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IDENTIFICATION OF THE PULSE AXISYMMETRIC LOAD ACTING ON A COMPOSITE CYLINDRICAL SHELL, INHOMOGENEOUS IN LENGTH, MADE OF DIFFERENT MATERIALS

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The problem of identifying the load acting on the elements of structures belongs to the class of inverse problems of the mechanics of a deformable solid, which are often incorrect. Solving such problems is associated with the instability of the calculation results, which requires the development of special methods for their research. This predetermines the relevance of this study.

The object of the study is a single-pass cylindrical shell consisting of two rigidly fastened butt-fastened sections made of different materials. Each of the shells is assumed to be elastic isotropic, having a cross-section of medium thickness. The equations of axisymmetric deformation of shells are used within the framework of Timoshenko hypotheses.

An approach to solving direct and inverse problems for such discretely heterogeneous objects is proposed, which implies the conditional separation of a discretely heterogeneous cylindrical shell along the length, followed by the addition of functions of fictitious loads. The main analytical relationships for building a system of integral Volterra equations are given, for which an analytic-numerical solution is derived.

The final ratios have been obtained, which make it possible to calculate the kinematic and force parameters of the study object in the process of non-stationary deformation. The inverse problem of identifying arbitrary loads acting on a shell that is heterogeneous in length is solved in a general form. An algorithm for the restoration of pulse loads has been developed, which is robust to errors in the initial data (about 5 %).

The material related to solving direct and inverse problems for shells that are discretely heterogeneous in length can significantly advance the methodology for identifying pulse loads acting on structural elements

Keywords: cylindrical shell, nonstationary deformation, inverse problem, integral equation, Tikhonov regularization

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1. Introduction

In many cases that involve designing a variety of structures, including the shell type, there are issues related to the correct choice of the value and nature of current loads that are time-variable, which often may be unknown.

Complete information about these loads makes it possible already at the stage of designing a structure to perform its strength calculation and determine the most successful parameters in order to ensure the predefined strength, bearing capacity, and durability. Another problem may also arise – the task of enabling the predetermined oscillation modes. This leads to the need to solve the so-called inverse problems, which involve reversing the causal relationship between deformations and loads.

The most general definition of the inverse problem in the mechanics of a deformable solid implies finding, at a given (in whole or in part) stressed-strained state of a structural element, a fixation system for this element and/or its force (kinematic) perturbation. At the same time, determining the stressed-strained state of a structural element with a known fixation of this element, as well as its known force or kinematic loading, is the result of solving the corresponding direct problem.

The following narrower formulations can be introduced:

1. A direct (dynamic) problem implies determining the dependences of the components of movement or deformation of a structural element over time under a known system of perturbation loads.

2. The inverse (dynamic) problem is to define the law of change over time of the perturbing force or the system of

forces acting on the element of the structure. In this case, it is considered that the change in movement (deformation) over time at some point (or several points) of this element is known.

The class of problems associated with determining (identification) of the fixation system (for example, by finding reactions at the boundary) in many sources, for example [1], is typically referred to as the inverse boundary problems of the mechanics of a deformable solid. However, these classifications often refer to static. In the case of studying the dynamics (especially non-stationary), the task of identifying boundary conditions (which in principle can change over time) is rarely set.

A significant number of inverse problems can be found in hydro- and gas dynamics, thermal physics, electrodynamics, and other branches of physics, including mechanics. The results of studying inverse problems in the mechanics of a deformable solid with non-stationary deformation of objects are very modest.

It is worth noting that solving inverse problems can be used at the initial stages of creating engineering products and in the process of their refinement, which accelerates their design and reduces the cost of implementation.

Mathematical methods and computer technology make it possible to solve complex inverse problems. However, the fundamental difficulties that arise in the process of solving such problems require the development of special approaches and methods for their research.

Fundamental work [2] considers the basics of the theory of deformation of shells, taking into consideration shear strains. As shown in one of the best reviews on non-classical theories of oscillations of rods, plates, and shells [3], they are all based on Timoshenko hypothesis (model).

2. Literature review and problem statement

A broad overview of the state of the issue of the theory of inverse problems and methods for solving them, among which a considerable part is occupied by inverse problems of mechanics of a deformable solid, can be found in monograph [4].

Paper [5] shows that inverse problems have properties that require special mathematical means of solving them. First, such problems are often nonlinear. Second, it is possible that the solution is not unique, and, third, the most unpleasant and difficult to solve property is their instability in relation to small changes in input information. For inverse problems, the error inherent in all measurements can have a very strong effect on the error of restoring any properties of an object. Small errors in the initial data lead to large deviations of approximate and accurate solutions. This leads to the fact that a successful result depends both on the quantity and quality of experimental information, and on the perfection of methods for its processing. Problems with such properties are usually called incorrect.

The concepts of correct and incorrect problems are key in modern mathematical physics. Regularization methods, in particular the Tikhonov method, became particularly widespread in solving incorrect problems [6]. The use of regularization methods, in combination with the correct selection of regulatory parameters, makes it possible to effectively solve the inverse incorrect non-stationary problems of the mechanics of a deformable solid.

Most studies tackling the non-stationary deformation of systems mainly contain solutions to direct problems only. Often, problems of this type for cylindrical shells are solved by numerical methods. The finite-element method (FEM) has become common, which today is one of the most universal ones.

For example, work [7] considers a vacuumed round cylindrical shell immersed in a liquid, subject to external pulse loading. The mathematical model of the system is based on the theory of Reissner-Mindlin shells (Timoshenko type). Numerical simulation is performed.

In [8], a procedure for studying non-stationary oscillations of shells of a heterogeneous structure under the action of short-term dynamic loads is given. The procedure is based on the finite-element model of a thin elastic heterogeneous shell and a reduced model built on its basis for dynamics problems.

Works [9–11] consider the non-stationary loading of multilayer shells. The dynamic behavior of shells is described within the framework of an advanced theory. An analytical approach to the analysis of the vibration of layered orthotropic shells is reported. Shell motion equations and boundary conditions are derived from Hamilton's variational principle. The analytical solution to the problem was obtained by the immersion method. The system of shell equations of motion is integrated by expansion into Taylor series. Thus, in work [9], a multilayer sloping shell is considered under pulsed loading. In [10], multilayer composite shells of complex shape with low-velocity impact are investigated. Separately, it is worth highlighting work [11], which solves the problem of minimizing the mass of layered orthotropic shells of constant thickness during pulse loading. The method of adaptive optimization with hybrid elements is used to solve the problem of optimal design of shells. The influence of geometric parameters on the optimal structure of a two-layer composite shell is investigated.

Works [12–15] are aimed at solving direct problems for cylindrical shells on an elastic base.

In [12], the procedure and software for studying the dynamics of a reinforced cylindrical shell on an elastic base have been developed. The shell is affected by local loads, distributed over small areas, and linearly changing over a short time. The effect of local load on shell deformation parameters is investigated in a test example. Conclusions are drawn about the influence of the pulse shape, the time of action of external forces, and reinforcement on the deformed state of the shell.

Paper [13] sets the problem of forced oscillations of a discretely reinforced cylindrical shell on an elastic base under distributed pulse loading. The dynamic behavior of a heterogeneous cylindrical shell is analyzed using the theory of shells of the Timoshenko type. The problem is solved by the method of finite differences.

Paper [14] gives an overview of experimental work to determine the dynamics of smooth and reinforced cylindrical shells in contact with the ground environment, at various non-stationary loads. The results of the study of three-layer shells of rotation, the equations of motion of which are obtained within the framework of the hypotheses of the geometrically nonlinear theory of Timoshenko, are reported. Numerical results for shells with a piece or discrete aggregate make it possible to analyze the effect of geometric and physical-mechanical parameters of structures on their dynamics and identify new mechanical effects. On the basis of the classical theory of shells and rods, the influence of the discrete arrangement of the edges and the coefficients of the elastic base of Winkler or Pasternak on the normal frequencies and modes of rectangular flat cylindrical and spherical shells is investigated. Using the geometrically nonlinear theory and Timoshenko hypotheses, the equations of oscillations of the ribbed shells of rotation on the elastic base of Winkler or Pasternak were derived. Using the integral-interpolation method, numerical algorithms are developed, and corresponding non-stationary problems are solved.

Paper [15] describes a method for analyzing the stressed-strained state of a ribbed cylindrical shell interacting with an elastic base under dynamic axisymmetric loading. The influence of the duration of loading and the parameters of the foundation on the sagging and bending moments of the shell is analyzed.

In [16], on the basis of the theory of shear deformation of the first order, taking into consideration the influence of shear deformation and the moment of inertia (the Timoshenko type theory), the equilibrium equations of a composite cylindrical shell with stiffeners reinforced with carbon nanotubes (CNTs) are derived. To determine the equivalent stiffness of a composite cylindrical shell supported by a mesh, the blurred stiffness method is used. The Galerkin method was used to solve the equilibrium equations of free and forced oscillations of a composite cylindrical shell supported by edges; the influence of mesh edges on the dynamic response of the shell was studied.

Transient processes caused by moving and simultaneous pulse loads of cylindrical shells have been studied in [17] using numerical modeling.

Paper [18] reports the results of a numerical study into the processes of dynamic deformation of a composite cylindrical shell protected from the inside by a cylindrically folded package of metal woven meshes. Internal loading is achieved by the explosion of a spherical charge. The proposed mathematical model of deformation of metal meshes is supplemented by an experimental diagram of dynamic compression in the direction perpendicular to the grid layers. In the circumferential direction, the value of the effective modulus of elasticity is selected in a series of calculations. The estimated data are compared with the experimental results. The presence of a gas-permeable cylindrical mesh package leads to a decrease in the maximum circumferential deformation of the outer shell.

Study [19] investigates the nonlinear dynamic reaction of layered composite cylindrical shells made of epoxy resin reinforced with glass fiber (GRE) under pulsed load based on the first-order shear deformation theory (FSDT – the Timoshenko type theory) of shells. It is assumed that the Greene deformation and the von K rm n conjecture take into consideration geometric nonlinearity due to the large deformation in the model. The solution procedure consists of a differential quadrature method (DQM) based on the direct projection of the Heaviside function, and a multi-step temporal integration scheme based on an uneven rational B-spline (NURBS). DQM and NURBS are used to discretize basic equations in the spatial and temporal domains, respectively. The approach is supported by showing its rapid rate of convergence and conducting comparative studies with available solutions in limited available cases. Then a comprehensive parametric study of the model is carried out and the influence of geometric parameters, number of layers, load location, time duration, and types of pulse load on nonlinear dynamic reactions of multilayer composite cylindrical shells from GRE is investigated.

Separately, it is worth highlighting those works that are related to the destruction of cylindrical shells. For example, paper [20] considers their detonation and fragmentation under conditions of high deformation rate and high-speed impact. The cited paper uses a numerical model of fluid dynamics of smoothed particles (SPH) to study this problem. The reliability of the model is initially checked by comparing the simulation results with experimental data. It is concluded that different modes of the fractures of cylindrical shells can be obtained using the same model with a single constitutive model and fracture parameters.

In [21], the nonlinear process of dynamic response of a completely pinched cylindrical shell subjected to a lateral local impact of a projectile with a hemispherical head was numerically simulated. The explicit nonlinear dynamic finite-element computer program LS-DYNA and the Plastic Kinematic Cowper-Symonds model were used. Dynamic modes of deformation and destruction of cylindrical shells under various impact conditions were obtained. The minimum impact velocity, which forms cracks in the thickness of the shell wall, termed the rate of destruction, was investigated. The results can be applied to engineering predictions of damage or safety of a cylindrical shell in a lateral local impact.

Studies [22–26] investigate direct and inverse incorrect problems of pulse axisymmetric deformation for cylindrical shells of finite length. In [22], on the basis of the theory of integral equations of Volterra, the problem of controlling non-stationary oscillations of a cylindrical shell of finite length is solved. It is carried out to ensure the fulfillment of the control criterion (a given deflection at some of its points) by introducing additional loads. The procedure for using the theory of incorrect problems of mathematical physics in relation to the problem of identification of control loads for shells is described.

Note works [23, 24], which describe an approximate technique for identifying an arbitrary axisymmetric load acting on a cylindrical shell. Moreover, in [24], not only the temporal but also the spatial component of the load is identified.

The current study employs an approach similar to that described in [22–24], however, unlike [22–24], non-stationary deformation of composite shells is studied as an object.

The following conclusions can be drawn from the above literature review [7–24]:

– most papers [7–24] contain solutions only to direct problems that are solved by numerical methods, often by special software systems based on FEM, for example, LS-DYNA;

– along with a large number of works [8–19] tackling layered, heterogeneous, or reinforced shells, composite cylindrical shells are considered extremely rarely, although they are used in practice;

– the issues of deformation of composite cylindrical shells made of various materials remain insufficiently studied. Moreover, with regard to direct problems, it is possible to obtain at least numerical solutions with the help of FEM systems. It is interesting to derive analytical or analytical-numerical solutions;

– the issues of identification of loads affecting composite shells made of various materials have not been studied at all.

Therefore, it is advisable to conduct a study on devising new methods for solving inverse problems in the field of non-stationary mechanics of a deformable solid for structures in the form of composite shells.

3. The aim and objectives of the study

The purpose of this study is to devise a procedure for solving in general the inverse problem of identifying the effect exerted on the cylindrical shell of the composite length by the axisymmetric non-stationary pulse load. This makes it possible to summarize the results obtained to solve the problems of deformation of complex cylindrical shells, thereby significantly advancing the methodology for identifying pulse loads affecting structural elements.

To accomplish the aim, the following tasks have been set:

– to build a mathematical model for solving the direct problem of non-stationary deformation of a cylindrical shell heterogeneous in length;

– to construct a mathematical model for solving the inverse problem of non-stationary deformation of a composite cylindrical shell;

– to conduct numerical modeling and obtain numerical results of solving a direct problem to study the stressed-strained state of a cylindrical shell heterogeneous in length;

– to perform numerical modeling and obtain numerical results of solving the inverse problem of identifying an axisymmetric non-stationary pulse load acting on a cylindrical shell of a composite length.

4. The study materials and methods

The object of this study is a single-span cylindrical shell of constant cross-section with a length l, radius a, and thickness h, consisting of two rigidly fastened sections made of different materials.

During the simulation, a refined model of a cylindrical shell of medium thickness was used. The model is linear; the shell operates under conditions of elastic deformations; within each site, the shell is homogeneous.

The system is exposed to some pulsed load distributed over a section of the surface with a length of $2b_p$ at a distance x_p from one of the ends with the law of change in time $P(t)$, as a result of which the shell executes oscillations. The integrity of the shell in the process of oscillations is enabled by the rigid fastening of its parts at the place of their docking.

A fairly detailed diagram of the composite shell and the load acting on it is shown in Fig. 1, *a*. Different parts of the shell are fastened at a point with an axial coordinate x_0. The ends of the shell are fixed and hinged with slippage along the fastening supports.

While considering such problems on the oscillations of the composite single-span cylindrical shell, we note the similarity in the formulation with the previously considered problem on the oscillations of the composite beam described in [27].

The solution will be sought using the method of fictitious loads, as well as some special techniques based on the conditional replacement of each of the mentioned parts of the shell with shells of other lengths. In this case, each of the shells is supposed to be made of different materials.

The solution to the problem is based on the method of artificial dismemberment of the deformable system into two subsystems. Let's conditionally separate the sections of the shell made of different materials, replacing the reaction in the bonding with some force, which we denote as a function. Fig. 1, *b* shows the diagram of the first beam of the system affected by the said load.

In the course of the study, the methods of mathematical physics, Fourier series theory, Laplace integral transformation, operator theory, Volterra's theory of integral equations, Tikhonov's regulating algorithm, matrix calculus were applied.

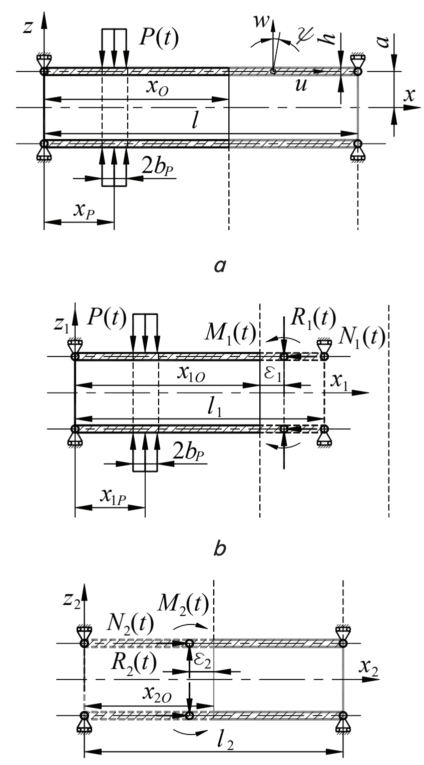


Fig. 1. Cylindrical shell: *a* – initial scheme of the shell and the loads acting on it; *b* – the first section of the disconnected shell; *b* – the second section of the disconnected shell

The reliability of the results of solving the direct problem was confirmed by a good agreement of the results when comparing with the data obtained with the help of FEM, as well as with the results of solving the problem for a homogeneous shell, reported in [24].

The reliability of the solution to the inverse problem is confirmed by the correspondence of the data obtained during identification with the results of solving the direct problem.

5. Results of investigating the non-stationary deformation of the cylindrical shell heterogeneous in length

5. 1. Solution to the direct problem of non-stationary deformation of the shell that is heterogeneous in length with its pulsed loading

We shall study the non-stationary deformation of the shell under consideration using a theory based on the Timoshenko model. Work [3] gives a system of three equations with respect to deflection w, axial displacement u, and the angle of transverse shear ψ:

$$\begin{cases} \dfrac{\partial^2 u}{\partial \xi^2} + \dfrac{\nu l}{a}\dfrac{\partial w}{\partial \xi} - \dfrac{\partial^2 u}{\partial t^2} = \dfrac{l^2}{E'} n(\xi;t); \\ \bar{k}^2\left(\dfrac{\partial^2 w}{\partial \xi^2} + l\dfrac{\partial \psi}{\partial \xi}\right) - \dfrac{1}{a}\left(\dfrac{l^2}{a}w + \nu l\dfrac{\partial u}{\partial \xi}\right) - \\ -\dfrac{\partial^2 w}{\partial t^2} = \dfrac{l^2}{E'} q(\xi;t); \\ \dfrac{h^2}{12}\dfrac{\partial^2 \psi}{\partial \xi^2} - \bar{k}^2\left(l\dfrac{\partial w}{\partial \xi} + \psi l^2\right) - \dfrac{h^2}{12}\dfrac{\partial^2 \psi}{\partial t^2} = \dfrac{l^2}{E'} m(\xi;t), \end{cases} \quad (1)$$

where E is the modulus of elasticity; ν is the Poisson coefficient; ρ is the density of the shell material; $E'=\frac{Eh}{1-\nu^2}$; $\xi=x/l$; $t=\frac{\bar{t}\sqrt{E}}{l\sqrt{\rho(1-\nu^2)}}$; $\bar{t}$ – time; $\bar{k}^2=\frac{1-\nu}{2}k^2$; k^2 is the coefficient of shear; $n(\xi;t)$, $q(\xi;t)$, $m(\xi;t)$ are the functions that characterize the loads from the effects of axial force, lateral force, and bending moment, respectively.

It is worth noting that the equations given in (1) are based on the assumption that the cylindrical shell is homogeneous, and they cannot be directly used in solving the problem for the composite shell. However, it is possible to apply this theory to the shell areas separately. To do this, it is necessary to carry out a conditional disconnection of the composite shell, replacing each of its parts with a single-span shell made of the appropriate material. The introduced single-span shells should be supplemented with sections of some length and support. The index j=1; 2 indicates the left and right parts of the composite shell. A detailed scheme of the transition to single-span shells is shown in Fig. 1, *a–c*.

The interaction of parts of a composite heterogeneous shell is simulated using concentrated dummy loads described by time functions. $N_j(t)$ and $R_j(t)$ are forces acting along the axis of the shells and perpendicular to the axis, respectively. $M_j(t)$ is the concentrated external moments applied to each of the introduced homogeneous shells at some distance $\varepsilon_j(t)$ from the plane of connection of the sections.

On the basis of the above scheme (Fig. 1, *a–c*) of non-stationary deformation of one composite shell, we proceed to solving the problem of deformation of two homogeneous shells, to which the perturbation load $P(t)$ and unknown functions of reactions $N_j(t)$, $R_j(t)$, and $M_j(t)$ are applied. Subsequently, the implementation of the contact of the sites is simulated by the appropriate selection of fictitious forces and moments.

The first shell of the system is shown in Fig. 1, *a*. As can be seen, the shell is affected by the perturbation load $P(t)$, as well as concentrated reactions $N_1(t)$, $R_1(t)$, and $M_1(t)$. The problem of deformation of this shell is solved on the assumption that the function of the perturbation load is predetermined while the resulting reactions are unknown.

In the system of deformation equations of the first shell, the index j=1 is omitted for convenience since the subsequent transformations contain parameters related only to the first shell. Then we come to equations of form (1).

The laws of change of loads included in the mentioned equations will be written in the following form:

$$n(\xi,t)=N(t)\delta(\xi-\xi_O);$$

$$q(\xi,t)=P(t)\begin{bmatrix}H(\xi-(\xi_P-\xi_b))-\\-H(\xi-(\xi_P+\xi_b))\end{bmatrix}+R(t)\delta(\xi-\xi_O); \quad (2)$$

$$m(\xi,t)=M(t)\delta(\xi-\xi_O),$$

where the coordinate ξ_O is the distance to the concentrated load, which, for the diagram shown in Fig. 1, *b*, corresponds to $\xi_O=\xi_{1O}+\varepsilon_1$. $\delta(\xi-\xi_O)$ is the Dirac delta function, which simulates the concentrated nature of the load that occurs in the bonding. $H(\xi)$ is a Heaviside function.

The dimensionless quantities ξ_O, ξ_P, and ξ_b in the expressions correspond to the coordinates $x_{1O}+l_1\varepsilon_1$, x_{1P} and the extent of section b_P.

The solution to the system of equations (1) with respect to the unknown functions u, w, and ψ will be looked for assuming zero initial conditions. Boundary conditions correspond to the articulated support of the shell ends with slippage, which will correspond in coordinates x=0 and $x=l$ to the conditions:

$$w=0;\ M=0;\ N=0. \quad (3)$$

The desired functions (1) are written as decompositions into the following trigonometric series:

$$u(\xi,t)=\sum_{k=1}^{\infty}u_k(t)\cos\lambda_k\xi;\quad w(\xi,t)=\sum_{k=1}^{\infty}w_k(t)\sin\lambda_k\xi;$$

$$\psi(\xi,t)=\sum_{k=1}^{\infty}\psi_k(t)\cos\lambda_k\xi, \quad (4)$$

where $\lambda_k=k\pi$.

After substituting expression (4) into the system of equations (1), the equation is integrated by the variable ξ in the interval [0; 1], and the orthogonality property of trigonometric functions is applied. As a result, we obtain

$$\begin{cases}\lambda_k^2u_k(t)-\frac{\nu l}{a}\lambda_kw_k(t)+\ddot{u}_k(t)=N_k(t);\\ -\bar{k}^2\left(\lambda_k^2w_k(t)+l\lambda_k\psi_k(t)\right)-\\ -\frac{1}{a}\left(\frac{l^2}{a}w_k(t)-\nu l\lambda_ku_k(t)\right)+\ddot{w}_k(t)=Q_k(t);\\ \frac{h^2\lambda_k}{12}\psi_k(t)+\bar{k}^2\left(l\lambda_kw_k(t)+l^2\psi_k(t)\right)+\\ +\frac{h^2}{12}\ddot{\psi}_k(t)=M_k(t),\end{cases} \quad (5)$$

where

$$c=\frac{2l}{E'};\quad c_k^N=-c\cdot\cos\lambda_k\xi_O;$$

$$c_k^P=c\cdot\frac{2l}{\lambda_k}\sin\lambda_k\xi_P\sin\lambda_k\xi_b;\quad c_k^R=c\cdot\sin\lambda_k\xi_O;$$

$$c_k^M=c\cdot\cos\lambda_k\xi_O;\quad N_k(t)=c_k^NN(t);$$

$$M_k(t)=c_k^MM(t);\quad Q_k(t)=c_k^PP(t)+c_k^RR(t).$$

Substituting decompositions (4) into a system of partial differential equations (1) and taking advantage of the orthogonality property of trigonometric functions, we come to the system of ordinary differential equations for the variable t.

The system of differential equations is solved as follows [28]: under zero initial conditions, the direct integral Laplace transform is performed [29]. In the space of images, on the basis of the solution to the system of algebraic equations, the desired coefficients of decomposition are u_k^L, w_k^L, ψ_k^L derived; the inverse Laplace transformation is performed. As a result, the following expression is obtained to determine the axial movements:

$$u_k(t)=c_k^N\int_0^t\sum_{i=1}^3 D_{ik}^{UN}\sin\omega_{ik}(t-\tau)N(\tau)\mathrm{d}\tau-$$
$$-c_k^P\int_0^t\sum_{i=1}^3 D_{ik}^{UQ}\sin\omega_{ik}(t-\tau)\mathrm{d}\tau P(\tau)-$$
$$-c_k^R\int_0^t\sum_{i=1}^3 D_{ik}^{UQ}\sin\omega_{ik}(t-\tau)R(\tau)\mathrm{d}\tau+$$
$$+c_k^M\int_0^t\sum_{i=1}^3 D_{ik}^{UM}\sin\omega_{ik}(t-\tau)M(\tau)\mathrm{d}\tau;$$

$$w_k(t)=c_k^N\int_0^t\sum_{i=1}^3 D_{ik}^{WN}\sin\omega_{ik}(t-\tau)N(\tau)\mathrm{d}\tau-$$
$$-c_k^P\int_0^t\sum_{i=1}^3 D_{ik}^{WQ}\sin\omega_{ik}(t-\tau)\mathrm{d}\tau P(\tau)-$$
$$-c_k^R\int_0^t\sum_{i=1}^3 D_{ik}^{WQ}\sin\omega_{ik}(t-\tau)R(\tau)\mathrm{d}\tau+$$
$$+c_k^M\int_0^t\sum_{i=1}^3 D_{ik}^{WM}\sin\omega_{ik}(t-\tau)M(\tau)\mathrm{d}\tau;$$

$$\psi_k(t)=c_k^N\int_0^t\sum_{i=1}^3 D_{ik}^{\Psi N}\sin\omega_{ik}(t-\tau)N(\tau)\mathrm{d}\tau-$$
$$-c_k^P\int_0^t\sum_{i=1}^3 D_{ik}^{\Psi Q}\sin\omega_{ik}(t-\tau)\mathrm{d}\tau P(\tau)-$$
$$-c_k^R\int_0^t\sum_{i=1}^3 D_{iki}^{\Psi Q}\sin\omega_{ik}(t-\tau)R(\tau)\mathrm{d}\tau+$$
$$+c_k^M\int_0^t\sum_{i=1}^3 D_{ik}^{\Psi M}\sin\omega_{ik}(t-\tau)M(\tau)\mathrm{d}\tau, \quad (6)$$

where

$$A_k=\bar{k}^2\lambda_k^2+l^2a^{-2};\quad B_k=\lambda_k^2;$$
$$C_k=\lambda_k^2+12\bar{k}^2l^2h^{-2};\quad e_k=A_k+B_k+C_k;$$
$$f_k=A_kB_k+A_kC_k++B_kC_k-B_k\left(\frac{\nu^2l^2}{a^2}+\frac{12\bar{k}^4l^2}{h^2}\right);$$
$$g_k=A_kB_kC_k-\frac{\nu^2l^2}{a^2}B_kC_k-\frac{12\bar{k}^4l^2}{h^2}B_k^2;$$
$$\Delta'_k=3\omega_{ik}^5-2e_k\omega_{ik}^3+f_k\omega_{ik};$$
$$D_{ik}^{UN}=\left(\left(A_k-\omega_{ik}^2\right)\left(C_k-\omega_{ik}^2\right)-\frac{12}{h^2}\bar{k}^4l^2\lambda_k^2\right)\Big/\Delta'_k(\omega_{ik});$$
$$D_{ik}^{UQ}=\left(\left(C_k-\omega_{ik}^2\right)\frac{\nu l\lambda_k}{a}\right)\Big/\Delta'_k(\omega_{ik});$$
$$D_{ik}^{UM}=\left(\frac{12}{h^2}\frac{\nu l^2\lambda_k^2\bar{k}^2}{a}\right)\Big/\Delta'_k(\omega_{ik});$$
$$D_{ik}^{WN}=D_{ik}^{UQ};\quad D_{ik}^{WQ}=\frac{\left(B_k-\omega_{ik}^2\right)\left(C_k-\omega_{ik}^2\right)}{\Delta'_k(\omega_{ik})};$$
$$D_{ik}^{WM}=\left(\frac{12}{h^2}\bar{k}^2l\lambda_k\left(B_k-\omega_{ik}^2\right)\right)\Big/\Delta'_k(\omega_{ik});$$
$$D_{ik}^{\Psi N}=D_{ik}^{UM};\quad D_{ik}^{\Psi Q}=D_{ik}^{WM};$$
$$D_{ik}^{\Psi M}=\left(\frac{12}{h^2}\left(A_k-\omega_{ik}^2\right)\left(B_k-\omega_{ik}^2\right)-\frac{\nu^2l^2\lambda_k^2}{a^2}\right)\Big/\Delta'_k(\omega_{ik}).$$

Note that ω_{ik} is the modules of the imaginary roots of the equation $\Delta_k(s)=0$;

$$\Delta_k(s)=\frac{h^2}{12}\left(s^6+e_ks^4+f_ks^2+g_k\right).$$

An expression to determine the desired values after substitution:

$$u(\xi,t)=\int_0^t K_P^U(\xi,t-\tau)P(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_N^U(\xi,t-\tau)N(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_R^U(\xi,t-\tau)R(\tau)\mathrm{d}\tau+$$
$$+\int_0^t K_M^U(\xi,t-\tau)M(\tau)\mathrm{d}\tau;$$

$$w(\xi,t)=\int_0^t K_P^W(\xi,t-\tau)P(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_N^W(\xi,t-\tau)N(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_R^W(\xi,t-\tau)R(\tau)\mathrm{d}\tau+$$
$$+\int_0^t K_M^W(\xi,t-\tau)M(\tau)\mathrm{d}\tau;$$

$$\psi(\xi,t)=\int_0^t K_P^\Psi(\xi,t-\tau)P(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_N^\Psi(\xi,t-\tau)N(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_R^\Psi(\xi,t-\tau)R(\tau)\mathrm{d}\tau+$$
$$+\int_0^t K_M^\Psi(\xi,t-\tau)M(\tau)\mathrm{d}\tau, \quad (7)$$

where

$$K_P^U(\xi,t)=\sum_{k=1}^\infty c_k^P\cos\lambda_k\xi\sum_{i=1}^3 D_{ki}^{UQ}\sin\omega_{ik}t;$$
$$K_N^U(\xi,t)=\sum_{k=1}^\infty c_k^N\cos\lambda_k\xi\sum_{i=1}^3 D_{ki}^{UN}\sin\omega_{ik}t;$$
$$K_R^U(\xi,t)=\sum_{k=1}^\infty c_k^R\cos\lambda_k\xi\sum_{i=1}^3 D_{ki}^{UQ}\sin\omega_{ik}t;$$
$$K_M^U(\xi,t)=\sum_{k=1}^\infty c_k^M\cos\lambda_k\xi\sum_{i=1}^3 D_{ki}^{UN}\sin\omega_{ik}t;$$
$$K_P^W(\xi,t)=\sum_{k=1}^\infty c_k^P\sin\lambda_k\xi\sum_{i=1}^3 D_{ki}^{WQ}\sin\omega_{ik}t;$$

$$K_N^W(\xi,t)=\sum_{k=1}^{\infty}c_k^N\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{UW}\sin\omega_{ik}t;$$

$$K_R^W(\xi,t)=\sum_{k=1}^{\infty}c_k^R\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{WQ}\sin\omega_{ik}t;$$

$$K_M^W(\xi,t)=\sum_{k=1}^{\infty}c_k^M\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{WN}\sin\omega_{ik}t;$$

$$K_P^\Psi(\xi,t)=\sum_{k=1}^{\infty}c_k^P\cos\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{\Psi Q}\sin\omega_{ik}t;$$

$$K_N^\Psi(\xi,t)=\sum_{k=1}^{\infty}c_k^N\cos\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{\Psi N}\sin\omega_{ik}t;$$

$$K_R^\Psi(\xi,t)=\sum_{k=1}^{\infty}c_k^R\cos\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{\Psi Q}\sin\omega_{ik}t;$$

$$K_M^\Psi(\xi,t)=\sum_{k=1}^{\infty}c_k^M\cos\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{UN}\sin\omega_{ik}t.$$

Specific forces, according to the accepted theory of deformation of the shell, can be written in the following form [3]:

$$N_x=E'\left(\frac{1}{l}\frac{\partial u}{\partial\xi}+\frac{\nu}{a}w\right);\quad Q=\bar{k}^2E'\left(\frac{1}{l}\frac{\partial w}{\partial\xi}+\psi\right);$$

$$M_x=\frac{E'h^2}{12l}\frac{\partial\psi}{\partial\xi}. \qquad (8)$$

After substitution (7) in (8), expressions are obtained to determine the desired specific forces:

$$N_x(\xi,t)=\int_0^t K_P^N(\xi,t-\tau)P(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_N^N(\xi,t-\tau)N(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_R^N(\xi,t-\tau)R(\tau)\mathrm{d}\tau+$$
$$+\int_0^t K_M^N(\xi,t-\tau)M(\tau)\mathrm{d}\tau+\frac{E'\nu}{a}w(\xi,t);$$

$$Q_x(\xi,t)=\int_0^t K_P^Q(\xi,t-\tau)P(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_N^Q(\xi,t-\tau)N(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_R^Q(\xi,t-\tau)R(\tau)\mathrm{d}\tau+$$
$$+\int_0^t K_M^Q(\xi,t-\tau)M(\tau)\mathrm{d}\tau+\bar{k}^2E'\cdot\psi(\xi,t);$$

$$M_x(\xi,t)=\int_0^t K_P^M(\xi,t-\tau)P(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_N^M(\xi,t-\tau)N(\tau)\mathrm{d}\tau-$$
$$-\int_0^t K_R^M(\xi,t-\tau)R(\tau)\mathrm{d}\tau+$$
$$+\int_0^t K_M^M(\xi,t-\tau)M(\tau)\mathrm{d}\tau, \qquad (9)$$

where

$$K_P^N(\xi,t)=-\frac{E'}{l}\sum_{k=1}^{\infty}\lambda_k c_k^P\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{UQ}\sin\omega_{ik}t;$$

$$K_N^N(\xi,t)=-\frac{E'}{l}\sum_{k=1}^{\infty}\lambda_k c_k^N\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{UN}\sin\omega_{ik}t;$$

$$K_R^N(\xi,t)=-\frac{E'}{l}\sum_{k=1}^{\infty}\lambda_k c_k^R\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{UQ}\sin\omega_{ik}t;$$

$$K_M^N(\xi,t)=-\frac{E'}{l}\sum_{k=1}^{\infty}\lambda_k c_k^M\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{UN}\sin\omega_{ik}t;$$

$$K_P^Q(\xi,t)=\frac{\bar{k}^2E'}{l}\sum_{k=1}^{\infty}\lambda_k c_k^P\cos\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{WQ}\sin\omega_{ik}t;$$

$$K_N^Q(\xi,t)=\frac{\bar{k}^2E'}{l}\sum_{k=1}^{\infty}\lambda_k c_k^N\cos\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{UW}\sin\omega_{ik}t;$$

$$K_R^Q(\xi,t)=\frac{\bar{k}^2E'}{l}\sum_{k=1}^{\infty}\lambda_k c_k^R\cos\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{WQ}\sin\omega_{ik}t;$$

$$K_M^Q(\xi,t)=\frac{\bar{k}^2E'}{l}\sum_{k=1}^{\infty}\lambda_k c_k^M\cos\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{WN}\sin\omega_{ik}t;$$

$$K_P^M(\xi,t)=-\frac{E'h^2}{12l}\sum_{k=1}^{\infty}\lambda_k c_k^P\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{\Psi Q}\sin\omega_{ik}t;$$

$$K_N^M(\xi,t)=-\frac{E'h^2}{12l}\sum_{k=1}^{\infty}\lambda_k c_k^N\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{\Psi N}\sin\omega_{ik}t;$$

$$K_R^M(\xi,t)=-\frac{E'h^2}{12l}\sum_{k=1}^{\infty}\lambda_k c_k^R\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{\Psi Q}\sin\omega_{ik}t;$$

$$K_M^M(\xi,t)=-\frac{E'h^2}{12l}\sum_{k=1}^{\infty}\lambda_k c_k^M\sin\lambda_k\xi\sum_{i=1}^{3}D_{ki}^{UN}\sin\omega_{ik}t.$$

It is easy to derive similar expressions for the second shell, the loading scheme of which is shown in Fig. 1, *c*.

Further transformations that were made when solving the problem necessitated the use of the dimensionless time parameter common to both shells. This is due to the fact that the dimensionless time contains parameters that depend on the length of the shell and its material:

$$t_1=\frac{\bar{t}\sqrt{E_1}}{l_1\sqrt{\rho_1\left(1-\nu_1^2\right)}};\quad t_2=\frac{\bar{t}\sqrt{E_2}}{l_2\sqrt{\rho_2\left(1-\nu_2^2\right)}}. \qquad (10)$$

The linear relationship between these two quantities is obvious:

$$t_2=C_t t_1;\quad C_t=\frac{l_1}{l_2}\sqrt{\frac{E_2\rho_1\left(1-\nu_1^2\right)}{E_1\rho_2\left(1-\nu_2^2\right)}}. \qquad (11)$$

The total dimensionless time was taken to be t_1, hereinafter denoted as t. The expressions corresponding to (7), (9) for the second shell are obtained under assumption (11) and taking into consideration the index j=2.

To comply with the condition of rigid bonding of the sections of the composite shell in the docking plane, six contact conditions must be met, namely, the coincidence of three kinematic and three force parameters in this plane. To this end, equate expressions sequentially for dynamic parameters at the specified point. Consider the condition of coincidence of the axial movements of the shells $u_1(\xi_{1O}, t)$ and $u_2(\xi_{2O}, t)$, that is, $u_1(\xi_{1O}, t)=u_2(\xi_{2O}, t)$. The expanded notation of this equality is as follows:

$$\begin{aligned}&-\int_0^t K_{1P}^U(\xi_{1O},t-\tau)P(\tau)\mathrm{d}\tau+\\&+\int_0^t K_{1N}^U(\xi_{1O},t-\tau)N_1(\tau)\mathrm{d}\tau-\\&+\int_0^t K_{1R}^U(\xi_{1O},t-\tau)R_1(\tau)\mathrm{d}\tau-\\&-\int_0^t K_{1M}^U(\xi_{1O},t-\tau)M_1(\tau)\mathrm{d}\tau=\\&=-\int_0^t K_{2N}^U(\xi_{2O},t-\tau)N_2(\tau)\mathrm{d}\tau-\\&-\int_0^t K_{2R}^U(\xi_{2O},t-\tau)R_2(\tau)\mathrm{d}\tau+\\&+\int_0^t K_{2M}^U(\xi_{2O},t-\tau)M_2(\tau)\mathrm{d}\tau.\end{aligned} \tag{12}$$

Using the approximation of the type of rectangles in time of functions dependent on t [30], we transform equation (12) using the matrix form of the notation. The perturbation load function $P(t)$ included in the equation, within the framework of the direct problem under consideration, is assumed to be known while the other functions of concentrated reactions are unknown. Given this fact, let's write in the left part of the equation terms containing only unknown vectors. Equation (12) is transformed to this form:

$$\begin{aligned}&\mathbf{A}_{11}\cdot\mathbf{N}_1+\mathbf{A}_{12}\cdot\mathbf{R}_1+\mathbf{A}_{13}\cdot\mathbf{M}_1+\\&+\mathbf{A}_{14}\cdot\mathbf{N}_2+\mathbf{A}_{15}\cdot\mathbf{R}_2+\mathbf{A}_{16}\cdot\mathbf{M}_2=\mathbf{A}_{17}\cdot\mathbf{P}.\end{aligned} \tag{13}$$

The equation contains the unknown vectors $\mathbf{N_j}$, $\mathbf{R_j}$, and $\mathbf{M_j}$ corresponding to the functions $N_j(t)$, $R_j(t)$, and $M_j(t)$, the vector $\mathbf{P}$ is the load function $P(t)$ and their corresponding matrix analogs of operators $\mathbf{A_{11}}$–$\mathbf{A_{17}}$.

The form of the other five equations, reflecting the conditions for fastening the shells, is similar. A detailed record of them is not given. Finally, the system of linear algebraic equations (SLAE) displaying the bonding conditions will take the form

$$\begin{cases}\mathbf{A}_{11}\cdot\mathbf{N}_1+\mathbf{A}_{12}\cdot\mathbf{R}_1+\mathbf{A}_{13}\cdot\mathbf{M}_1+\\+\mathbf{A}_{14}\cdot\mathbf{N}_2+\mathbf{A}_{15}\cdot\mathbf{R}_2+\mathbf{A}_{16}\cdot\mathbf{M}_2=\mathbf{A}_{17}\cdot\mathbf{P};\\\qquad\vdots\\\mathbf{A}_{61}\cdot\mathbf{N}_1+\mathbf{A}_{62}\cdot\mathbf{R}_1+\mathbf{A}_{63}\cdot\mathbf{M}_1+\\+\mathbf{A}_{64}\cdot\mathbf{N}_2+\mathbf{A}_{65}\cdot\mathbf{R}_2+\mathbf{A}_{66}\cdot\mathbf{M}_2=\mathbf{A}_{67}\cdot\mathbf{P}.\end{cases} \tag{14}$$

The subsequent solution to SLAE (14) regarding the unknowns involved a generalized Gaussian algorithm [30]. In addition, when determining the unknowns, there is a need to use a regulating algorithm since it is necessary to numerically solve the integral equations of Volterra of the first kind. Tikhonov's regulating algorithm was used [6].

The reactions obtained in solving SLAE, in addition to information for their analysis, make it possible to determine the kinematic and force parameters of the composite cylindrical shell during deformation by substituting them into appropriate expressions.

5. 2. Solving the inverse problem of non-stationary deformation of a composite cylindrical shell

The problem of finding an unknown temporary change in the load acting on the composite cylindrical shell is considered. The initial data for load recovery are the law of deflection change $\tilde{w}(t)$, given at some point x_S of the system depicted in Fig. 1, *a*. Point x_S is located on the shell section j=1.

At the first stage, we solve the inverse problem in a similar way to solving the direct problem involving the methods and techniques described above, up to the construction of SLAE (14), reflecting the conditions for fastening the shell sections. The difference in solving the inverse problem from the direct problem at this stage is that the perturbation load function (14) $P(t)$ included in the equation is now unknown.

The considered system of equations is supplemented by an expression reflecting the condition for setting the deflection $\tilde{w}(t)$, which made it possible to obtain the following SLAE:

$$\begin{cases}A_{11}\cdot N_1+A_{12}\cdot R_1+A_{13}\cdot M_1+\\+A_{14}\cdot N_2+A_{15}\cdot R_2+A_{16}\cdot M_2+A_{17}\cdot P=0;\\\qquad\vdots\\A_{61}\cdot N_1+A_{62}\cdot R_1+A_{63}\cdot M_1+\\+A_{64}\cdot N_2+A_{65}\cdot R_2+A_{66}\cdot M_2+A_{67}\cdot P=0;\\A_{71}\cdot N_1+A_{72}\cdot R_1+A_{73}\cdot M_1+A_{77}\cdot P=\tilde{w}.\end{cases} \tag{15}$$

The last SLAE equation (15) contains a vector $\tilde{\mathbf{w}}$, corresponding to a given deflection $\tilde{w}(t)$, and matrix analogs of the operators $\mathbf{A}_{71}$, $\mathbf{A}_{72}$, $\mathbf{A}_{73}$, $\mathbf{A}_{77}$ under unknown loads.

The solution to SLAE (15) with respect to the desired vector $\mathbf{P}$ is found in the same way as in the case of a direct problem, using a generalized Gaussian algorithm and applying a regulating algorithm. Before employing the generalized Gaussian algorithm to solve SLAE (15), it is advisable to use equivalent matrix transformations and normalize the values of the matrices. This will minimize the errors that occur when performing calculations.

5. 3. Obtaining numerical results from solving a direct problem

The process of non-stationary deformation of a composite shell was simulated, made of steel (ρ_1=7850 kg/m^3; E_1=2.1·10^{11} Pa; ν_1=0.3) and titanium (ρ_2=4600 kg/m^3; E_2=1.1·10^{11} Pa; ν_2=0.33), with the following geometric parameters: a=0.3 m; h=0.043 m; l=1.5 m; x_p=0.6 m; x_O=0.9 m; b_P=0.15 m. Shear coefficient was taken to be k=5/6.

Fig. 2, *a–c* depicts the result of solving a direct problem, namely, a graph of the change in deflection w, axial displacement u, and the angle of transverse shift ψ for the point of contact of sections x_O.

In Fig. 2, *a–c*, curve 1 corresponds to the function of the perturbing load, curves 2 are the specified kinematic parameters calculated using the proposed method. For comparison, in Fig. 2, curves 3 show the corresponding curves calculated using the finite-element method.

The above plots in Fig. 2, *a–c* indicate a good agreement between the results obtained using different methods (the average error was up to 10 % in phase and up to 8 % in amplitude between curves 2 and 3). Note that, as in the case of the problem of the composite beam, the accuracy of the solution mainly depends on the correct selection of regularization parameters introduced during the implementation of the corresponding regulatory algorithm. The number of such parameters, in this case, correspond to the number of unknown reactions in SLAE (15).

The results of comparing the deflections *w* for the two shell points are shown in Fig. 3, *a, b*. Curves 2 and 3 in Fig. 3, *a, b* correspond to the deflections reported in paper [24] and the resulting deflections. Curve 1 is the temporal dependence of the non-stationary load function, taken in these two calculations to be the same.

It should be noted that during numerical modeling, we superimposed a random "noise" of 5 % on the values of the deflection function, which had to be introduced according to the specificity of the loading of the shell described in [24].

We note the good agreement between the values of deflections reported in [24] and our deflections (Fig. 3, *a, b*), which indicates the reliability of the results obtained using the procedure described here.

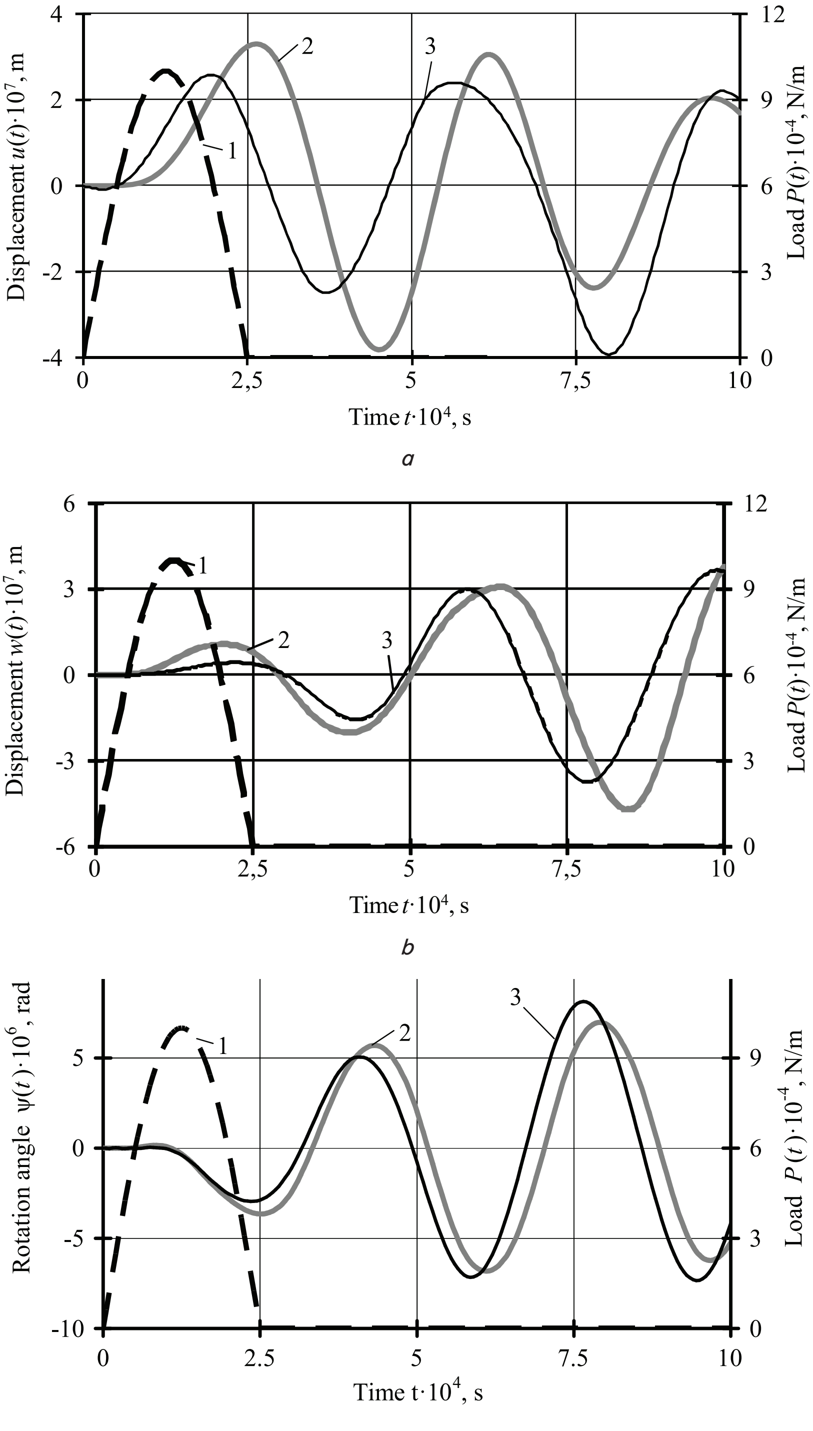
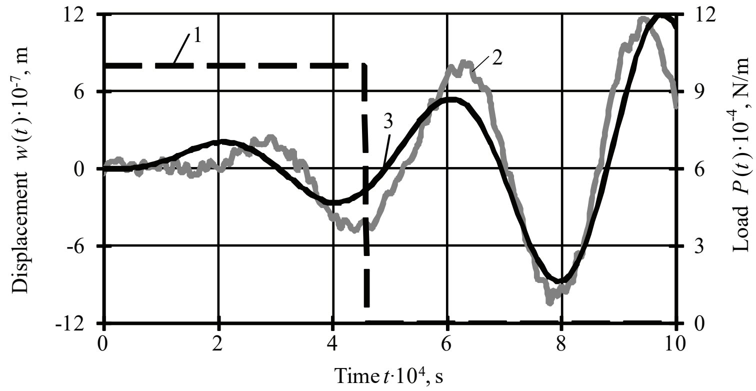
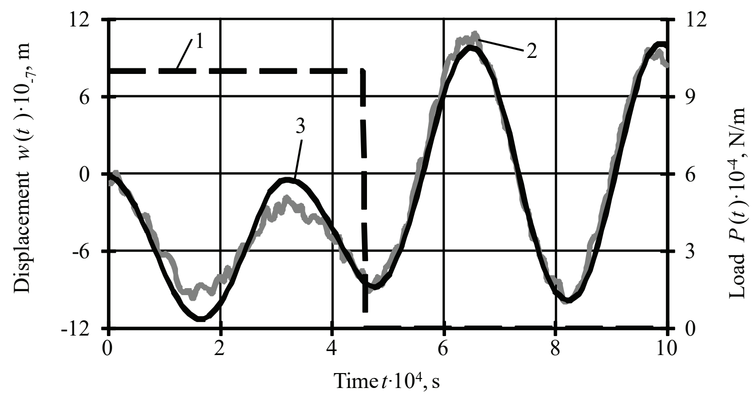


Fig. 2. Change in the kinematic parameters of the composite shell: *a* — displacement *u*; *b* — displacement *w*; *c* — angle of rotation ψ



a



b

Fig. 3. Change in the deflection *w* of the shell: *a* — at point ξ=0.25; *b* — at point ξ=0.5

5. 4. Obtaining numerical results from solving the inverse problem

Numerical simulations to find the time dependence of the evenly distributed load were carried out for the shell already considered earlier, made of steel and titanium. The mechanical and geometric parameters of the shell were also given above. Two variants of pulse loads that cause oscillations of the composite shell were considered, namely, the "half-wave of the sinus" and the "step". The results of such numerical studies are shown in Fig. 4, *a–d*.

Curve 1 in Fig. 4, *a–d* corresponds to the initial deflection values at point x_S obtained as a result of solving a direct problem using FEM. Curves 2, 3 are the load given during the simulation of the corresponding direct problem and the load identified in the process of solving the inverse problem. Curves obtained using FEM play the role of values of functions recorded, for example, experimentally. Load recovery was executed by the deflection recorded at the point with the coordinate ξ=0.55 (Fig. 4, *a, b*) and directly under load at ξ=0.35 (Fig. 4, *b, d*).

Trigonometric series in expressions to obtain the desired parameters, which contain the sum of the terms from 1 to ∞, were replaced by the K-partial sum of the series in the numerical solution. When performing calculations, the expected finite number of terms in the series was estimated to determine each of the parameters. The number of terms in the corresponding series was assumed to be 100. The choice of the value of the step in time Δt in numerical calculation, as is the case in the transition from (12) to (13) and other similar transformations, is largely due to the nature of the change in the curve under study over time. When calculating, the values of Δt corresponding to M=400 partitions of the studied time interval were taken. Such studies were performed for each specific calculation.

It should be noted that the average error of identification at the time interval in question was: Fig. 4, *a* – 14 %; Fig. 4, *b* – 6 %; Fig. 4, *c* – 5 %; Fig. 4, *d* – 4 %. This makes it possible to conclude that the accuracy of the identification results is significantly affected by the nature of the identifiable load. The error of the load identification in the form of a "step" in Fig. 4, *a* is larger than the error in the identification of the load in the form of a "half-wave sine wave" in Fig. 4, *b*. Identification error in Fig. 4, *c* is larger than the identification error in Fig. 4, *d*. In addition, the identification error is affected by the distance between the load application point ξ=0.35 and the point at which the deflections ξ are set. The identification error in the deflection values specified at the point ξ=0.55 in Fig. 4, *a, b* is greater than the identification error by the values of deflections set at the point ξ=0.35 in Fig. 4, *c, d*.

Fig. 5 shows the results of load identification in the form of a "sine half-wave" (curve 1) calculated at different values of the regularization parameter α (curves 2, 3).

Load values in Fig. 4, *a–d* are obtained at the values of the normalized regularization parameter, respectively, $\alpha=10^{-4}$; 10^{-3}; 10^{-2}; 10^{-2}.

Load values are obtained at the value of the regularization parameter $\alpha=10^{-2}$ curve 2 and $\alpha=10^{-3}$ curve 3.

It is obvious that curve 2 (Fig. 5), which corresponds to the optimal value of the regularization parameter α, more accurately describes the desired load, in contrast to curve 3, obtained at a non-optimal regularization parameter α. The average identification error with the optimal regularization parameter α was 6 %. Thus, the use of the Tikhonov regularization method [6] has made it possible to obtain a stable solution to the incorrectly posed inverse problem.

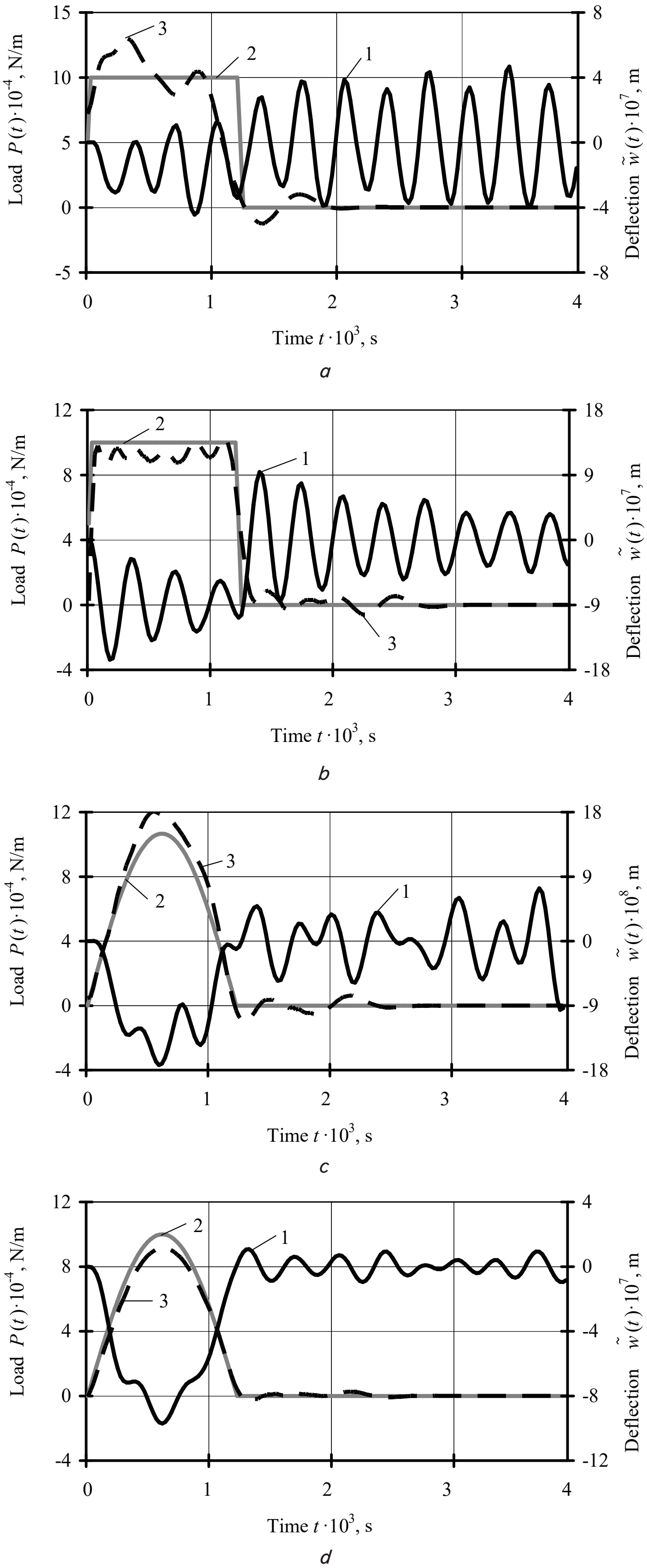


Fig. 4. Results of load identification in the form: *a* – "step" at ξ=0.55; *b* – "half-wave of the sinus" at ξ=0.55; *b* – "step" at ξ=0.35; *d* – "half-wave of the sinus" at ξ=0.35

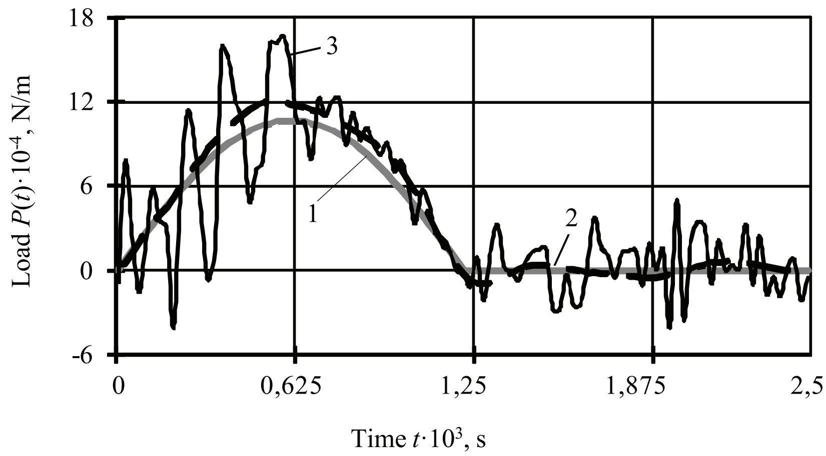


Fig. 5. Load identification at different α values

6. Discussion of results of solving the direct and inverse problem

As part of advancing the theory of inverse problems of mechanics of a deformable solid, a technique for solving the direct and inverse problem for a composite cylindrical shell is proposed. The problem solving is based on the method of artificial dismemberment of the deformable system into two subsystems. To solve the set problems, a method of fictitious loads was used, as well as special techniques based on the conditional replacement of each of the mentioned parts of the shell with shells of other lengths. In this case, each of the shells is supposed to be made of different materials.

The advantage, in comparison with works [22, 23], is the choice of a more complex object of study. Compared to [27], the method of studying complex structures was transferred from beam systems to structural elements in the form of complex cylindrical shells.

Paper [24] reports a number of results related to the non-stationary deformation of a round homogeneous cylindrical shell. The paper contains analytically obtained plots of the deflection of a homogeneous steel shell when exposed to a distributed pulse load. The results given in [24] were compared with similar calculations according to the described procedure, performed for a composite shell of the same exact dimensions, both parts of which are made of steel.

A good agreement between the deflection values reported in paper [24] and the deflections in Fig. 3, *a*, *b* indicates the reliability of the results obtained according to the procedure described here.

When solving the inverse problem, a number of results were obtained, and the efficiency of calculating numerical identification data depending on the location of the deflection registration point was investigated.

If the sensor is installed at the force impact site, the results of direct and inverse problems are better coordinated, and when the sensor is removed from the source of the disturbance, there is a slight deterioration in the identification results.

In addition, the accuracy of identification is affected by the nature of the perturbing load. The results obtained for the "half-wave sine" load agree better than the results for a less "smooth" function in the form of a "step". It is natural to assume that among the factors affecting the effectiveness of load identification, an important role is played by the choice of the regularization method and the subsequent correct selection of the value of the regulatory parameter. In the Tikhonov method used to solve this problem, it is necessary to pay special attention to the selection of the value of the regularization parameter α. An important role in this can be played by the technique of non-binding for selecting the regularization parameter.

It should be emphasized that in this work a refined model of a cylindrical shell of medium thickness is used. Among the accepted hypotheses that limit the application of this model, one can note the following: the model is linear; the shell works under conditions of elastic deformations; the shell is homogeneous within each site.

The practical value of the results obtained in this study relates to the fact that they can significantly advance the methodology for identifying pulse loads acting on structural elements. The identification results obtained for the considered composite cylindrical shell can be generalized to solve the problems of deformation of cylindrical shells composed of three or more parts.

A disadvantage of this work worth noting is that it is mainly theoretical in nature. The reliability of the results was verified by comparison with the authors of other papers and by using a numerical experiment.

With the further development of the current research, it is planned to conduct experimental studies.

7. Conclusions

1. A mathematical model has been built to solve the direct problem of non-stationary deformation of a cylindrical shell heterogeneous in length. In analytical form, functions describing the stressed-strained state of the composite cylindrical shell are obtained. The mathematical solution is reduced to solving the Volterra system of integral equations of the 1st kind.

2. A mathematical model has been constructed to solve the inverse problem of non-stationary deformation of a composite cylindrical shell. Due to the incorrectness of this problem, the Tikhonov regularization method was used to obtain a solution with the choice of the regularization parameter by the residual method. When deriving functional

dependences, the system of integral equations was represented in matrix form.

3. Numerical modeling of non-stationary oscillations of the composite shell was carried out and numerical results of solving the direct problem of studying its stressed-strained state were obtained. The comparison of the obtained results with the results of modeling using the finite-element method is given. The average error in determining the kinematic parameters was up to 10 % in phase and up to 8 % in amplitude.

4. Numerical modeling was carried out and numerical results of solving the inverse problem of identification of the axisymmetric non-stationary pulse load acting on the composite cylindrical shell were obtained. The average identification error at the time interval under consideration ranged from 4 % to 14 %.

The results of identification of various changes in the loads over time are obtained: in the form of a "half-wave of the sinus" and a "step". The possibility of identifying loads based on the parameters of the stressed-strained state at various points of the shell is shown. The devised methodology is robust to significant "noises" of the initial data. A noise level of about 5 % gives a slight error of results and makes it possible to derive a stable solution.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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