Surrogate model-based sensitivity analysis of a one-dimensional arterial pulse wave propagation model with correlated input

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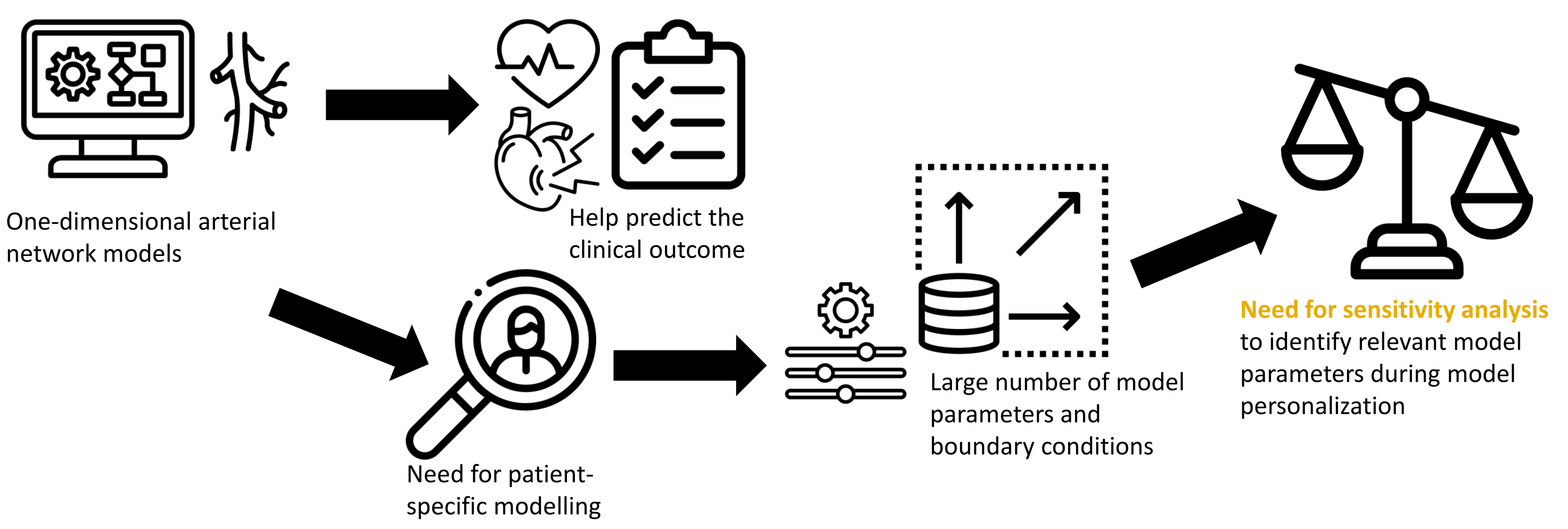
Background



One-dimensional arterial network models

Help predict the clinical outcome

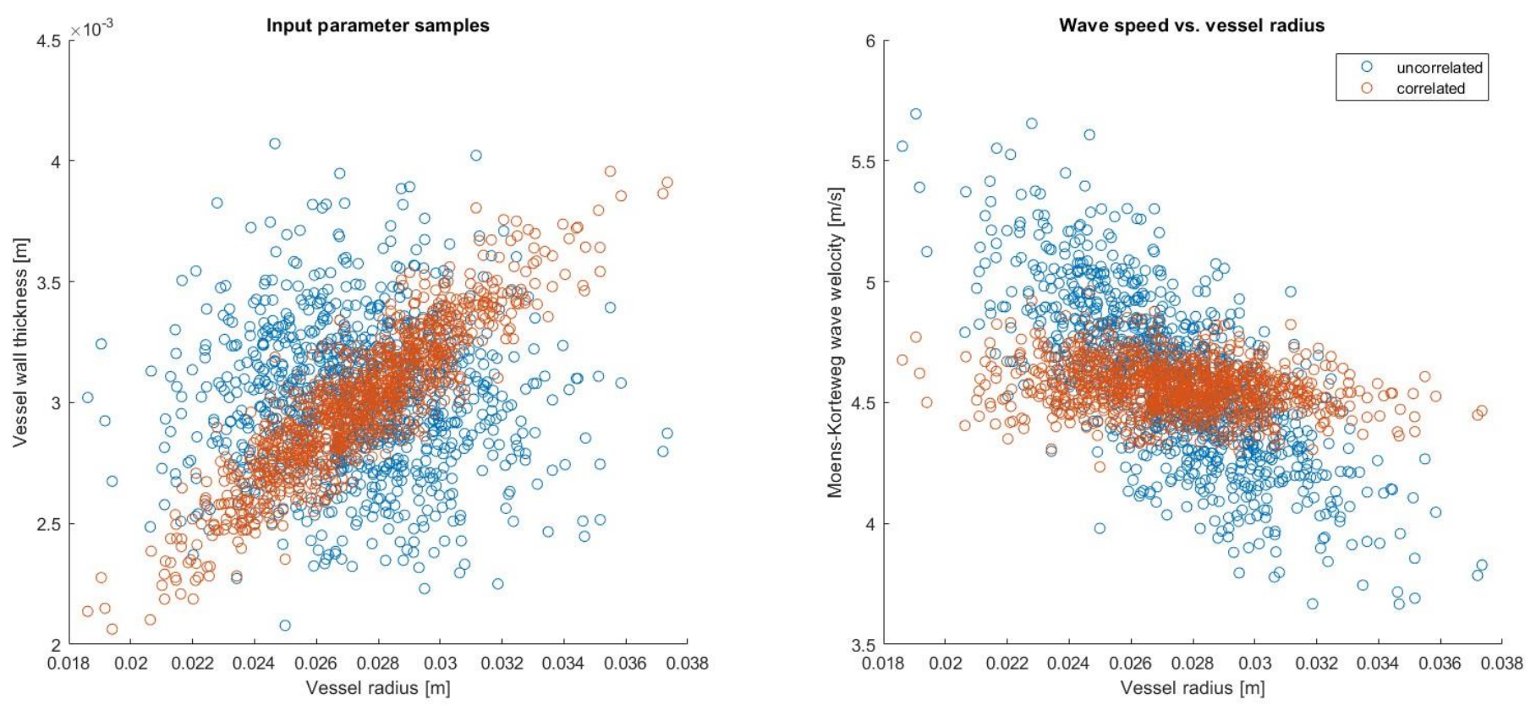
Need for patient-specific modelling

Large number of model parameters and boundary conditions

Need for sensitivity analysis to identify relevant model parameters during model personalization

Aim

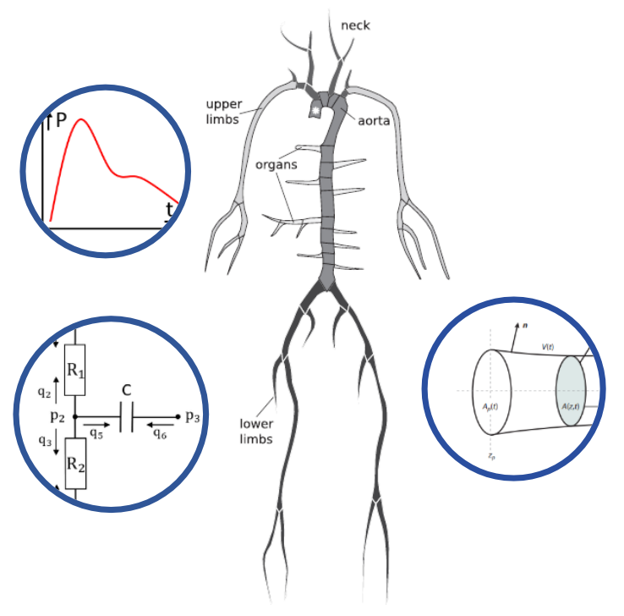
However statistical independency between parameters is often assumed incorrectly. The input-output relation might differ as you can see in the MC experiments conducted for the pulse wave velocity:



The aim is to investigate the effects of considering correlations on the importance ranking of model parameters.

Methods

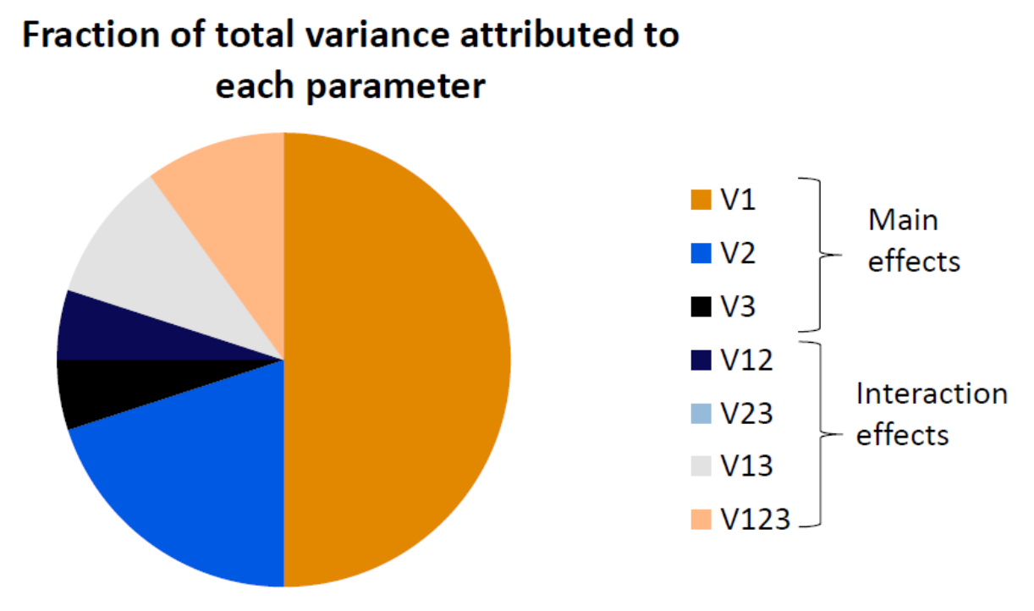
1. **Create a fast-evaluating surrogate model** of the 1D pulse wave propagation model of Boileau et al.[1] using the vectorial kernel orthogonal greedy algorithm by Santin et al.[2]



$$S(x_i) := \sum_{j=1}^{n} \alpha_j K(x_j, x_i)$$

$$K(x_j, x\,) = \exp(-\frac{\left\|x_j - x\right\|^2}{2\sigma^2})$$

2. **Perform sensitivity analysis** on the surrogate model whilst **taking correlations into account** based on the method of Li et al.[3]



$$S_i \rightarrow S_i^{TC} = \frac{V(E(Y|X_i))}{V(Y)} = S_i^U + S_i^C$$

$$S_i^T \rightarrow S_{T,i}^{TU} = \frac{E(V(Y|X_{-i}))}{V(Y)} = S_i^U + S_i^{IU}$$

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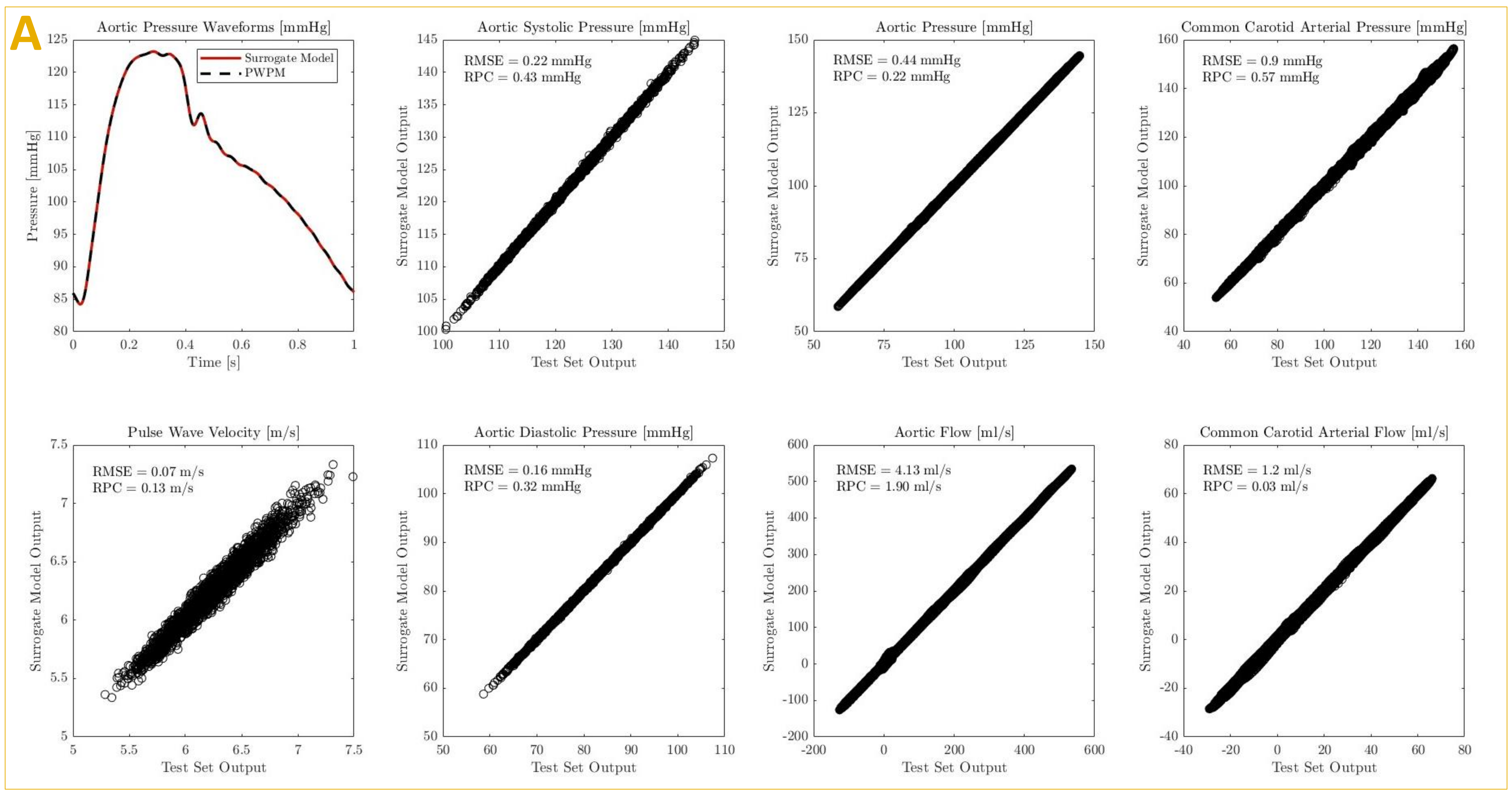
[1] Boileau, E., et al. (2015). A benchmark study of numerical schemes for one-dimensional arterial blood flow modelling. *International Journal for Numerical Methods in Biomedical Engineering*

[2] G. Santin and B. Haasdonk, *Kernel Methods for Surrogate Modeling*, ArXiv preprint 1907.10556 (2019).

[3] Li, L., et al. (2016) General validation and decomposition of the variance-based measures for models with correlated inputs. *Aerospace Science and Technology*,

Results



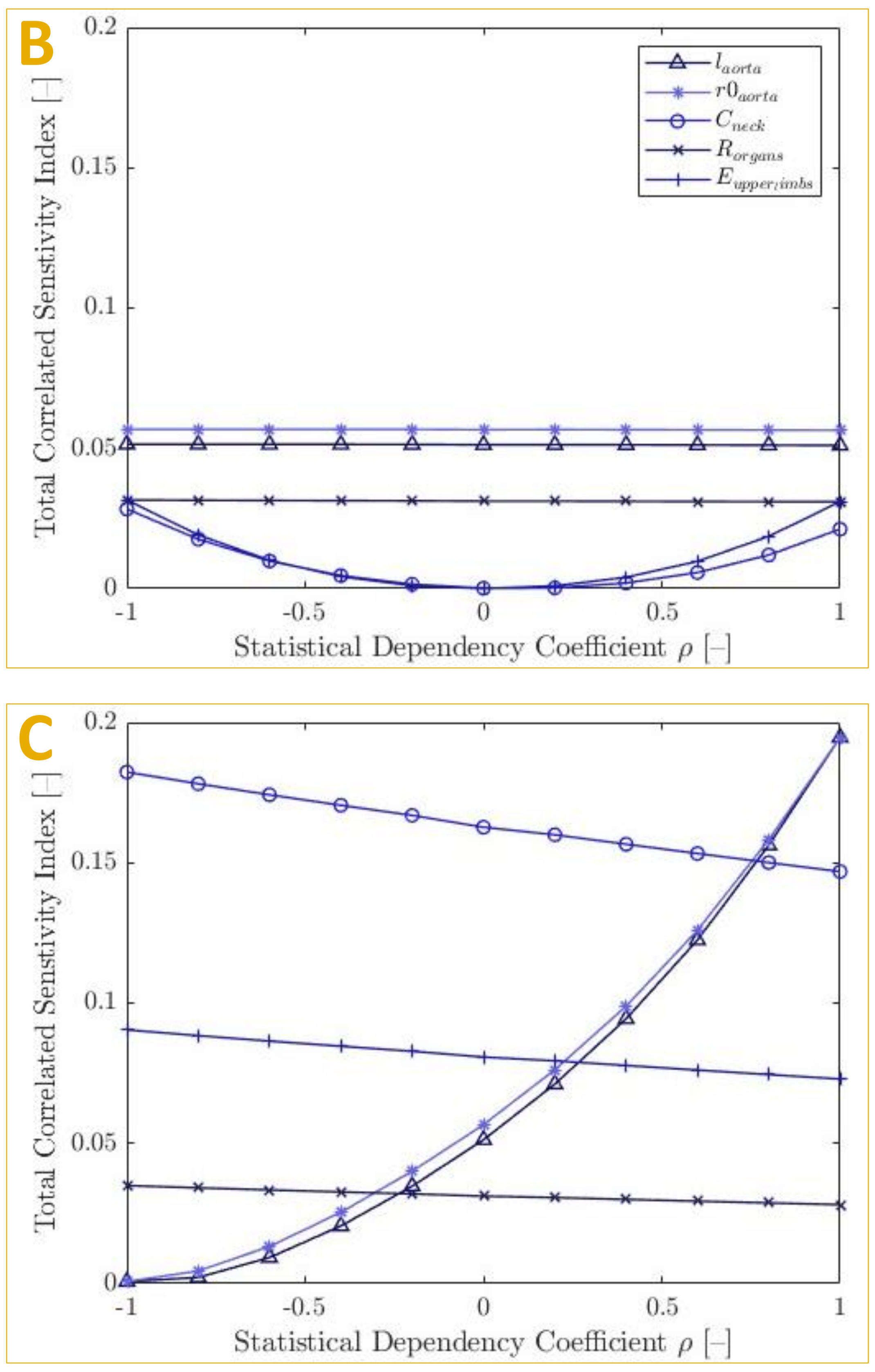


Figure A: Surrogate model performance results showing the overlapping pressure waveforms and the agreement between the surrogate output and test dataset output when training the surrogate model on different outputs. **Figure B & C:** Sensitivity indices of a subset of input parameters, representing their contribution to the variance in the pulse wave velocity, whilst having varying statistical dependency coefficients (Pearson correlation). In **Figure B** the correlation is defined between various parameters that were expected to be correlated to a low extent. In **Figure C** the correlation is defined between the aortic radius and length, which were expected to be highly correlated.

Conclusions

This study involved a methodology of two consecutive parts. The first part, the generation of the surrogate model, resulted in surrogate models that were able to **accurately generate similar results** as that of the one-dimensional pulse wave propagation model. The surrogate models could **evaluate the one-dimensional pulse wave propagation model within 0.05 milliseconds**. In the second part, the sensitivity analysis was performed and the method, whilst assuming uncorrelated input parameters, was benchmarked against the established agPCE method by Quicken et al.[4] Due to the immensely fast evaluation time, the correlated sensitivity analysis could be **performed within an hour**, even though it required more than **70 million model evaluations**. The correlated sensitivity analysis showed that considering correlations can significantly affect the computed sensitivity indices. Input parameters that are evidently correlated are affected to a higher extent than uncorrelated parameters. Therefore, when performing sensitivity analysis within the field of cardiovascular biomechanics it is **important to consider correlations** because they can significantly alter the parameters their importance ranking and thus seriously affect your decision concerning input prioritization during model personalization.

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[4] Quicken, J. A. C., et al. (2016). Application of an adaptive polynomial chaos expansion on computationally expensive three-dimensional cardiovascular models for uncertainty quantification and sensitivity analysis. *Journal of Biomechanical Engineering*

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