



UNDERSTANDING OF GENERAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

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Abstract: This article covers the concept of general solutions of partial differential equations. Also, as in real differential equations, partial differential equations have infinitely many solutions. These solutions are called general solutions. Private solutions are separated from general solutions based on certain conditions. It is scientifically justified that these additional conditions are given in the general limit of the equation of the field under consideration.

Key words: differential equations with particular derivatives, general solutions, differential equations, additional conditional equations

An equation that necessarily involves the derivative or differential of an unknown function is called a differential equation. If the unknown function has one argument, the corresponding equation is called an ordinary differential equation, and if it has multiple arguments, it is called a differential equation with particular derivative.

Various processes occurring in nature (physical, chemical, mechanical, biological, etc.) have their own laws of action. Some processes may occur according to the same law, in such cases, their study is much easier. But it is not always possible to directly find the laws describing the processes. Finding relationships between characteristic quantities and their derivatives is naturally easy. The solution of many natural and technical problems is brought to the search for such unknown functions, where this function represents a given event or process, and certain relationships and connections are given between this unknown function and its derivatives. Expressions connected based on such relationships and laws are examples of differential equations. Differential equations and their systems are used in the construction of mathematical models of many dynamic processes. The set of solutions of such differential equations or their systems is infinite, and the solutions differ from each other by fixed numbers. Initial or boundary conditions are additionally set for single-valued determination of the solution. The number of such conditions should be consistent with the order of the differential equation or their system. Depending on the provision of additional conditions, differential equations are divided into the following two types of problems:

- Cauchy's problem - if one point (starting point) of the interval is given as an additional condition;
- Boundary problem - if the additional condition is given on the boundaries of the interval.

1 - definition. A differential equation is an equation that expresses the connection between the arbitrary variable x , the unknown function $y=f(x)$ and its derivatives u' , u'' , $u(n)$. If the desired function $y=f(x)$ is a function of one arbitrary variable, then the differential equation is an ordinary differential equation, if it is a function of several variables $u=U(x_1, x_2, \dots, x_n)$ is called a partial differential equation.

Definition 2. The order of the differential equation is the highest order of the derivative in the equation.

Definition 3. The solution or integral of a differential equation is any function $y=f(x)$ that transforms the differential equation into an integral.

The first-order differential equation in general has the following form.

$$F(x, y, y') = 0 \quad (1)$$

If this equation can be solved with respect to the first-order derivative, then we have equation

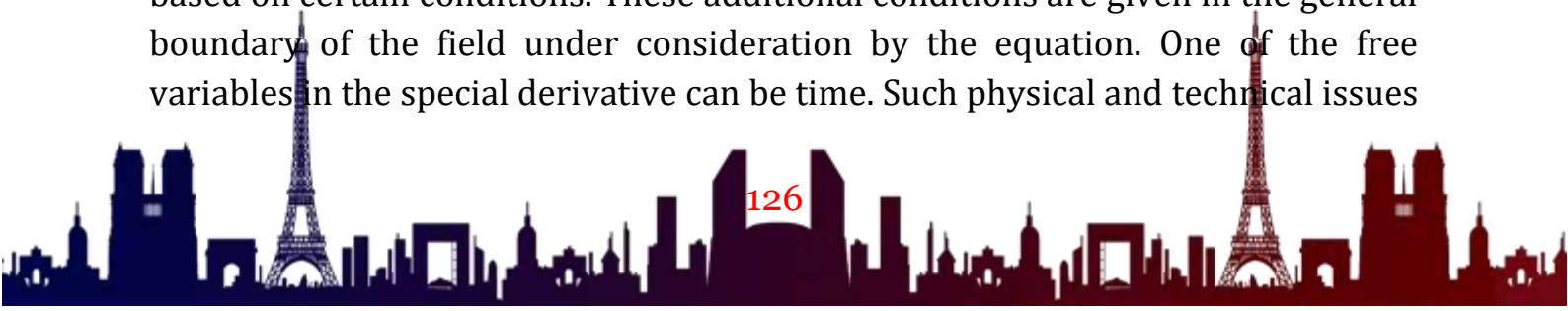
$$y' = f(x, y) \quad (2)$$

Usually, equation (2) is called an equation solved with respect to the derivative. The theorem about the existence and uniqueness of the solution for equation (2) is valid:

Theorem. If the function $f(x, y)$ in equation (2) and the special derivative df/dy obtained from it with respect to y are continuous functions in an area containing the point (x_0, y_0) in the XOY plane, then y of the given equation There is only one solution $y=f(x)$ satisfying the condition $(x_0)=y_0$. The condition that the function $y(x)$ must be equal to the number y_0 at $x=x_0$ is called the initial condition:

$$y(x_0) = y_0$$

If the unknown function in the differential equation depends on two or more arguments, it is called a partial differential equation. It is clear from the name of such equations that the special derivatives of the function with respect to arbitrary arguments are involved in them. As in ordinary differential equations, partial differential equations have infinitely many solutions. These solutions are called general solutions. Private solutions are separated from general solutions based on certain conditions. These additional conditions are given in the general boundary of the field under consideration by the equation. One of the free variables in the special derivative can be time. Such physical and technical issues



are common in practice. As additional conditions, the values of the sought-after function at a specified value of time are used for such equations. For example, the condition can be given at the initial time $t=0$ (or generally $t = 0$, $t = \text{const}$). We call such a condition an initial condition.

If additional conditions are given at the boundary of the field, such a problem is called a boundary problem. If only the initial condition is given without the boundary conditions, such a problem is called the Cauchy problem for partial differential equations. In this case, the matter is considered in an infinite field. If the problem involves both initial and boundary conditions, such a problem is called a mixed problem. Solutions of differential equations can be exact (analytical) and approximate (numerical). If some differential equations can be solved exactly, in practice there are such equations, especially their systems, that cannot be found exact solutions. Even for equations that have an analytical solution, in some cases it is necessary to find numerical solutions with previously given values. That is why numerical methods for solving ordinary differential equations have been developed. Solutions of differential equations can be exact (analytical) and approximate (numerical). While some differential equations can be solved exactly, in practice there are such equations, especially their systems, whose exact solutions cannot be found. Even for equations that have an analytical solution, in some cases it is necessary to find numerical solutions with previously given values. That is why numerical methods for solving ordinary differential equations have been developed.

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