Fitting the dipole amplitude with collinearly improved JIMWLK equation

Piotr Korcyl

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

in collaboration with L. Motyka and K. Cichy

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S. Cali *et al.*, Eur.Phys.J.C 81 (2021) 663 arXiv:2104.14254,
PK, Eur. Phys. J. C 82 (2022) 369, arXiv:2111.07427

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JIMWLK evolution equation

Basic facts

- JIMWLK equation describes the non-linear small-x evolution
- it uses Wilson lines as fundamental degrees of freedom
- two-point correlation function $\langle U^\dagger(x)U(y)\rangle$ gives the dipole amplitude
- two-point correlation functions with derivatives provide a basis for small-x TMD structure functions
- initial condition corresponds to a configuration of Wilson lines
- numerically useful reformulation as a Langevin equation

Langevin formulation

$$U(\mathrm{x}, s+\delta s) = \exp\left(-\sqrt{\delta s}\sum_{\mathrm{y}} U(\mathrm{y},s)(\mathrm{K}(\mathrm{x}-\mathrm{y})\cdot\xi(\mathrm{y}))U^\dagger(\mathrm{y},s)\right)\times$$
$$\times\, U(\mathrm{x},s)\times\exp\left(\sqrt{\delta s}\sum_{\mathrm{y}}\mathrm{K}(\mathrm{x}-\mathrm{y})\cdot\xi(\mathrm{y})\right).$$

Initial condition from the McLerran-Venugopalan model

Numerical prescription

- discretize the transverse plane with a lattice spacing a
- introduce finite volume L by imposing periodic boundary conditions
- on each lattice site generate $\rho(\mathrm{x}) = \rho(\mathrm{x})^a \lambda^a$ such that $g^2 \langle \rho(\mathrm{x})^a \rho(\mathrm{y})^b \rangle = \delta^{ab} \delta^{nl} \delta(\mathrm{x} - \mathrm{y}) g^4 \mu^2$
- solve the Yang-Mills equations (Poisson equation) to get the Wilson lines

$$U^{ab}(\mathrm{x}) = \exp\left(-igA^{ab}(\mathrm{x})\right) = \exp\left(-i \frac{g\rho^{ab}(\mathrm{x})}{\nabla^2 - m^2}\right),$$

Parameters

- dimensionful physical parameters: $g^2\mu$, L, m, a
- dimensionless combinations:
  - $a\ g^2\mu$
  - $L\ g^2\mu$
  - $m/g^2\mu$
- use $g^2\mu$ as units, later should be obtained from fit to data as $\Lambda/g^2\mu$

Initial condition from the McLerran-Venugopalan model

MV model on a small torus

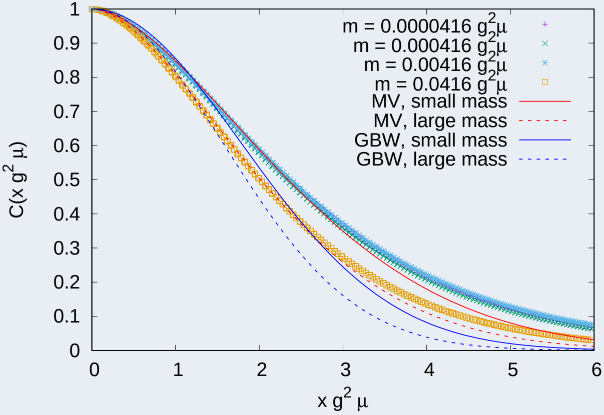


Figure: Dipole amplitude in the MV model on a small torus. $Lg^2\mu = 30.72$, $L/a = 512$.

Initial condition from the McLerran-Venugopalan model

MV model on a large torus

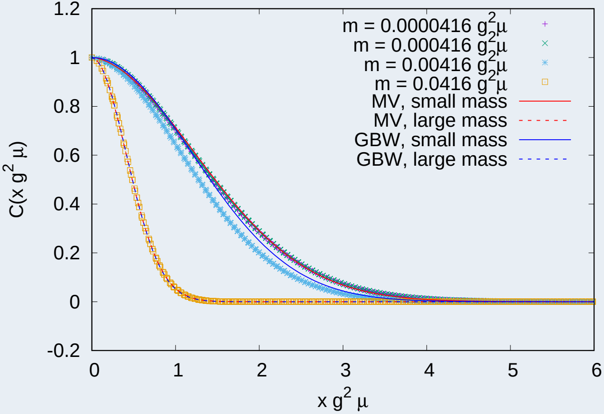


Figure: Dipole amplitude in the MV model on a large torus. $Lg^2\mu = 983.04$, $L/a = 16384$.

Initial condition from the McLerran-Venugopalan model

Convergence of the dipole amplitude with volume

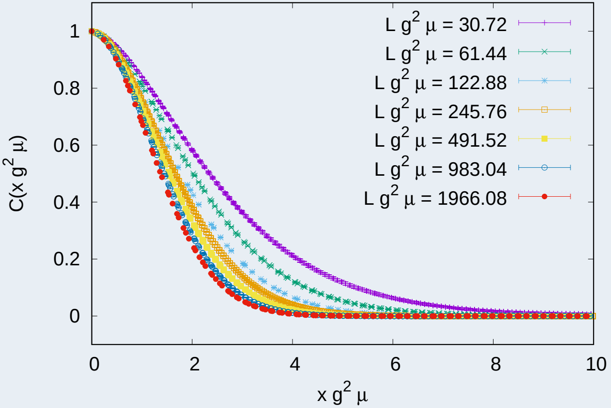


Figure: Volume dependence of the dipole amplitude in the MV model on the torus. Increasing torus size pushes the distribution to the left.

Initial condition from the McLerran-Venugopalan model

Convergence of the saturation radius with volume

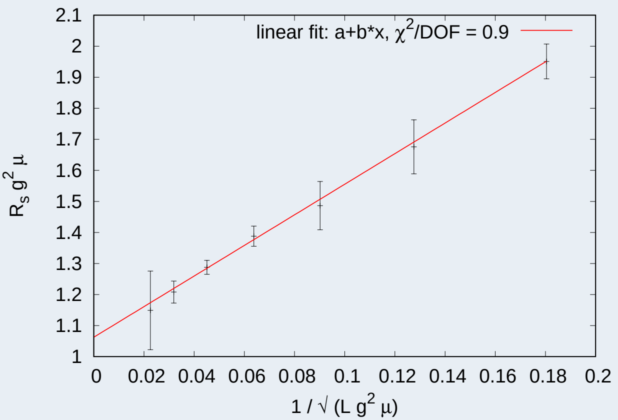


Figure: Volume dependence of the dipole amplitude in the MV model on the torus. Extrapolation of the saturation radius to the infinite volume limit.

Initial condition from the McLerran-Venugopalan model

Improved implementation

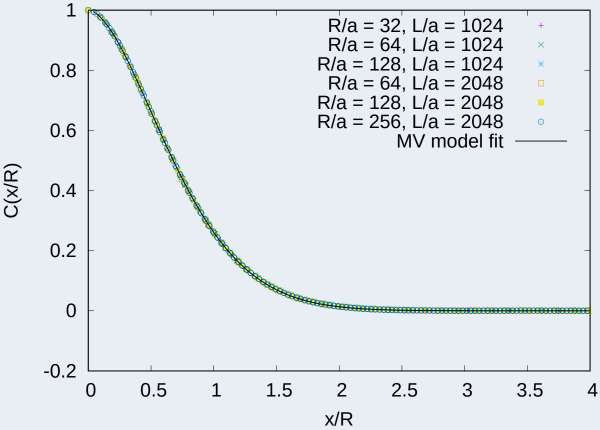


Figure: Volume dependence of the dipole amplitude in the MV model on the torus. The new method shows negligible finite size and lattice spacing effects.

Including the evolution

Including the running coupling constant

$$U(\mathrm{x}, s+\delta s) = \exp\left(-\sqrt{\delta\sigma}\sum_{\mathrm{y}} U(\mathrm{y}, s)(\sqrt{\alpha}\mathrm{K}(\mathrm{x}-\mathrm{y})\cdot\xi(\mathrm{y}))U^{\dagger}(\mathrm{y}, s)\right)\times$$

$$\times U(\mathrm{x}, s)\times\exp\left(\sqrt{\delta\sigma}\sum_{\mathrm{y}}\sqrt{\alpha}\mathrm{K}(\mathrm{x}-\mathrm{y})\cdot\xi(\mathrm{y})\right)$$

where $\sqrt{\delta s} = \sqrt{\delta\sigma}\sqrt{\alpha(|\mathrm{x}-\mathrm{y}|)}$.

Coupling constant

$$\alpha_s(\mathrm{r}) = \frac{4\pi}{\beta_0 \ln\left\{\left[\left(\frac{R_{\mathrm{initial}}^2\mu_0^2}{R_{\mathrm{initial}}^2\Lambda_{\mathrm{QCD}}^2}\right)^{\frac{1}{c}} + \left(\frac{R_{\mathrm{initial}}^2}{\mathrm{r}^2}\frac{4e^{-2\gamma_E}}{R_{\mathrm{initial}}^2\Lambda_{\mathrm{QCD}}^2}\right)^{\frac{1}{c}}\right]^c\right\}},$$

Summary of parameters

$R_{\mathrm{initial}}\Lambda$ with $R_{\mathrm{initial}}g^2\mu \approx 1$, $R_{\mathrm{initial}}\mu_0$, and Λ which provides the units.

JIMWLK with running coupling

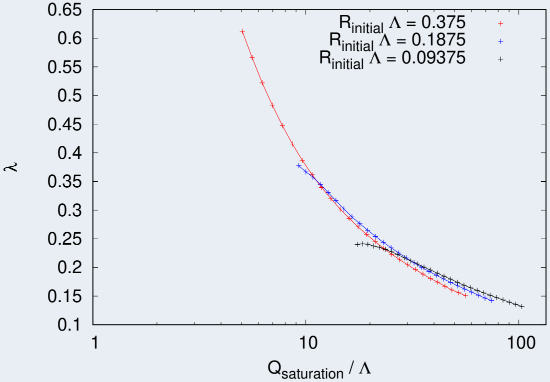


Figure: $R_{\mathrm{initial}}\Lambda$ is the only parameter of the initial condition and of the evolution. Coinciding data from evolution for different values of $R_{\mathrm{initial}}\Lambda$ corresponds to geometrical scaling.

JIMWLK evolution equation with collinear improvement

Collinear improvement

All order resummation of corrections enhanced by kinematical constraints. Known from BFKL studies to be important to correctly describe phenomenology.

Langevin equation formulation from Hatta, Iancu (2016)

At each point of the discretized transverse plane a Wilson line exists with an additional index: the scale at which the final correlator is evaluated.

$$U(\mathrm{x}, R, s+\delta s) =$$

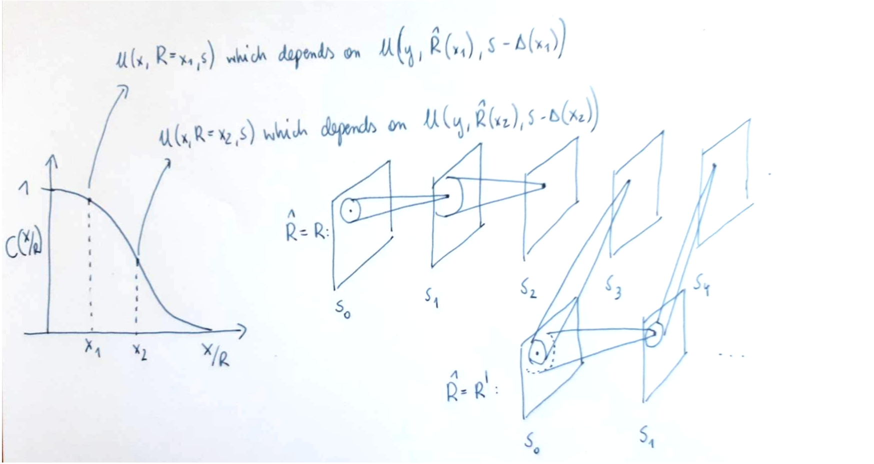
$$\exp\left(-\sqrt{\delta\varepsilon}\sum_{\mathrm{y}}\sqrt{\alpha_s}\,\theta(s-\rho^{R}_{\mathrm{xy}})\,U(\mathrm{y},\hat{R},s-\Delta^{R}_{xy})\left[\mathrm{K}_{\mathrm{xy}}\cdot\xi(\mathrm{y})\right]U^{\dagger}(\mathrm{y},\hat{R},s-\Delta^{R}_{xy})\right)$$

$$\times\, U(\mathrm{x},R,s)\times$$

$$\exp\left(\sqrt{\delta\varepsilon}\sum_{\mathrm{y}}\sqrt{\alpha_s}\,\theta(s-\rho^{R}_{\mathrm{xy}})\mathrm{K}_{\mathrm{xy}}\cdot\xi(\mathrm{y})\right),$$

$\rho^{R}_{\mathrm{xy}} = \ln\frac{(\mathrm{x}-\mathrm{y})^2}{R^2}$, $\Delta^{R}_{xy} = \theta\left(|x-y|-R\right)\rho^{R}_{\mathrm{xy}}$, $\hat{R} = \max(|x-y|, R)$, $s = \varepsilon\alpha_s$.

Saturation scale evolution speed



Saturation scale evolution speed

JIMWLK with collinear improvement

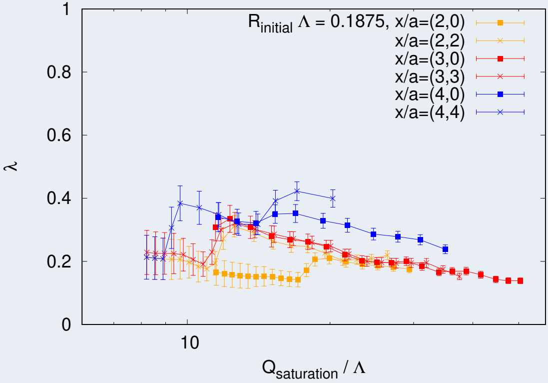


Figure: Preliminary results for the saturation scale evolution speed at $R_{\text{initial}}\Lambda = 0.1875$ for different discretizations. Much lower intercept than without the collinear improvement. Evolution at scales shorter than a should be performed with another approach.

Conclusions

Summary

- JIMWLK equation provides a way to describe DIS data deep in the low-x regime
- numerical implementation and solution possible using the reformulation in terms of Langevin equation
- many systematic effects/ambiguities have to be studied and understood
- collinear resummation in place; rapidity evolution rate of the saturation scale factor 3 smaller!

Outlook

- collinear resummation/kinematical constraint in place and under testing
- phenomenological implications/applications in progress!
- porting to GPGPU using openMP 5.0 offloading in progress!