Emergence of social hierarchies in a society of two competitive classes

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Introduction

In recent years, the statistical physics approach is giving us a unique position for understanding a large variety of socioeconomic systems and, in particular, cities. The need for better interpretable models for cities is critical: understanding the micro-motives behind human behaviour is a necessary step to explain their macroscopic social behaviour and to be useful in the decision-making process. One of the topics which is worth tackling is the observation, study and understanding of the formation of different kinds of hierarchies which could appear inside cities [1, 2]. The hierarchy of social organization is an omnipresent property of animal and human aggregations, related to many features of the system such as collective decisions, intrinsic properties of the individuals, spatial characteristics, and so on.

The mathematical model

We here follow the footsteps of the Bonabeau model introduced by E. Bonabeau et al. in 1995 [3] and add a second class of agents into the society. The variation of an agent fitness can change by competition and only a pairwise interactions between agents of opposite classes are allowed. To proceed with a simple interaction the fitness of each involved agent is standardized under the minimum value of its class, that is to say, it could be understood as the prestige/reputation which an agent has in its own community.

1. Winning probability:

$$P_{ij}(t) = \frac{1}{1+\exp\left(\eta\left(\hat{F}_j(t)-\hat{f}_i(t)\right)\right)} \tag{1}$$

where $\hat{f}_i$ is the standardized agent fitness of one class, capital letter $\hat{F}_j$ the fitness of the other one and $\eta > 0$ is a free parameter which controls the strength of the interaction.

2. Normalization:

$$\hat{F}_i(t) = \frac{F_i - F_{min}}{F_{max}-F_{min}}\ ,\ [0,1] \tag{2}$$

And the same for the other class.

3. Fitness exchange: The exchange of fitness is fixed with a given proportion x of the opponent:

$$\begin{aligned} F_j' &\to F_j + x f_i \\ f_i' &\to f_i(1-x) \end{aligned} \tag{3}$$

And the contrary if the other class wins.

4. MonteCarlo Setup:

- N_f and N_F individuals performing a stochastic random walk on a 2D square lattice.
- A residence time algorithm to reproduce the time that a jump happens is applied.
- Initially the fitnesses are setted at $f_i(0) = F_j(0) = F_{total}/N_f + N_F$ with $F_{total} = 1000$: the so-called egalitarian situation.
- The fight just happens for the moving agent and if the site is occupied by one or more agents of the other class.

Fitness temporal evolution

Numerical simulations show that for a broad range of values of η, the fitnesses of the agents of each class show a clear decays in time except for one or very few agents which capture almost all the fitness of the system. Furthermore, the system size L does not change the fixed point of several studied observables, and it just makes increase the time when the second to last agent survives. Basically, this time scales with a potential behaviour.

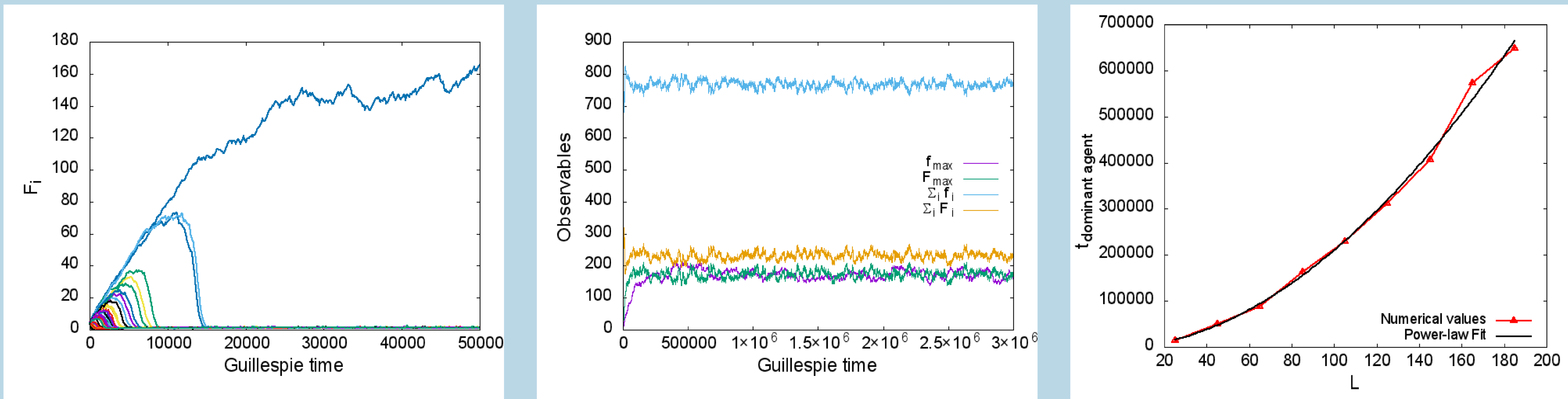

Figure 1. The first plot correspond to the time evolution of all the fitnesses of one class and the second one to four studied observables of the system, both for $L = 25$. The third plot is the time when the penultimate survivor decays as a function of the system size L. This plot is obtained averaging over 50 different runs. All the simulations are done for $N_f = 500$, $N_F = 50$, $\eta = 5$ and $x = 0.01$.

Phase Transition and Universal Scaling Function

The studied observables at the stationary regime do not depend on the system size L and they just depend on the number of agents in the society:

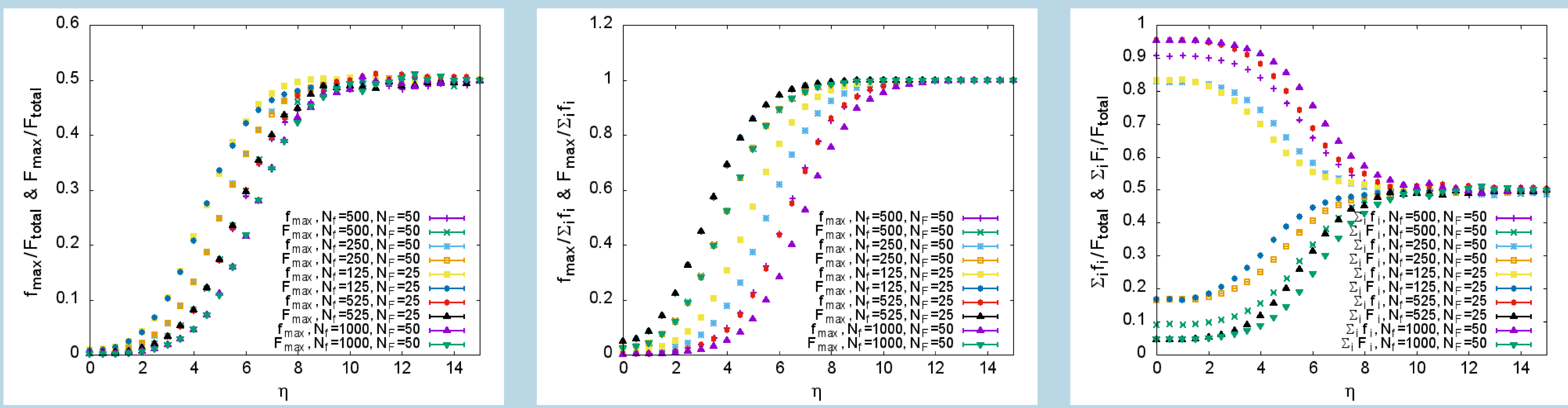

Figure 2. The first and second plot correspond to the maximum agent fitness now normalized by the total fitness in the society f_{max}/F_{total} and the maximum agent fitness of each class normalized by the sum of all agent fitness of the same class $f_{max}/\sum_i f_i$, respectively. The third plot is the total fitness of each class under the total fitness in the society $\sum_i f_i/F_{total}$. All observables are plotted as a function of the control parameter η and for several values of individuals. The exchanged proportion x has been fixed at 0.01. Results are computed at the stationary regime and averaging under time evolution together with many different runs.

In addition, a universal scaling function for the maximum agent fitness normalized by the sum of all agent fitness, is a good candidate for the data collapsing on the same curve independent of the agents' quantity:

$$\frac{f_{max}}{\sum_i f_i}(\eta, N_f) = \frac{1}{1+e^{-a_0(N)(\eta-\eta_0(N))}} = \frac{1}{1+e^{-\eta}(N_f-1)}, \tag{4}$$

where $a_0 \approx 1\ \forall N$ and $\eta_0(N) = ln(N-1)$. The same appears for the other class. This sigmoidal behavioural change could be understood as a phase transition from egalitarian to hierarchical society for each class as a function of the control parameter η.

Conclusions

1. MonteCarlo simulations show a clear behavioural change in several observables of the system as a function of the control parameter η, which it can be understood as a phase transition from an egalitarian to hierarchical society.
2. The observables at the stationary regime are invariant in the system size, and they just depend on the number of agents of each class.
3. A universal scaling function for the maximum agent fitness normalized by the sum of all agent fitness, is a good candidate for the data collapsing on the same curve independent of the agents' quantity.
4. For a fixed class, the results do also not vary.
5. Further variations of the model, such as other winners' probabilities, different kind of interactions between agents and even additional more classes could also be implemented and studied in detail.

References

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