

CHARACTERIZATION OF DROUGHTS IN SERBIA USING STANDARDIZED PRECIPITATION INDEX AND MARKOV CHAINS¹

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SUMMARY: This paper presents an application of homogeneous Markov chain model to time series of SPI in order to characterize droughts in Serbia. The long-term probabilities of occurrence of different drought severity classes, recurrence times, expected residence time in each drought class and expected first passage time to reach the non-drought state were computed. Results are presented in a form of maps and the drought-prone areas are identified. Results of the performed analysis are useful in the context of drought early warning systems and in a planning of drought mitigation practices.

Key words: drought vulnerability map, SPI, Markov chain.

INTRODUCTION

Agricultural production in Serbia over the last decades has been affected by frequent occurrence of drought periods. In the most part of Serbia, climate can be characterized as a moderate continental climate with varying meteorological conditions, particularly quantity and temporal distribution of precipitation. Since Serbia is predominantly an agricultural region, especially Vojvodina region with over 75% of arable land, drought can have considerably negative economic impact. As a consequence of the insufficiently developed irrigation systems, the atmospheric precipitations are still the major factor in providing water to the soil and crops. In a greater or lesser intensity drought occurs regularly in different parts of Serbia and it is the limiting factor of high yields (Dragović, 2001; Spasov, 2003; Dodig et al., 2006; Benka et al., 2010; Rajić et al., 2010).

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Drought is a complex natural disaster that causes serious environmental, social, and economic consequences worldwide (Wilhite et al., 2000). Complex drought phenomenon is usually explained by drought index, which identifies drought characteristics like magnitude, duration, severity and spatial extent of drought. Drought indices are used in analysis of occurrence probability and predictability characteristics of drought events. There are many drought indices and one of the most frequently used is standardized precipitation index – SPI (McKee et al., 1993). Guttman (1997) recommended that the SPI could be used as the primary drought index because it is simple, probability based and spatially invariant in its interpretation, so that it could be used in risk and decision analysis. Although SPI index is not directly related to parameters which determine agricultural drought conditions (such as soil water balance) it was recommended also by Commission of Agrometeorology of the World Meteorological Organization to implement SPI worldwide (WMO, 2009). Analysis of drought occurrences and identification of drought prone regions are necessary for developing long-term water resources management policies and drought mitigation strategies. The Markov chains are commonly used in climatology to evaluate predictability characteristic of meteorological droughts (Lohani et al., 1998; Steinemann, 2003; Abreu et al., 2008). In recent years Paulo et al. (2005) and Paulo and Pereira (2007) presented a Markov chain approach, using both homogeneous and nonhomogeneous formulations, to characterize the stochastic nature of drought and to evaluate the predictability characteristics of drought represented by the SPI. Khalili et al. (2011), in their research, presented a comparability analysis of predictability characteristics of the SPI and more recently developed reconnaissance drought index (RDI).

In this paper the Markov chain approach was applied to time series of SPI in order to characterize droughts in Serbia. Markov chains are used to estimate: the probability of occurrence of different drought severity classes, recurrence time of drought classes, expected residence time in each class of severity and expected first passage time to reach non-drought class.

MATERIAL AND METHODS

The Standardized Precipitation Index (SPI) was developed by McKee et al. (1993) for the purpose of defining and monitoring drought. For its calculation only precipitation data are needed. Its main feature is that it could be computed at different time scales (1, 3, 6, 9, 12, 24 and 48 months) to monitor droughts with respect to different usable water resources. Short-term SPI could be used to detect agricultural drought, and long-term SPI could be used for water supply management. The SPI is computed by fitting historical precipitation data to a probability density function for a specific time period and location. Typically, the gamma distribution is applied. The cumulative distribution function of a Gamma distribution is then transformed to a normal distribution with a mean of zero and standard deviation of one. This procedure enables to simplify interpretation of SPI values as measure of departure of precipitation accumulations from its normal values in units of standard deviation, although precipitation accumulations do not follow normal distribution. Negative values of the SPI are showing severity of dryness and positive values of are showing degree of wetness. Since distribution of SPI fits normal distribution, SPI values between -1 and 1 have 68% probability of occurrence, between -2 and 2 have 95% probability of occurrence and between -3 and 3 have 99%

probability of occurrence. The categories of the SPI, according to McKee et al. (1993) are shown in Table 1.

Table 1. Categorization of moisture condition by SPI, McKee et al. (1993)

Tabela 1. Kategorizacija uslova vlažnosti na osnovu SPI, McKee et al. (1993)

SPI value <i>Vrednost SPI</i>	Moisture conditions <i>Uslovi vlažnosti</i>	Cumulative probability <i>Kumulativna verovatnoća</i>
+2,00 ≤ SPI	Extremely wet / <i>Ekstremno kišno</i>	0.000 - 0.023
+1,50 ≤ SPI ≤ +1,99	Severely wet / <i>Vrlo kišno</i>	0.023 - 0.067
+1,00 ≤ SPI ≤ +1,49	Moderately wet / <i>Umereno kišno</i>	0.067 - 0.159
-0,99 ≤ SPI ≤ +0,99	Near normal / <i>U granicama normale</i>	0.159 - 0.841
-1,00 ≤ SPI ≤ -1,49	Moderate drought / <i>Umereno sušno</i>	0.841 - 0.933
-1,50 ≤ SPI ≤ -1,99	Severe drought / <i>Vrlo sušno</i>	0.933 - 0.977
SPI ≤ -2,00	Extreme drought / <i>Ekstremno sušno</i>	0.977 - 1.000

Monthly precipitation data, period from 1971 to 2010, were used for calculation of the SPI. Data were collected from 27 principal meteorological stations in Serbia. Calibration period from 1971 to 2000 was used to determine parameters of the gamma distribution. The SPI was calculated at 3 month scale (SPI3) for each month.

The Markov chain model was used to investigate the stochastic behavior of drought.

A Markov chain is a stochastic process $\{X_t : t \in T\}$ in which the probability distribution of the random variable X_t at the time $t=t_n$ depends only on the value x_{n-1} at time t_{n-1} , and not on the values of the process at previous times (Winston, 2003). Since the current value is fully determined by the knowledge of only one past period, this Markov chain is said to be of order one. The first-order Markov chain can be expressed as:

$$P[X_{t_n} \leq x_n | X_{t_{n-1}} = x_{n-1}, \dots, X_{t_1} = x_1] = P[X_{t_n} \leq x_n | X_{t_{n-1}} = x_{n-1}] \quad (1)$$

A Markov chain can be viewed as system of states which are describing the status of each random variable. The changes of state of the system are called transitions, and the probabilities associated with various state-changes are called transition probabilities. The set of all states and transition probabilities completely characterizes a Markov chain. The transition probability p_{ij} is the probability of going to state j for the next time period, given that the present state is i . It can be mathematically expressed as:

$$P = [p_{ij}] = P\{X_{t+1} = j | X_t = i\} \quad (2)$$

Estimation of the transition probabilities can be done by calculating the conditional relative frequencies of the transition counts, i.e. counting the number of times that SPI passes from state i to state j as follows:

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}} \quad (3)$$

where, \hat{p}_{ij} is transition probability, n_{ij} is number of transitions from state i to state j and $\sum_j n_{ij}$ is number of occurrences of state i .

If the transition probabilities are independent of the time period under

consideration, then the Markov chain is stationary or homogeneous in time and if the transition probabilities depend on the time period then the Markov chain is nonstationary (nonhomogeneous). The transition probabilities are most conveniently represented by a matrix P , called the transition matrix. The transition matrix of homogeneous first-order Markov chain is:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1j} & \cdots \\ p_{21} & p_{22} & \cdots & p_{2j} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \ddots \\ p_{i1} & p_{i2} & \cdots & p_{ij} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \ddots \end{pmatrix} \quad (4)$$

The transition matrix has a property of the right stochastic matrix since each row of the transition matrix P sums to one and all elements are non-negative. Number of elements of the transition matrix depends on the number of states.

In this study long-term probabilities (steady state), recurrence time, expected residence time and expected first passage time to reach non-drought class were analyzed.

Steady state probabilities are the long-term probabilities of each drought class and they are independent of the initial state of the Markov chain. Steady state probabilities, π_j , are computed as the non-negative solution of the system of linear equations:

$$\pi_j = \sum_{k=1}^S \pi_k p_{kj} \quad , \quad \sum_{j=1}^S \pi_j = 1 \quad , \quad \forall k, j \in S \quad (5)$$

where π_j is the steady state probability of drought class j and S is number of drought classes.

The mean recurrence time, t_{ii} , which is the average time to return to the same drought class, can be computed from the steady state probability π_i as:

$$t_{ii} = \frac{1}{\pi_i} \quad (6)$$

The probability of uninterrupted residence time, duration of drought class i in m month, can be computed as:

$$\begin{aligned} &P(X_1 = i | X_0 = i)P(X_2 = i | X_1 = i) \cdots P(X_{m-2} = i | X_{m-1} = i)P(X_m \neq i | X_{m-1} = i) = \\ &= P_i^{m-1}(1 - P_i) \end{aligned}$$

The expected uninterrupted residence time for class i is given by:

$$E(T_i | X_0) = \sum_k P(m = k | X_0 = 1) \quad (8)$$

The expected first passage time is the average time period taken for the process to move for the first time from some class i to a class j . It is computed as the non-negative

solution of the system of linear equations:

$$t_j = 1 + \sum_{k \neq j} p_k t_k, \quad \forall i, j \in S \quad (9)$$

RESULTS

Values of SPI3 for every month, period from 1971 to 2010, were computed for 27 principal meteorological stations in Serbia. SPI3 was chosen because research conducted by Szalai (2000) showed that considering agricultural drought, there is a significant correlation between 3 month SPI and ground layer soil moisture. Also, Labeledzki (2007) stated that SPI3 well reflect agricultural drought.

The four state first-order homogenous Markov chains were applied to the SPI. Four drought categories (states) were considered: non-drought – “state 1” (SPI>0), mild drought – “state 2” (0≥SPI≥-0.99), moderate drought – “state 3” (-1≥SPI≥-1.5) and severe/extreme drought – “state 4” (SPI<-1.5). Steady state probabilities, recurrence times and residence times were computed for each drought category for every meteorological station. Expected times to reach non-drought state from any drought state are also computed. Results are presented in Table 2.

In order to test the adequacy of application of Markov chain model, the Nash-Sutcliffe efficiency test of hydrological model was performed between empirical steady state probabilities and Markovian steady state probabilities. Test result was 0.999, which is very close to the optimum value of 1.0, confirming the adequacy of using Markov chains.

Table 2. Steady state probabilities, residence times and mean first passage times

Tabela 2. Stacionarne verovatnoće stanja, vremena trajanja i vremena prvog relaza u normalno stanje vlažnosti

Station <i>Stanica</i>	Steady state probability <i>Stacionarne verovatnoće stanja</i>				Residence time (month) <i>Vreme trajanja (mesec)</i>				Mean first passage time to state 1 (month) <i>Vreme prvog prelaza u stanje 1 (mesec)</i>		
	1	2	3	4	1	2	3	4	2	3	4
Becej	0.55	0.29	0.09	0.07	3.8	2.1	1.2	1.8	3.8	5.6	5.8
Beograd	0.55	0.30	0.08	0.07	3.5	1.7	1.1	1.8	3.2	4.6	5.2
Crni Vrh	0.55	0.30	0.08	0.07	3.8	2.0	1.1	1.6	3.9	4.5	5.6
Cuprija	0.55	0.31	0.09	0.06	3.5	1.8	1.2	1.7	3.2	4.2	4.8
Dimitrovgrad	0.61	0.24	0.08	0.07	3.7	1.6	1.1	2.4	2.6	3.5	4.8
Kikinda	0.57	0.28	0.08	0.08	3.8	2.1	1.1	1.7	3.3	4.6	5.1
Kragujevac	0.53	0.32	0.08	0.07	3.6	2.0	1.2	2.0	3.8	4.9	5.3
Kraljevo	0.53	0.32	0.09	0.07	3.4	1.8	1.1	1.5	3.3	3.8	5.0
Krusevac	0.56	0.28	0.09	0.07	3.7	1.7	1.2	1.6	3.5	4.5	5.3
Leskovac	0.56	0.30	0.05	0.08	3.6	1.7	1.1	1.8	3.1	4.1	4.5
Loznica	0.57	0.27	0.09	0.08	3.6	1.6	1.1	1.8	3.0	4.1	5.2
Negotin	0.52	0.31	0.10	0.07	3.7	1.9	1.1	1.8	3.9	4.9	6.1
Nis	0.55	0.31	0.07	0.07	3.7	1.8	1.1	1.3	3.6	4.7	5.1
Palic	0.53	0.34	0.06	0.08	3.7	2.5	1.1	2.0	4.0	5.0	5.5

Pozega	0.52	0.32	0.09	0.07	3.4	1.7	1.2	1.5	3.3	4.5	5.0
Rimski Sancevi	0.55	0.28	0.10	0.07	3.8	1.9	1.3	2.1	4.0	5.5	6.6
Sjenica	0.53	0.32	0.10	0.05	3.2	1.7	1.2	1.2	3.0	3.9	4.0
Sremska Mitrovica	0.54	0.31	0.07	0.08	3.7	2.0	1.1	1.9	3.7	4.9	5.7
Sombor	0.52	0.32	0.09	0.06	3.6	2.0	1.1	1.8	3.7	4.3	5.6
Smederevska Palanka	0.53	0.32	0.08	0.07	3.4	1.9	1.0	1.6	3.2	4.5	5.6
Valjevo	0.56	0.30	0.07	0.08	3.6	1.7	1.1	1.8	3.2	4.0	4.5
Veliko Gradiste	0.55	0.29	0.10	0.07	3.8	2.0	1.1	1.8	3.7	4.9	5.7
Vranje	0.55	0.29	0.10	0.06	3.8	1.8	1.2	1.6	3.7	4.9	5.0
Vrsac	0.54	0.30	0.08	0.08	3.6	1.7	1.2	2.0	3.4	4.8	5.9
Zajecar	0.53	0.33	0.06	0.08	3.6	2.1	1.1	2.4	3.5	4.5	5.6
Zlatibor	0.55	0.32	0.07	0.07	3.4	1.7	1.0	1.6	3.0	4.4	5.1
Zrenjanin	0.52	0.32	0.08	0.08	3.6	2.0	1.1	2.2	3.7	4.6	6.4

Spatial distributions of the results of Markov chain modeling, regarding severe droughts, are presented on Figures 1 - 3. Apart from result values from Markov chain modeling, it was necessary to spatially define station's position. For each station, coordinates are determined in state coordinate system applying Gauss-Krüger projection. Spatial interpolation was performed using Inverse Distance Weighting method. Recurrence times of the severe droughts vary from 12.2 to 19.1 months. Regions of most frequent drought occurrences are Banat, North Backa, West part of Srem, Macva and Kolubara district and East and South-East Serbia. Residence times of severe drought are ranging from 1.2 to 2.4 months. Longest durations of severe droughts are in Central Banat and South-East Serbia. Period needed to reach non-drought state from state of severe drought is from 4 to 6.6 month. Most time needed to reach non-drought state is in Central Vojvodina and in East Serbia surrounding Negotin. Drought vulnerability map, presented on Figure 4, was created based on previous three maps. In the first step, maps of recurrence times, residence times and passage times from drought state 4 to statel were reclassified into five classes and values from 1 to 5 were assigned to each class. Value 1 represents non-vulnerable class, value 5 represents most vulnerable class. Then, all three maps were added together and reclassified again into five classes. Results showed that most vulnerable regions to drought are North and Central Vojvodina and Eastern Serbia.

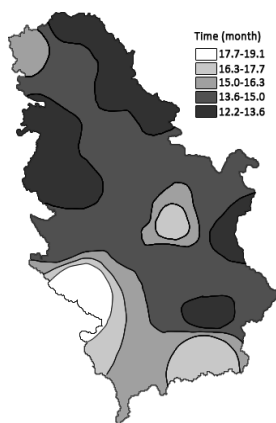


Figure 1. Recurrence time of severe drought
Slika 1. Povratni period jake suše

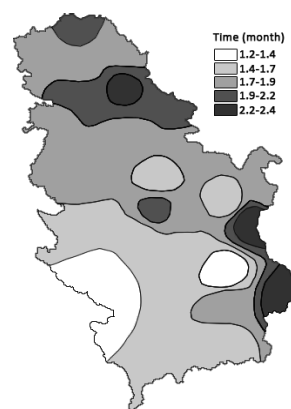


Figure 2. Residence time of severe drought
Slika 2. Vreme trajanja jake suše

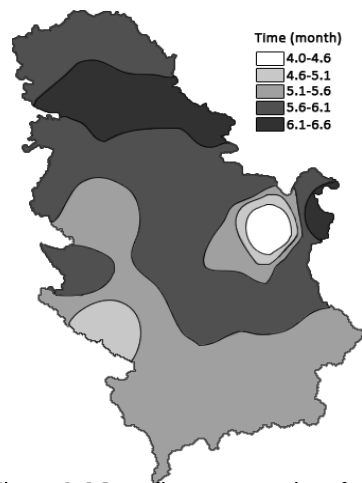


Figure 3. Mean first passage time from severe drought to non-drought state
Slika 3. Potrebno vreme za prelaz iz jake suše u normalno stanje

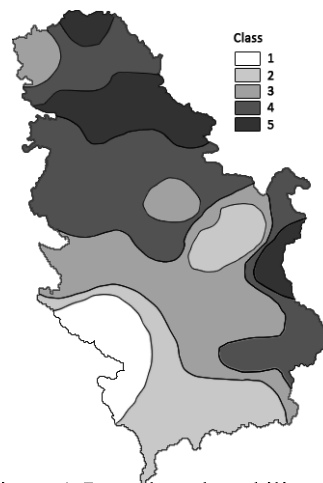


Figure 4. Drought vulnerability map
Slika 4. Karta ugroženosti od suše

DISCUSSION

As in presented paper, the homogeneous formulation of Markov chain has been successfully used by other authors to evaluate predictability characteristics of droughts (Khalili et al., 2011; Abreu et al. 2008; Steinemann, 2003). Non-homogeneous formulation of Markov chains has been used by Lohani et al. (1998), Paulo et al. (2005) and Paulo and Pereira (2007) in order to account the nonstationarity, i.e. dependency of transition probabilities on the initial month. In general, results of application of both formulations agree well, except in short-time forecast of drought classes where the non-homogeneous formulation has some advantages.

Drought occurrences in Serbia have been examined by many authors. Spasov and Zelenhasić (1990) concluded that longest durations of dry spells are in Eastern Serbia. Also, Dodig et al. (2006) stated that drought in Serbia occurs most frequently and it is the most intensive in the region of Eastern Serbia. Those results are in agreement with result presented in this paper. However, results in this paper also show that Central and North part of Vojvodina are equally vulnerable to drought as region of Eastern Serbia and that the spatial extent of drought is much larger in Vojvodina region.

CONCLUSION

This paper presents an application of Markov chains to time series of SPI3 in order to characterize droughts in Serbia. The four state first-order homogenous Markov chains were used to compute long-term probabilities of occurrence of different drought severity classes, recurrence times, expected residence time in each drought class and expected first passage time to reach the non-drought state. Historical time series of precipitation data collected form 27 principal meteorological stations in Serbia were analyzed and results are presented in a form of thematic maps which are convenient to examine the spatial extent of drought. Results indicate that the most drought-prone areas are regions of North and Central Vojvodina and Eastern Serbia.

Drought analysis based on stochastic methods, such as Markov chain model, are

quite useful in the context of drought early warning systems, identification of drought prone areas and in a planning of drought mitigation practices.

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KARAKTERIZACIJA SUŠA U SRBIJI KORIŠĆENJEM STANDARDIZOVANOG INDEKSA PADAVINA I MARKOVLJEVIH LANACA

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Izvod

U radu je prikazana primena modela homogenih Markovljevih lanaca na vremenske serije indeksa SPI za potrebe karakterizacije suša u Srbiji. Proračunate su verovatnoće pojave različitih kategorija jačine suše, povratni periodi, vreme trajanja određene kategorije suše i trajanje prvog prelaza u stanje normalne vlažnosti. Rezultati su predstavljeni u formi karata i identifikovane su oblasti ugrožene sušom. Rezultati sprovedene analize su korisni za potrebe sistema rane najave suše i za potrebe planiranja mera za ublažavanje posledica od suše.

Ključne reči: karta ugroženosti sušom, SPI, Markovljevi lanci.

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