An Inclusive Study on Fundamentals of Hypersoft Set

Muhammad Saeed [2], Atiqe Ur Rahman[2,*], Muhammad Ahsan[3], and Florentin Smarandache[4]
[1,2,3] University of Management and Technology, Lahore, Pakistan.
muhammad.saeed@umt.edu.pk, aurkhb@gmail.com, ahsan1826@gmail.com
[4] Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA ; smarand@unm.edu

Abstract. Smarandache developed hypersoft set theory as an extension of soft set theory, to adequate the existing concept of soft set for multi-attribute function. In this study, essential elementary properties e.g. not set, subset, absolute set, and aggregation operations e.g. union, intersection, complement, AND, OR, restricted union, extended intersection, relevant complement, restricted difference, restricted symmetric difference, are characterized under hypersoft set environment with illustrated examples. New notions of relation, function and their basic properties are also discussed for hypersoft sets. Moreover, matrix representation of hypersoft set is presented along with different operations. Lastly, some new hybrids of hypersoft set are also presented for further extension in different directions.

Keywords: Hypersoft Set; Hypersoft Relation; Hypersoft Function; Hypersoft Matrix.

1. Introduction

The theories like theory of probability, theory of fuzzy sets, and the interval mathematics, are considered as mathematical means to tackle many intricate problems involving various uncertainties in different fields of mathematical sciences. These theories have their own complexities which restrain them to solve these problems successfully. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. A mathematical tool is needed for dealing with uncertainties which should be free of all such impediments. In 1999, Molodtsov [1] introduced a mathematical tool called soft sets in literature as a new parameterized family of subsets of the universe of discourse. In 2003, Maji et al. [2] extended the concept and introduced some fundamental terminologies and operations like equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, absolute soft set, AND, OR and also the operations of union and intersection. They verified De Morgan's laws and a number of other results too. In 2005, Pei et al. [3] discussed the relationship between soft sets and information systems. They showed the soft sets as a class of special information systems. In 2009, Ali et al. [4] pointed out several assertions in previous work of Maji et al. and proposed new notions such as the restricted intersection, the restricted union, the restricted difference

and the extended intersection of two soft sets. In 2010, 2011, Babitha et al. [5,6] introduced the concept of soft set relation as a sub soft set of the Cartesian product of the soft sets and also discussed many related concepts such as equivalent soft set relation, partition, composition and function. In 2011, Sezgin et al. [7], Ge et al. [8], Fuli [9] gave some modifications in the work of Maji et al. and also established some new results. In 2020, Saeed et al. [10] performed an extensive inspection of the concept of soft elements and soft members in soft sets. Many researchers [11–22] developed certain hybrids with soft sets to get more generalized results for implementation in decision making and other related disciplines. In 2018, Smarandache [23] introduced the concept of hypersoft set as a generalization of soft set.
Motivating from above literature, in this study, some essential fundamentals (i.e. elementary properties, set theoretic operations, basic laws, set relations, set function and matrix representation) are conceptualized under hypersoft set environment. The rest of this article is structured as follows: Section 2 gives some basic definitions and results on hyper soft sets. Section 3 presents elementary properties of hypersoft sets. Section 4 describes set theoretic operations of hypersoft sets. Section 5 provides some basic properties, results and laws on hypersoft sets. Section 6 discusses hypersoft relations and hypersoft functions. Section 7 presents the matrix representation of hypersoft sets with some operations. Section 8 presents some hybrids of hypersoft sets and then last section 9 concludes the paper.

2. Preliminaries

Here we recall some basic terminologies regarding soft set and hypersoft set. Throughout the paper, $\mathcal{U}$ denotes the universe of discourse.

Definition 2.1. [1]
A pair (ζ_S, Λ) is called a *soft set* over $\mathcal{U}$, where $\zeta_S : \Lambda \to P(\mathcal{U})$ and Λ be a subset of set of attributes E.

Definition 2.2. [2]
A soft set (ζ_{S_1}, Λ_1) is a *soft subset* of another soft set (ζ_{S_2}, Λ_2) if

(i) $\Lambda_1 \subseteq \Lambda_2$, and
(ii) $\zeta_{S_1}(\omega) \subseteq \zeta_{S_2}(\omega)$ for all $\omega \in \Lambda_1$.

Definition 2.3. [2]
Union of two soft sets (ζ_{S_1}, Λ_1) and (ζ_{S_2}, Λ_2) is a soft set (ζ_{S_3}, Λ_3) with $\Lambda_3 = \Lambda_1 \cup \Lambda_2$ and for $\omega \in \Lambda_3$,

$$\zeta_{S_3}(\omega) = \begin{cases} \zeta_{S_1}(\omega) & \omega \in (\Lambda_1 \setminus \Lambda_2) \\ \zeta_{S_2}(\omega) & \omega \in (\Lambda_2 \setminus \Lambda_1) \\ \zeta_{S_1}(\omega) \cup \zeta_{S_2}(\omega) & \omega \in (\Lambda_1 \cap \Lambda_2) \end{cases}$$

Definition 2.4. [2]
Intersection of two soft sets (ζ_{S_1}, Λ_1) and (ζ_{S_2}, Λ_2) is a soft set (ζ_{S_3}, Λ_3) with $\Lambda_3 = \Lambda_1 \cap \Lambda_2$ and for $\omega \in \Lambda_3$,

$$\zeta_{S_3}(\omega) = \zeta_{S_1}(\omega) \cap \zeta_{S_2}(\omega)$$

For more details on soft set can be found in [1–9]

3. **Hypersoft Set (HS-set)**

In this section, some fundamentals of hypersoft set are presented. Some of the definitions given in [24] are modified.

Definition 3.1. [21]
The pair (Ψ, G) is called a *hypersoft set* over $\mathcal{U}$, where G is the cartesian product of n disjoint attribute-valued sets $G_1, G_2, G_3, ..., G_n$ corresponding to n distinct attributes $g_1, g_2, g_3, ..., g_n$ respectively and $\Psi : G \to P(\mathcal{U})$. The collection of all hypersoft sets is denoted by $\Omega_{(\Psi,G)}$.

Example 3.2. Suppose that Mr. X wants to buy a mobile from a mobile market. There are eight kinds of mobiles (options) which form the set of discourse $\mathcal{U} = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8\}$. The best selection may be evaluated by observing the attributes i.e. $a_1 =$ Company, $a_2 =$ Camera Resolution, $a_3 =$ Size, $a_4 =$ RAM, and $a_5 =$ Battery power. The attribute-valued sets corresponding to these attributes are:
$B_1 = \{b_{11}, b_{12}\}$
$B_2 = \{b_{21}, b_{22}\}$
$B_3 = \{b_{31}, b_{32}\}$
$B_4 = \{b_{41}, b_{42}\}$
$B_5 = \{b_{51}\}$
then $G = B_1 \times B_2 \times B_3 \times B_4 \times B_5$
$G = \{g_1, g_2, g_3, g_4, ..., g_{16}\}$ where each $g_i, i = 1, 2, ..., 16$, is a 5-tuple element.
The hypersoft set (Ψ, G) is given as

$$(\Psi, G) = \left\{ \begin{array}{l} (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), (g_3, \{m_2, m_3, m_4\}), (g_4, \{m_4, m_5, m_6\}), \\ (g_5, \{m_6, m_7, m_8\}), (g_6, \{m_2, m_3, m_4, m_7\}), (g_7, \{m_1, m_3, m_5, m_6\}), \\ (g_8, \{m_2, m_3, m_6, m_7\}), (g_9, \{m_2, m_3, m_6, m_7, m_8\}), (g_{10}, \{m_1, m_3, m_6, m_7, m_8\}), \\ (g_{11}, \{m_2, m_4, m_6, m_7, m_8\}), (g_{12}, \{m_1, m_2, m_3, m_6, m_7, m_8\}), \\ (g_{13}, \{m_2, m_3, m_5, m_7, m_8\}), (g_{14}, \{m_1, m_3, m_5, m_7, m_8\}), \\ (g_{15}, \{m_1, m_2, m_3, m_5, m_7, m_8\}), (g_{16}, \{m_4, m_5, m_6, m_7, m_8\}) \end{array} \right\}$$

Definition 3.3. Let $\mathcal{F}(\mathcal{U})$ be the collection of all fuzzy sets over $\mathcal{U}$. Let $a_1, a_2, a_3, ..., a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $G_1, G_2, G_3, ..., G_n$, with $G_i \cap G_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., n\}$. Then a *fuzzy hypersoft set* (Ψ_{fhs}, G) over $\mathcal{U}$ is defined by the set of ordered pairs as follows,

$$(\Psi_{fhs}, G) = \{(g, \Psi_{fhs}(g)) : g \in G, \Psi_{fhs}(g) \in \mathcal{F}(\mathcal{U})\}$$

where $\Psi_{fhs} : G \to \mathcal{F}(\mathcal{U})$ and for all $g \in G = G_1 \times G_2 \times G_3 \times ..... \times G_n$

$$\Psi_{fhs}(g) = \{\mu_{\Psi_{fhs}(g)}(u)/u : u \in \mathcal{U}, \mu_{\Psi_{fhs}(g)}(u) \in [0, 1]\}$$

is a fuzzy set over $\mathcal{U}$.

Above definition is a modified version of fuzzy hypersoft set given in [21] and [23].

Example 3.4. Considering the example 3.2, we have
Fuzzy hypersoft set (Ψ_{fhs}, G) is given as

$$(\Psi_{fhs}, G) = \left\{ \begin{array}{l} (g_1, \{0.1/m_1, 0.2/m_2\}), (g_2, \{0.1/m_1, 0.2/m_2, 0.3/m_3\}), \\ (g_3, \{0.2/m_2, 0.3/m_3, 0.4/m_4\}), \\ (g_4, \{0.4/m_4, 0.5/m_5, 0.6/m_6\}), \\ (g_5, \{0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_6, \{0.2/m_2, 0.3/m_3, 0.4/m_4, 0.7/m_7\}), \\ (g_7, \{0.1/m_1, 0.3/m_3, 0.5/m_5, 0.6/m_6\}), \\ (g_8, \{0.2/m_2, 0.3/m_3, 0.6/m_6, 0.7/m_7\}), \\ (g_9, \{0.2/m_2, 0.3/m_3, 0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_{10}, \{0.1/m_1, 0.3/m_3, 0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_{11}, \{0.2/m_2, 0.4/m_4, 0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_{12}, \{0.1/m_1, 0.2/m_2, 0.3/m_3, 0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_{13}, \{0.2/m_2, 0.3/m_3, 0.5/m_5, 0.7/m_7, 0.8/m_8\}), \\ (g_{14}, \{0.1/m_1, 0.3/m_3, 0.5/m_5, 0.7/m_7, 0.8/m_8\}), \\ (g_{15}, \{0.1/m_1, 0.2/m_2, 0.3/m_3, 0.5/m_5, 0.7/m_7, 0.8/m_8\}), \\ (g_{16}, \{0.4/m_4, 0.5/m_5, 0.6/m_6, 0.7/m_7, 0.8/m_8\}) \end{array} \right\}$$

Definition 3.5. Let $(\Psi_1, G_1), (\Psi_2, G_2) \in \Omega_{(\Psi, G)}$ then (Ψ_1, G_1) is said to be *hypersoft subset* of (Ψ_2, G_2) if

(i) $G_1 \subseteq G_2$
(ii) $\forall g \in G_1, \Psi_1(g) \subseteq \Psi_2(g)$

Example 3.6. Considering example 3.2, if
$(\Psi_1, G_1) = \left\{ \begin{array}{l} (g_1, \{m_1\}), (g_2, \{m_1, m_2\}), (g_3, \{m_2, m_3\}) \end{array} \right\}$
$(\Psi_2, G_2) = \left\{ \begin{array}{l} (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), \\ (g_3, \{m_2, m_3, m_4\}), (g_4, \{m_4, m_5, m_6\}) \end{array} \right\}$
then
$(\Psi_1, G_1) \subseteq (\Psi_2, G_2)$

Definition 3.7. A set $G = G_1 \times G_2 \times G_3 \times ..... \times G_n$ in hypersoft set (Ψ, G) is said to be *Not set* if it has the representation as

$$\ltimes G = \{\ltimes g_1, \ltimes g_2, \ltimes g_3, \ltimes g_4, ..., \ltimes g_m\}$$

where $m = \prod_{i=1}^{n} |G_i|$, each $\ltimes g_i, i = 1, 2, ..., m$, is a *Not* n-tuple element.

Example 3.8. Taking sets $G_1, G_2, G_3, G_4, , G_5$ from example 3.2, we have

$$\ltimes G = \{\ltimes g_1, \ltimes g_2, \ltimes g_3, \ltimes g_4, ..., \ltimes g_{16}\}$$

where each $\ltimes g_i, i = 1, 2, ..., 16$, is a *Not* 5-tuple element.

Definition 3.9. A hypersoft set (Ψ, G_1) is called a *relative null hypersoft set* w.r.t $G_1 \subseteq G$, denoted by $(\Psi, G_1)_\Phi$, if $\Psi(g) = \emptyset, \forall g \in G_1$.

Example 3.10. Considering example 3.2, if

$$(\Psi, G_1)_\Phi = \left\{ \ (g_1, \emptyset), (g_2, \emptyset), (g_3, \emptyset) \ \right\}$$

where $G_1 \subseteq G$.

Definition 3.11. A hypersoft set (Ψ, G_1) is called a *relative whole hypersoft set* w.r.t $G_1 \subseteq G$, denoted by $(\Psi, G_1)_\mathcal{U}$, if $\Psi(g) = \mathcal{U}, \forall g \in G_1$.

Example 3.12. Considering example 3.2, if

$$(\Psi, G_1)_\mathcal{U} = \left\{ \ (g_1,\mathcal{U}), (g_2,\mathcal{U}), (g_3,\mathcal{U}) \ \right\}$$

where $G_1 \subseteq G$.

Definition 3.13. A hypersoft set (Ψ, G) is called a *absolute whole hypersoft set* over $\mathcal{U}$, denoted by $(\Psi, G)_\mathcal{U}$, if $\Psi(g) = \mathcal{U}, \forall g \in G$.

Example 3.14. Considering example 3.2, if

$$(\Psi, G)_\mathcal{U} = \left\{ \begin{array}{l} (g_1,\mathcal{U}), (g_2,\mathcal{U}), (g_3,\mathcal{U}), (g_4,\mathcal{U}), \\ (g_5,\mathcal{U}), (g_6,\mathcal{U}), (g_7,\mathcal{U}), (g_8,\mathcal{U}), \\ (g_9,\mathcal{U}), (g_{10},\mathcal{U}), (g_{11},\mathcal{U}), (g_{12},\mathcal{U}), \\ (g_{13},\mathcal{U}), (g_{14},\mathcal{U}), (g_{15},\mathcal{U}), (g_{16},\mathcal{U}) \end{array} \right\}$$

Proposition 3.15. *Let* $(\Psi_1, G_1), (\Psi_2, G_2), (\Psi_3, G_3) \in \Omega_{(\Psi,G)}$ *with* $G_1, G_2, G_3 \subseteq G$ *then*

(i) $(\Psi_1, G_1) \subseteq (\Psi_1, G_1)_\mathcal{U}$

(ii) $(\Psi_1, G_1)_\Phi \subseteq (\Psi_1, G_1)$

(iii) $(\Psi_1, G_1) \subseteq (\Psi_1, G_1)$

(iv) *If* $(\Psi_1, G_1) \subseteq (\Psi_2, G_2)$ *and* $(\Psi_2, G_2) \subseteq (\Psi_3, G_3)$ *then* $(\Psi_1, G_1) \subseteq (\Psi_3, G_3)$

(v) *If* $(\Psi_1, G_1) = (\Psi_2, G_2)$ *and* $(\Psi_2, G_2) = (\Psi_3, G_3)$ *then* $(\Psi_1, G_1) = (\Psi_3, G_3)$

Definition 3.16. The *complement* of a hypersoft set (Ψ, G), denoted by $(\Psi, G)^{\ominus}$, is defined as

$$(\Psi, G)^{\ominus} = (\Psi^{\ominus}, \ltimes G)$$

where

$$\Psi^{\ominus} : \ltimes G \to P(\mathcal{U})$$

with

$$\Psi^{\ominus}(\ltimes g) = \mathcal{U} \setminus \Psi(g), \forall g \in G$$

Example 3.17. Assuming data from example 3.2, we have

$$(\Psi, G)^{\ominus} = \left\{ \begin{array}{l} (\ltimes g_1, \{m_3, m_4, m_5, m_6, m_7, m_8\}), (\ltimes g_2, \{m_4, m_5, m_6, m_7, m_8\}), \\ (\ltimes g_3, \{m_1, m_5, m_6, m_7, m_8\}), (\ltimes g_4, \{m_1, m_2, m_3, m_7, m_8\}), \\ (\ltimes g_5, \{m_1, m_2, m_3, m_4, m_5\}), (\ltimes g_6, \{m_2, m_3, m_4, m_7\}), \\ (\ltimes g_7, \{m_2, m_4, m_7, m_8\}), (\ltimes g_8, \{m_1, m_4, m_5, m_8\}), \\ (\ltimes g_9, \{m_1, m_4, m_5\}), (\ltimes g_{10}, \{m_2, m_4, m_5\}), \\ (\ltimes g_{11}, \{m_1, m_3, m_5\}), (\ltimes g_{12}, \{m_4, m_5\}), \\ (\ltimes g_{13}, \{m_1, m_4, m_6\}), (\ltimes g_{14}, \{m_2, m_4, m_6\}), \\ (\ltimes g_{15}, \{m_4, m_6\}), (\ltimes g_{16}, \{m_1, m_2, m_3\}) \end{array} \right\}$$

Definition 3.18. The *relative complement* of a hypersoft set (Ψ, G), denoted by $(\Psi, G)^{\circledast}$, is defined as

$$(\Psi, G)^{\circledast} = (\Psi^{\circledast}, G)$$

where

$$\Psi^{\circledast} : G \to P(\mathcal{U})$$

with

$$\Psi^{\circledast}(g) = \mathcal{U} \setminus \Psi(g), \forall g \in G$$

Example 3.19. Assuming data from example 3.2, we have

$$(\Psi, G)^{\circledast} = \left\{ \begin{array}{l} (g_1, \{m_3, m_4, m_5, m_6, m_7, m_8\}), (g_2, \{m_4, m_5, m_6, m_7, m_8\}), \\ (g_3, \{m_1, m_5, m_6, m_7, m_8\}), (g_4, \{m_1, m_2, m_3, m_7, m_8\}), \\ (g_5, \{m_1, m_2, m_3, m_4, m_5\}), (g_6, \{m_2, m_3, m_4, m_7\}), \\ (g_7, \{m_2, m_4, m_7, m_8\}), (g_8, \{m_1, m_4, m_5, m_8\}), \\ (g_9, \{m_1, m_4, m_5\}), (g_{10}, \{m_2, m_4, m_5\}), \\ (g_{11}, \{m_1, m_3, m_5\}), (g_{12}, \{m_4, m_5\}), \\ (g_{13}, \{m_1, m_4, m_6\}), (g_{14}, \{m_2, m_4, m_6\}), \\ (g_{15}, \{m_4, m_6\}), (g_{16}, \{m_1, m_2, m_3\}) \end{array} \right\}$$

Proposition 3.20. *Let* $(\Psi, G) \in \Omega_{(\Psi, G)}$ *then*

(i) $((\Psi, G)^{\ominus})^{\ominus} = (\Psi, G)$

(ii) $((\Psi, G)^{\circledast})^{\circledast} = (\Psi, G)$

(iii) $((\Psi_1, G_1)_{\mathcal{U}})^{\ominus} = (\Psi_1, G_1)_{\Phi} = ((\Psi_1, G_1)_{\mathcal{U}})^{\circledast}$ *where* $G_1 \subseteq G$

(iv) $((\Psi_1, G_1)_{\Phi})^{\ominus} = (\Psi_1, G_1)_{\mathcal{U}} = ((\Psi_1, G_1)_{\Phi})^{\circledast}$ *where* $G_1 \subseteq G$

4. **Set Theoretic Operations on Hypersoft Set**

In this section, set theoretic operations i.e. union, intersection, difference, AND, OR etc., are discussed under hypersoft set environment.

Definition 4.1. *Union of two hypersoft sets* (π, G_1) and (λ, G_2) , denoted by $(\pi, G_1) \cup (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cup G_2$ and for $g \in G_3$,

$$\mu(g) = \begin{cases} \pi(g) & g \in (G_1 \setminus G_2) \\ \lambda(g) & g \in (G_2 \setminus G_1) \\ \pi(g) \cup \lambda(g) & g \in (G_1 \cap G_2) \end{cases}$$

Example 4.2. Let

$(\pi, G_1) = \left\{ \ (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), (g_3, \{m_2, m_3, m_4\}) \ \right\}$

$(\lambda, G_2) = \left\{ \ (g_3, \{m_1, m_2\}), (g_4, \{m_4, m_5, m_6\}), (g_5, \{m_2, m_4, m_6\}) \ \right\}$

then

$$(\mu, G_3) = \left\{ \begin{array}{l} (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), (g_3, \{m_1, m_2, m_3, m_4\}), \\ (g_4, \{m_4, m_5, m_6\}), (g_5, \{m_2, m_4, m_6\}) \end{array} \right\}$$

Definition 4.3. *Intersection of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \cap (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cap G_2$ and for $g \in G_3$,

$$\mu(g) = \pi(g) \cap \lambda(g).$$

Example 4.4. Consider the example 4.2, we have

$$(\mu, G_3) = \left\{ \ (g_3, \{m_2\}) \ \right\}$$

Definition 4.5. *Extended Intersection of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \cap_{\varepsilon} (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cup G_2$ and for $g \in G_3$,

$$\mu(g) = \begin{cases} \pi(g) & g \in (G_1 \setminus G_2) \\ \lambda(g) & g \in (G_2 \setminus G_1) \\ \pi(g) \cap \lambda(g) & g \in (G_1 \cap G_2) \end{cases}$$

Example 4.6. Assuming sets given in example 4.2, we have

$$(\mu, G_3) = \left\{ \begin{array}{l} (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), (g_3, \{m_2\}), \\ (g_4, \{m_4, m_5, m_6\}), (g_5, \{m_2, m_4, m_6\}) \end{array} \right\}$$

Definition 4.7. *OR-operation of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1)\bigvee(\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \times G_2$ and for $(g_i, g_j) \in G_3, g_i \in G_1, g_j \in G_2$,

$$\mu(g_i, g_j) = \pi(g_i) \cup \lambda(g_j).$$

Example 4.8. Consider the example 4.2, we have

$$G_1 \times G_2 = \left\{ \begin{array}{l} h_1 = (g_1, g_3), h_2 = (g_1, g_4), h_3 = (g_1, g_5), h_4 = (g_2, g_3), h_5 = (g_2, g_4), \\ h_6 = (g_2, g_5), h_7 = (g_3, g_3), h_8 = (g_3, g_4), h_9 = (g_3, g_5) \end{array} \right\}$$

then

$$(\mu, G_3) = \left\{ \begin{array}{l} (h_1, \{m_1, m_2\}), (h_2, \{m_1, m_2, m_4, m_5, m_6\}), \\ (h_3, \{m_1, m_2, m_4, m_6\}), (h_4, \{m_1, m_2, m_3\}), \\ (h_5, \{m_1, m_2, m_3, m_4, m_5, m_6\}), (h_6, \{m_1, m_2, m_3, m_4, m_6\}), \\ (h_7, \{m_1, m_2, m_3, m_4\}), (h_8, \{m_2, m_3, m_4, m_5, m_6\}), \\ (h_9, \{m_2, m_3, m_4, m_6\}), \end{array} \right\}$$

Definition 4.9. *AND-operation of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1)\bigwedge(\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \times G_2$ and for $(g_i, g_j) \in G_3, g_i \in G_1, g_j \in G_2$,

$$\mu(g_i, g_j) = \pi(g_i) \cap \lambda(g_j).$$

Example 4.10. Considering data from examples 4.2 and 4.10, we have

$$(\mu, G_3) = \left\{ \begin{array}{l} (h_1, \{m_1, m_2\}), (h_2, \{\}), (h_3, \{m_2\}), (h_4, \{m_1, m_2\}), \\ (h_5, \{\}), (h_6, \{m_2\}), (h_7, \{m_2\}), (h_8, \{m_4\}), (h_9, \{m_2, m_4\}), \end{array} \right\}$$

Definition 4.11. *Restricted Union of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \cup_{\mathcal{R}} (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cap G_2$ and for $g \in G_3$,

$$\mu(g) = \pi(g) \cup \lambda(g).$$

Example 4.12. For sets given in example 4.2, we have

$$(\mu, G_3) = \left\{ \ (g_3, \{m_1, m_2, m_3, m_4\}) \ \right\}$$

Definition 4.13. *Restricted Difference of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \setminus_{\mathcal{R}} (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cap G_2$ and for $g \in G_3$,

$$\mu(g) = \pi(g) - \lambda(g).$$

Example 4.14. For sets given in example 4.2, we have

$$(\mu, G_3) = \left\{ \ (g_3, \{m_3, m_4\}) \ \right\}$$

Definition 4.15. *Restricted Symmetric Difference of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1)\blacktriangle(\lambda, G_2)$, is a hypersoft set (μ, G_3) defined by

$$(\mu, G_3) = \left\{ \ ((\pi, G_1) \cup_{\mathcal{R}} (\lambda, G_2)) \setminus_{\mathcal{R}} ((\pi, G_1) \cap (\lambda, G_2)) \ \right\}$$

or

$$(\mu, G_3) = \left\{ \ ((\pi, G_1) \setminus_{\mathcal{R}} (\lambda, G_2)) \cup_{\mathcal{R}} ((\lambda, G_2) \setminus_{\mathcal{R}} (\pi, G_1)) \ \right\}$$

Example 4.16. For sets given in example 4.2, we have

$$((\pi, G_1) \setminus_{\mathcal{R}} (\lambda, G_2)) = \left\{ \ (g_3, \{m_3, m_4\}) \ \right\}$$

and

$$((\lambda, G_2) \setminus_{\mathcal{R}} (\pi, G_1)) = \left\{ \ (g_3, \{m_1\}) \ \right\}$$

then

$$(\mu, G_3) = \left\{ \ (g_3, \{m_1, m_3, m_4\}) \ \right\}$$

5. Basic Properties and Laws of Hypersoft Set Operations

In this section, some basic properties and laws are discussed for hypersoft set theoretic operations. All hypersoft sets in $\Omega_{(\psi,\mathcal{G})}$ satisfy the following properties, results and laws.

(a) **Idempotent Laws**
  (i) $(\psi, \mathcal{G}) \cup (\psi, \mathcal{G}) = (\psi, \mathcal{G}) = (\psi, \mathcal{G}) \cup_{\mathcal{R}} (\psi, \mathcal{G})$
  (ii) $(\psi, \mathcal{G}) \cap (\psi, \mathcal{G}) = (\psi, \mathcal{G}) = (\psi, \mathcal{G}) \cap_{\varepsilon} (\psi, \mathcal{G})$

(b) **Identity Laws**
  (i) $(\psi, \mathcal{G}) \cup (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G}) = (\psi, \mathcal{G}) \cup_{\mathcal{R}} (\psi, \mathcal{G})_{\Phi}$
  (ii) $(\psi, \mathcal{G}) \cap (\psi, \mathcal{G})_{\mathcal{U}} = (\psi, \mathcal{G}) = (\psi, \mathcal{G}) \cap_{\varepsilon} (\psi, \mathcal{G})_{\mathcal{U}}$
  (iii) $(\psi, \mathcal{G}) \setminus_{\mathcal{R}} (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G}) = (\psi, \mathcal{G})\blacktriangle(\psi, \mathcal{G})_{\Phi}$
  (iv) $(\psi, \mathcal{G}) \setminus_{\mathcal{R}} (\psi, \mathcal{G}) = (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G})\blacktriangle(\psi, \mathcal{G})$

(c) **Domination Laws**
  (i) $(\psi, \mathcal{G}) \cup (\psi, \mathcal{G})_{\mathcal{U}} = (\psi, \mathcal{G})_{\mathcal{U}} = (\psi, \mathcal{G}) \cup_{\mathcal{R}} (\psi, \mathcal{G})_{\mathcal{U}}$
  (ii) $(\psi, \mathcal{G}) \cap (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G}) \cap_{\varepsilon} (\psi, \mathcal{G})_{\Phi}$

(d) **Property of Exclusion**

$$(\psi, \mathcal{G}) \cup (\psi, \mathcal{G})^{\circledast} = (\psi, \mathcal{G})_{\mathcal{U}} = (\psi, \mathcal{G}) \cup_{\mathcal{R}} (\psi, \mathcal{G})^{\circledast}$$

(e) **Property of Contradiction**

$$(\psi, \mathcal{G}) \cap (\psi, \mathcal{G})^{\circledast} = (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G}) \cap_{\varepsilon} (\psi, \mathcal{G})^{\circledast}$$

(f) **Absorption Laws**
  (i) $(\zeta, \mathcal{G}_1) \cup ((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2)) = (\zeta, \mathcal{G}_1)$
  (ii) $(\zeta, \mathcal{G}_1) \cap ((\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2)) = (\zeta, \mathcal{G}_1)$

  - (iii) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} ((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2)) = (\zeta, \mathcal{G}_1)$
  - (iv) $(\zeta, \mathcal{G}_1) \cap_{\varepsilon} ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)) = (\zeta, \mathcal{G}_1)$
- (g) **Commutative Laws**
  - (i) $(\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \cup (\zeta, \mathcal{G}_1)$
  - (ii) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \cup_{\mathcal{R}} (\zeta, \mathcal{G}_1)$
  - (iii) $(\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \cap (\zeta, \mathcal{G}_1)$
  - (iv) $(\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \cap_{\varepsilon} (\zeta, \mathcal{G}_1)$
  - (v) $(\zeta, \mathcal{G}_1) \blacktriangle (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \blacktriangle (\zeta, \mathcal{G}_1)$
- (h) **Associative Laws**
  - (i) $(\zeta, \mathcal{G}_1) \cup ((\xi, \mathcal{G}_2) \cup (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2)) \cup (\psi, \mathcal{G}_3)$
  - (ii) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} ((\xi, \mathcal{G}_2) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3)$
  - (iii) $(\zeta, \mathcal{G}_1) \cap ((\xi, \mathcal{G}_2) \cap (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2)) \cap (\psi, \mathcal{G}_3)$
  - (iv) $(\zeta, \mathcal{G}_1) \cap_{\varepsilon} ((\xi, \mathcal{G}_2) \cap_{\varepsilon} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2)) \cap_{\varepsilon} (\psi, \mathcal{G}_3)$
  - (v) $(\zeta, \mathcal{G}_1) \bigvee ((\xi, \mathcal{G}_2) \bigvee (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \bigvee (\xi, \mathcal{G}_2)) \bigvee (\psi, \mathcal{G}_3)$
  - (vi) $(\zeta, \mathcal{G}_1) \bigwedge ((\xi, \mathcal{G}_2) \bigwedge (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \bigwedge (\xi, \mathcal{G}_2)) \bigwedge (\psi, \mathcal{G}_3)$
- (i) **De Morgans Laws**
  - (i) $((\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2))^{\ominus} = (\zeta, \mathcal{G}_1)^{\ominus} \cap_{\varepsilon} (\xi, \mathcal{G}_2)^{\ominus}$
  - (ii) $((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2))^{\ominus} = (\zeta, \mathcal{G}_1)^{\ominus} \cup (\xi, \mathcal{G}_2)^{\ominus}$
  - (iii) $((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2))^{\circledast} = (\zeta, \mathcal{G}_1)^{\circledast} \cap (\xi, \mathcal{G}_2)^{\circledast}$
  - (iv) $((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2))^{\circledast} = (\zeta, \mathcal{G}_1)^{\circledast} \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)^{\circledast}$
  - (v) $((\zeta, \mathcal{G}_1) \bigvee (\xi, \mathcal{G}_2))^{\ominus} = (\zeta, \mathcal{G}_1)^{\ominus} \bigwedge (\xi, \mathcal{G}_2)^{\ominus}$
  - (vi) $((\zeta, \mathcal{G}_1) \bigwedge (\xi, \mathcal{G}_2))^{\ominus} = (\zeta, \mathcal{G}_1)^{\ominus} \bigvee (\xi, \mathcal{G}_2)^{\ominus}$
  - (vii) $((\zeta, \mathcal{G}_1) \bigvee (\xi, \mathcal{G}_2))^{\circledast} = (\zeta, \mathcal{G}_1)^{\circledast} \bigwedge (\xi, \mathcal{G}_2)^{\circledast}$
  - (viii) $((\zeta, \mathcal{G}_1) \bigwedge (\xi, \mathcal{G}_2))^{\circledast} = (\zeta, \mathcal{G}_1)^{\circledast} \bigvee (\xi, \mathcal{G}_2)^{\circledast}$
- (j) **Distributive Laws**
- (i) $(\zeta, \mathcal{G}_1) \cup ((\xi, \mathcal{G}_2) \cap (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2)) \cap ((\zeta, \mathcal{G}_1) \cup (\psi, \mathcal{G}_3))$
- (ii) $(\zeta, \mathcal{G}_1) \cap ((\xi, \mathcal{G}_2) \cup (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2)) \cup ((\zeta, \mathcal{G}_1) \cap (\psi, \mathcal{G}_3))$
- (iii) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} ((\xi, \mathcal{G}_2) \cap_{\varepsilon} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)) \cap_{\varepsilon} ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3))$
- (iv) $(\zeta, \mathcal{G}_1) \cap_{\varepsilon} ((\xi, \mathcal{G}_2) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2)) \cup_{\mathcal{R}} ((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\psi, \mathcal{G}_3))$
- (v) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} ((\xi, \mathcal{G}_2) \cap (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)) \cap ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3))$
- (vi) $(\zeta, \mathcal{G}_1) \cap ((\xi, \mathcal{G}_2) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2)) \cup_{\mathcal{R}} ((\zeta, \mathcal{G}_1) \cap (\psi, \mathcal{G}_3))$

6. Relations and Functions on Hypersoft Sets

Here we present the notions of relations and functions for hypersoft sets.

Definition 6.1. *Cartesian Product* of two hypersoft sets (Ψ_1, G_1) and (Ψ_2, G_2), denoted by $(\Psi_1, G_1) \times (\Psi_2, G_2)$, is a hypersoft (Ψ_3, G_3) where $G_3 = G_1 \times G_2$ and

$$\Psi_3 : G_3 \to P(\mathcal{U} \times \mathcal{U})$$

defined by

$$\Psi_3(g_i, g_j) = \Psi_1(g_i) \times \Psi_2(g_j) \ \forall \ (g_i, g_j) \in G_3$$

that is

$$\Psi_3(g_i, g_j) = \{(h_i, h_j) : h_i \in \Psi_1(g_i), h_j \in \Psi_2(g_j)\}$$

Definition 6.2. If $(\Psi_1, G_1), (\Psi_2, G_2) \in \Omega_{(\Psi,G)}$ then a relation from (Ψ_1, G_1) to (Ψ_2, G_2) is called *hypersoft set relation* $(\mathfrak{R}, G_4)$ (simply $\mathfrak{R}$) which is the hypersoft subset of $(\Psi_1, G_1) \times (\Psi_2, G_2)$ where $G_4 \subseteq G_1 \times G_2$ and $\forall \ (h_1, h_2) \in G_4, \mathfrak{R}(h_1, h_2) = \Psi_3(h_1, h_2)$, where $(\Psi_3, G_3) = (\Psi_1, G_1) \times (\Psi_2, G_2)$.

Definition 6.3. Let $\mathfrak{R}$ be a hypersoft set relation from (Ψ_1, G_1) to (Ψ_2, G_2) such that $(\Psi_3, G_3) = (\Psi_1, G_1) \times (\Psi_2, G_2)$. Then

(i) Domain of $\mathfrak{R}$ (Dom $\mathfrak{R}$) is a hypersoft set $(\psi, K) \subset (\Psi_1, G_1)$ where

$$K = \{g_i \in G_1 : \Psi_3(g_i, g_j) \in \mathfrak{R} \ forsome \ g_j \in G_2\}$$

and

$$\psi(g_1) = \Psi_1(g_1), \forall \ g_1 \in K$$

(ii) Range of $\mathfrak{R}$ (Range $\mathfrak{R}$) is a hypersoft set $(\xi, L) \subset (\Psi_2, G_2)$ where $L \subset G_2$ and

$$L = \{g_j \in G_2 : \Psi_3(g_i, g_j) \in \mathfrak{R} \ forsome \ g_i \in G_1\}$$

and

$$\xi(g_2) = \Psi_1(g_2), \forall \ g_2 \in L$$

(iii) The inverse of $\mathfrak{R}$ ($\mathfrak{R}^{-1}$) is a hypersoft set relation from (Ψ_2, G_2) to (Ψ_1, G_1) defined by

$$\mathfrak{R}^{-1} = \{\Psi_2(q_j) \times \Psi_1(q_i) : \Psi_1(q_i) \mathfrak{R} \Psi_2(q_j)\}$$

Example 6.4. Let

$$(\Psi_1, G_1) = \left\{ \ \Psi_1(g_1), \Psi_1(g_2), \Psi_1(g_3) \ \right\}, (\Psi_2, G_2) = \left\{ \ \Psi_2(g_4), \Psi_2(g_5), \Psi_2(g_6) \ \right\}$$

$$(\Psi_1, G_1) \times (\Psi_2, G_2) = \left\{ \begin{array}{c} (\Psi_1(g_1) \times \Psi_2(g_4)), (\Psi_1(g_1) \times \Psi_2(g_5)), (\Psi_1(g_1) \times \Psi_2(g_6)), \\ (\Psi_1(g_2) \times \Psi_2(g_4)), (\Psi_1(g_2) \times \Psi_2(g_5)), (\Psi_1(g_2) \times \Psi_2(g_6)), \\ (\Psi_1(g_3) \times \Psi_2(g_4)), (\Psi_1(g_3) \times \Psi_2(g_5)), (\Psi_1(g_3) \times \Psi_2(g_6)) \end{array} \right\}$$

then

$$\mathfrak{R} = \left\{ \ (\Psi_1(g_1) \times \Psi_2(g_4)), (\Psi_1(g_1) \times \Psi_2(g_6)), (\Psi_1(g_2) \times \Psi_2(g_6)), (\Psi_1(g_3) \times \Psi_2(g_6)) \ \right\}$$

(i) $Dom\,\mathfrak{R} = (\psi, K)$ where $K = \{g_1, g_2, g_3\} \subseteq G_1$ and $\psi(g_i) = \Psi_1(g_i) \forall\ g_i \in K$

(ii) $Range\,\mathfrak{R} = (\xi, L)$ where $L = \{g_4, g_6\} \subset G_2$ and $\xi(g_j) = \Psi_2(g_j) \forall\ g_j \in L$

(iii) $\mathfrak{R}^{-1} = \left\{\ (\Psi_2(g_4) \times \Psi_1(g_1)), (\Psi_2(g_6) \times \Psi_1(g_1)), (\Psi_2(g_6) \times \Psi_1(g_2)), (\Psi_2(g_6) \times \Psi_1(g_3))\ \right\}$

Definition 6.5. Let $\mathfrak{R}$ and $\mathfrak{S}$ are two hypersoft set relations on hypersoft set (Ψ, K), then we have

(i) $\mathfrak{R} \subset \mathfrak{S}$, if for all $u, v \in K, \Psi(u) \times \Psi(v) \in \mathfrak{R}$ then $\Psi(u) \times \Psi(v) \in \mathfrak{S}$

(ii) The Complement of $\mathfrak{R}$, denoted by $\mathfrak{R}^{©}$, is defined as

$$\mathfrak{R}^{©} = \{\Psi(u) \times \Psi(v) : \Psi(u) \times \Psi(v) \notin \mathfrak{R}, \forall\ u, v \in K\}$$

(iii) The union of $\mathfrak{R}$ and $\mathfrak{S}$, denoted by $\mathfrak{R} \cup \mathfrak{S}$, defined as

$$\mathfrak{R} \cup \mathfrak{S} = \{\Psi(u) \times \Psi(v) : \Psi(u) \times \Psi(v) \in \mathfrak{R}\ or\ \Psi(u) \times \Psi(v) \in \mathfrak{S}, \forall\ u, v \in K\}$$

(iv) The intersection of $\mathfrak{R}$ and $\mathfrak{S}$, denoted by $\mathfrak{R} \cap \mathfrak{S}$, defined as

$$\mathfrak{R} \cap \mathfrak{S} = \{\Psi(u) \times \Psi(v) : \Psi(u) \times \Psi(v) \in \mathfrak{R}\ and\ \Psi(u) \times \Psi(v) \in \mathfrak{S}, \forall\ u, v \in K\}$$

Example 6.6. Let

$$(\Psi, K) = \left\{\ \Psi(g_1), \Psi(g_2), \Psi(g_3)\ \right\}$$

$$(\Psi, K) \times (\Psi, K) = \left\{ \begin{array}{l} (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_1) \times \Psi(g_3)), \\ (\Psi(g_2) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_3)), \\ (\Psi(g_3) \times \Psi(g_1)), (\Psi(g_3) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_3)) \end{array} \right\}$$

then we have

$$\mathfrak{R} = \left\{\ (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_3)), (\Psi(g_2) \times \Psi(g_3)), (\Psi(g_3) \times \Psi(g_3))\ \right\}$$

and

$$\mathfrak{S} = \left\{\ (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_2))\ \right\}$$

now

(1)

$$\mathfrak{R}^{©} = \left\{\ (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_1)), (\Psi(g_3) \times \Psi(g_2))\ \right\}$$

$$\mathfrak{S}^{©} = \left\{\ (\Psi(g_1) \times \Psi(g_3)), (\Psi(g_2) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_3)), (\Psi(g_3) \times \Psi(g_3))\ \right\}$$

(2)

$$\mathfrak{R} \cup \mathfrak{S} = \left\{ \begin{array}{l} (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_1) \times \Psi(g_3)), (\Psi(g_2) \times \Psi(g_2)), \\ (\Psi(g_2) \times \Psi(g_3)), (\Psi(g_3) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_3)) \end{array} \right\}$$

(3)

$$\mathfrak{R} \cap \mathfrak{S} = \left\{\ (\Psi(g_1) \times \Psi(g_1))\ \right\}$$

Definition 6.7. Let $\mathfrak{R}$ be a hypersoft set relation on (Ψ, K), then

(i) $\mathfrak{R}$ is *reflexive* if $\Psi(u) \times \Psi(u) \in \mathfrak{R}$ for all $u \in K$, e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_1)) \ \Big\}$$

(ii) $\mathfrak{R}$ is *symmetric* if $\Psi(u) \times \Psi(v) \in \mathfrak{R}$ then $\Psi(v) \times \Psi(u) \in \mathfrak{R}$ for all $u, v \in K$, e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_1)) \ \Big\}$$

(iii) $\mathfrak{R}$ is *transitive* if $\Psi(u) \times \Psi(v) \in \mathfrak{R}$ and $\Psi(v) \times \Psi(w) \in \mathfrak{R}$ then $\Psi(u) \times \Psi(w) \in \mathfrak{R}$ for all $u, v, w \in K$, e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_1) \times \Psi(g_3)), (\Psi(g_2) \times \Psi(g_3)) \ \Big\}$$

(iv) $\mathfrak{R}$ is called *equivalence relation* if it is reflexive, symmetric and transitive. e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_2)) \ \Big\}$$

(v) $\mathfrak{R}$ is called *identity* if $\Psi(u) \times \Psi(v) \in \mathfrak{R}$ then $u = v$ for all $u, v \in K$, e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_3)) \ \Big\}$$

Definition 6.8. If $\mathfrak{R}$ is a hypersoft set relation from (Ψ_1, G_1) to (Ψ_2, G_2) and $\mathfrak{S}$ is a hypersoft set relation from (Ψ_2, G_2) to (Ψ_3, G_3) then composition of $\mathfrak{R}$ and $\mathfrak{S}$, denoted by $\mathfrak{R} \circ \mathfrak{S}$, is also a hypersoft set relation $\mathfrak{T}$ from (Ψ_1, G_1) to (Ψ_3, G_3) defined as
if $\Psi_1(u) \in (\Psi_1, G_1)$ and $\Psi_3(w) \in (\Psi_3, G_3)$ then $\Psi_1(u) \times \Psi_3(w) \in \mathfrak{R} \circ \mathfrak{S}$
i.e.
$\Psi_1(u) \times \Psi_3(w) \in \mathfrak{R} \circ \mathfrak{S}$ iff $\Psi_1(u) \times \Psi_2(v) \in \mathfrak{R}$ and $\Psi_2(v) \times \Psi_3(w) \in \mathfrak{R}$

Example 6.9. Let

$$\mathfrak{R} = \Big\{ \ (\Psi_1(g_1) \times \Psi_2(g_1)), (\Psi_1(g_1) \times \Psi_2(g_3)), (\Psi_1(g_2) \times \Psi_2(g_3)), (\Psi_1(g_3) \times \Psi_2(g_3)) \ \Big\}$$

and

$$\mathfrak{S} = \Big\{ \ (\Psi_2(g_1) \times \Psi_3(g_1)), (\Psi_2(g_1) \times \Psi_3(g_2)), (\Psi_2(g_2) \times \Psi_3(g_2)), (\Psi_2(g_3) \times \Psi_3(g_2)) \ \Big\}$$

then

$$\mathfrak{R} \circ \mathfrak{S} = \Big\{ \ (\Psi_1(g_1) \times \Psi_3(g_1)), (\Psi_1(g_1) \times \Psi_3(g_2)), (\Psi_1(g_2) \times \Psi_3(g_2)), (\Psi_1(g_3) \times \Psi_3(g_2)) \ \Big\}$$

Definition 6.10. A hypersoft set relation $\mathfrak{F}$ from (Ψ_1, G_1) to (Ψ_2, G_2), represented by $\mathfrak{F} : (\Psi_1, G_1) \to (\Psi_2, G_2)$, is said to be hypersoft function if

(i) domain of $\mathfrak{F} = G_1$

(ii) there is no repetition of elements in $Dom\, \mathfrak{F}$

(iii) there is a unique element in $Range\, \mathfrak{F}$ corresponding to every element in $Dom\, \mathfrak{F}$.
i.e. if $\Psi_1(u)\, \mathfrak{F}\, \Psi_2(v)$ (or $\Psi_1(u) \times \Psi_2(v) \in \mathfrak{F}$) then $\mathfrak{F}(\Psi_1(u)) = \Psi_2(v)$.

Example 6.11. Let $G_1 = \{u_1, u_2, u_3\}$ and $G_2 = \{v_1, v_2, v_3, v_4\}$ then
$(\Psi_1, G_1) = \left\{ \ \Psi_1(u_1), \Psi_1(u_2), \Psi_1(u_3) \ \right\}$
$(\Psi_2, G_2) = \left\{ \ \Psi_2(v_1), \Psi_2(v_2), \Psi_2(v_3), \Psi_2(v_4) \ \right\}$
so hypersoft functions is

$$\mathfrak{F} = \left\{ \ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_3)), (\Psi_1(u_3) \times \Psi_2(v_4)) \ \right\}$$

Definition 6.12. A hypersoft function $\mathfrak{F} : (\Psi_1, G_1) \to (\Psi_2, G_2)$ is said to be

(i) *into-hypersoft function* if *Range* $\mathfrak{F} \subset G_2$.
e.g. Let $G_1 = \{u_1, u_2, u_3\}$ and $G_2 = \{v_1, v_2, v_3, v_4\}$ then
$\mathfrak{F} = \left\{ \ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_3)), (\Psi_1(u_3) \times \Psi_2(v_4)) \ \right\}$

(ii) *into-hypersoft function (or surjective hypersoft function)* if *Range* $\mathfrak{F} = G_2$.
e.g. Let $G_1 = \{u_1, u_2, u_3, u_4\}$ and $G_2 = \{v_1, v_2, v_3, v_4\}$ then
$\mathfrak{F} = \left\{ \ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_3)), (\Psi_1(u_3) \times \Psi_2(v_4)), (\Psi_1(u_4) \times \Psi_2(v_2)) \ \right\}$

(iii) *one-to-one hypersoft function (or injective hypersoft function)* if $\Psi_1(u_1) \neq \Psi_1(u_2)$ then $\mathfrak{F}(\Psi_1(u_1)) \neq \mathfrak{F}(\Psi_1(u_2))$.
e.g.
$\mathfrak{F} = \left\{ \ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_4)), (\Psi_1(u_3) \times \Psi_2(v_2)), (\Psi_1(u_4) \times \Psi_2(v_3)) \ \right\}$

(iv) *bijective hypersoft function (or one-to-one hypersoft correspondence)* if it is both injective and surjective.
e.g.
$\mathfrak{F} = \left\{ \ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_2)), (\Psi_1(u_3) \times \Psi_2(v_3)), (\Psi_1(u_4) \times \Psi_2(v_4)) \ \right\}$

Definition 6.13. The identity hypersoft set function on hypersoft soft set (Ψ, L) is defined by $\mathfrak{I} : (\Psi, L) \to (\Psi, L)$ such that $\mathfrak{I}(\Psi(l)) = \Psi(l) \ \forall \ \Psi(l) \in (\Psi, L)$.
e.g. Let $L = \{l_1, l_2, l_3, l_4\}$ then
$\mathfrak{I} = \left\{ \ (\Psi(l_1) \times \Psi(l_1)), (\Psi(l_2) \times \Psi(l_2)), (\Psi(l_3) \times \Psi(l_3)), (\Psi(l_4) \times \Psi(l_4)) \ \right\}$

7. Matrix Representation of Hypersoft Set

In this section, matrix representation of hypersoft set is presented.

Definition 7.1.

(i) Let (ζ, H) be a hypersoft set over $\mathcal{U}$. A subset $\mathbb{R}_H$ of $\mathcal{U} \times H$ is said to be relation form of (ζ, H) if it is uniquely represented as

$$\mathbb{R}_H = \left\{ \ (u, h) : h \in H, u \in \zeta(h) \ \right\}.$$

(ii) *The characteristic function* $\mathcal{X}_{\mathbb{R}_H}$ is defined by $\mathcal{X}_{\mathbb{R}_H} : \mathcal{U} \times H \to \{0, 1\}$, where

$$\mathcal{X}_{\mathbb{R}_H}(u, h) = \begin{cases} 1 & ; \quad (u, h) \in \mathbb{R}_H \\ 0 & ; \quad (u, h) \notin \mathbb{R}_H \end{cases}$$

(iii) If $|\mathcal{U}| = m$ and $|H| = n$ then hypersoft set (ζ, H) can be represented by a matrix (α_{ij}) called an $m \times n$ hypersoft matrix of (ζ, H) over $\mathcal{U}$ as given below

$$(\alpha_{ij})_{m\times n} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & .... & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & .... & \alpha_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha_{m1} & \alpha_{m2} & .... & \alpha_{mn} \end{pmatrix}$$

Note: The collection of all $m \times n$ hypersoft matrices over $\mathcal{U}$ is denoted by $HSM(\mathcal{U})_{m\times n}$.

Example 7.2. Let $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5\}$ and $H = \{h_1, h_2, h_3, h_4, h_5\}$ where each h_i is a i^{th} tuple, for $i =$ number of attribute-valued sets. Then

$$\zeta(h_1) = \{u_1, u_2\},\ \ \zeta(h_2) = \emptyset,\ \ \zeta(h_3) = \{u_4, u_5\},\ \ \zeta(h_4) = \{u_2, u_3, u_4, \},\ \ \zeta(h_5) = \emptyset,$$

therefore we have

$$(\zeta, H) = \left\{\ (h_1, \{u_1, u_2\}), (h_3, \{u_4, u_5\}), (h_4, \{u_2, u_3, u_4, \})\ \right\}$$

and

$$\mathbb{R}_H = \left\{\ (u_1, h_1), (u_2, h_1), (u_4, h_3), (u_5, h_3), (u_2, h_4), (u_3, h_4), (u_4, h_4)\ \right\}.$$

Hence hypersoft matrix is given as

$$(\alpha_{ij})_{5\times 5} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} i, j = 1, 2, 3, 4, 5.$$

Definition 7.3. Let $(\alpha_{ij})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$ then $(\alpha_{ij})_{m\times n}$ is said to be:

(i) *A zero (or null) hypersoft matrix*, denoted by $(0)_{m\times n}$, if $\alpha_{ij} = 0\ \forall\ i, j$ e.g.

$$(0)_{5\times 5} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} i, j = 1, 2, 3, 4, 5.$$

(ii) An H_1-*universal hypersoft matrix*, denoted by $(\alpha_{ij})^{H_1}_{m\times n}$, if $\alpha_{ij} = 1, \forall j \in J_{H_1} = \{j : h_j \in H_1\}$ and i.
e.g. Let H be as given in 7.2 and $H_1 = \{h_2, h_4, h_5\} \subseteq H$ with $\zeta(h_2) = \zeta(h_4) = \zeta(h_5) =$

$\mathcal{U}$ then

$$(\alpha_{ij})^{H_1}_{5\times 5} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} i,j = 1,2,3,4,5.$$

(iii) A *universal hypersoft matrix*, denoted by $(\alpha_{ij})^{\mathcal{U}}_{m\times n}$, if $\alpha_{ij} = 1, \forall\ i,j$.

e.g. Let H be as given in 7.2 with $\zeta(h_1) = \zeta(h_2) = \zeta(h_3) = \zeta(h_4) = \zeta(h_5) = \mathcal{U}$ then

$$(\alpha_{ij})^{\mathcal{U}}_{5\times 5} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} i,j = 1,2,3,4,5.$$

Definition 7.4. Let $L_1 = (\alpha_{ij})_{m\times n}, L_2 = (\beta_{ij})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$ then

(i) L_1 is said to be *hypersoft sub-matrix* of L_2, denoted by $L_1 \subseteq L_2$ if $\alpha_{ij} \leq \beta_{ij}$ e.g.

$$L_1 = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \text{ and } L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Note: We may also say that L_1 is dominated by L_2 or L_2 dominates L_1.

(ii) L_1 and L_2 are said to be comparable, denoted by $L_1 \parallel L_2$, if $L_1 \subseteq L_2$ or $L_2 \subseteq L_1$.

(iii) L_1 is said to be *proper hypersoft sub-matrix* of L_2, denoted by $L_1 \subset L_2$ if for atleast one term $\alpha_{ij} \leq \beta_{ij}$ e.g. $L_1 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$ and $L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Note: We may also say that L_1 is dominated properly by L_2 or L_2 properly dominates L_1.

(iv) L_1 is said to be *strictly hypersoft sub-matrix* of L_2, denoted by $L_1 \subsetneqq L_2$ if for each term $\alpha_{ij} \leq \beta_{ij}$ e.g. $L_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ and $L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Note: We may also say that L_1 is dominated strictly by L_2 or L_2 strictly dominates L_1

(v) *union* of L_1 and L_2, denoted by $L_1 \cup L_2$, is also a hypersoft matrix $L_3 = (\delta_{ij})_{m\times n}$ if $\delta_{ij} = max\{\alpha_{ij}, \beta_{ij}\}\ \forall\ i, j$ e.g.

$$\text{Let}\quad L_1 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \text{then}$$

$$L_3 = L_1 \cup L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(vi) *intersection* of L_1 and L_2, denoted by $L_1 \cap L_2$, is also a hypersoft matrix $L_3 = (\delta_{ij})_{m\times n}$ if $\delta_{ij} = min\{\alpha_{ij}, \beta_{ij}\}\ \forall\ i, j$ e.g.

$$\text{Let}\quad L_1 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \text{then}$$

$$L_3 = L_1 \cap L_2 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

(vii) *complement* of $L = (\alpha_{ij})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$, denoted by $L^{\copyright}\,(\mu_{ij})_{m\times n}$, is also a hypersoft matrix if $\mu_{ij} = 1 - \alpha_{ij}\ \forall\ i, j$ e.g.

$$\text{Let}\quad L = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \quad \text{then} \quad L^{\copyright} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

(viii) *difference* of L_1 from L_2, denoted by $L_2 \setminus L_1$, is also a hypersoft matrix L_3 such that $L_3 = L_2 \cap L_1^{\copyright}$ e.g.

$$L_1 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \text{then}$$

$L_3 = L_2 \cap L_1^{©}$

$$L_3 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \cap \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Proposition 7.5. *For* $X = (\alpha_{ij})_{m\times n}, Y = (\beta_{ij})_{m\times n}, Z = (\alpha_{ij})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$*, we have following characteristics properties, operations and laws:*

(i) $X \cup X = X,\ \ X \cap X = X$

(ii) $X \cup (0)_{m\times n} = X,\ \ X \cap (\alpha_{ij})^{\mathcal{U}}_{m\times n} = X$

(iii) $X \cap (0)_{m\times n} = (0)_{m\times n},\ \ X \cup (\alpha_{ij})^{\mathcal{U}}_{m\times n} = (\alpha_{ij})^{\mathcal{U}}_{m\times n}$

(iv) $((0)_{m\times n})^{©} = (\alpha_{ij})^{\mathcal{U}}_{m\times n},\ \ ((\alpha_{ij})^{\mathcal{U}}_{m\times n})^{©} = (0)_{m\times n}$

(v) $X \cup X^{©} = (\alpha_{ij})^{\mathcal{U}}_{m\times n},\ \ X \cap X^{©} = (0)_{m\times n}$

(vi) $(X \cup Y)^{©} = X^{©} \cap Y^{©},\ \ (X \cap Y)^{©} = X^{©} \cup Y^{©}$

(vii) $(X^{©})^{©} = X$

(viii) $X \cup Y = Y \cup X,\ \ X \cap Y = Y \cap X$

(ix) $X \cup (Y \cup Z) = (X \cup Y) \cup Z,\ \ X \cap (Y \cap Z) = (X \cap Y) \cap Z$

(x) $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z),\ \ X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

Definition 7.6. Let $P = (p_{ij})_{m\times n},\ \ Q = (q_{ik})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$, then

(i) *AND- product* of P and Q, denoted by $P \wedge Q$, is defined as $\wedge : HSM(\mathcal{U})_{m\times n} \times HSM(\mathcal{U})_{m\times n} \to HSM(\mathcal{U})_{m\times n^2}$ with $(p_{ij}) \wedge (q_{ik}) = (r_{il})$ where $r_{il} = min\{p_{ij}, q_{ik}\}$ and $l = n(j-1)+k$.

(ii) *OR- product* of P and Q, denoted by $P \vee Q$, is defined as $\vee : HSM(\mathcal{U})_{m\times n} \times HSM(\mathcal{U})_{m\times n} \to HSM(\mathcal{U})_{m\times n^2}$ with $(p_{ij}) \vee (q_{ik}) = (r_{il})$ where $r_{il} = max\{p_{ij}, q_{ik}\}$ and $l = n(j-1)+k$.

(iii) *AND - NOT - product* of P and Q, denoted by $P \overline{\wedge} Q$, is defined as $\overline{\wedge} : HSM(\mathcal{U})_{m\times n} \times HSM(\mathcal{U})_{m\times n} \to HSM(\mathcal{U})_{m\times n^2}$ with $(p_{ij}) \overline{\wedge} (q_{ik}) = (r_{il})$ where $r_{il} = min\{p_{ij}, 1-q_{ik}\}$ and $l = n(j-1)+k$.

(iv) *OR - NOT - product* of P and Q, denoted by $P \veebar Q$, is defined as $\veebar : HSM(\mathcal{U})_{m\times n} \times HSM(\mathcal{U})_{m\times n} \to HSM(\mathcal{U})_{m\times n^2}$ with $(p_{ij}) \veebar (q_{ik}) = (r_{il})$ where $r_{il} = max\{p_{ij}, 1-q_{ik}\}$ and $l = n(j-1)+k$.

Example 7.7. Let $P = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ then

$$\text{(i)}\ \ P \wedge Q = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\text{(ii)}\ \ P \vee Q = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\text{(iii)}\ \ P \barwedge Q = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \wedge \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{(iv)}\ \ P \veebar Q = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \vee \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

8. **Hybrids of Hypersoft Sets**

Smarandache [23] defined some hybrids of hypersoft set. Here we give some more hybrids of hypersoft set. In this section $\mathcal{J} = \mathcal{J}_1 \times \mathcal{J}_2 \times ..... \times \mathcal{J}_m$ with $\mathcal{J}_p \cap \mathcal{J}_q = \emptyset \ \forall \ p, q = 1, 2, ..., m$ where $\mathcal{J}_p$ are attribute-valued sets corresponding to m distinct attributes $j_1, j_2, ..., j_m$ respectively.

Definition 8.1. Let $\mathcal{F}_{ivf}(\mathcal{U})$ be a collection of interval-valued fuzzy sets over $\mathcal{U}$ then a *interval-valued fuzzy hypersoft set*$(ifhs - set)$ $(\Gamma, \mathcal{J})$ over $\mathcal{U}$ is defined as,

$$(\Gamma, \mathcal{J}) = \left\{ \ (j, \Gamma(j)) : j \in \mathcal{J}, \Gamma(j) \in \mathcal{F}_{ivf}(\mathcal{U}) \ \right\}$$

where $\Gamma : \mathcal{J} \rightarrow \mathcal{F}_{ivf}(\mathcal{U})$ and

$$\Gamma(j) = \left\{ \ \mu_{\Gamma(j)}(u)/u : u \in \mathcal{U}, \mu_{\Gamma(j)}(u) \in [0, 1] \ \right\}$$

is an interval-valued fuzzy set over $\mathcal{U}$.

Example 8.2. Let $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ and $\mathcal{J} = \{j_1, j_2, j_3, j_4, j_5, j_6, j_7, j_8\}$ where each j_i is r^{th}-tuple, r is the product of orders of $\mathcal{J}_i$, we have
Interval-Valued fuzzy hypersoft set $(\Gamma, \mathcal{J})$ is given as

$$(\Gamma, \mathcal{J}) = \left\{ \begin{array}{l} (j_1, \{[0.1, 0.2]/u_1, [0.2, 0.3]/u_2, [0.4, 0.5]/u_4, [0.5, 0.6]/u_5\}), \\ (j_2, \{[0.1, 0.3]/u_1, [0.2, 0.4]/u_2, [0.3, 0.4]/u_3, [0.6, 0.8]/u_6\}), \\ (j_3, \{[0.2, 0.3]/u_2, [0.3, 0.4]/u_3, [0.4, 0.5]/u_4, , [0.5, 0.7]/u_5\}), \\ (j_4, \{[0.4, 0.5]/u_4, [0.5, 0.6]/u_5, [0.6, 0.7]/u_6, [0.7, 0.8]/u_7\}), \\ (j_5, \{[0.3, 0.6]/u_3, [0.6, 0.7]/u_6, [0.7, 0.8]/u_7, [0.8, 0.9]/u_8\}), \\ (j_6, \{[0.2, 0.4]/u_2, [0.3, 0.5]/u_3, [0.4, 0.6]/u_4, [0.7, 0.8]/u_7\}), \\ (j_7, \{[0.1, 0.4]/u_1, [0.3, 0.4]/u_3, [0.5, 0.7]/u_5, [0.6, 0.8]/u_6\}), \\ (j_8, \{[0.2, 0.5]/u_2, [0.3, 0.6]/u_3, [0.6, 0.8]/u_6, [0.7, 0.8]/u_7\}) \end{array} \right\}$$

Definition 8.3. A *fuzzy parameterized hypersoft set* ($fphs$-set) $(\mathcal{D}, \mathcal{J})$ over $\mathcal{U}$ is defined as

$$(\mathcal{D}, \mathcal{J}) = \left\{ \ (\zeta_{\mathcal{F}}(j)/j, \psi_{\mathcal{F}}(j)) : j \in \mathcal{J}, \ \psi_{\mathcal{F}}(j) \in P(\mathcal{U}), \ \zeta_{\mathcal{F}}(j) \in [0, 1] \ \right\}$$

where $\mathcal{F}$ is a fuzzy set with $\zeta_{\mathcal{F}} : \mathcal{J} \to [0, 1]$ as membership function of $fphs$-set and $\psi_{\mathcal{F}} : \mathcal{J} \to P(\mathcal{U})$ is called approximate function.

Example 8.4. From example 8.2, we have

$$(\mathcal{D}, \mathcal{J}) = \left\{ \begin{array}{l} (0.1/j_1, \{u_1, u_2\}), (0.2/j_2, \{u_1, u_2, u_3\}), (0.3/j_3, \{u_2, u_3, u_4\}), \\ (0.4/j_4, \{u_4, u_5, u_6\}), (0.5/j_5, \{u_6, u_7, u_8\}), (0.6/j_6, \{u_2, u_3, u_4, u_7\}), \\ (0.7/j_7, \{u_1, u_3, u_5, u_6\}), (0.8/j_8, \{u_2, u_3, u_6, u_7\}) \end{array} \right\}$$

Definition 8.5. An *interval-valued fuzzy parameterized hypersoft set* (iv-$fphs$-set) $(\mathcal{E}, \mathcal{J})$ over $\mathcal{U}$ is defined as

$$(\mathcal{E}, \mathcal{J}) = \left\{ \ (\eta_{\mathcal{F}_{iv}}(j)/j, \gamma_{\mathcal{F}_{iv}}(j)) : j \in \mathcal{J}, \ \gamma_{\mathcal{F}_{iv}}(j) \in P(\mathcal{U}), \ \eta_{\mathcal{F}_{iv}}(j) \in [0, 1] \ \right\}$$

where $\mathcal{F}_{iv}$ is an interval-valued fuzzy set with $\eta_{\mathcal{F}_{iv}} : \mathcal{J} \to [0, 1]$ as membership function of $fphs$-set and $\gamma_{\mathcal{F}_{iv}} : \mathcal{J} \to P(\mathcal{U})$ is called approximate function.

Example 8.6. From example 8.2, we have

$$(\mathcal{E}, \mathcal{J}) = \left\{ \begin{array}{l} ([0.1, 0.2]/j_1, \{u_1, u_2\}), ([0.2, 0.3]/j_2, \{u_1, u_2, u_3\}), ([0.3, 0.4]/j_3, \{u_2, u_3, u_4\}), \\ ([0.4, 0.5]/j_4, \{u_4, u_5, u_6\}), ([0.5, 0.6]/j_5, \{u_6, u_7, u_8\}), ([0.6, 0.7]/j_6, \{u_2, u_3, u_4, u_7\}), \\ ([0.7, 0.8]/j_7, \{u_1, u_3, u_5, u_6\}), ([0.8, 0.9]/j_8, \{u_2, u_3, u_6, u_7\}) \end{array} \right\}$$

Definition 8.7. An *intuitionistic fuzzy parameterized hypersoft set* ($ifphs$-set) $(\mathcal{H}, \mathcal{J})$ over $\mathcal{U}$ is defined as

$$(\mathcal{H}, \mathcal{J}) = \left\{ \ (< \eta_T(j), \eta_F(j) > /j, \gamma_{\mathcal{IF}}(j)) : j \in \mathcal{J}, \ \gamma_{\mathcal{IF}}(j) \in P(\mathcal{U}), \ \eta_T(j) \in [0, 1], \eta_F(j) \in [0, 1] \ \right\}$$

where $\mathcal{IF}$ is an intuitionistic fuzzy set with $\eta_T(j), \eta_F(j) : \mathcal{J} \to [0, 1]$ as truth and falsity membership functions of $ifphs$-set and $\gamma_{\mathcal{IF}} : \mathcal{J} \to P(\mathcal{U})$ is called approximate function.

Example 8.8. From example 8.2, we have

$$(\mathcal{H},\mathcal{J})=\left\{\begin{array}{l}(<0.1,0.2>/j_1,\{u_1,u_2\}),(<0.2,0.3>/j_2,\{u_1,u_2,u_3\}),\\(<0.3,0.4>/j_3,\{u_2,u_3,u_4\}),(<0.4,0.5>/j_4,\{u_4,u_5,u_6\}),\\(<0.5,0.6>/j_5,\{u_6,u_7,u_8\}),(<0.6,0.7>/j_6,\{u_2,u_3,u_4,u_7\}),\\(<0.7,0.8>/j_7,\{u_1,u_3,u_5,u_6\}),(<0.8,0.9>/j_8,\{u_2,u_3,u_6,u_7\})\end{array}\right\}$$

Definition 8.9. A *neutrosophic parameterized hypersoft set* (*nphs*-set) $(\mathcal{N},\mathcal{J})$ over $\mathcal{U}$ is defined as

$$(\mathcal{N},\mathcal{J})=\left\{\begin{array}{l}(<\eta_T(j),\eta_I(j),\eta_F(j)>/j,\gamma_{\mathcal{N}}(j)):j\in\mathcal{J},\gamma_{\mathcal{N}}(j)\in P(\mathcal{U}),\\\eta_T(j)\in[0,1],\eta_I(j)\in[0,1],\eta_F(j)\in[0,1]\end{array}\right\}$$

where $\mathcal{IF}$ is an intuitionistic fuzzy set with $\eta_T(j),\eta_I(j),\eta_F(j):\mathcal{J}\rightarrow[0,1]$ as truth, indeterminacy and falsity membership functions of *nphs*-set and $\gamma_{\mathcal{N}}:\mathcal{J}\rightarrow P(\mathcal{U})$ is called approximate function.

Example 8.10. From example 8.2, we have

$$(\mathcal{N},\mathcal{J})=\left\{\begin{array}{l}(<0.1,0.2,0.2>/j_1,\{u_1,u_2\}),(<0.2,0.3,0.3>/j_2,\{u_1,u_2,u_3\}),\\(<0.3,0.4,0.4>/j_3,\{u_2,u_3,u_4\}),(<0.4,0.5,0.5>/j_4,\{u_4,u_5,u_6\}),\\(<0.5,0.6,0.6>/j_5,\{u_6,u_7,u_8\}),(<0.6,0.7,0.7>/j_6,\{u_2,u_3,u_4,u_7\}),\\(<0.7,0.5,0.8>/j_7,\{u_1,u_3,u_5,u_6\}),(<0.8,0.4,0.9>/j_8,\{u_2,u_3,u_6,u_7\})\end{array}\right\}$$

Definition 8.11. A hypersoft set $(\mathcal{B},\mathcal{J})$ is said to be *bijective hypersoft set* (*bhs*-set) over $\mathcal{U}$ if

(i) $\bigcup_{j\in\mathcal{J}}\mathcal{B}(j)=\mathcal{U}$

(ii) for $j_p,j_q\in\mathcal{J},p\neq q,\mathcal{B}(j_p)\cap\mathcal{B}(j_q)=\emptyset$

Example 8.12. Taking data from example 8.2, we have

$$(\mathcal{B},\mathcal{J})=\left\{(j_1,\{u_1\}),(j_2,\{u_2\}),(j_3,\{u_3\}),(j_4,\{u_4\}),(j_5,\{u_5\}),(j_6,\{u_6\}),(j_7,\{u_7\}),(j_8,\{u_8\})\right\}$$

Definition 8.13. A fuzzy hypersoft set $(\mathcal{B}_f,\mathcal{J})$ is said to be *bijective fuzzy hypersoft set* (*bfhs*-set) over $\mathcal{U}$ if

(i) $\bigcup_{j\in\mathcal{J}}\mathcal{B}_f(j)=\mathcal{U}$ with $\sum_{u\in\mathcal{U}}\mu_f(u)\in[0,\,1]$ where $\mu_f(u)$ is a fuzzy membership for each $u\in\mathcal{U}$

(ii) for $j_p,j_q\in\mathcal{J},p\neq q,\mathcal{B}_f(j_p)\cap\mathcal{B}_f(j_q)=\emptyset$

Example 8.14. Assuming example 8.2, we have

$$(\mathcal{B}_f,\mathcal{J})=\left\{\begin{array}{l}(j_1,\{0.1/u_1\}),(j_2,\{0.2/u_2\}),(j_3,\{0.13/u_3\}),(j_4,\{0.14/u_4\}),\\(j_5,\{0.05/u_5\}),(j_6,\{0.06/u_6\}),(j_7,\{0.07/u_7\}),(j_8,\{0.08/u_8\})\end{array}\right\}$$

Definition 8.15. An interval-valued fuzzy hypersoft set $(\mathcal{B}_{ivf},\mathcal{J})$ is said to be *bijective interval-valued fuzzy hypersoft set* (*biv-fhs*-set) over $\mathcal{U}$ if

(i) $\bigcup\limits_{j\in\mathcal{J}} \mathcal{B}_{ivf}(j) = \mathcal{U}$ with $\sum\limits_{u\in\mathcal{U}} Sup(\mu_f(u)) \in [0,\, 1]$ where $\mu_f(u)$ is an interval-valued fuzzy membership for each $u \in \mathcal{U}$

(ii) for $j_p, j_q \in \mathcal{J}, p \neq q, \mathcal{B}_{ivf}(j_p) \cap \mathcal{B}_{ivf}(j_q) = \emptyset$

Example 8.16. Suppose sets given in example 8.2, we have

$$(\mathcal{B}_{ivf},\mathcal{J}) = \left\{ \begin{array}{l} (j_1, \{[0.01, 0.1]/u_1\}), (j_2, \{[0.02, 0.2]/u_2\}), (j_3, \{[0.03, 0.13]/u_3\}), (j_4, \{[0.04, 0.14]/u_4\}), \\ (j_5, \{[0.03, 0.05]/u_5\}), (j_6, \{[0.02, 0.06]/u_6\}), (j_7, \{[0.03, 0.07]/u_7\}), (j_8, \{[0.04, 0.08]/u_8\}) \end{array} \right\}$$

Definition 8.17. An intuitionistic fuzzy hypersoft set $(\mathcal{B}_{if},\mathcal{J})$ is said to be *bijective intuitionistic fuzzy hypersoft set* (*bifhs*-set) over $\mathcal{U}$ if

(i) $\bigcup\limits_{j\in\mathcal{J}} \mathcal{B}_{if}(j) = \mathcal{U}$ with $\sum\limits_{u\in\mathcal{U}} T_{if}(u) \in [0,\, 1]$ and $\sum\limits_{u\in\mathcal{U}} F_{if}(u) \in [0,\, 1]$ where $T_{if}(u)$ and $F_{if}(u)$ are truth and false membership for each $u \in \mathcal{U}$

(ii) for $j_p, j_q \in \mathcal{J}, p \neq q, \mathcal{B}_{if}(j_p) \cap \mathcal{B}_{if}(j_q) = \emptyset$

Example 8.18. Let the sets provided in example 8.2, we have

$$(\mathcal{B}_{if},\mathcal{J}) = \left\{ \begin{array}{l} (j_1, \{< 0.01, 0.1 > /u_1\}), (j_2, \{< 0.02, 0.2 > /u_2\}), (j_3, \{< 0.03, 0.13 > /u_3\}), \\ (j_4, \{< 0.04, 0.14 > /u_4\}), (j_5, \{< 0.03, 0.05 > /u_5\}), (j_6, \{< 0.02, 0.06 > /u_6\}), \\ (j_7, \{< 0.03, 0.07 > /u_7\}), (j_8, \{< 0.04, 0.08 > /u_8\}) \end{array} \right\}$$

Definition 8.19. An neutrosophic hypersoft set $(\mathcal{B}_{\mathcal{N}},\mathcal{J})$ is said to be *bijective neutrosophic hypersoft set* (*bnhs*-set) over $\mathcal{U}$ if

(i) $\bigcup\limits_{j\in\mathcal{J}} \mathcal{B}_{\mathcal{N}}(j) = \mathcal{U}$ with $\sum\limits_{u\in\mathcal{U}} T_{\mathcal{N}}(u) \in [0,\, 1]$, $\sum\limits_{u\in\mathcal{U}} I_{\mathcal{N}}(u) \in [0,\, 1]$ and $\sum\limits_{u\in\mathcal{U}} F_{\mathcal{N}}(u) \in [0,\, 1]$ where $T_{\mathcal{N}}(u), I_{\mathcal{N}}(u)$ and $F_{\mathcal{N}}(u)$ are truth, indeterminacy and false membership for each $u \in \mathcal{U}$

(ii) for $j_p, j_q \in \mathcal{J}, p \neq q, \mathcal{B}_{\mathcal{N}}(j_p) \cap \mathcal{B}_{\mathcal{N}}(j_q) = \emptyset$

Example 8.20. Considering example 8.2, we have

$$(\mathcal{B}_{\mathcal{N}},\mathcal{J}) = \left\{ \begin{array}{l} (j_1, \{< 0.01, 0.02, 0.1 > /u_1\}), (j_2, \{< 0.02, 0.03, 0.2 > /u_2\}), \\ (j_3, \{< 0.03, 0.04, 0.13 > /u_3\}), (j_4, \{< 0.04, 0.05, 0.14 > /u_4\}), \\ (j_5, \{< 0.03, 0.04, 0.05 > /u_5\}), (j_6, \{< 0.02, 0.05, 0.06 > /u_6\}), \\ (j_7, \{< 0.03, 0.04, 0.07 > /u_7\}), (j_8, \{< 0.04, 0.05, 0.08 > /u_8\}) \end{array} \right\}$$

9. **Conclusions**

In this study, fundamental properties, aggregation operations, basic set laws, relations and functions are characterized under hypersoft set environment. Moreover, essential concepts of matrices and their basic operations are discussed for hypersoft sets. Future work may include the development of hybrids of hypersoft set with fuzzy set, rough set, expert set, cubic set etc. and algebraic structures like hypersoft topological spaces, hypersoft functional spaces, hypersoft groups, hypersoft vector spaces, hypersoft ring, hypersoft measure etc.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Molodtsov, D., Soft Set Theory - First Results, *Comput. Math. with Appl.* **1999**, 37, 19-31.
2. Maji, P.K.; Biswas, R., and Roy, A. R., Soft Set Theory, *Comput. Math. with Appl.* **2003**, 45, 555-562.
3. Pei. D., and Miao. D., From soft set to information system, *In international conference of granular computing, IEEE.* **2005**, 2, 617-621.
4. Ali, M. I.; Feng, F.; Liu, X.; Min, W. K., and Sabir, M., On some new operations in soft set theory, *Comput. Math. with Appl.* **2009**, 57, 1547-1553.
5. Babitha K.V., and Sunil J.J., Soft set relations and functions, *Comput. Math. with Appl.* **2010**, 60, 1840-1849.
6. Babitha K.V., and Sunil J.J., Transitive closure and ordering in soft set, *Comput. Math. with Appl.* **2011**, 61, 2235-2239.
7. Sezgin, A., and Atagün, A. O., On operations of soft sets, *Comput. Math. with Appl.* **2011**, 61(5), 1457-1467.
8. Ge X., and Yang S., Investigations on some operations of soft sets, *World Academy of Science Engineering and Technology* **2011**, 75, 1113-1116.
9. Li, F., Notes on soft set operations, *ARPN Journal of systems and softwares* **2011**, 205-208.
10. Saeed, M., Hussain, M., and Mughal, A. A., A Study of Soft Sets with Soft Members and Soft Elements: A New Approach, *Punjab University Journal of Mathematics* **2020**, 52(8).
11. Khalid, A. and Abbas, M., Distance measures and operations in intuitionistic and interval-valued intuitionistic fuzzy soft set theory, *Int J Fuzzy Syst.* **2005**, 17(3), 490-497.
12. Hassan, N., Sayed, O. R., Khalil, A. M. and Ghany, M. A., Fuzzy soft expert system in prediction of coronary artery disease, *Int J Fuzzy Syst.* **2017**, 19(5), 1546-1559.
13. Feng, F., Li, C., Davvaz, B. and Ali, M. I., Soft sets combined with fuzzy sets and rough sets: a tentative approach, *Soft Computing* **2010**, 14(9), 899-911.
14. Guan, X., Li, Y. and Feng, F., A new order relation on fuzzy soft sets and its application, *Soft Computing* **2013**, 17(1), 63-70.
15. Khameneh, A. Z. and Kilicman, A., Parameter reduction of fuzzy soft sets: An adjustable approach based on the three-way decision, *Int J Fuzzy Syst.* **2018**, 20(3), 928-942.
16. Zhan, J. and Zhu, K., A novel soft rough fuzzy set: Z-soft rough fuzzy ideals of hemirings and corresponding decision making, *Soft Computing* **2017**, 21(8), 1923-1936.
17. Paik, B. and Mondal, S. K., A distance-similarity method to solve fuzzy sets and fuzzy soft sets based decision-making problems, *Soft Computing* **2020**, 24(7), 5217-5229.
18. Akram, M., Ali, G. and Alcantud, J. C. R., New decision-making hybrid model: intuitionistic fuzzy N-soft rough sets, *Soft Computing* **2019**, 23(20), 9853-9868.
19. Zhang, J., Wu, X. and Lu, R., Decision Analysis Methods Combining Quantitative Logic and Fuzzy Soft Sets, *Int J Fuzzy Syst.* **2020**, 1-14.
20. Alshehri, H. A., Abujabal, H. A. and Alshehri, N. O., New types of hesitant fuzzy soft ideals in BCK-algebras, *Soft Computing* **2018**, 22(11), 3675-3683.
21. Çağman, N., Çitak, F., and Enginoğlu, S., Fuzzy parameterized fuzzy soft set theory and its applications, *Turkish Journal of Fuzzy System* **2010**, 1(1), 21-35.
22. Enginoğlu, S. and Çağman, N., Fuzzy parameterized fuzzy soft matrices and their application in decision-making, *TWMS Journal of Applied and Engineering Mathematics* **2020**, 10(4), 1105-1115.
23. Smarandache, F., Extension of Soft Set of Hypersoft Set, and then to Plithogenic Hypersoft Set, *Neutrosophic Sets Syst.* **2018**, 22, 168-170.
24. Saeed, M., Ahsan, M., Siddique, M.k. and Ahmad, M.R., A Study of the Fundamentals of Hypersoft Set Theory, *International Journal of Scientific and Engineering Research* **2020**, 11.