Tensors

Kronecker symbol

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else} \end{cases}$$

Permutation symbol

$$\varepsilon_{ijk} = \begin{cases} +1 & [[1,2,3], [2,3,1], [3,1,2]] \\ -1 & [[1,3,2], [2,1,3], [3,2,1]] \\ 0 & \text{else} \end{cases}$$

Ricci-tensor

$$\boldsymbol{\varepsilon}_{ijk} = \varepsilon_{ijk} \, e_i \otimes e_j \otimes e_k$$

$$\mathbb{1} = \delta_{ij} \, e_i \otimes e_j$$

$$\mathbb{1}_{<4>} = \mathbb{1} \,\square\, \mathbb{1} = \delta_{ik} \cdot \delta_{jm} \, e_i \otimes e_j \otimes e_k \otimes e_m$$

$$\mathbb{1}^{S}_{<4>} = \frac{1}{2}\left(\mathbb{1} \,\square\, \mathbb{1} + (\mathbb{1} \,\square\, \mathbb{1})^{TR}\right) = \frac{1}{2}\left(\delta_{ik}\delta_{jm} + \delta_{im}\delta_{jk}\right) e_i \otimes e_j \otimes e_k \otimes e_m$$

$$\mathbb{1}^{A}_{<4>} = \mathbb{1}_{<4>} - \mathbb{1}^{S}_{<4>}$$

$$= \frac{1}{2}\left(\mathbb{1} \,\square\, \mathbb{1} - (\mathbb{1} \,\square\, \mathbb{1})^{TR}\right) = \frac{1}{2}\left(\text{''} \quad - \quad \text{''} \right) \quad \text{''}$$

$$\mathbb{P}_1 = \frac{1}{3} \, \mathbb{1} \otimes \mathbb{1} = \frac{1}{3} \, \delta_{ij} \, \delta_{km} \, e_i \otimes e_j \otimes e_k \otimes e_m$$

$$\mathbb{P}_2 = \mathbb{1}^{S}_{<4>} - \mathbb{P}_1$$