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Review of Supports for Learners with Learning Difficulties in Mathematics, with Dyscalculia and Developmental Disabilities

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Summary of key findings

- Strong core pedagogy in the whole-class setting has a significant impact on learners who experience difficulties learning mathematics (Dennis et al., 2016).
- Targeted small group interventions have been highlighted in a number of reviews as being beneficial: "As such, the results in these studies indicated the efficacy of the interventions by extrapolating that even the most struggling students can benefit from small group intervention that is intensive, strategic and explicit" (Monei and Pedro, 2017, p.286).
- Prevention and early intervention for mathematics difficulties are vital (Charitaki et al.,2021). Nelson and Powell (2018, p.536) argue based on their review that "without targeted interventions and early determination of difficulty with math, students as early as kindergarten who display math difficulty may be at risk for poor secondary and adulthood outcomes."
- The importance of counting with one-to-one correspondence in early interventions is underlined by the fact that it accounted for 51% of the variance in treatment effects (Nelson and McMaster, 2019).
- The following elements have been identified as features of successful interventions: assessment of the learners' current mathematical performance and developing a profile of strengths and areas for development; adopting a developmental approach and using knowledge of learning trajectories for key areas; careful planning; professional development for staff involved and use of games to increase motivation and reduce fear and anxiety (Dowker, 2019).
- The evidence base of the use of technology that builds on students' strengths is growing but studies also highlighted that the quality of instruction is more important than the learning environment. The role of the teacher facilitating the process is crucial (Monei and Pedro, 2017).
- The benefits of cross-age peer tutoring have a strong evidence base in relation to supporting learners in mathematics (Alegre et al., 2019).
- While there is less evidence in relation to upper primary there is evidence of the benefit of specific interventions for fraction learning given its key role in understanding further mathematics (Ennis and Losinski, 2019).
- There is little research on interventions at post primary level but an increased focus on algebra is recommended given its central importance as an educational gatekeeper and because difficulties with algebra are not so easily resolved by use of a calculator (Lewis and Fisher, 2016).
- For learners with more complex developmental needs techniques such as systematic instruction, task-analytic instruction, system of least prompts, constant time delay, simultaneous prompting have been shown to be successful teaching strategies. The use of concrete materials and for some learners the use of virtual manipulatives have also been shown to be effective supports for learning key mathematical concepts. The use of instructional technology, computer assisted, augmented reality and video modelling have an emerging research base (Bowman et al., 2019; Cox & Jimenez, 2020).

Recommendations: Pillar; Students with additional learning needs in mathematics

1. Increase the emphasis on the need for strong core pedagogy in the whole-class setting which has a significant impact on learners who experience difficulties learning mathematics (Dennis et al., 2016).

2.Continue to provide for targeted small group interventions which have been highlighted in a number of reviews as being beneficial: "As such, the results in these studies indicated the efficacy of the interventions by extrapolating that even the most struggling students can benefit from small group intervention that is intensive, strategic and explicit" (Monei and Pedro, 2017, p.286).

3. Increase the time and support for prevention and early intervention for mathematics difficulties. Nelson and Powell (2018, p.536) argue based on their review that "without targeted interventions and early determination of difficulty with math, students as early as kindergarten who display math difficulty may be at risk for poor secondary and adulthood outcomes."

4. Ensure that counting with one-to-one correspondence is emphasised in early interventions as its importance is underlined by the fact that it accounted for 51% of the variance in treatment effects (Nelson and McMaster, 2019).

5. Interventions should consist of elements identified as leading to success: assessment of the learners' current mathematical performance and developing a profile of strengths and areas for development; adopting a developmental approach and using knowledge of learning trajectories for key areas; careful planning; professional development for staff involved and use of games to increase motivation and reduce fear and anxiety (Dowker, 2019).

6.Greater attention should be paid to the role of the teacher in facilitating the use of technology. The evidence base of the use of technology that builds on students' strengths is growing but studies also highlighted that the quality of instruction is more important than the learning environment. The role of the teacher facilitating the process is crucial (Monei and Pedro, 2017).

7. While there is less evidence in relation to upper primary there is evidence of the benefit of specific interventions for fraction learning given its key role in understanding further mathematics (Ennis and Losinski, 2019).

8. The benefits of cross-age peer tutoring has a strong evidence base in relation to supporting learners in mathematics (Alegre et al., 2019).

9. At post primary level an increased focus on algebra is recommended given its central importance as an educational gatekeeper and because difficulties with algebra are not so easily resolved by use of a calculator (Lewis and Fisher, 2016).

10. For learners with more complex developmental needs techniques such as systematic instruction, task-analytic instruction, system of least prompts, constant time delay, simultaneous prompting have been shown to be successful teaching strategies. The use of concrete materials and for some learners the use of virtual manipulatives have also been shown to be effective supports for learning key mathematical concepts. The use of

instructional technology, computer assisted, augmented reality and video modelling have an emerging research base (Bowman et al., 2019; Cox & Jimenez, 2020).

Introduction

Where a child or young person experiences difficulties in learning mathematics or numeracy skills and knowledge it can be very impactful in a negative manner in many aspects of their lives. It could be argued that the impact has increased as mathematical skills become more critical in society and the economy. It affects self-esteem, mental health and attainment levels in school; Some children develop an early dislike for mathematics or mathematics anxiety and low motivation for the area. Data from the *Growing up In Ireland* study suggests that twice as many children with special educational needs at age 9 dislike mathematics compared to reading (Cosgrove et al., 2018). The impacts can be long term as they affect career prospects and opportunities for employment (Benavides-Varela et al., 2020).

While the reality and seriousness of mathematical difficulties are very clear there is still no agreed definition or classification and the terminology used in the research literature varies hugely when referring to levels of difficulty and possible causes (Butterworth, 2019). The Diagnostic and Statistical Manual of Mental Disorders (DSM), of the American Psychiatric Association, refers to dyscalculia or Mathematical Learning Disability –MLD- in the fifth Edition of the manual - DSM 5, as a neurodevelopmental disorder with specific learning impairment in mathematics. Children with dyscalculia have impairment in processing numerical information, learning arithmetic facts, and have poor calculation and math reasoning abilities. These difficulties are below expectation for the individual's age, intelligence, and educational experience, and occur in the absence of visual or hearing impairments, mental, neurological disorders or psycho-social difficulty or language differences (American Psychiatric Association, 2013). It can be evident at a preschool level in the learning of tasks such as counting and subitising (Butterworth, 2019). Some researchers use the term “at risk” of experiencing mathematical difficulties which they link to

Research questions

- How can learners who experience difficulties learning mathematics be supported in developing numeracy skills within mathematics and across the curriculum?
- How can learners with an intellectual disability be supported in developing numeracy skills within mathematics and across the curriculum?

the interaction of environmental, economic or cultural disadvantaged contexts with a disposition to number difficulties (Benevedes-Varela, 2020).

Methodology

This review will firstly analyse the findings from existing systematic and meta-analysis reviews of the literature concerned with dyscalculia or mathematical learning disability and then with interventions to support learners with cognitive or developmental disabilities since 2011, the publication of the *National Literacy and Numeracy Strategy* (DES, 2011). Since 2011 a number of international reviews of numeracy and mathematics interventions have been completed. While the volume of research and the evidence base is not as strong as in relation to literacy a number of key findings have emerged that can inform policy and practice in the area. Also, there are more studies of primary aged learners with difficulties than of post primary. An area of growth since 2011 has been in the use of technology to support development. Two separate searches were conducted, firstly for mathematics/numeracy and specific difficulties in mathematics such as dyscalculia and low attainment mathematics/numeracy and then for intellectual or developmental disability. The first search used the following search terms: ("numeracy" OR "math*" OR "arithmetic*" OR “problem solving” OR “math fluency” OR “number sense”) AND (“dyscalculia” OR “math* disability” OR “math* difficult*” OR “low attainment in math*”) AND ("intervention" OR “instruction”) AND (“systematic review” OR “meta-analysis” OR “research synthesis”). The second search replaced the second line with AND ("intellectual disability*" OR "developmental disability*" OR “cognitive disability*” OR “special education”). The exclusion criteria used were pre 2011, post-secondary, non-numeracy/non-mathematics focus. The EBSCO and SCOPUS databases were used and *The International Handbook of Mathematical Learning Difficulties: From Laboratory to the Classroom* (Fritz et al. 2019) was also consulted as a summary of research in the field. In relation to the first search excluding duplication 57 articles were assessed and 23 satisfied the inclusion criteria (Appendix, Figure 1). For the second search and initial search using numeracy only and developmental disabilities yielded no articles of a review nature. Including mathematics yielded 53 articles (excluding duplication) post 2011. Applying the exclusion criteria, nine articles satisfied the inclusion criteria of being a systematic review in the area of mathematics and developmental disabilities (Appendix, Figure 2).

Pedagogical interventions

Dennis et al (2016, p163) in a meta-analysis of empirical research on teaching students with mathematics learning difficulties stress the "importance of core instruction in the whole class settings (i.e., Tier 1 instruction), as a strong core instruction can boost the math performance of students struggling at learning math. For studies using smaller instructional groups, the ad hoc comparison showed that there was no significant difference between small group instruction and one-to-one instruction. These findings suggested that both types of small group instruction were effective, and these findings were consistent with earlier work in the area of reading".

Beginning with dyscalculia, following a review of 11 studies Monei and Pedro's (2017) results indicated that individualised and small group teaching that was given to children with dyscalculia was beneficial as the children improved significantly after intervention in mental and written calculation. Strategy instruction interventions with an emphasis on understanding and learning new material and skills were effective. These included explicit practice and feedback related to the use of the strategy. Strategies included verbal cognitive strategies on how to identify key numerical, relational and question elements in a problem, visual strategies using diagrams depicting the underlying structure of the problem and strategies for efficient counting.

Targeted small group interventions have been highlighted in a number of reviews as being beneficial: "As such, the results in these studies indicated the efficacy of the interventions by extrapolating that even the most struggling students can benefit from small group intervention that is intensive, strategic and explicit" (Monei and Pedro, 2017, p.286). The participants in the studies represented many different ethnic groups so the findings are generalisable and point to the overall effectiveness of interventions for children presenting with dyscalculia (Monei and Pedro, 2017).

Contrary to some previous research a “neuropsychological intervention study revealed that neuropsychological interventions showed that reinforcing active memory (auditory and visual memory) by undergoing practice with words, numbers, and recalling them had a positive effect on mathematics performance of elementary students with mathematics learning disability (Faramarzi & Sadri, 2014) (Monei and Pedro, 2017).”

A key finding across a number of review is the benefit from focussing on counting and number sense tasks: “Moreover, the findings in this study also indicate that lessons that are specifically on number-sense tasks (for example, number concepts) and fluency building with arithmetic combinations provided students with the added “boost” they needed to become more proficient in these areas.” (Monei and Pedro, 2017, p.291). Successful intervention in problem solving integrated cognitive modelling to support identification of the problem structure.

Given the frequent advice on the use of manipulatives for mathematics teaching and learning it is surprising that until recently a systematic review of this area for students with disabilities did not exist. Two recent reviews of the use of manipulatives, Lafay et al. (2019) and Bouck and Park (2018) provide evidence of their usefulness in teaching and learning for students with difficulties in mathematics. Virtual manipulatives can be less stigmatising for older students.

Furlong et al. (2016, p.165) in a review of interventions to improve mathematical performance for children with mathematical learning difficulties (MLD) found that “sequencing tasks from easy to difficult, task analysis, combining with teachers providing explicit explanation of concepts and procedures, were more likely to benefit students with MD. Moreover, we found that intervention delivered in a form of small-group instruction was a more effective and efficient instructional delivery mode for this group of students.” McKenna et al., (2015) in a synthesis of a small number of observational studies found infrequent use of key strategies such as cognitive strategy instruction, student verbalisation and discourse, independent practice and checking for understanding.

An important area of research given the emphasis on inclusive education are approaches that support learners in mainstream classes. A number of reviews have highlighted the benefits of peer tutoring in this regard. Alegre et al (2019) in a review of peer tutoring in mathematics in primary education found that cross-age tutoring has better results than same-age tutoring. Interestingly those studies with 30 or less participants and which lasted no more than 8 to 10 weeks were more effective than those with a greater number of participants and of a longer duration.

Genesis et al. (2021) conducted a selective meta-analysis of language focused intervention on the mathematics performance for English learners. They found little empirical work in the area with most studies published in the last five years. The following pedagogical components were identified in the interventions reviewed: instructional scaffolding, explicit instruction; visual representations; use of manipulatives; range of examples; mathematical communication; connection to daily life experiences; feedback; peer modelling/pairing; advance organisers; questioning; one-on-one instruction; use of technology; small-group instruction; strategy cues; large group instruction; practice opportunities and dynamic assessment.

Dowker (2019) refers to the key role that emotions and attitudes play in learning mathematics and that some learners may develop severe anxiety and fear of the subject. While some will need more intensive psychology interventions there is evidence that interventions that lead to an improvement in performance also help to reduce anxiety and help develop more positive attitudes (Dowker, 2019).

In reviewing interventions, particularly those targeted at learners who require more intensive support, Dowker (2019) has identified the features of the most effective. These include effective assessment of the learners' current mathematical performance and developing a profile of strengths and areas for development; adopting a developmental approach and using knowledge of learning trajectories for key areas. Careful planning is also a hallmark of successful approaches and professional development for staff involved. In addition, programmes using games to motivate learners and not associate mathematics with fear and anxiety are an important feature of successful approaches. The National Educational Psychological Service (2020) have put together a very useful guide for teachers around evidence based interventions.

Use of technology

The evidence base of the use of technology that builds on students' strengths is growing but studies also highlighted that the quality of instruction is more important than the learning environment. The role of the teacher facilitating the process is crucial (Monei and Pedro, 2017). Küçükalkan et al. (2019, p.920) in a meta-analysis examination of the effects of computer-based mathematics instruction methods in children with mathematical learning difficulties found that "in four categories: Computer-Assisted Instruction (CAI), Computer Enriched Instruction (CEI), Computer-Managed Instruction (CMI) and Computer-Simulated Instruction (CSI). All CBI methods implemented in educational settings have small or medium effect size." The conclusion drawn from the review is that computer based instruction had positive effects on learners experiencing difficulties in mathematics.

Benavides-Varela et al (2020) meta-analyzed empirical evidence about the effectiveness of digital-based interventions for students with mathematical learning difficulties. A systematic search of randomized controlled studies published between 2003 and 2019 was conducted. A total of 15 studies were included. A random effects meta-analysis indicated that digital-based interventions generally improved mathematical performance (mean ES = 0.55). There was no evidence that videogames offered additional advantages over digital-based drilling and tutoring approaches. The effect size was not moderated when interventions were delivered in primary school or in preschool.

Nelson and Powell (2017) in a systematic review of longitudinal studies of mathematics difficulty indicate that students with math difficulty demonstrate growth in mathematics but continue to perform at a lower level than peers. They may continue to struggle in later grades. In summary, in the 35 studies included, more than 15 methods of identification were used to identify students in just the first category of math difficulty. For these reasons, practitioners and researchers would benefit from a consensus on what defines math difficulty.

Early intervention

A key finding across reviews is the importance of prevention and early intervention for mathematics difficulties (Charitaki et al., 2021). Nelson and Powell (2017, p.536) argue based on their review that "without targeted interventions and early determination of difficulty with math, students as early as kindergarten who display math difficulty may be at risk for poor secondary and adulthood outcomes."

The researchers found very different profiles for learners in terms of the difficulties they were experiencing and cautioned that interventions that may work for some may not work for others. They recommend frequent monitoring of progress of learners with difficulties so changes can be made to teaching interventions.

While early intervention is critical it leads to the question of what should the focus of that intervention be. Nelson and McMaster (2018, p.1001) in a meta-analysis of the effects of early numeracy interventions for students in preschool and early elementary found that "the results of this meta-analysis indicated that interventions for preschool, kindergarten, and first-grade students that focused on teaching early numeracy skills were moderately effective. Interventions were more effective when they included counting with correspondence skills. Due to the number of students who enter school with limited proficiency in whole number understanding, the results of this meta-analysis are encouraging and suggest that preschool through first-grade represents a critical window for math intervention."

The importance of including counting with one-to-one correspondence in early interventions is underlined by the fact that it accounted for 51% of the variance in treatment effects (Nelson and McMaster, 2018). Interestingly, counting is not one of the named early activities in the primary mathematics curriculum but it does get increased emphasis in the revised drafts published by the NCCA. The influence of Piaget on the 1999 curriculum who believed in focussing on sorting, matching, classifying and ordering to develop skills of logic before counting may have contributed to this gap. Earlier research from the 1970s had outlined a really beneficial developmental trajectory of counting through identified principles of counting (one-one, stable order, cardinality, abstraction and order-irrelevance) (Gelman and Gallistel, (1978). These form a really useful framework for observing, assessing, planning and teaching early number skills to children.

Contrary to Piaget, sorting, matching, classifying and ordering though important skills in their own right do not automatically lead to the development of counting skills. Nelson and McMaster (2018) reiterate how counting is the gateway to other further mathematical skills such as cardinality and early addition and subtraction. Other areas of promise in early intervention include an emphasis on subitising, ordinal numbers and number line estimation. (Nelson and McMaster, 2018). They also point to the benefits of explicitly teaching mathematics vocabulary which requires few additional resources. Mnemonic strategies and games-based pedagogy are suggested as other options for vocabulary instruction (Nelson and McMaster, 2018). In the Charitaki et al., (2021) review combining instructional strategies such as Concrete-Representation-Abstract (CRA) methods, explicit instruction, corrective feedback, the use of concrete manipulatives and visual representation were found to be effective in 5-8 year olds experiencing low attainment in mathematics.

Another interesting finding from this review was that for many children brief interventions were successful in improving achievement levels. Given what we know about performance as early as preschool predicting later mathematics performance, and the gaps that develop without intervention, the case for a strong focus on counting and the language of mathematics in the early years is irrefutable.

Upper primary and post-primary

In a review taking stock of 40 Years of research on mathematical learning disability Lewis et al. (2016, p.338) looked at 164 studies on MLD published between 1974 and 2013. Findings indicate that (a) there was great variability in the classification methods used, (b) studies rarely reported demographic differences between the MLD and typically achieving groups, and (c) studies overwhelmingly focused on primary-aged students engaged in basic arithmetic calculation.

While there is evidence of less interventions in upper primary there is some recent promising work in supporting students' understanding of fractions. This is an area that has caused difficulty for many learners but is essential as a basis for understanding decimals and percentages and for progress in algebra at second level. Roesslein and Codding (2018) in a review found “that all interventions included multiple evidence-based instructional components (e.g., concrete and visual representations, range and sequence of examples, etc.). These multicomponent interventions improved performance on a variety of proximal fraction outcome measures”.

Ennis and Losinski (2019, p.381) in a review of instructional interventions to improve fraction skills for students with disabilities found that “explicit instruction possessed the widest literature base, with six studies utilizing the method, yielding large effects”. This was followed by graduated instruction, “which uses components of explicit instruction when moving from a concrete representation of a problem to an abstract one”; “strategy instruction, which uses explicit instruction to teach specific approaches, also had a large effect within three studies” and “anchored instruction, which entails a real-world problem-solving approach (e.g., building a hovercraft), had modest effects”.

Stevens et al. in a review focussing on post primary students with mathematical difficulties (2018, p.335) found “no studies singularly targeted geometry, measurement and data, statistics and probability, or algebraic expressions and equations, all of which are necessary skills to succeed in middle and high school courses.” Lewis and Fisher (2016) recommend an increased focus on algebra given its central importance as an educational gatekeeper and because difficulties with algebra are not so easily resolved by use of a calculator.

There are mixed views on the length of interventions and much depends on the individual progress of the particular learners. There is evidence that short interventions in early primary can make a large difference while interventions with older learners required greater intensity and duration (Stevens et al. 2018). Researchers suggest that this might be due to the difficulties that older learners experience being more complex and entrenched and that they may have significant knowledge gaps across a number of mathematics areas (Stevens et al. 2018).

Learners with complex needs

In reviewing the literature in mathematics for children and young people with developmental disabilities including moderate/severe general learning disabilities approaches underpinned by a behavioural approach currently have the strongest evidence base. Techniques such as systematic instruction, task-analytic instruction, system of least prompts, constant time delay, simultaneous prompting have been shown to be successful teaching strategies (Bowman et al.

2019). The most recent review, that of Schnepel & Aunio (2021, p.1) found that "consistent with previous studies, the analysis showed that interventions with systematic and explicit instruction with feedback and the use of manipulatives are effective instructional approaches and strategies for students with ID". Effective interventions had well-structured, high intensity learning sequences that were adapted to the students' achievement level.

As referred to above, the use of concrete materials has also been shown to be effective supports for learning key mathematical concepts. The use of instructional technology, computer assisted, augmented reality and video modelling have an emerging research base (King et al., 2016). There is also promise in the teaching of cognitive strategies and schema based instruction. Clausen et al., (2021) systematically reviewed the used of modified schema-based instruction to teach mathematical word problem solving to students with moderate and severe disabilities. While interventions made a difference they concluded that it did not yet meet the criteria for an evidence-based practice and recommended further research. The use of graphic organisers and peer support/mediated have also been highlighted. Maths stories and board games have been shown to increase understanding.

A popular US programme *Touch Maths* which incorporates touch points on numerals in a multi-sensory approach has also been shown to be effective in teaching number and number operations for learners with general learning disabilities (Cox et al., 2020). The importance of teaching for generalisation has been stressed and being conscious of the demands of attention, memory and language. In addition, it has been shown that colour can compete for attention in number tasks (Bowman et al 2019; Cox & Jimenez, 2020). A review in relation to Autism and co-morbid learning disability found little specific outside of the recommendations above for learners with general learning disabilities and recommended further research in this area (King et al., 2016).

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Appendices

Research questions

- How can learners who experience difficulties learning mathematics be supported in developing numeracy skills within mathematics and across the curriculum?
- How can learners with an intellectual disability be supported in developing numeracy skills within mathematics and across the curriculum?

Search terms

The first search used the following search terms: ("numeracy" OR "math*" OR "arithmetic*" OR "problem solving" OR "math fluency" OR "number sense") AND ("dyscalculia" OR "math* disability" OR "math* difficult*" OR "low attainment in math*") AND ("intervention" OR "instruction") AND ("systematic review" OR "meta-analysis" OR "research synthesis"). The second search replaced the second line with AND ("intellectual disability*" OR "developmental disability*" OR "cognitive disability*" OR "special education"). The exclusion criteria used were pre 2011, post-secondary, non-numeracy/non-mathematics focus. The EBSCO and SCOPUS databases were used and *The International Handbook of Mathematical Learning Difficulties: From Laboratory to the Classroom* (Fritz et al. 2019) was also consulted as a summary of research in the field.

Figure 1. PRISMA flow diagram for review of reviews for supports for numeracy/mathematics (learning difficulties/dyscalculia/low attainment)

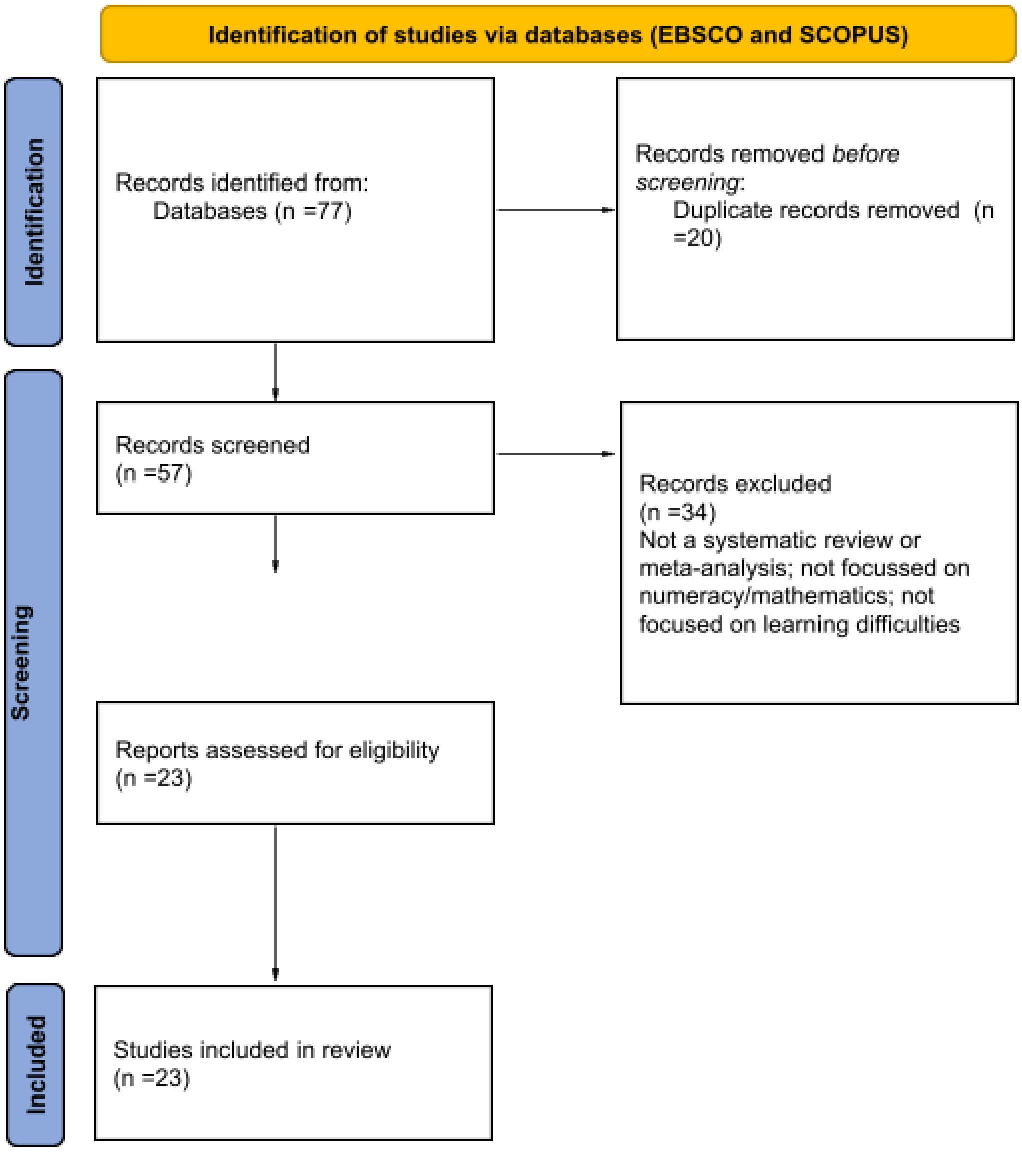

Figure 2. PRISMA flow diagram for review of reviews for supports for numeracy/mathematics (intellectual/cognitive/developmental disabilities)

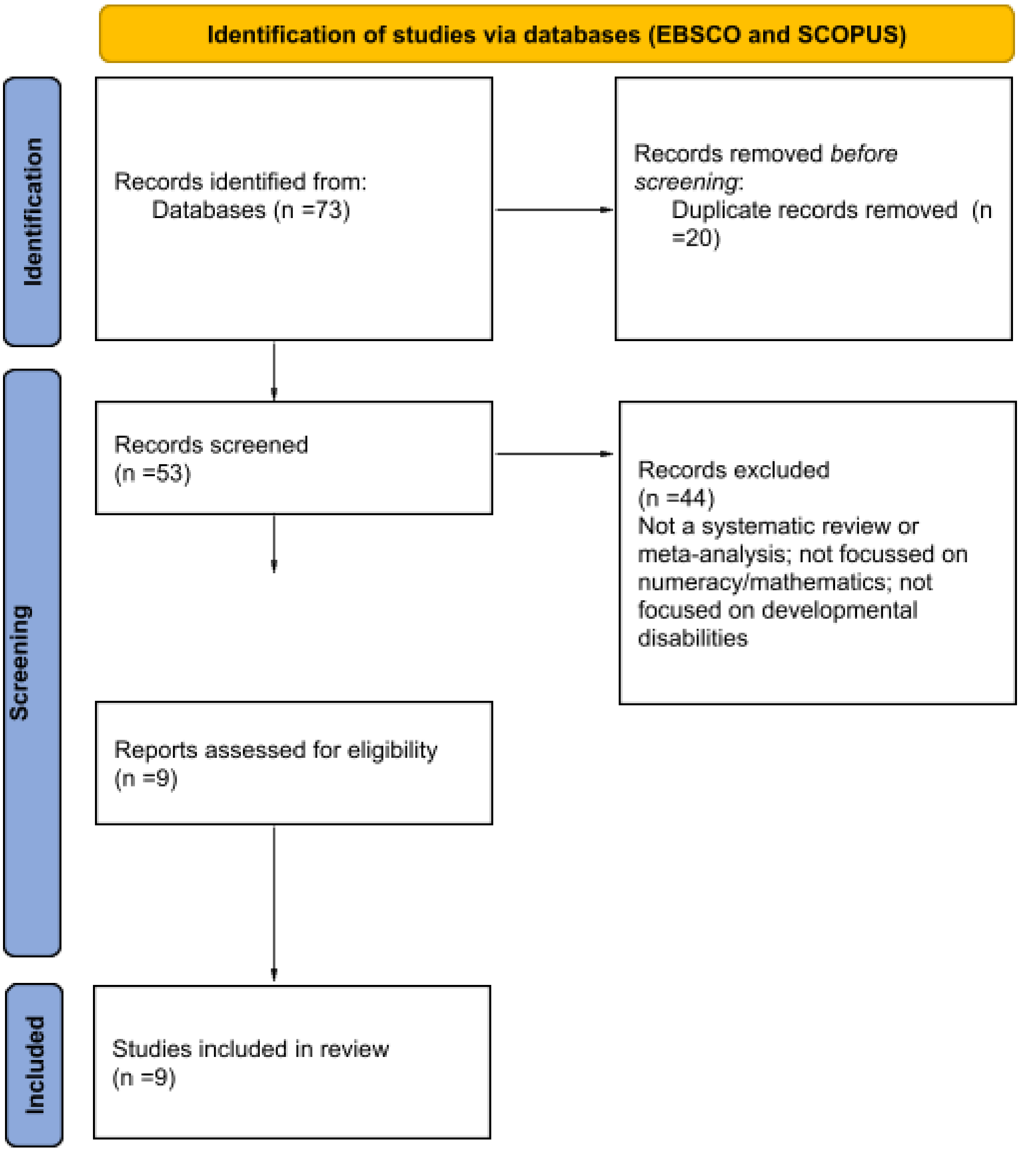

Table 1. Tabulation of findings including effect sizes for dyscalculia/math disability

Review	Number of studies	Effect size (If available)	SEN/ Disability	Age range	Findings
Alegre et al., (2019)	50	The average effect size was reported to be large to very large (Cohen's d = .89; standard deviation = .60).		Primary	Cross-age tutoring has better results than same-age tutoring
Arizmendi et al., (2021)	175	Hedges' g of 0.26	Maths difficulties	Primary	If an approach to math intervention is language-focused, then those parameters related to an EL's specific academic needs in the language of math need to be specified. If an approach to math intervention is language-focused, then those parameters related to the learner's specific academic needs in the language of math need to be specified.
Benavides-Varela et al., (2020)	15	ES 0.55	Maths disabilities and difficulties	Pre-school and primary	Technological tools positively impact maths achievement There was no evidence that videogames offer additional advantages with respect to digital-based drilling and tutoring approaches.

Bouck and Park, (2018)	26		Maths difficulties	Primary	Manipulatives had a positive impact on student learning
Charitaki et al., (2021)	20	0.67	Low attainers in Maths	Pre-school and primary 5-8	"On average, the interventions included instructional strategies such as explicit instruction, corrective feedback, CRA, concrete manipulatives, and visual representation, and one-to-one instructional arrangement are moderately effective for children aged 5–8."
Chodura et al., (2015)	35	0.83	Maths difficulties	Primary	Direct and assisted instruction effective for learners at risk of dyscalculia
Dennis et al., (2016)	25	The mean adjusted ES was 0.53 across all studies, with the 95 percent confidence interval ranging between 0.36 and 1.07.	Maths learning difficulties	Primary	Peer assisted learning; explicit teacher-led instruction; both small group and one-to-one equally effective; importance of strong core teaching in the whole class setting
Ennis and Losinski, (2019)	21	Effect sizes ranged from g = 0.42 to 11.51		Primary	Explicit instruction; graduated instruction (concrete to abstract); strategy instruction and anchored instruction (real world problem solving) all effective

Furlong et al., (2016)			Maths difficulties	Primary	"Sequencing tasks from easy to difficult, task analysis, combining with teachers providing explicit explanation of concepts and procedures, were more likely to benefit students with MD. Moreover, we found that intervention delivered in a form of small-group instruction was a more effective and efficient instructional delivery mode for this group of students."
Genesis et al., (2021)			English language learners with difficulties in maths	Primary	The following pedagogical components were identified in the interventions reviewed: instructional scaffolding, explicit instruction; visual representations; use of manipulatives; range of examples; mathematical communication; connection to daily life experiences; feedback; peer modelling/pairing; advance organisers; questioning; one-on-one instruction; use of technology; small-group instruction; strategy cues; large group instruction; practice opportunities and dynamic assessment.
Küçükalkan et al., (2019)	33	Effect size Computer-Based Instruction g 0.606 lower limit0.488 upper limit 0.724 p*0.000	Maths learning disability	Children	CBI has been examined in four categories: Computer-Assisted Instruction (CAI), Computer Enriched Instruction (CEI), Computer-Managed Instruction (CMI) and Computer-Simulated Instruction (CSI). All CBI methods implemented in educational settings have small or medium effect size.
Lafay et al., (2019)	38		Maths disabilities	Primary	"Interventions using manipulatives were reported to be effective for a range of learning objectives (e.g., conceptual understanding and computational fluency)"

Lein et al., (2020)	33	g = 0.56, 95% confidence interval [0.40, 0.72	Learning disabilities	K-12	Intervention effects were larger for elementary grades than secondary grades; results support mathematics word problem solving interventions for students with LD and/or MD. Interventions were more effective when they focused on the underlying problem structure and directly targeted teaching for transfer.
Lewis and Fisher (2016)	164			Post-primary	An increased focus on algebra given its central importance as an educational gatekeeper and because difficulties with algebra are not so easily resolved by use of a calculator.
McKenna et al., (2015)	5		Learning disabilities	Elementary and primary	Infrequent use of key strategies: Explicit instruction, cognitive strategy instruction, student verbalisation and discourse, independent practice, visual representation, range and sequence of examples, checking for understanding.
Monei and Pedro, (2017)	11 studies		dyscalculia	Elementary and primary	Specific, individualised training Computer-assisted instruction (CAI) Strategy instruction on solution accuracy) Preventative tutoring on the math problem solving (Schema-broadening tutoring) Neuropsychological intervention on performance The adaptive use of approximate calculation in an addition verification task Tier 3 intervention Computer-mediated instruction (CMI) and teacher mediated instruction (TMI) Tier 2 intervention in a multi tiered model Strategic counting instruction with and without deliberate practice with those counting strategies, on number combination (NC) Strategy instruction and working memory capacity (WMC)

Nelson and McMaster, (2019)	34	The average weighted effect size for numeracy interventions with two outliers removed was moderate (g 0.64), and the 95% confidence interval did not include zero [0.52, 0.76].	Young students, including students with disabilities and those at risk for MD	Pre-school to 1st grade	"Interventions were more effective when they included counting with correspondence skills. Due to the number of students who enter school with limited proficiency in whole number understanding, the results of this meta-analysis are encouraging and suggest that preschool through first-grade represents a critical window for math intervention." p.1001
Nelson and Powel, (2018)	35				"Without targeted interventions and early determination of difficulty with math, students as early as kindergarten who display math difficulty may be at risk for poor secondary and adulthood outcomes." P.536
Park et al., (2021)	16		Learning disabilities	K-12	Overall, students with learning disabilities demonstrated mathematical improvement after receiving interventions involving virtual manipulatives.
Peltier, C., et al. (2019)	53	0.34-1.00 range with	Learning disability	K-12	Positive effects for virtual manipulatives as well as concrete

		omnibus ES 0.91 (CI 0.87-0.95)		grade	which may be less stigmatising for older students
Powell et al., (2019)	65		Maths difficulties	primary	Interventions in mathematics improved maths outcomes; students with co-morbid difficulties in reading have a greater need for word problem intervention
Roesslein and Codding, (2018)	12			Elementary students struggling with maths	Multi-component evidence based strategies (concrete and visual representations, range and sequence of examples improved fraction knowledge
Stevens et al., (2018)	25	0.49		Secondary students with mathematical difficulties	Fractions interventions significantly improved students' mathematics outcomes more than interventions in operations

Table 2. Tabulation of findings including effect sizes for developmental disabilities

Review	Number of studies	Effect size (if available)	SEN/ Disability	Age range	Findings
Bowman, J.A., McDonnell, J., Ryan, J.H., Fudge-Coleman, O. (2019)	24		Moderate and severe disabilities	5-21	Concrete representation Anchored instruction Instructional technology Teaching for generalisation
Clausen et al., (2021)	12		Moderate and severe intellectual disabilities	Middle school	While interventions made a difference they concluded that it did not yet meet the criteria for an evidence-based practice and recommended further research.
Cox, S.K. and Jimenez, B.A. (2020)	Review of 7 reviews		Autism and learning disability	K-12 grade	System of least prompts Constant time delay Simultaneous prompting Schema based instruction Cognitive strategies Video modelling Augmented reality

					Touch Maths programme Use of Maths stories Virtual or concrete materials Peer supports Graphic organisers
Hudson et al., (2018)	33		Moderate or severe disability	3-22 years	Systematic instruction In vivo instruction System of least prompts strategy Constant time delay strategy Task-analytic instruction
King, S. A.; Lemons, C. J.; Davidson, K. A. (2016)	14		Autism and learning disability	5-17 years	Explicit instruction with prompts and positive consequences and manipulatives Emerging: Video modelling Computer assisted Peer mediated
Lemons, C. J., Powell, S. R., King, S. A., and	9		Down syndrome	6-20 years	Direct instruction, Modelling, Guided practice, Concrete materials Games Graphic organisers, Prompting and Feedback

Davidson, K. A. (2015)					Computer assisted
Porter, J. (2019)	8		Down Syndrome	12 mths. - 35 years	More discriminating between quantities over acquisition of number string Colour can compete for attention in number tasks Board games foster understanding Be mindful of the demands of attention, memory and language
Schnepel, S. & Aunio, P. (2021).	20		Intellectual disabilities	5-12	"Consistent with previous studies, the analysis showed that interventions with systematic and explicit instruction with feedback and the use of manipulatives are effective instructional approaches and strategies for students with ID" (p.1)
Spooner, F., Root, J. R., Saunders, A. F., & Browder, D. M. (2019)	36		Moderate and severe developmental disability	3-21 years	Systematic instruction Technology-aided instruction Graphic organisers Manipulatives