

Method

1. The oriented elliptic correlation scales

The correlation scale of the oriented ellipse is established as the basic shape unit for OI analysis, and its correlation with distance is modeled as follows,

$$F_{d,\theta} = \exp\left[-\frac{d}{D(\theta)}\right] \quad (1)$$

$F_{d,\theta}$ is a function with the distance d and the azimuth θ in polar coordinates. The elliptic equation express the shape of correlation scales, which can be described as

$$D(\theta) = \frac{L_{\max} L_{\min}}{\sqrt{L_{\max}^2 \sin^2(\theta - \varphi) + L_{\min}^2 \cos^2(\theta - \varphi)}} \quad (2)$$

φ is the rotation angle for the ellipse. L_{\max} and L_{\min} stand for the semi-major axis and semi-minor axis of the ellipse, which are calculated by using the Gauss-Newton iteration method.

2. Estimation of SST observation error

The estimated standard deviation $\rho_{i,j}$ can be calculated using the following equations,

$$\begin{cases} \rho = [(\mathbf{R} - \langle \mathbf{R} \rangle)^T \mathbf{D}_{\text{local}}^{-1} (\mathbf{R} - \langle \mathbf{R} \rangle)]^{0.5} \\ \mathbf{D}_{\text{local}} = \langle (\mathbf{R} - \langle \mathbf{R} \rangle)(\mathbf{R} - \langle \mathbf{R} \rangle)^T \rangle \\ \mathbf{R} = [T_s, T_s - T_c, (T_s - T_c)(\sec \theta - 1)] \end{cases} \quad (3)$$

Here, \mathbf{R} is a vector of regressors obtained from $\mathbf{MD}_{\text{local}}$, $\langle \cdot \rangle$ which denotes averaging of the vector, superscript T stands for the transpose of a vector, and $\mathbf{D}_{\text{local}}$ is a covariance matrix of regressors. The schematic diagram of the SST observation error estimation as shown in Figure 1.

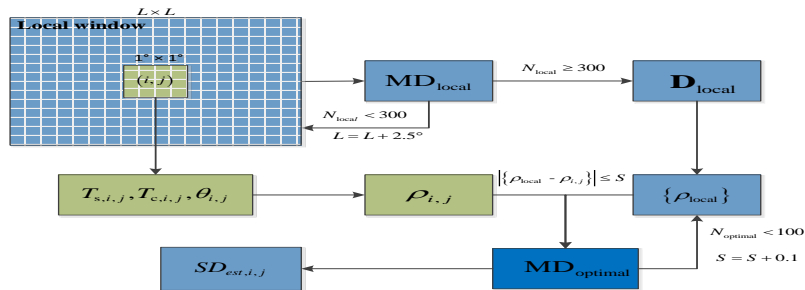


Figure 1. Schematic diagram of the error standard deviation estimation SDeST using the optimal matched datasets (MDoptimal) for visible and infrared radiometer (VIRR) surface sea temperature (SST) products. $T_{s,i,j}$, $T_{c,i,j}$, $\theta_{i,j}$, and $\rho_{i,j}$ separately stand for the original VIRR SST, climatology SST, view zenith angle, and estimated standard deviation in the position of SST matrix (i,j) , respectively.

3. Estimation of SST background error

According to the Kalman filtering equations, the analysis field error $\sigma_{k,t}^a$ in the current iteration t can be used to estimate the background field errors $\sigma_{k,t+1}^b$ in the next iteration $(t + 1)$, and a proportional relationship exists between the estimated analysis field error in the current iteration and the background field error in the next iteration.

$$\sigma_{k,t+1}^b = \kappa \sigma_{k,t}^a \quad (4)$$

where k denotes the target position and κ is the coefficient of proportionality between the two errors. In this case, the magnitude of the κ value varies with the iteration time. It can be expressed as the ratio of the average error of the current observation field $\bar{\sigma}_t^o$ to the average error of the current SST

analysis field $\bar{\sigma}_t^a$:

$$\kappa = \frac{\bar{\sigma}_t^o}{\bar{\sigma}_t^a} \quad (5)$$

and the error of the current analysis field is as follows:

$$\sigma_{k,t}^a = \left| (\sigma_{k,t}^b)^2 - \sum_{i=1}^N w_{ik} \sigma_{ik}^b \right|^{0.5} \quad (6)$$

Results

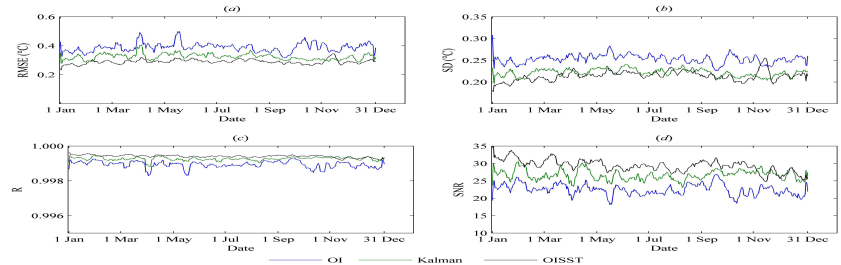


Figure 2. Time series of (a) root-mean-square error (RMSE), (b) standard deviation (SD), (c) correlation coefficient (R), and (d) signal-to-noise ratio (SNR) from OI, Kalman and OISST results for 2016.

Table 1 Error statistics for OI, Kalman and OISST results for 2016

	RMSE (°C)	SD (°C)	R	SNR	NUM
OI	0.3911	0.2539	0.9989	22.41	82441
Kalman	0.3243	0.2214	0.9993	26.64	82441
OISST	0.2897	0.2140	0.9994	29.31	82441