

# Fractal Analysis of Rain Fields for Communications System Design

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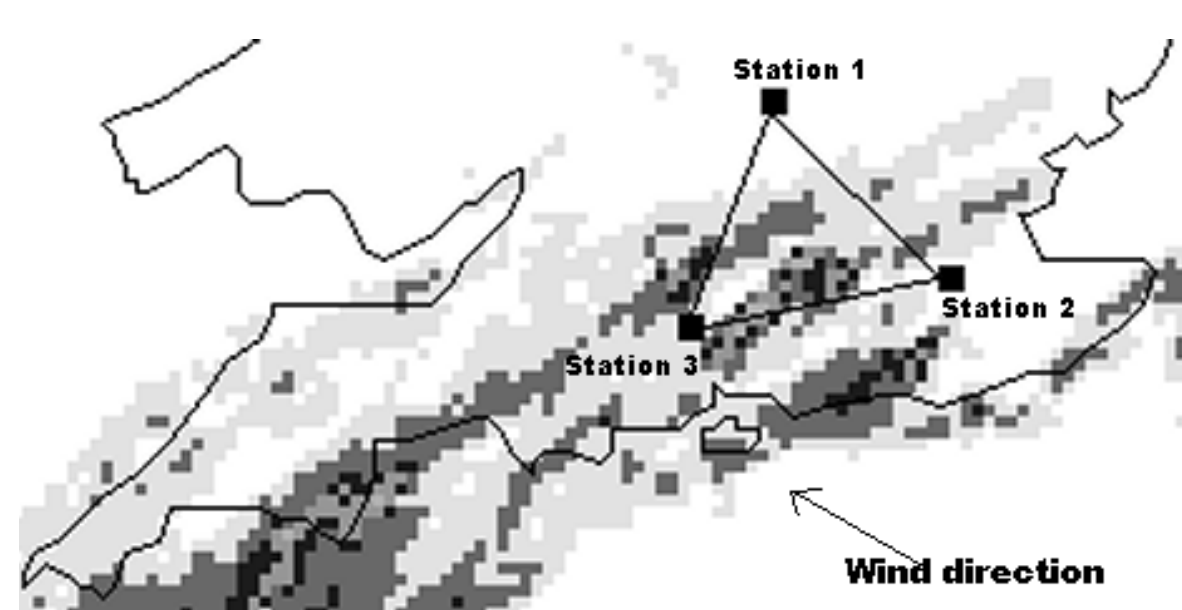
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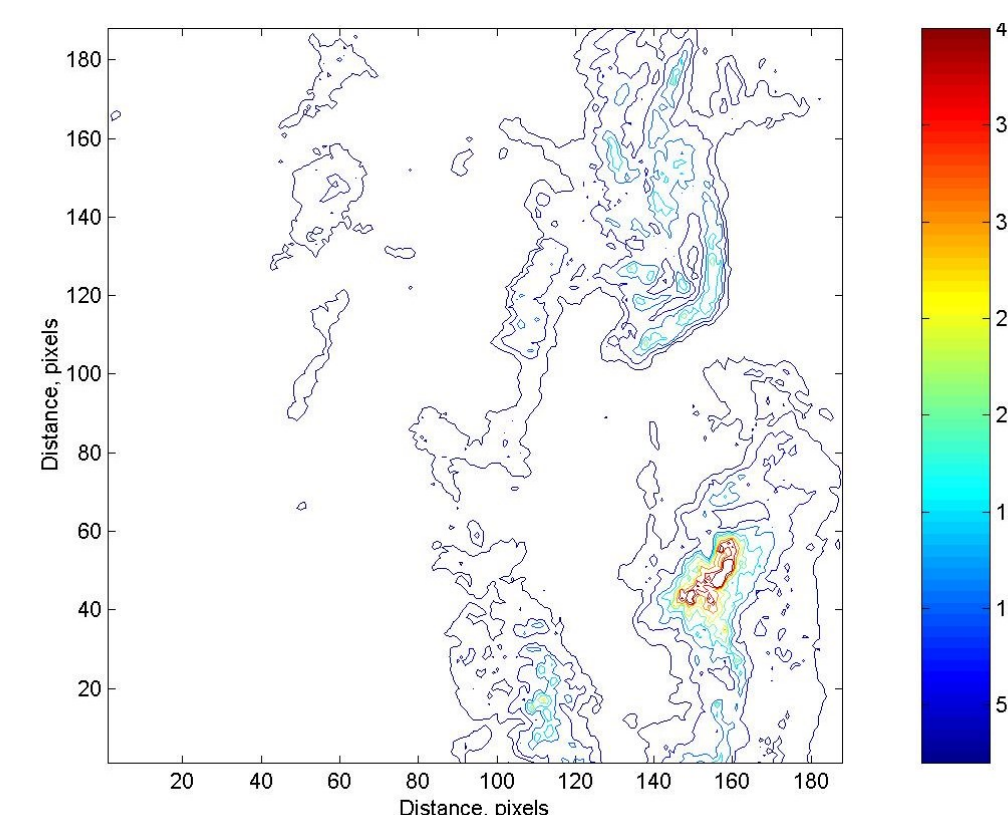


Chilbolton Advanced Meteorological Radar (CAMRa)

Communications systems operating at frequencies above 10 GHz suffer from severe attenuation due to rain, clouds and atmospheric gases, though rain is the dominant factor responsible for most of the time-variant fading experienced. Systems currently in operation deal with this problem by allocating a fixed fade margin. However, for higher frequencies this is economically unfeasible as the fade margin required to provide a set quality of service becomes too high. For this reason, much attention is being paid to methods of dealing with fades on an adaptive and reactive basis. Such methods are collectively known as Fade Mitigation Techniques (FMTs). One FMT, which concerns the (time-variant) two dimensional distribution of the precipitation rate  $R(x,y,t)$  (mm/h), is called Space Diversity.



Example layout of a 3 site communications system in the South of England with measured radar reflectivities of a frontal rain event. The greyscale values show the reflectivity strengths



Example of contour lines produced by a typical rain field as recorded by the radar (above). The contour lines enclose areas of rain greater than or equal to a specified threshold (mm/hr)

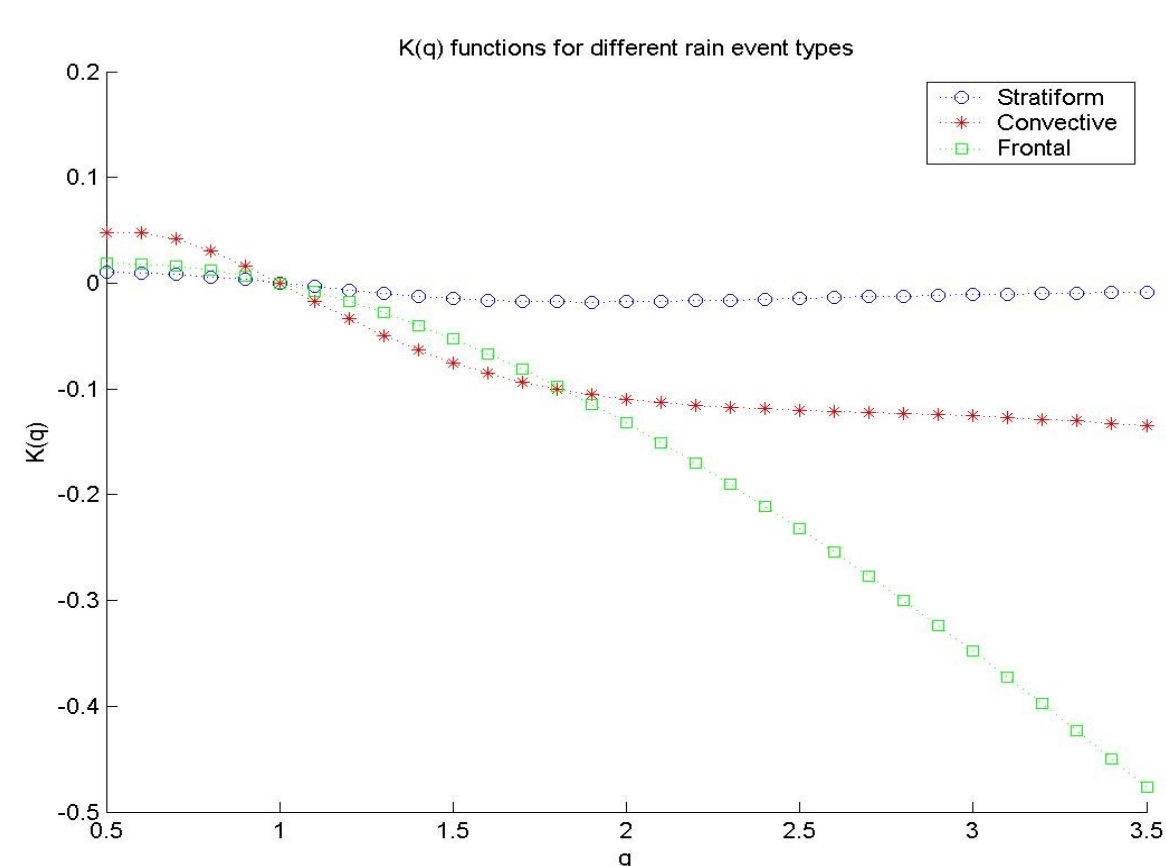
## Measurements, Simulations and Applications: Diversity Gain

Space diversity in a satellite system (also known as site diversity) employs two or more ground stations receiving the same satellite signal with a separation distance sufficiently large so that the rain at the two sites is de-correlated. The sites, in a properly configured arrangement, encounter intense rainfall at different times, and switching to the site experiencing the least fading improves system performance considerably.

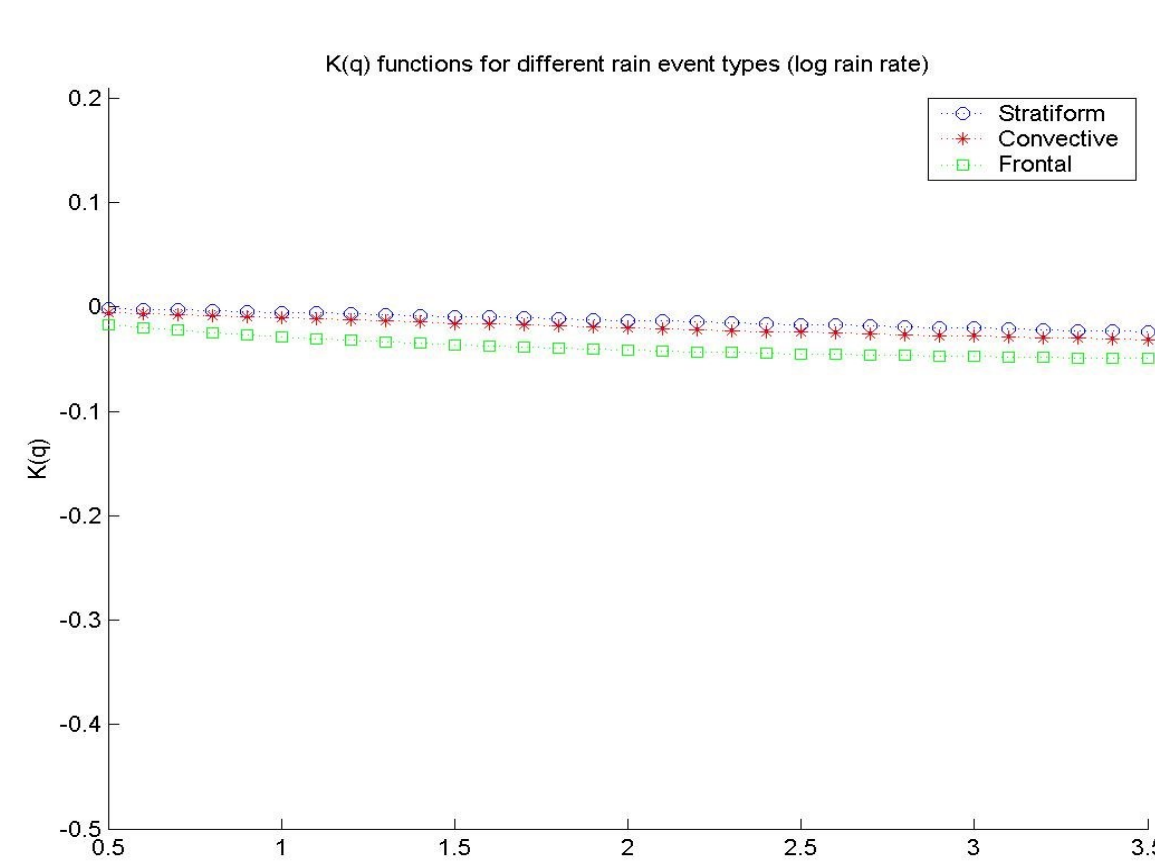
To properly configure such a system requires knowledge of the temporal and spatial variations in rain fields, information which can be provided through a number of methods, including fractal techniques. Simple fractal analysis, such as the area-perimeter method to determine the fractal dimension of rain rate contours, can be used to determine some of the spatial quantities of the rain field which are of interest to systems designers. Similarly, the power laws that govern the distribution of the number of contours with respect to their enclosed area (also known as the Korcak distribution) are useful as inputs in rain variation models, including synthetic storm models, in their own right.

Fractal analysis of rain fields also leads us to the use of fractal methods to simulate visually and statistically realistic rain fields. These then can be used by systems designers in place of the expensive and difficult to obtain radar measurements of rain fields in order to test their proposed systems before deployment. The simulated rain fields presented here have been used in this context, in a case study of a switching algorithm for a two site Earth-space system using site diversity as a FMT.

## Mono- versus multi-fractal analysis for communications system design



$K(q)$  functions calculated from radar derived rain rate measurements of a stratiform, convective and frontal rain event.



$K(q)$  functions calculated from radar derived log rain rate measurements of a stratiform, convective and frontal rain event.

Most fractal models of rain are multifractal, and include times/areas where no rain actually occurs. However, a simplified fractal model of rain suitable for communications engineering purposes can be created by only considering the times when it is raining, and modelling **log rain rate** rather than rain rate.

Multifractal analysis of **rain rate** fields shows a  $K(q)$  function that is curved, a characteristic of multifractal fields. However, multifractal analysis of **log rain rate** fields shows  $K(q)$  functions that can be approximated by straight lines, meaning that monofractal methods may be used to accurately describe them.

This transformation of the representation of the observable, from **rain rate** to **log rain rate** is consistent with the "world-view" of engineers, who are accustomed to dealing with parameters that naturally use logarithmic scales, such as transmit and receive power (dBm), antenna gain (dBi) and attenuation (dB).

## Rain Field Simulators for Communications System Engineering

Rain cell models currently in use in the communications area are often statistical in nature and do not enable the construction of typical two dimensional rain-rate fields, assuming that rain can be adequately described as a field of individual rain cells. Other excellent models have disadvantages in that they only deal with the spatial variation of the rain-rate within a rain cell, or assume regular shapes to the rain cells, such as ellipses, or Gaussian functions of position centred on the area of maximum rain rate.

The rain field simulator used in this research is based on an additive discrete cascade process to simulate fractional Brownian motion, and produces a monofractal field  $X(x,y)$ , which can be used as a power to the base  $b$  (a parameter governed by the required maximum rain rate in the resulting simulated field) in order to convert to a simulated rain rate field  $R(x,y)$  as below:

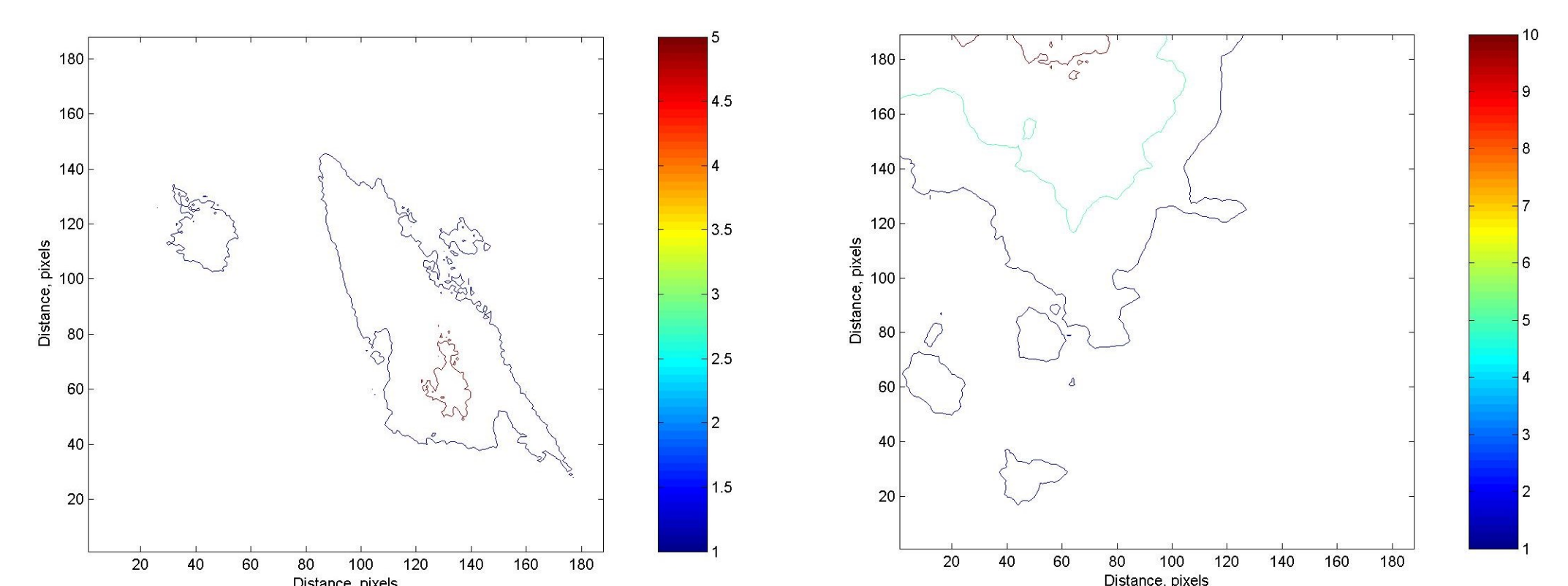
$$R(x, y) = b^{X(x,y)}$$

## Conclusions and Further Work

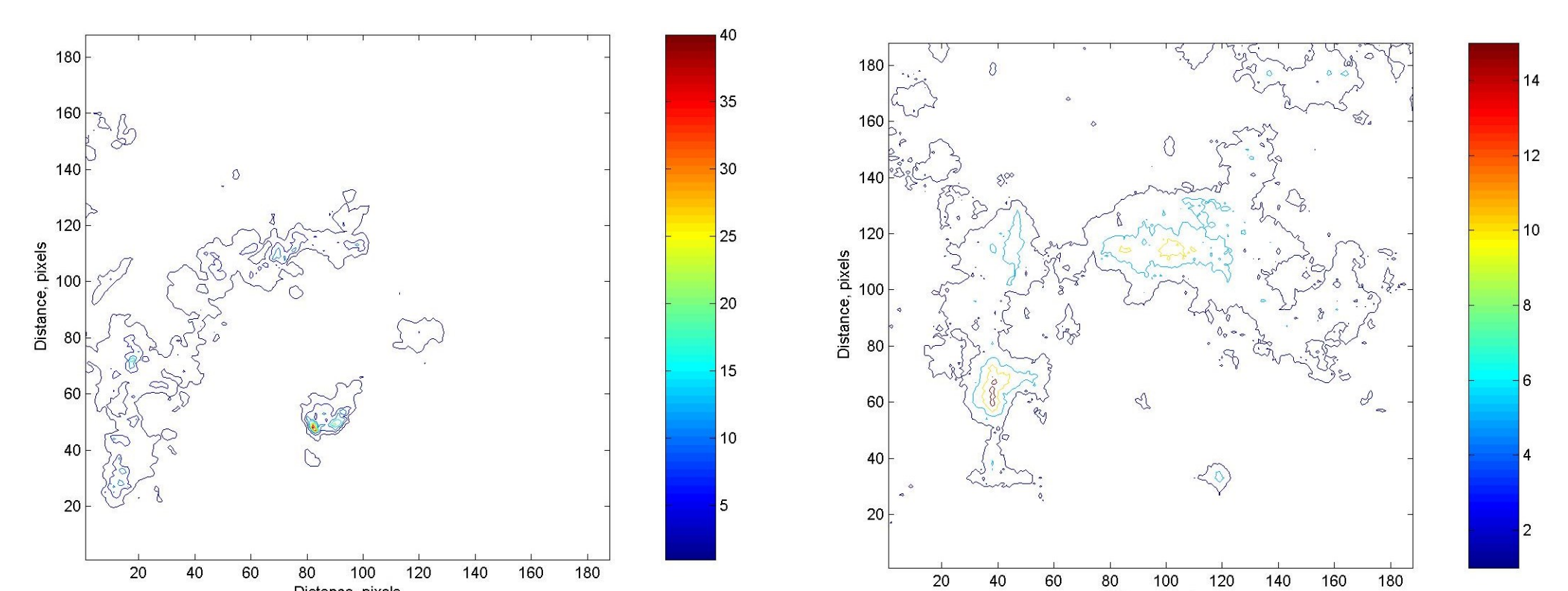
The algorithm used in this research produces rain fields that are visually realistic, as well as having the same statistical properties as are measured in real rain fields. These simulated fields can then be applied to systems design in order to test the dynamic behaviour of a system without the need for costly rain rate measurements.

Further work would involve modifying the present method for introducing temporal variation into the simulator to take better account of dynamic evolution and movement of measured rain fields.

## Example measured rain fields (left) in comparison with simulated rain fields (right)



Stratiform rain fields



Convective rain fields

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