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A Rigorous Experimental Technique to Measure the Thermal Diffusivity of Metals in Different 3D Forms

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Abstract:- This present work is a continuation and validation of the results explained in a previous paper titled A Rigorous Experimental Technique for Measuring the Thermal Diffusivity of Metals and goes further to describe the notion of dimensionless time t_D practical for solving the energy-density field distribution in 4D space. Moreover, the notion of dimensionless time and statistical characteristic length of the 3D material object is introduced, defined and proved effective.

We have carried out a preliminary experimental investigation and a theoretical analysis on five 3D geometric objects of different shapes in aluminum and steel and the results obtained for the thermal diffusivity are in good agreement with the thermal tables.

I. INTRODUCTION

This article is a generalization to non-cubic forms of the theory and experiment explained in a previous article entitled A rigorous experimental technique for measuring the thermal diffusivity of metals [1,2] and goes further by describing the notion of dimensionless time t_D practical for solving energy density distribution in 4D space (x, y, z, t).

In reference 1 we limited the proposed experimental technique to experimental measurements of thermal diffusivity in aluminum and steel in cubic shapes, while in the present work we go further in other shapes. Regular shapes such as cylinders, hemispheres and pyramidal shapes have been studied.

To be precise, the previous works [1,5,6] are based on the numerical statistical method called Cairo technique which predicts an exponential decay of the energy density in a bounded medium and relates the exponent to the physical and geometric properties of the object, under test.

Moreover, we assume that the general heat diffusion PDE (Eq 1) cannot practically be solved numerically in real time. Finite difference computation (FDM) methods of real-time numerical solutions are extremely time-consuming and prone to instability and inaccuracy, while the same in dimensionless time t_D are short, fast, and the stability and accuracy are assured.

In the present experimental technique proposed to measure the thermal diffusivity of metals in different 3D forms, we assume that, *The spatio-temporal average of the energy density called the center of the energy density field U(x,y,z,t) in the object under test coincides with its center of mass CM along the time evolution of its cooling curve. In other words, the total thermal energy stored in an object during its cooling curve is equal to the temperature at its CM multiplied by the total number of free nodes in the grid (n).*

This suggests the extension of the proposed experimental technique from cubic shaped objects to other regular shapes such as cylinders, hemispheres, pyramids, etc. by finding the cooling curve of the tested object at its CM and by relating its exponent to the thermal diffusivity as explained in the theoretical part.

Recall that the so-called Cairo numerical technique transforms continuous real time t into dimensionless discrete time t_D. t_D is equal to N f where N is the number of iterations performed on the transition matrix B through its chain and f is a statistical factor.

The dimensionless diffusive time is equal to the number of iterations N multiplied by a statistical factor f.

The transformation from real continuous time to the dimensionless discrete time domain via the matrix B and vice versa requires the introduction of four parameters depending on the geometric shape of the body and its thermal diffusivity.

II. THEORY

Below is the general form of the partial differential equation for the time evolution of the energy density U in 3D geometric space,

d / dt (partial) U (x,y,z,t) = D Nabla^2 U (x,y,z,t) + S (x,y,z,t) .. . . (1)

In normal conventions. Equation (1) is subjected to Dirichlet boundary conditions BC and arbitrary initial conditions IC.

In fact, equation (1) characterizes the time evolution of the energy density in real time t and in the 3D geometric space x,y,z where in the SI system (MKS) the unit of t is the second (s), that of x,y,z is meter (m) and that of thermal diffusivity is m^2/s.

Our task is to show how to describe the solution in dimensionless time tD. In the proposed numerical method called Cairo technique, this is done via B-Matrix strings where the real time t is completely lost.

The notion of dimensionless time t_D was recently introduced and described in signal processing theory [7].

In the phenomena of diffusion in bounded objects, the dimensionless time is defined equal to f N where N is the whole number of operations or time step iterations carried out on the transition matrix B 1,2,3 . .. N and f is a scalar quantity depending on the physical and geometric properties of the tested object.

The proposed experimental technique itself is not complicated and can be summarized in the following five consecutive steps,
i-Perform the experimental results of the temperature cooling curve at the center of mass CM of the tested object and thus find the real time - half-time decay value, i.e. T1/2, T1/4, T1/8 etc [1,2].

ii- Calculate the statistical characteristic length of the tested object Lc via the semi-imperial formula (2), [1,2]
Lc = {6*Volume of object V / Area of object A} . . . .(2)
The statistical factor f emerges from another semi-imperial formula,
f = Pie /2. = 1.571

In fact, the characteristic length is of great importance in itself since the experimental temperature of the real-time cooling curve at the center of mass CM is described by,
T(t)=T(0).Exp (- D . f . t /Lc^2) . . . . . (3)

Equation 3 is simply a consequence of defining the exponent of the cooling curve as the heat left per second dU/dt divided by the heat stored U.

Equations 2 and 3 suggest an important geometric physical rule,

Two 3D bodies of different shapes cannot have the same volume to area ratio (V/A) unless both have exactly the same volume and area.
It is simple to show that the half-time decay interval is given by,
T1/2= Log 2. Lc^2/D f . . . (4)
Obviously Log 2. = 0.693

In other words, the required thermal diffusivity will be given by,
D= 0.693 *Lc^2 / (T1/2 * f) . . . (5)

Note that the statistical characteristic length Lc can be found mathematically or experimentally as explained in references 1,2.
iv- Plot the experimental real-time cooling curve T(t)=T(0) .Exp(- D f t /Lc^2),

Know the value of T1/2 and therefore calculate the equivalent thermal diffusivity D using formula 5.
v- Also plot the dimensionless cooling time curve proposed by the transition matrix B chains by choosing the appropriate value of RO and compare their fit with the experimental results.

In this paper, we have arbitrarily chosen to apply the B 27X27 transition matrix as the transition to the dimensionless time domain. The statistical transition matrix B which contains all the information to solve Equation 1 in the time-dependent 3D geometry of the cube in Figure 1 is specified via a procedure similar to that followed in previous work where the entries in the matrix B27X27 must be expressed in the following form [1,2,5,6],

27X27 B-Matrix inputs
Line1: RO 1/6-RO/6 0.0000 1/6- RO 1/6-RO/6 0.0000 0.0000 0.0000 0.00001/6-RO/6 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.0000 0.0000 0.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000
Line 2: 1/6-RO/6 RO 1/6-RO/6 0.0000 1/6-RO/6 0.0000 0.0000 0.0000 0.0000.0000 1/6-RO/6 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
Line 3: 0.0000 1/6-RO/6 RO 0.0000 0.0000 1/6RO/6 0.0000 0.0000 0.00000.00000.0000 1/6-RO/6 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.0000 0.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
. . . .\
Line 14: 0.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.0000 0.0000 0.00000.00000.0000 1/6-RO/6 0.0000 1/6-RO/6 RO 1/6-RO/6 0.0000 1/6-RO/60.00000.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.0000 0.0000 0.0000 0.0000. . . . .
Line 25: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.00000.0000 0.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.0000 0.00000.0000 0.00000.0000 1/6-RO/6 0.0000 0.0000 RO 1/6-RO/6 0.0000
Line 26: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.00000.0000 0.00000.0000 0.0000 1/6-RO/6 0.0000 1/6-RO/6 RO 1/6-RO/6
Line 27: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1/6-RO/60.0000 0.00000.0000 0.0000 0.0000 1/6-RO/6 0.0000 1/6-RO/6 RO
with RO = 0.22 for steel and 0.13 for aluminum as shown in references 1 and 2.
In order not to worry too much about the details of the theory, let us present the following five illustrative experimental applications with their experimental setups and experimental results.

III. EXPERIMENTAL SETUP AND EXPERIMENTAL RESULTS

We move on to five different applications on different 3D shapes, cubic and non-cubic, Al and steel where the tank cold water temperature is zero centigrade.

The experimental setup is described in detail in Reference 1 along with the composition of steel and aluminum used as the test material.

In all five experiments, the hot water reservoir was maintained at 76 C and the cold reservoir at 0 C.

III(a)- Steel cube of 10 cm side Fig 1.

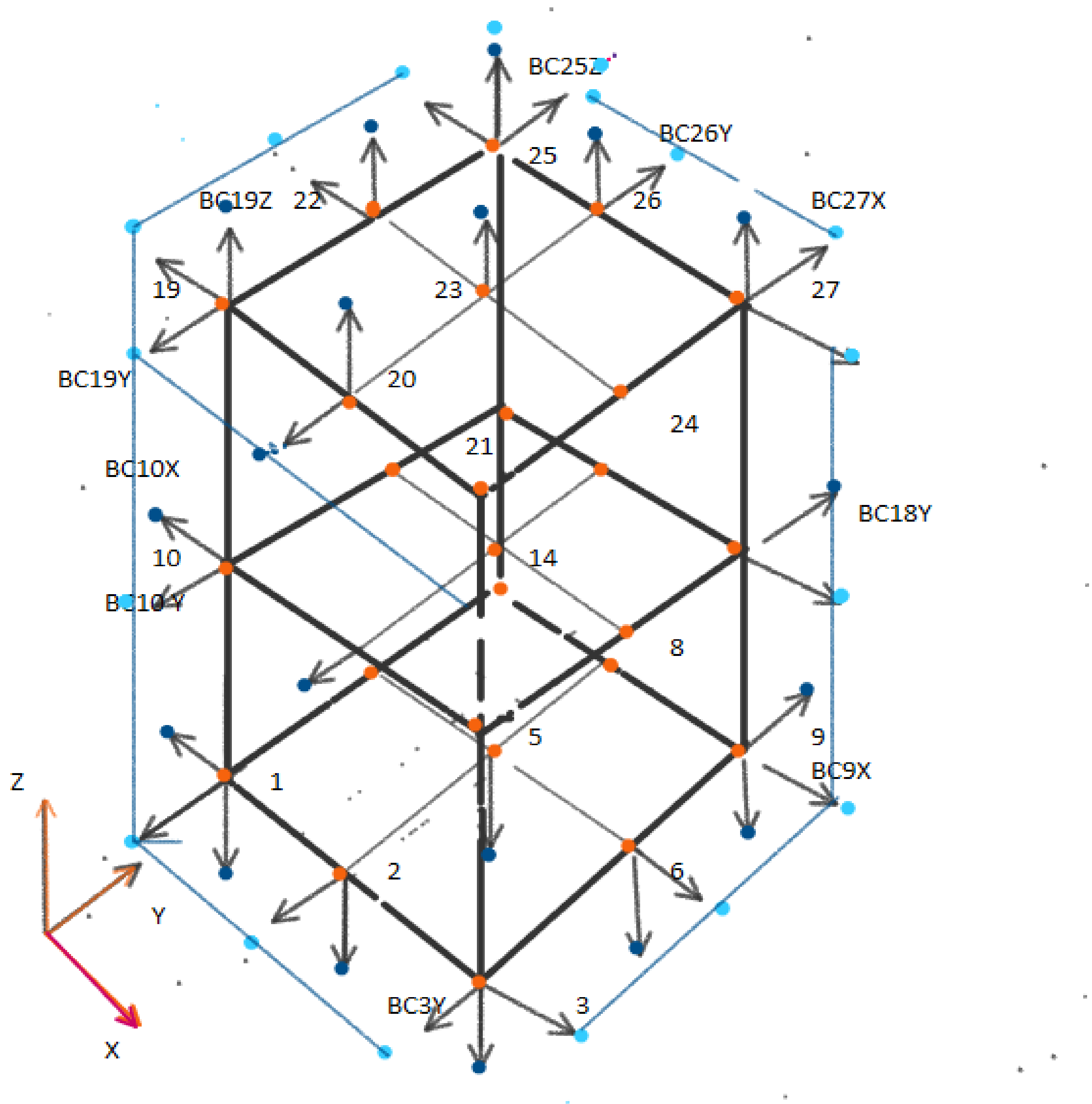

Fig 1. Steel cube with sides of 10 cm with holes and resistance thermometers.

The results of temperature T in centigrade at CM vs time in seconds is presented in Table III-a.

Table III-a , Cooling curve for steel cube 10 cm side length

t(sec)	**0**	**30**	**60**	**90**	**120**	**150**	**180**	**210**	**240**	**300**	**360**	**420**	**480**	**540**	**600**
T(c)	**76**	**58**	**48**	**39**	**31**	**25.2**	**20.2**	**15.8**	**12.9**	**10.9**	**9.1**	**8.4**	**7.9**	**7.4**	**---**

We conclude from table III-a that T1/2 is close to 100 s.
Eq 2 yields Lc =10 cm for steel cube ie, equal to its side length
Finally , using Eq 5, ,then the value of the thermal diffusivity for steel equals
D= Lc^2. Log 2/ (T1/2.*Pie/2)
D=1 E-2*0.693 / (100*1.57) =44.2 E-6 m^2/s in good agreement with the thermal tables [8].

III(b)-Aluminum cube of 10 cm side Fig 2.

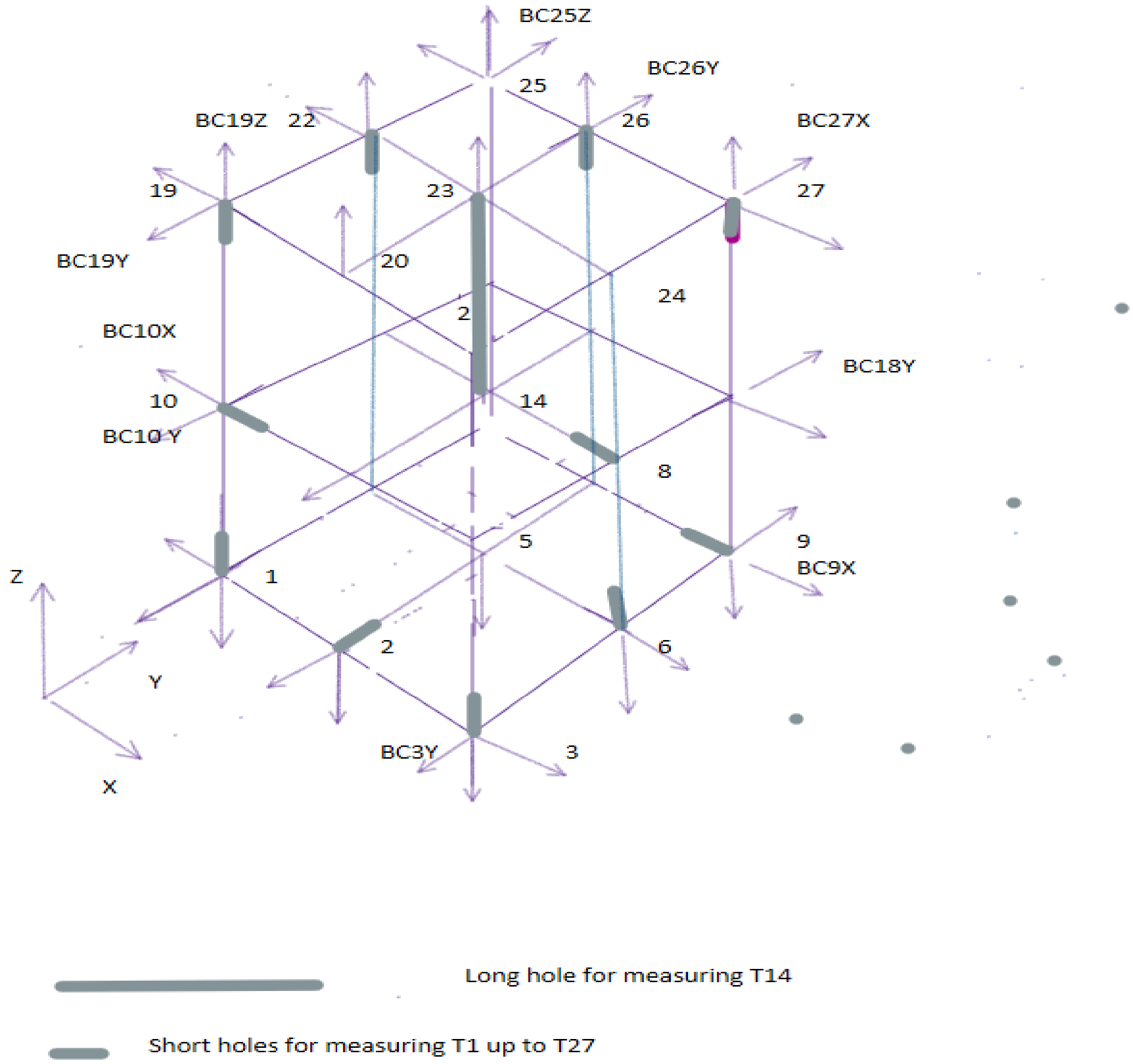

Fig 2. Aluminum Cube of side length 10 cm.

The results of temperature T in centigrade at CM vs time in seconds is presented in Table III-b.

Table III-b , Cooling curve for Aluminum cube 10 cm side length

t(sec)	0	30	60	90	120	150	180	210	240	300	360	420	480	540	600
T(c)	76	45	33	26.5	23	20	17.6	15.5	13.8	11.9	10.	8.2	7.95	7.4	6.95

We conclude from table III-b that T1/2 is close to 45 s. Equation 2 gives Lc = 10 cm for an aluminum cube, i.e. equal to the length of its side.
Finally , using Eq 5, ,then the value of the thermal diffusivity for Aluminum equals

D= Lc^2. Log 2/ (T1/2.*Pie/2)
D=1 E-2*0.693 /(45*1.57) =98 E-6 m^2/s
in good agreement with thermal tables[7].

III(c). Aluminum cylinder of mass 2.61 Kg, a radius R of 14.8 and a length L of 14.8 cm.Fig.3

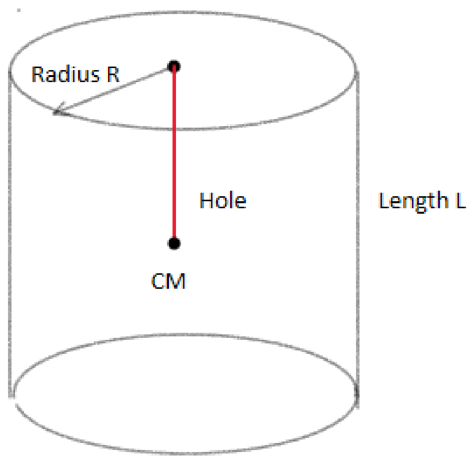

Fig.3 Regular cylinder with circular base of radius R and length L.

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The results of temperature T in centigrade at CM vs time in seconds is presented in Table III-c

Table III-c , Cooling curve for Aluminum cylinder 14.0 cm diameter and 6.5 cm length.

t(sec)	**0**	**30**	**60**	**90**	**120**	**150**	**180**	**210**	**240**	**300**	**360**	**420**	**480**	**540**	**600**
T(c)	**76**	**44**	**32**	**26.**	**23**	**20**	**17.2**	**15.6**	**13.8**	**11.6**	**9.8**	**8.6**	**7.9**	**7.4**	**6.9**

We conclude from table III-c that T1/2 is close to 45 s close to that of the Aluminum cube as expected.

Easy to calculate the surface area of the cylinder as nearly 600 cm^2 and

Volume of the cylinder is nearly 1000 cm^3 which are the same as those of the preceding cube of 10 cm side length.

Cooling curve for Aluminum cylinder (Table III-c)14.0 cm diameter and 6.5 cm length is similar to that of the cube (Table III-b) as Equation 3 predicts.

Obviously the calculated thermal diffusivity D is the same , D Alumium= 0.98 E-4 m^2/s.

III(d). Aluminum pyramid with a mass of 6.5 Kg, a square base of 20 cm and a height of 19 cm.Fig.4

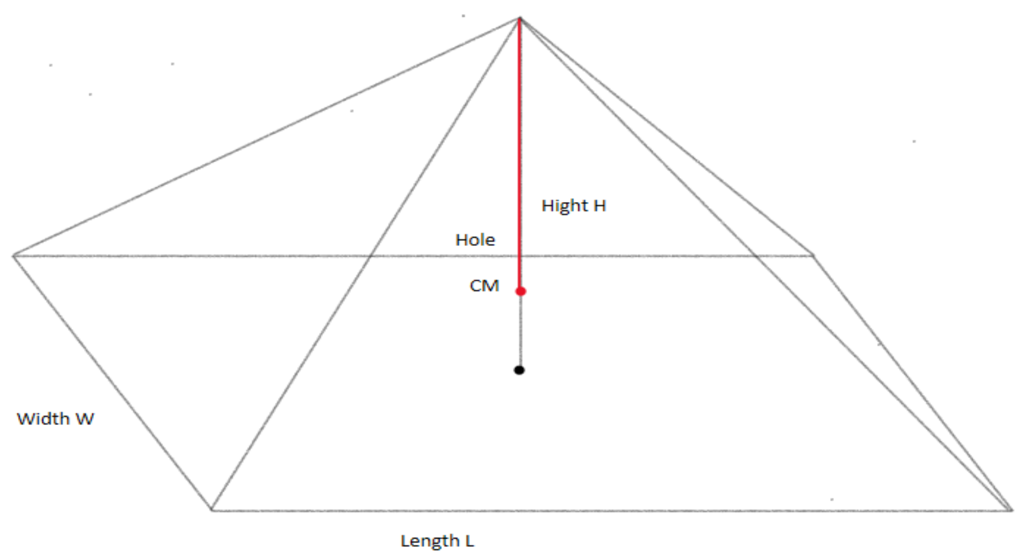

Fig.4 Regular pyramid with square base

The results of temperature T in centigrade at CM vs time in seconds is presented in Table III-d.

Table III-d , Cooling curve for Aluminum pyramid with a square base of 20 cm and a height of 19 cm.Fig.4

t(sec)	**0**	**30**	**60**	**90**	**120**	**150**	**180**	**210**	**240**	**300**	**360**	**420**	**480**	**540**	**600**
T(c)	**76**	**49**	**38.5**	**28**	**26**	**22.6**	**18.3**	**16.8**	**15.7**	**13.5**	**12.6**	**8.6**	**12**	**11.3**	**9.9**

We conclude from table III-d that T1/2 is close to 62 s

Easy to calculate the surface area of the pyramid as nearly 1160 cm^2 and
Volume of the pyramid is nearly 2500 cm^3 .
The characteristic length for the pyramid is 6 V/A=13.6 cm.
Cooling curve for Aluminum pyramid (Table III-d) at its CM is similar to that of an equivalent cube of side length 13.6 as Equation 3 predicts.

It is simple to calculate thermal diffusivity D from Eq. 5,
D= 0.693 *Lc^2 /(T1/2*f) = 132 E-6 m^2/s
which is slightly higher than that of thermal tables.

III(e). Aluminum half-sphere with a mass of 1.8 Kg and a diameter of 14.5 cm.Fig.5

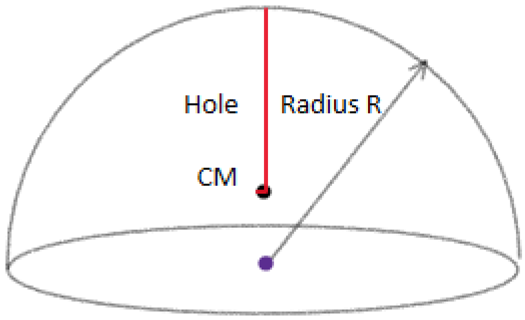

Fig.5 Regular aluminum hemisphere with circular base

The results of temperature T in centigrade at CM vs time in seconds is presented in Table III-e.

Table III-e , Cooling curve for Aluminum hemisphere with a mass of 1.8 Kg and circular base of 14.5 cm.

t(sec)	0	30	60	90	120	150	180	210	240	300	360	420	480	540	600
T(c)	76	36	25.5	20.5	17	14.6	12.5	11	9.7	8.7	7.6	6.6	5.9	4.9	4.3

We conclude from table III-e that T1/2 is close to 29 s
Easy to calculate the area of the hemisphere to nearly 495 cm^2 and
its volume is nearly 700 cm^3.The characteristic length Lc for the hemisphere is,
Lc= 6 V/A=8.48 cm.
Cooling curve for Aluminum halfsphere (Table III-e) at its CM is similar to that of an equivalent cube of side length 8.48 as Equation 3 predicts.

It is simple to calculate thermal diffusivity D from Eq. 5,
D= 0.693 *Lc^2 /(T1/2*f) = 109 E-6 m^2/s
which is close to that given by thermal tables [8].

IV. CONCLUSION

The presented experimental results and mathematical calculations provide a rigorous experimental basis for measuring the thermal diffusivity of metals and thus prove the accuracy and usefulness of the numerical method called the Cairo technique.

The introduction of the so-called characteristic length Lc has proven to be useful in predicting the time dependence of temperature in the cooling phase of regularly shaped objects and, therefore, in finding their thermal diffusivity.

The dimensionless time tD inherent in the B-Matrix chain solution of time-dependent energy density scattering in 3D geometric objects has been shown to be consistent, stable, fast, and accurate.

The theoretical and experimental results produced in this article are consistent and suggest to introduce and develop a generalized or unified theory to solve the problems of energy density diffusion (thermal energy, electric potential energy, sound kinetic energy, etc.) in 4D bounded media.

NB. All calculations in this article were produced through the author's double precision algorithm to ensure maximum accuracy, as followed by Ref. 10 for example

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