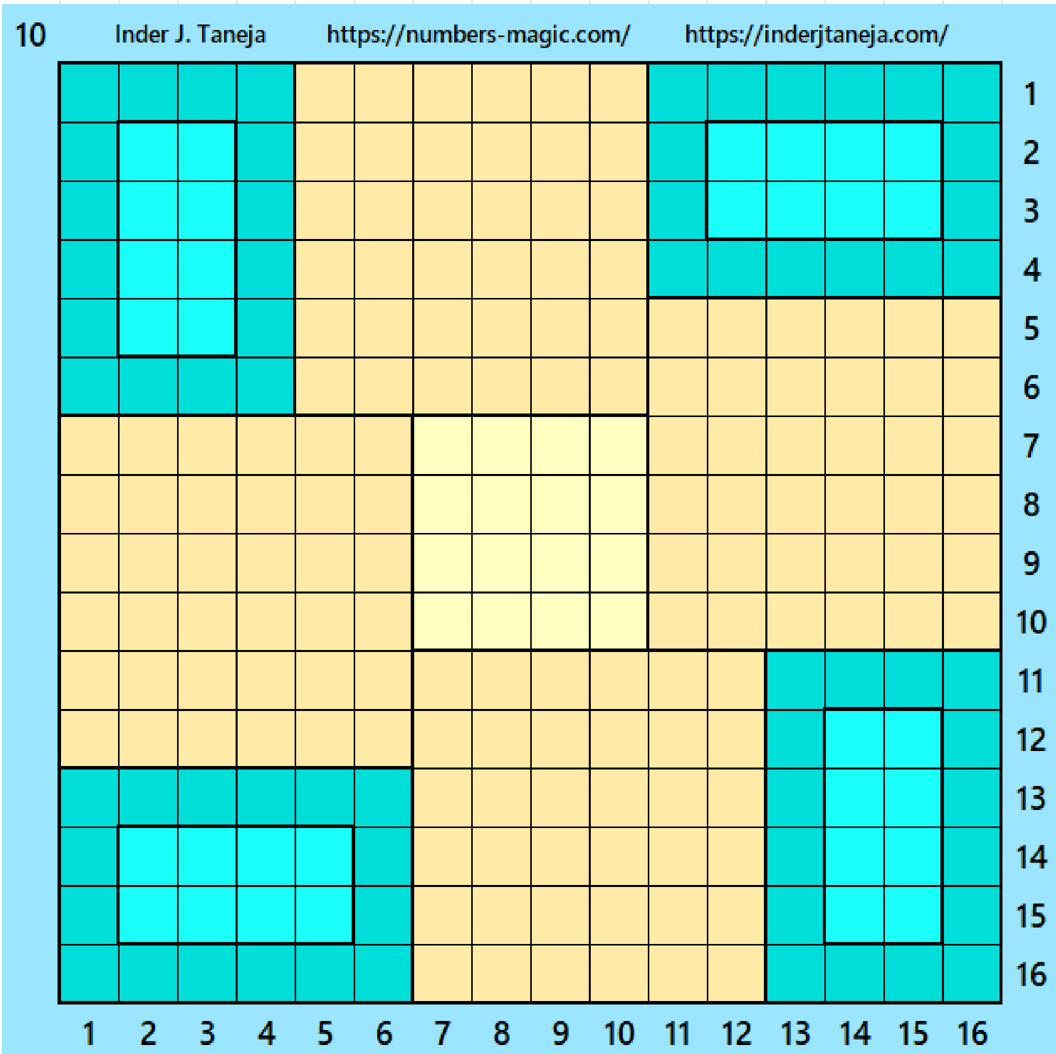
Let's consider 4 magic squares of order 6 and four BMRs of order 4×6. It gives us external border of order 6. We are left with inner block of order 4. See below examples:

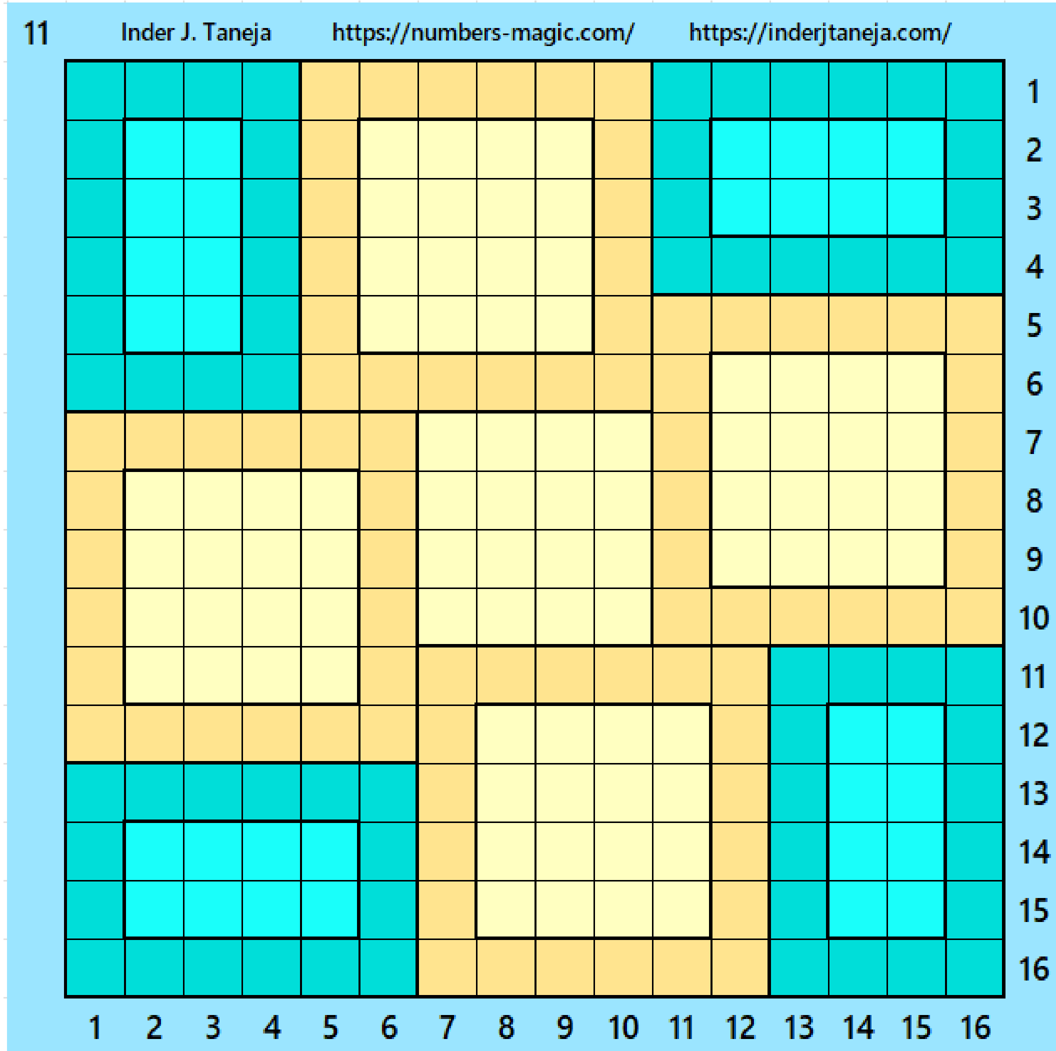




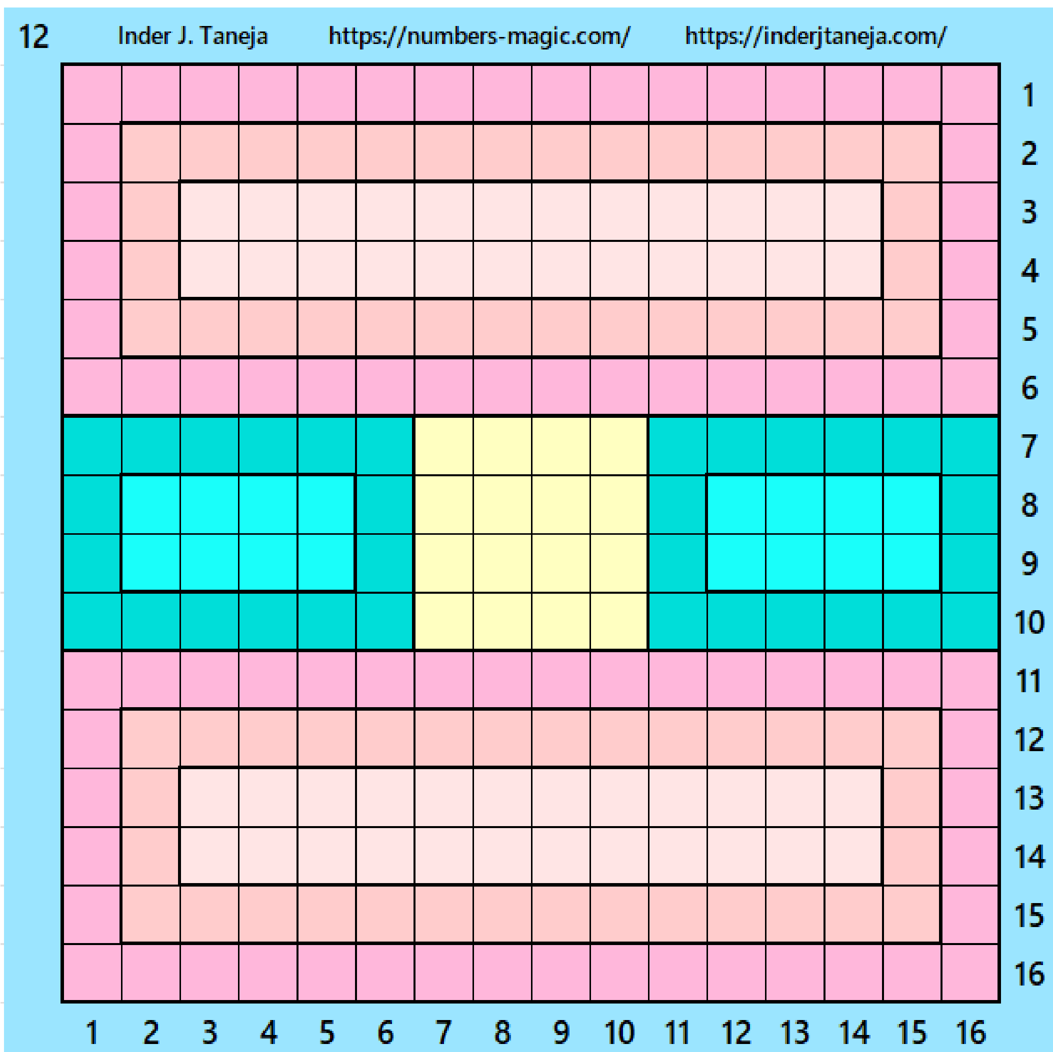
7.4 Closed Border of Order 6

Inverting the order of bordered magic rectangles and magic squares of order 6 in subsection 7.3, we get another conjecture of external border of order 6 See below examples:



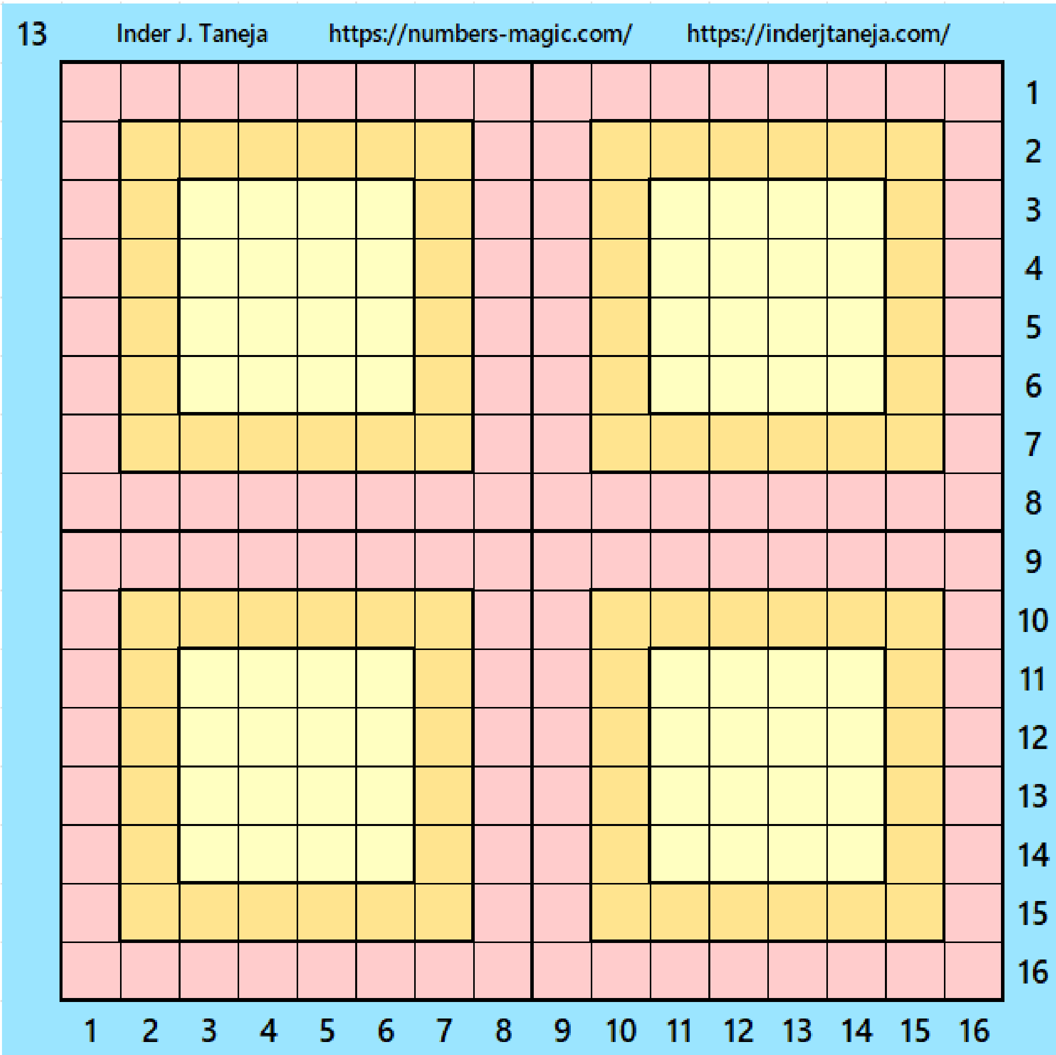


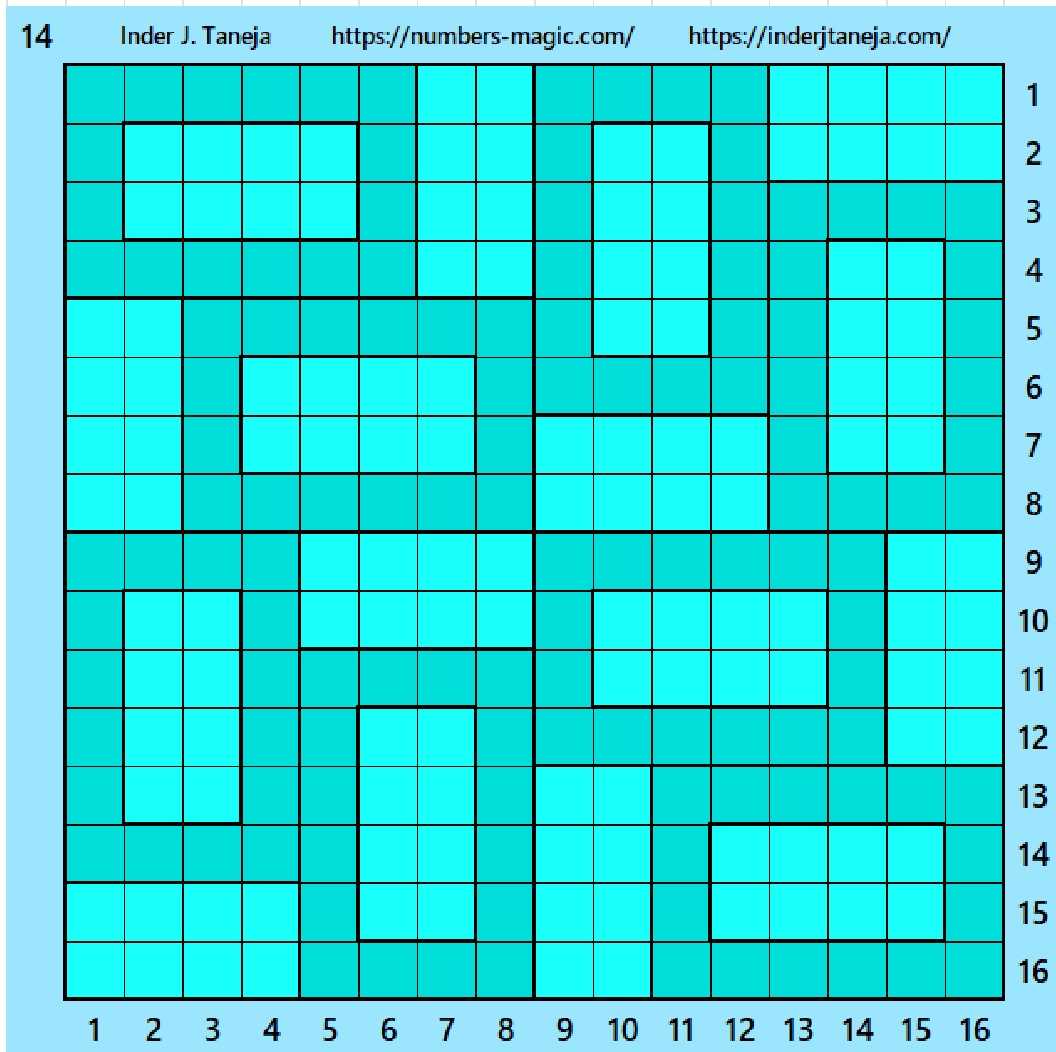
Still there is another possibility of making closed external border. It is made with 2 BMRs of order 6×16 and two BMRs of order 4×6. This conjecture give little different figure. See below:

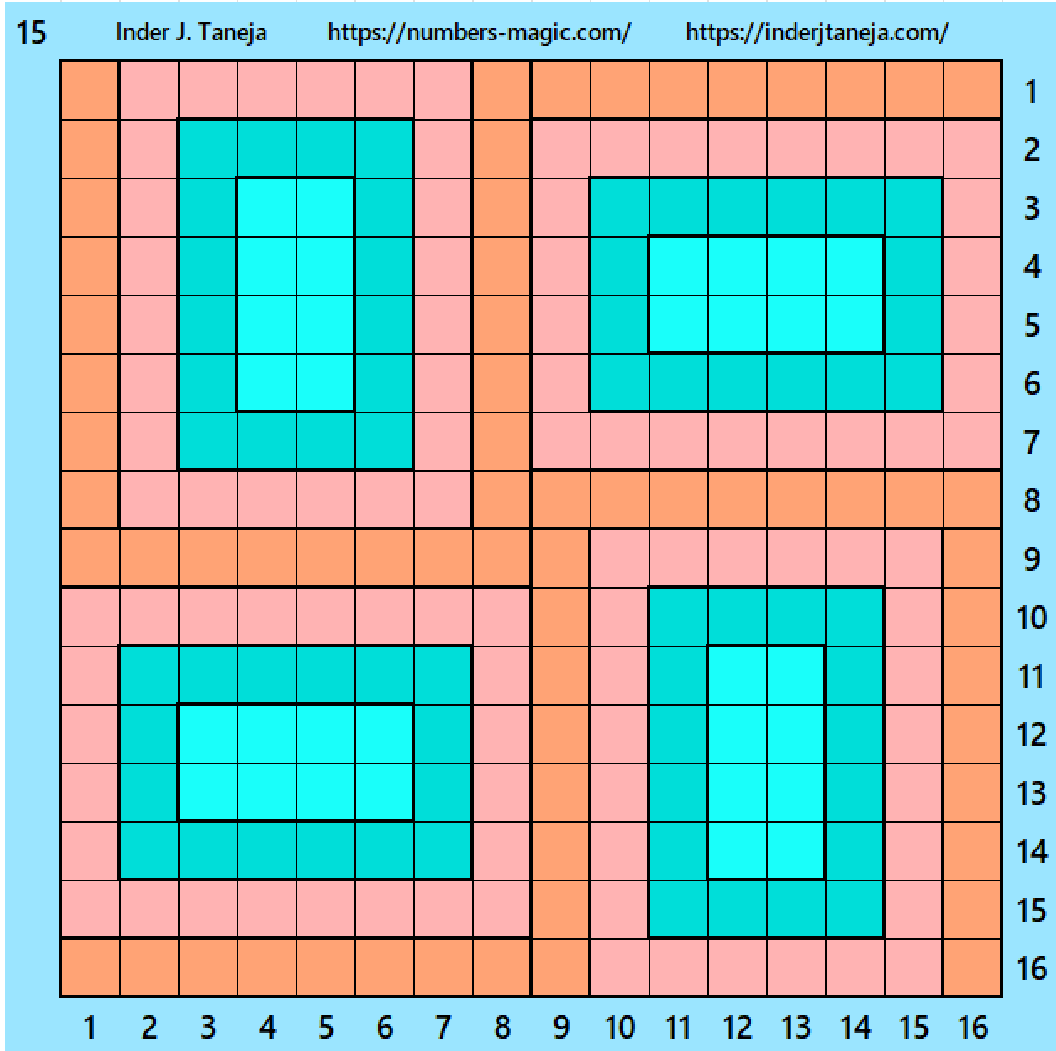


7.5 Blocks of Order 8

The magic square of order 16 can be written as equal sums magic squares of order 8. See below few examples:

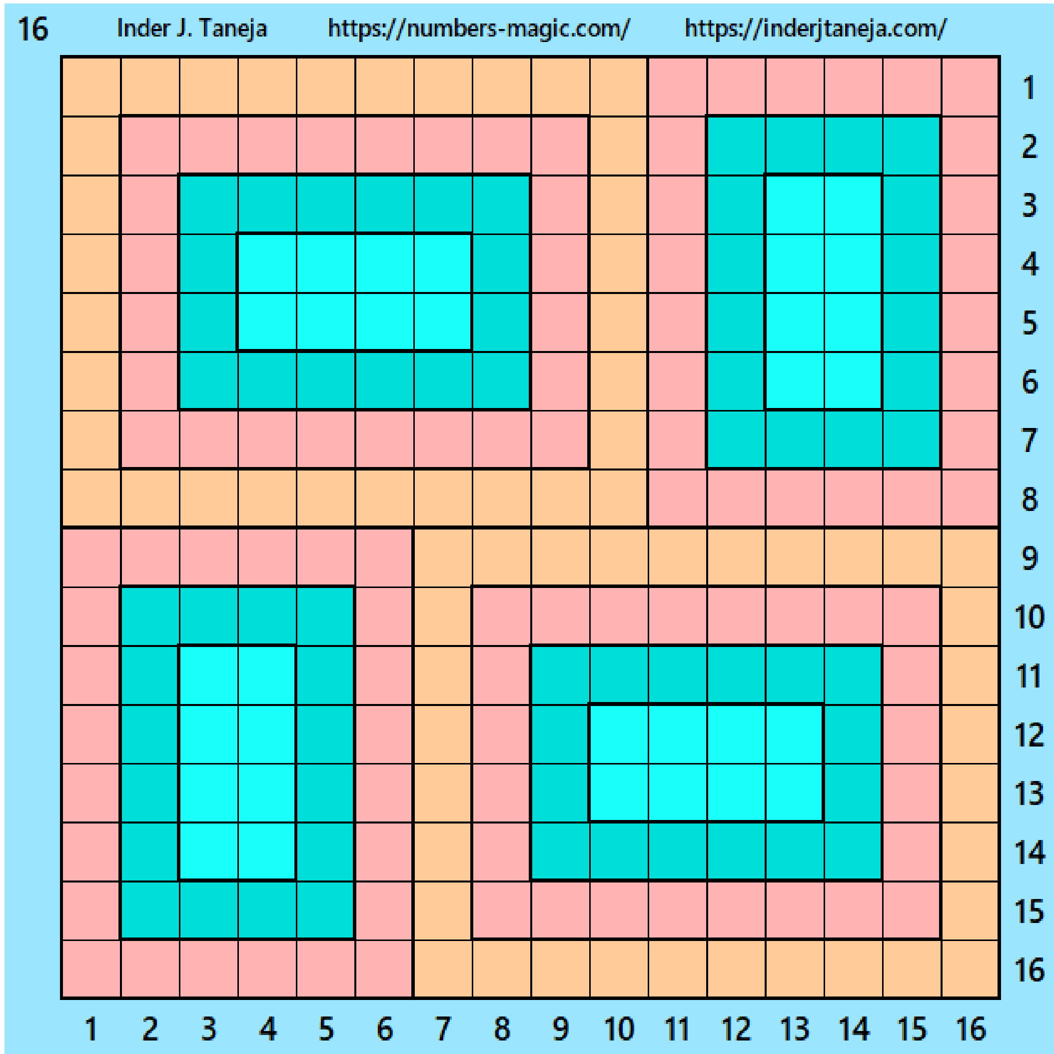






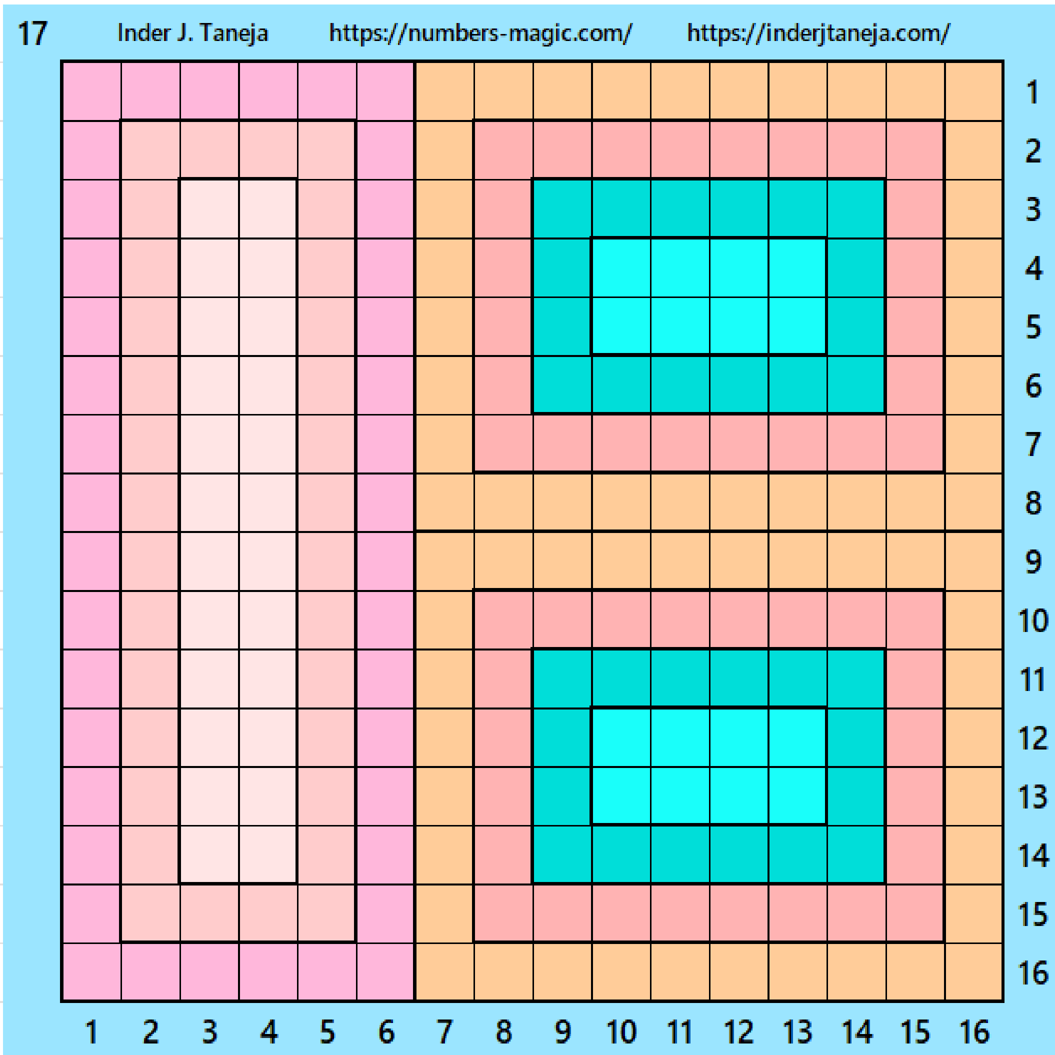
7.6 Consecutive BMRs

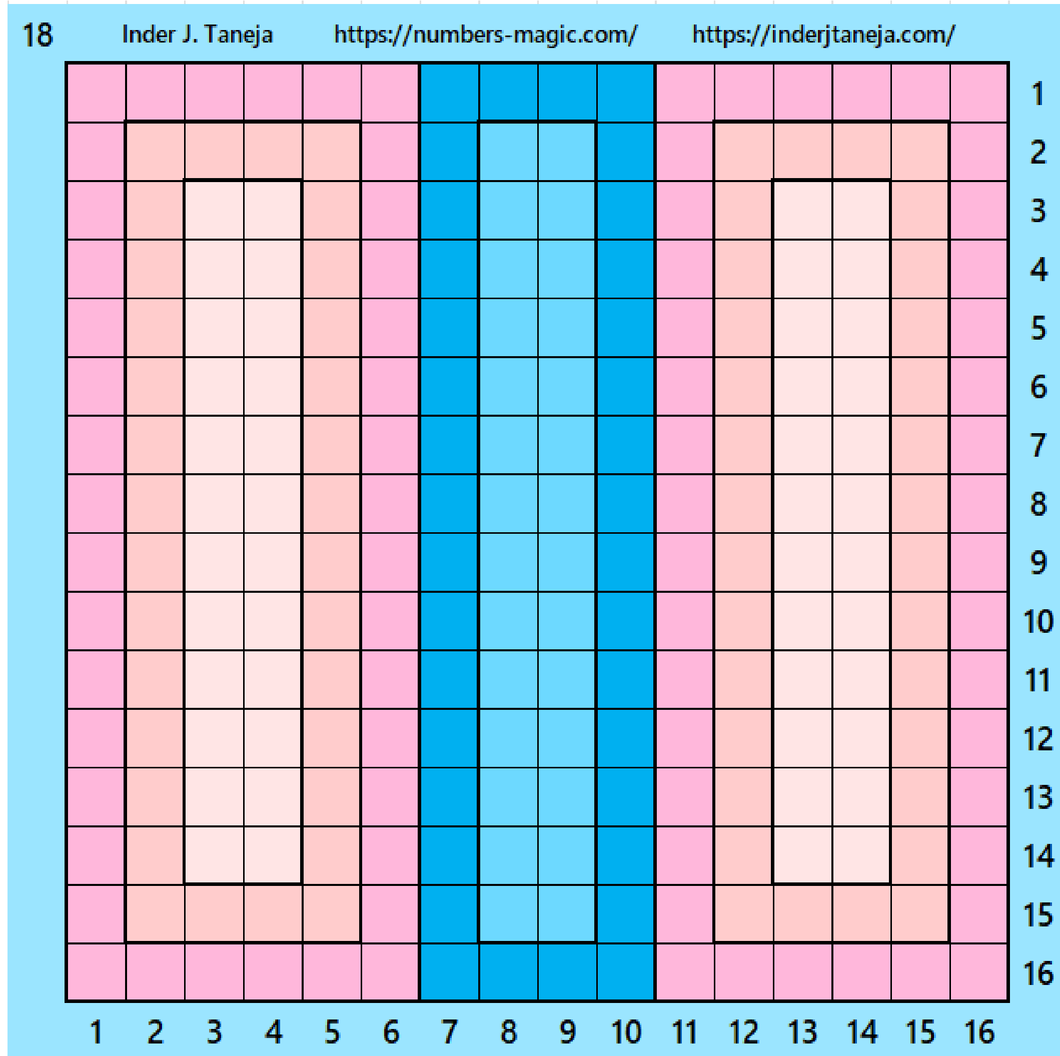
Let's consider two BMRs of orders 6×8 and 8×10. It gives us following magic square of order 16:

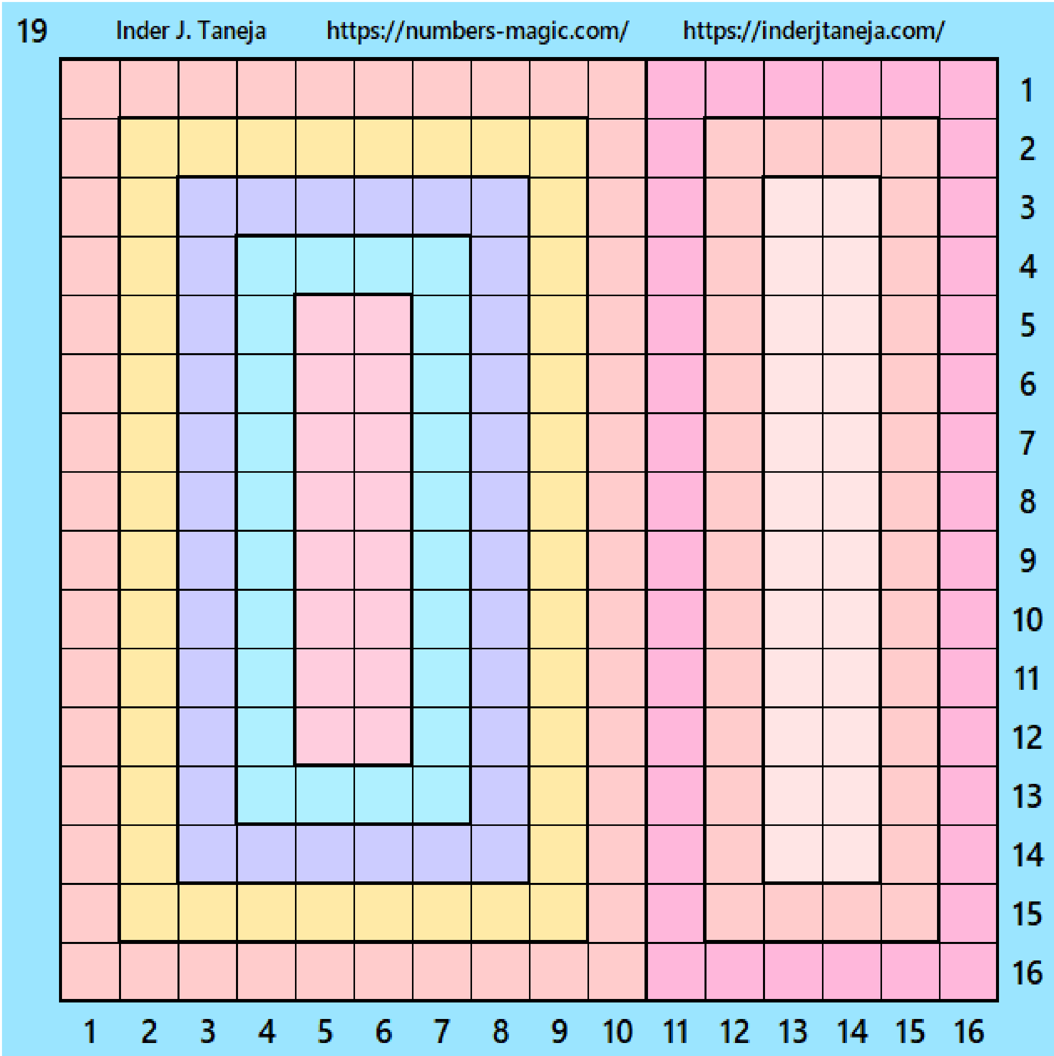


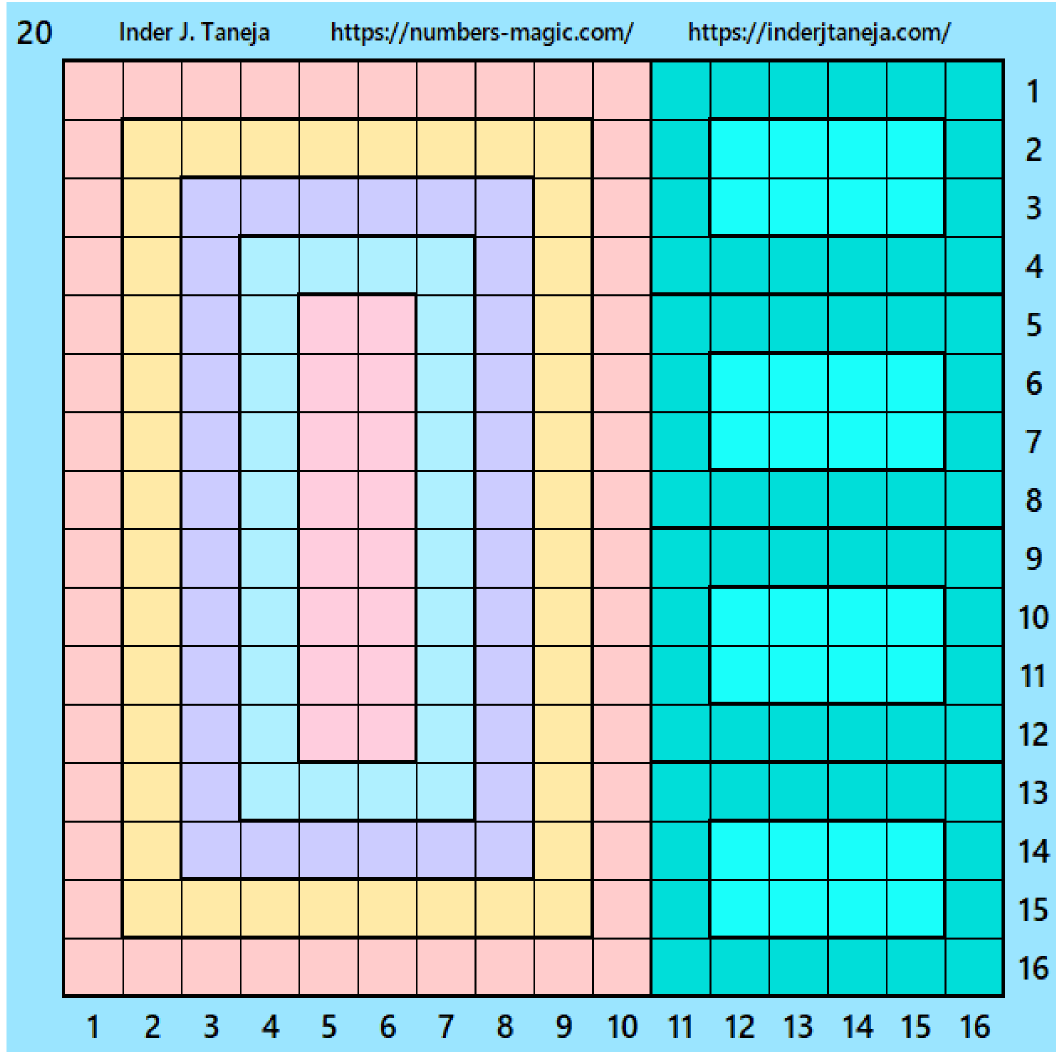
7.7 Extra Examples

Below are few examples of mixed types magic squares of order 16. In each case the construction is individual.


17
Inder J. Taneja
https://numbers-magic.com/
https://inderjtaneja.com/


18
Inder J. Taneja
https://numbers-magic.com/
https://inderjtaneja.com/





8 Appendix

Below are tables giving the existence of **bordered magic rectangles** for **half-sequential** numbers.

Order of Magic Square	Bordered Magic Rectangles
6	4×6
8	6×8
10	4×10, 8×10
12	6×12, 10×12
14	4×14, 8×14, 12×14
16	6×16, 10×16, 14×16
18	4×18, 8×18, 12×18
20	6×20, 10×20, 14×20, 18×20
22	4×22, 8×22, 12×22, 16×22, 20×22
24	6×24, 10×24, 14×24, 18×24, 22×24
26	4×26, 8×26, 12×26, 16×26, 20×26, 24×26
28	6×28, 10×28, 14×28, 18×28, 22×28, 26×28
30	4×30, 8×30, 12×30, 16×30, 20×30, 24×30, 28×30
32	6×32, 10×32, 14×32, 18×32, 22×32, 26×32, 30×32
34	4×34, 8×34, 12×34, 16×34, 20×34, 24×34, 28×34, 32×34

Order of Magic Square	Bordered Magic Rectangles
36	6×36, 10×36, 14×36, 18×36, 22×36, 26×36, 30×36, 34×36
38	4×38, 8×38, 12×38, 16×38, 20×38, 24×38, 28×38, 32×38, 36×38
40	6×40, 10×40, 14×40, 18×40, 22×40, 26×40, 30×40, 34×40, 38×40
42	4×42, 8×42, 12×42, 16×42, 20×42, 24×42, 28×42, 32×42, 36×42, 40×42
44	6×44, 10×44, 14×44, 18×44, 22×44, 26×44, 30×44, 34×44, 38×44,42×44
46	4×46, 8×46, 12×46, 16×46, 20×46, 24×46, 28×46, 32×46, 36×46, 40×46, 44×46
48	6×48, 10×48, 14×48, 18×48, 22×48, 26×48, 30×48, 34×48, 38×48, 42×48, 46×48
50	4×50, 8×50, 12×50, 16×50, 20×50, 24×50, 28×50, 32×50, 36×50, 40×50, 44×50, 48×50

9 Author's Contribution to Magic Squares and Recreation Numbers

For author's contribution to **magic squares** and **recreation numbers** please see the links below:

- **Inder J. Taneja**, Magic Squares, *https://inderjtaneja.com/2019/06/27/publications-magic-squares/*
- **Inder J. Taneja**, Recreation of Numbers, *https://inderjtaneja.com/2019/06/27/publications-recreation-of-numbers/*

Acknowledgement

The **bordered magic rectangles** are constructed based on the software produced by H. White [1]. The author is thankful to H. White for his valuable help.

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