ISSN: 2507-7589 ○ EISSN: 2507-7619

Recueil De Mécanique
El Wancharissi University
of Tissemsilt

Research Paper DOI : 10.5281/zenodo.7207454 Open access

Numerical correlations for Herschel bulkley fluid flow through an axisymmetric sudden expansion

***Mokhtar BEKHADRA*[a], *Nouredine SAD CHEMLOUL*[b], *Mohamed Amin MAKBOUL*[c]**

[a,b] *Laboratory of industrial technologies, Department of mechanical engineering, Faculty of Applied Sciences, University Ibn Khaldoun of Tiaret, 14000 Tiaret, Algeria.*

[c] *Laboratoire MECACOMP, Département de Mécanique*

ARTICLE INFO

Article history :

Received 21 May 22

Received in revised form 30 Spetember 22

Accepted 03 October 22

Keywords:

Herschel-Bulkley fluid; Sudden expansion; Recirculation length; Pressure loss

ABSTRACT

A numerical study carried out for the laminar flow of viscoplastic fluid (Herschel Bulkley model) through an axisymmetric sudden expansion of aspect ratio $Er = 1 : 2$ and for four values of Reynolds number confined between $Re = [50 \sim 200]$, Bingham $Bn = [0 \sim 2]$ respectively and the power low index $n = 0.6$, 0.8. The numerical simulation was performed by the commercial code ANSYS-Fluent. The results show the influence of the different forces on the structure of the flow in which the high plasticity of the flow reduces the length of the recirculation zone and increase the un-yielded zones in comparison with that of the Newtonian fluid, on the other side, the increase in inertial forces and the power low index increase the length of the recirculation zones as well as the redevelopment length. Based on these results, we present numerical correlations for the length of the circulation zone for the Herschel-Bulkley fluid as a function of the Reynolds number Re and the power low index n, these correlations show good agreement with the previous studies with maximum error 1.35%. The present study shows also the variation of local loss coefficient through the sudden expansion, for a Herschel-Bulkley fluid, the increases in Bingham number values increase the local loss coefficient, at higher Bingham number and for the Reynolds number values studied, the fluid becomes similar to Newtonian fluid flow at a very low Reynolds number, while high values of the power low index n slightly decrease the value of the local loss coefficient.

1 Introduction

The flow of viscoplastic fluid through the sudden expansion is common in the oil and gas industry, the fluid passing through these channels undergoes several changes, one of these changes is the fluid pressure which is reduced by the sudden change in the pipe section, to maintain the fluid pressure, it has to know the coefficient of local pressure loss due to the sudden change in the section.

Over time, many studies have focused on these types of flows, the experimental study of *Macango and Hung* [1] for a viscous Newtonian flow through an axisymmetric sudden expansion has reported that the eddy has a small role in energy exchange and is presented according to the Reynolds number. Numerically *Scott and Mirza* [2] create dimensionless functions to calculate the length and the intensity of the recirculation zones as a function of the Reynolds number, later *Badekas and Knight* [3] developed their equations as function of Reynolds number and the aspect ratios of the geometry as well. For

---

** Corresponding author. Tel.: +213 656 585 218.*

E-mail address: mokhtar.bekhadra@univ-tiaret.dz

viscoplastic flow *Scott et al* [4] numerically studied a fluid of Casson and Bingham models through a planer and axisymmetric sudden expansion, their results show a reduction in the length and intensity of the recirculation zones for the flow of viscoplastic fluid in comparison of the Newtonian fluid, the same results presented by *Vradis and Otugen* [5] in which the higher yield stress produces small zones of recirculation, generally , it has the inverse effect of the Reynolds number, this result was also provided by *Hammad and Vradis* [6] for a Herschel Bulkley fluid. The aspect ratio has an effect, according to *K. Hammad* [7] the aspect ratio of $Er = 5$ showed intensive and large recirculation zones then thus of $Er = 2$.

Pak et al [8] studied the flow of Newtonian and non-Newtonian fluid through axisymmetric sudden expansion, they announced that the length of recirculation is also a function of the concentration of non-Newtonian fluid in which the length of recirculation decreases when the concentration of non-Newtonian fluid Newtonian increases and it is smaller than those of Newtonian fluid for laminar flow. However, in turbulent flow, the recirculation length doubles two or three times that of the Newtonian flow and increases gradually as the fluid concentration increases.

Jay et al [9] presented a numerical study of viscoplastic fluid through 1 : 4 axisymmetric sudden expansion, therefore, when the yield stress of fluid decreases the un-yielded zones decrease, and the recirculation zones increase and vice versa. Regardless of the number of Reynolds *Mitsoulis and Huilgol* [10] confirmed the absence of the recirculation zone when the Bingham number goes to an infinite values.

The pressure loss coefficient through the sudden axisymmetric expansion was analysed by *Oliveira and Pinho* [11], *Oliveira et al* [12] a numerical correlation was presented based on numerical results for a Newtonian fluid. Another correlation introduced by *Kfuri et al* [13] for a viscoplastic fluid as a function of the fluid yield stress.

Recently, *Bekhadra et al* [14] introduced numerical correlations for the flow of Bingham fluid through a sudden expansion, this correlations shows the coefficient of pressure loss K as a function of rheological parameters and geometric conditions.

2 Mathematical Formulation

2.1 Problem Description

The geometry studied is an axisymmetric sudden expansion of different aspect ratios was presented in Figure 1. The entrance length of the sudden expansion $L_u = d_1$, where the downstream length $L_d = 120d_1$.

At the entrance, the boundary condition is set to be velocity inlet with a fully developed velocity profile; this boundary condition is introduced by using a separated geometry of considerable length in order to obtain the fully developed profile.

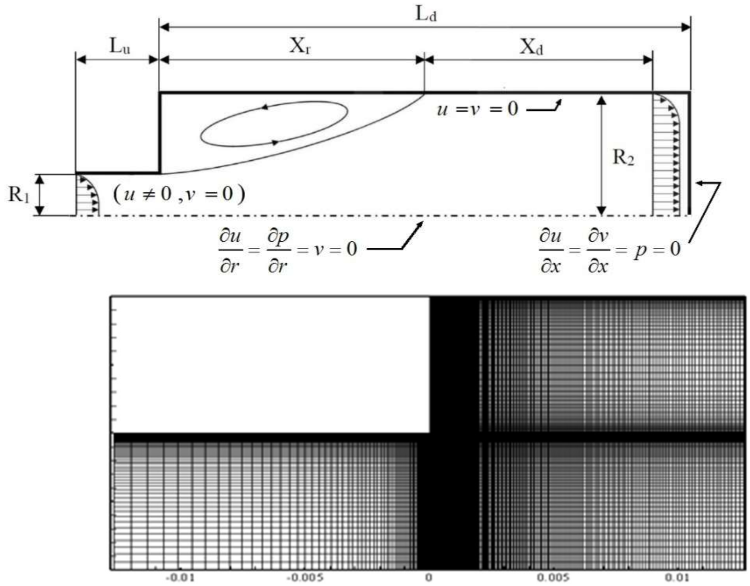

Fig. 1– Sudden expansion geometry and mesh distribution near the 1:2 sudden expansion.

2.2 Governing Equations

The conservation of mass is given by:

$$\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial v}{\partial r}=0 \tag{1}$$

While the conservation of momentum is given by

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial x}+\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rx}\right)\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial p}{\partial r}+\left(\frac{\partial \tau_{rx}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rr}\right)\right) \tag{3}$$

The extra stress tensor for power law with yield stress defined by $\tau_{ij}=2\eta(\dot{\gamma})D_{ij}$ while $\dot{\gamma}$ is the rate of deformation:

$$\dot{\gamma}=\frac{1}{2}(D_{ij}D_{ij})=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{v}{r}\right)^2+\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right)^2\right] \tag{4}$$

The plastic materials equation

$$\tau=\tau_y+K\dot{\gamma} \quad \text{for} \begin{cases}\tau>\tau_y\rightarrow\dot{\gamma}>0, \quad \tau<\tau_y\rightarrow\dot{\gamma}=0\end{cases} \tag{5}$$

3 Numerical Procedure

The numerical solution is obtained by using the commercial code ANSYS-Fluent, the SIMPLE algorithm was used to solve the pressure-velocity coupling. To discretize the convective terms, a quadratic upwind differencing scheme (QUICK) was used, three configurations of meshes are tested for absolute residual values of the continuity, and the axial velocity and the radial velocity are set at 10^{-6}.

3.1 Validation

The comparison of the recirculation length of our numerical solution and the previous studies is shown in Figures 2 and 3. For n = 0.6, 0.8, and for a value of Reynolds number Re = 50, the results showed good agreement with the previous study by *K. Hammad et al* in cases of no inlet section. In case of entrance length the recirculation length decreases a little.

4 Results and Discussion

The zones of recirculation formulated at the downstream flow are affected by several parameters, the Reynolds number has an effect where in the recirculating length doubles when the Reynolds number doubles this is very clear in the previous studies of *Scott and Mirza* [2] for a Newtonian fluid Bn = 0 and *K. Hammad and Vradis* [6], *K. Hammad* [7] for Bingham fluid n = 1. On the other hand, the increase in the power low index n brings the fluid closer to the behaviour of the Bingham fluid which increases the vortex length value as shown in Figures 4 and 5.

The plastic forces represent by the Bingham number have an effect that reduced the length of the recirculation zone without being zero, on the other hand, each increase in the Bingham number will enlarge the in-yielded zones. In total, there is an inverse effect between the forces of inertia and the behaviour index on the one hand and the plastic forces on the other hand.

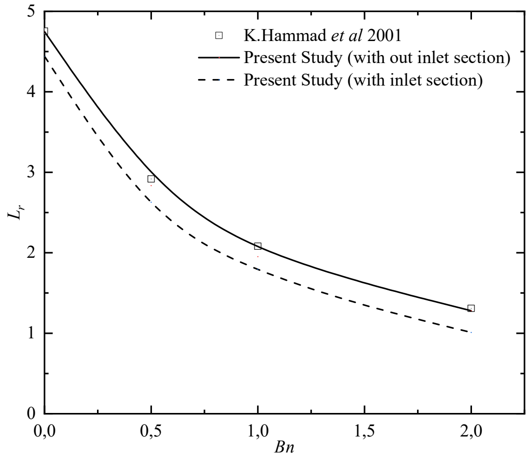

Fig.2 – Comparison of L_r vs *Bn* for *Re = 50* and *n = 0.6*

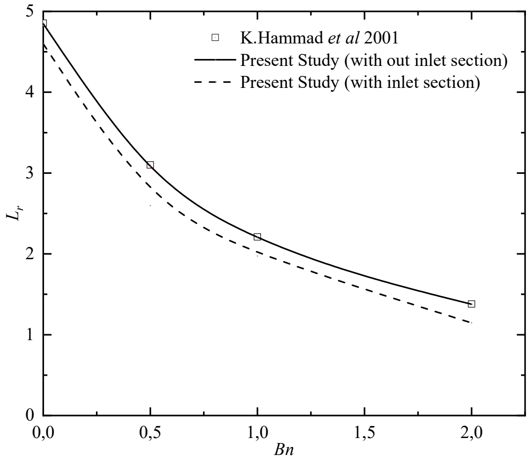

Fig.3 – Comparison of L_r vs *Bn* for *Re = 50* and *n = 0.8*

The effect of Reynolds number and the power low index *n* on the recirculation length show by numerical correlations (5), (6) and (7) for the Bingham numbers *Bn* = 0.5, 1 and 2 respectively, the maximum error value reached 1.15%.

Bn = 0.5 $$\left((0,0165n+0,0127)Re+(-0,347n+0,3)\right) \quad (5)$$

Bn = 1 $$\left((0,0085n+0,0091)Re+(0,226n-0,033)\right) \quad (6)$$

Bn = 2 $$\left((0,0003n+0,0081)Re+(0,729n-0,346)\right) \quad (7)$$

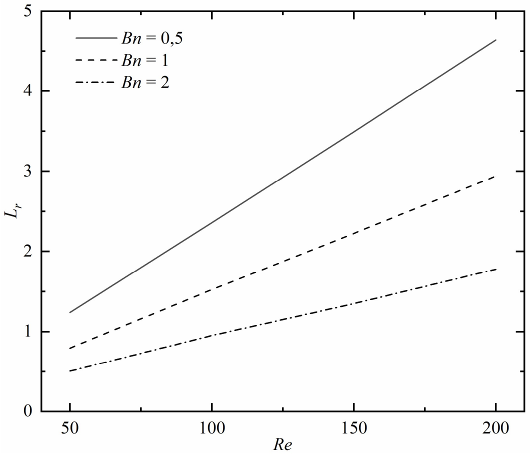

Fig.4 – L_r vs *Re* for *n* = 0.6.

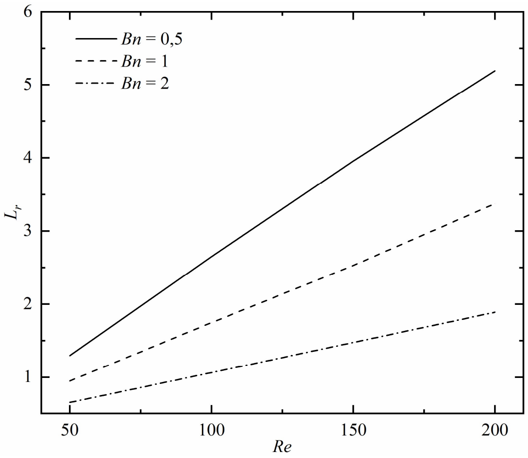

Fig.5 – L_r vs *Re* for *n* = 0.8.

4.2 Local loss coefficient K

The variation of the local loss coefficient *K* through the sudden expansion as a function of Reynolds number shown in Figures 6 and 7.

The increase in the Reynolds number reduces the value of the local loss coefficient regardless of the Newtonian or viscoplastic fluid *n* = 1 as proved in the study *Oliveira et al* [12] and *Bekhadra et al* [14].

In contrast, whenever the fluid has a large Bingham number value, it gives a larger value to the local loss coefficient *K* where the same effect appears in each of the studies by *Kfuri et al* [13] and *Bekhadra et al* [14] this increase caused by the un-yielded zones that formed in the enlargement.

Figure 8 shows the effect of the power low index *n* on the local loss coefficient *K* it shows a slight increase in the value of the local loss coefficient with an increase in the power low index.

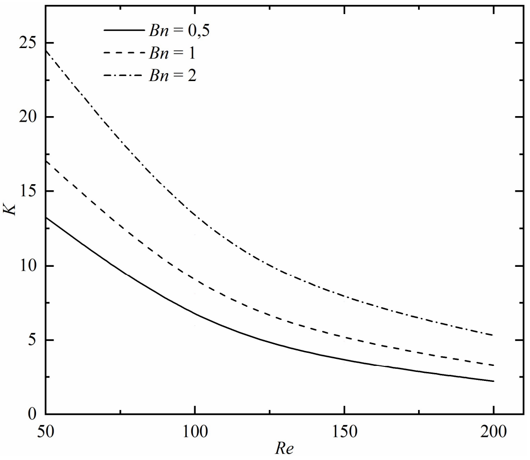

Fig.6 – *K* vs *Re* for *n* = 0.6

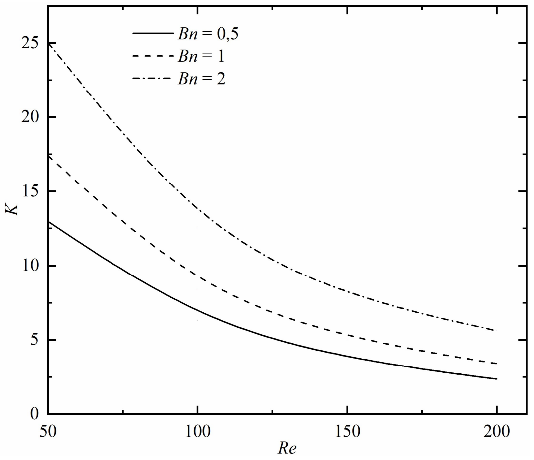

Fig.7 – *K* vs *Re* for *n* = 0.8

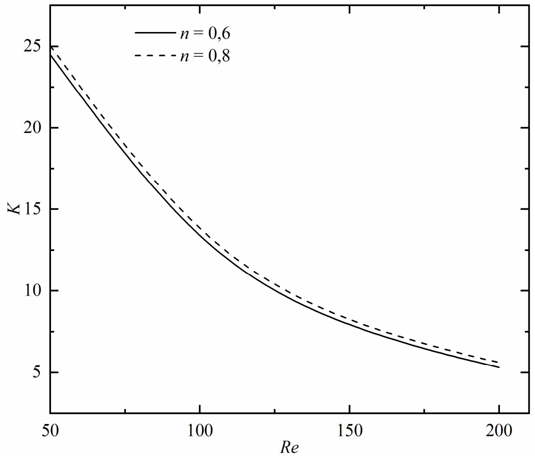

Fig.8 – *K* vs *Re* for *Bn* = 2.

Using the numerical results the correlation (8) was formulated to calculate the value of *K* in the range of the Reynolds number and the rheological conditions of the flow studied.

The present model shows good compatibility with *Bekhadra et al* [14] in Newtonian fluid flow and predicts the local loss coefficient for Herschel-Bulkley fluid flows for the rheological and geometric conditions studied. This model can help in the calculations of local loss coefficient due to sudden expansion in the oil industry and also help in selecting the appropriate reduction in piping design.

$$K=A+B\log(Re)+C((\log(Re))^2) \qquad (8)$$

Table 1– Values of coefficients appearing in equation (8)

Bn	*n=0.6*		
	A	*B*	*C*
0.5	118,29	-94,00	18,93
1	144,1	-113,19	22,61
2	198,72	-154,72	30,74
	n=0.8		
0,5	100,73	*-76,77*	*14,78*
1	*144,17*	*-112,6*	*22,35*
2	*199,71*	*-154,9*	*30,65*

5 Conclusion

A numerical simulation was performed by the commercial code ANSYS-Fluent for Herschel-Bulkley fluid flow through an axisymmetric sudden expansion of aspect ratio $Er = 2$. Based on the numerical results, dimensionless equations have been constructed for the recirculation length and the local loss coefficient; these equations show the effect of rheological conditions on the recirculation zone as well as on the singular loss coefficient due to the sudden change in section.

Acknowledgements

The authors are thankful for the anonymous reviews and the editor for the valuable comments that helped to improve the quality of the presented work. We also would like to thank all the members of the Industrial Technologies Lab.

REFERENCES

[1] E. O. Macagno and T.-K. Hung, Computational and experimental study of a captive annular eddy, Journal of Fluid Mechanics, vol. 28 N° 1 (1967) pp. 43–64.

[2] P. S. Scott, F. A. Mirza, and J. Vlachopoulos, A finite element analysis of laminar flows through planar and axisymmetric abrupt expansions, Computers & Fluids, vol. 14 N° 4 (1986), pp. 423–432.

[3] D. Badekas and D. D. Knight, Eddy Correlations for Laminar Axisymmetric Sudden Expansion Flows, Journal of Fluids Engineering, vol. 114 N°1 (1992) pp. 119–121.

[4] P. S. Scott, F. Mirza, and J. Vlachopoulos, Finite ‐ Element Simulation of Laminar Viscoplastic Flows with Regions of Recirculation, Journal of Rheology, vol. 32 N° 4 (1988) pp. 387 ‐ 400.

[5] G. C. Vradis and M. V. O¨tu¨gen, The Axisymmetric Sudden Expansion Flow of a Non-Newtonian Viscoplastic Fluid, Journal of Fluids Engineering, vol. 119 N° 1 (1997) pp. 193–200.

[6] K. J. Hammad, G. C. Vradis, and M. V. O¨tu¨gen, Laminar Flow of a Herschel-Bulkley Fluid Over an Axisymmetric Sudden Expansion, Journal of Fluids Engineering, vol. 123 N° 3 (2001) pp. 588–594.

[7] K. J. Hammad, Suddenly expanding recirculating and non-recirculating viscoplastic non-newtonian flows, J Vis, vol. 18 N° 4 (2015) pp. 655–667.

[8] B. Pak, Y. I. Cho, and S. U. S. Choi, Separation and reattachment of non-newtonian fluid flows in a sudden expansion pipe, Journal of Non-Newtonian Fluid Mechanics, vol. 37 N° 2 (1990) pp. 175–199.

[9] P. Jay, A. Magnin, and J. M. Piau, Viscoplastic fluid flow through a sudden axisymmetric expansion, AIChE Journal, vol. 47 N° 10 (2001) pp. 2155–2166,.

[10] E. Mitsoulis and R. R. Huilgol, Entry flows of Bingham plastics in expansions, Journal of Non-Newtonian Fluid Mechanics, vol. 122 N° 1 (2004) pp. 45–54.

[11] P. J. Oliveira and F. T. Pinho, Pressure drop coefficient of laminar Newtonian flow in axisymmetric sudden expansions, International Journal of Heat and Fluid Flow, vol. 18 N°5 (1997) pp. 518–529.

[12] P. J. Oliveira, F. T. Pinho, and A. Schulte, A general correlation for the local loss coefficient in Newtonian axisymmetric sudden expansions, International Journal of Heat and Fluid Flow, vol. 19 N° 6 (1998) pp. 655–660.

[13] S. L. D. Kfuri, E. J. Soares, R. L. Thompson, and R. N. Siqueira, Friction Coefficients for Bingham and Power-Law Fluids in Abrupt Contractions and Expansions, Journal of Fluids Engineering, vol. 139 N° 021203(2016).

[14] Bekhadra, M., Chemloul, N.E.S., Menouer, A., Chaib, K, Numerical study of laminar Bingham fluid in axisymmetric sudden expansion. International Journal of Heat and Technology, Vol. 40 N° 1(2022) pp. 45-52.