

Mapping CDO^{2&3} to Risk Equivalent CDO

Mapping CDO² and CDO³ trades to risk equivalent CDO (RE-CDO) trades can be done by matching the b/e spread a RE-CDO is calculated and taken as the proxy of a target CDO² or CDO³ trade. Then the MTM of the CDO² or CDO³ trade can be calculated using the market observed correlations by finding the MTM of the proxy via index base correlations.

At the present time there is no generally accepted methodology of applying market observed correlation information to a bespoke CDO² or CDO³ trade. The GSP mapping methodology is similar to the idea of a roughly delta hedging of a CDO² and CDO³ trade with a CDO trade. The tests have shown that the methodology does capture the most of the market risk of credit spread and correlation if calibrated frequently and has been implemented correctly.

However, we have also indicated that such mapping is not based on a sound theoretical foundation and can only be viewed as an approximation. The approximation is acceptable only when each child mezzanine tranches has very similar risk exposure of correlation and credit.

The model serves the purpose of finding a proxy CDO for a CDO² or CDO³ trade by matching b/e spread. The proxy CDO has a collateral pool with the same reference names as that of the CDO² or CDO³ trade and the notional amount of each name is the average of that of all child CDO pools. It has the same thickness of the tranche as that of the CDO² or CDO³ trade and its attachment is calculated such that the b/e spread matches that of the CDO² or CDO³ trade. The MTM of the CDO² or CDO³ trade can then be calculated using the market observed correlations by finding the MTM of the proxy via index base correlations.

The methodology can be illustrated through the following example. As shown in Figure 1, a CDO² trade has two child pools with the reference names 8~12 being shared. This CDO² trade can be mapped to another risk equivalent CDO (RE-CDO) with the pool shown in Fig. 1. In this pool the notional of each reference name is the average of that of two child CDOs.

Figure 1

CDO Squared			Risk Equivalent CDO	
Name	Pool A	Pool B	Name	Pool
1	Notl_A_1		1	0.5 X Notl_A_1
2	Notl_A_2		2	0.5 X Notl_A_2
3	Notl_A_3		3	0.5 X Notl_A_3
4	Notl_A_4		4	0.5 X Notl_A_4
5	Notl_A_5		5	0.5 X Notl_A_5
6	Notl_A_6		6	0.5 X Notl_A_6
7	Notl_A_7		7	0.5 X Notl_A_7
8	Notl_A_8	Notl_B_8	8	0.5 X (Notl_A_8+Notl_B_8)
9	Notl_A_9	Notl_B_9	9	0.5 X (Notl_A_9+Notl_B_9)
10	Notl_A_10	Notl_B_10	10	0.5 X (Notl_A_10+Notl_B_10)
11	Notl_A_11	Notl_B_11	11	0.5 X (Notl_A_11+Notl_B_11)
12	Notl_A_12	Notl_B_12	12	0.5 X (Notl_A_12+Notl_B_12)
13		Notl_B_13	13	0.5 X Notl_B_13
14		Notl_B_14	14	0.5 X Notl_B_14
15		Notl_B_15	15	0.5 X Notl_B_15
16		Notl_B_16	16	0.5 X Notl_B_16
17		Notl_B_17	17	0.5 X Notl_B_17
18		Notl_B_18	18	0.5 X Notl_B_18
19		Notl_B_19	19	0.5 X Notl_B_19
20		Notl_B_20	20	0.5 X Notl_B_20

We define the loss function of the two child pools as

$$(1) \quad L^A(t) = \sum_{n \in \{A\}} LGD_n 1_{t > \tau_n}, \quad L^B(t) = \sum_{n \in \{B\}} LGD_n 1_{t > \tau_n},$$

where $LGD_i = Notional \times (1 - R_i)$ is the loss given default (LGD) for the i^{th} reference name and τ_i is the default time. The child CDO trades have the attachment and detachment points K_A^a, K_A^d for pool A and K_B^a, K_B^d for pool B. Then the loss function of a CDO² trade with attachment point K^{CDO^2} and thickness T can be expressed as:

$$(2) \quad L^{CDO^2}(t) = (L^A(t) - K_A^a)^+ - (L^A(t) - K_A^d)^+ + (L^B(t) - K_B^a)^+ - (L^B(t) - K_B^d)^+,$$

and the loss function of RE-CDO reads

$$(3) \quad L^{RE-CDO}(t) = \frac{L^A(t) + L^B(t)}{2}$$

The expected losses of the CDO² trade and the RE-CDO trade can then have the form

$$(4) \quad \begin{aligned} EL[CDO^2] &= E[(L^{CDO^2}(t) - K^{CDO^2})^+ - (L^{CDO^2}(t) - K^{CDO^2} - T)^+]; \\ EL[CDO] &= E[(L^{RE-CDO}(t) - K^{CDO})^+ - (L^{RE-CDO}(t) - K^{CDO} - T)^+]. \end{aligned}$$

Here the attachment point of this RE-CDO tranche K^{CDO} is unknown. The mapping is to solve this attachment point such that the b/e spread of a CDO² and a RE-CDO matches. The following procedure is employed:

1. Setting correlation be 0.2, find the b/e spread of the CDO² tranche using the valuation model available in the GSP modelling framework.
2. Taking the b/e spread calculated in the first step; find the attachment point by using CDO valuation model in reverse.

For CDO³ trades, the exactly same procedure is used except that a CDO³ valuation model (see <https://finpricing.com/lib/FiBond.html>) is employed instead in the first step. As shown in the Eq.(4), all the complicated structures in the CDO² and CDO³ trades are simply ignored. However, the value and sensitivities of the CDO² and CDO³ trades are the functions of these structures. For example, the overlapping effect cannot be captured in the RE-CDO. This can be elaborated with the following scenarios:

1. If we assume all reference names have the same credit spread level, two default events in a CDO² would have six possible cases: both in pool A but not sharing, both in B but not sharing, etc. Each case has a different effect due to the subordination of each child CDO and the overlapping feature. However, it is indistinguishable in the RE-CDO trade and we treat all six cases as one case.
2. If we set $K_A^a = K_B^a = 0$ and $K_A^d = K_B^d = \infty$, then we have

$$(5) \quad L^{CDO^2}(t) = L^A(t) + L^B(t).$$

We essentially change the CDO² into a CDO, which has a different loss function from that of the RE-CDO as shown in Eq. (3). We won't expect that they will have identical risk profile using whatever mapping methodology in this scenario.

3. If two child CDOs have the same collateral pools $L^A(t) = L^B(t)$ and $K_A^d = K_B^a$, we would write the loss function as

$$(6) \quad \begin{aligned} L^{CDO^2}(t) &= (L^A(t) - K_A^a)^+ - (L^A(t) - K_B^d)^+ \\ L^{RE-CDO}(t) &= L^A(t) \end{aligned}$$

Apparently one is a linear function of loss function and the other is a nonlinear one. We might be able to achieve a good delta equivalent CDO and CDO², but the nonlinear effect cannot be matched.

Although theoretically speaking it is the approximation, practically the mapping seems to work well for two reasons. First, because we match the b/e spreads of a CDO trade and a CDO² trade, the MTM calculated by proxy can be reasonably used for CDO². Further, as our tests have shown in the next section, the credit spread risk and correlation risk of the RE-CDO and CDO² agree with each other if the child mezzanine trades have very similar risk profile. The change of the MTM due to the market change of credit spread and implied correlation can be successfully reflected through the proxy CDO.

Hence as far as capturing daily market change of credit spread and correlation is concerned, the GSP methodology is acceptable if a frequent mapping is performed.

At the present time there is no generally accepted methodology of applying market observed correlation information to a bespoke CDO² and CDO³ trade. The GSP mapping methodology can be viewed as a roughly delta hedging of a CDO² or CDO³ trade with a CDO trade. In an ideal case in which each mezzanine tranches has identical risk exposure of correlation and credit shown in this report, it is found that the approximation works very well.

In general, the approximation will deteriorate when the heterogeneity of the risk exposure of the child mezzanine tranches becomes large and the complexity of the overlapping increases. However, if we only care the small and local movement of the credit spread and correlation and do the mapping frequently, the approximation is acceptable. For the same reason, the mapping cannot address the scenarios of large and overall shocks such as stress test.

Tests have shown that, given the current market environment, the current choices of weekly mapping and correlation fixed at 0.2 in the mapping are appropriate. However, they should be reviewed frequently. If changing the mapping correlation has a material effect on the implied MTM through proxy CDO, a risk control should be enforced.

