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Überführung von $\mathcal{H}$ in Symmetrisierung

equation (2)

$$a_{ij} \otimes a_{jk} \otimes a_{ki} = [e_i \otimes e_j] \otimes [e_j \otimes e_k] \otimes [e_k \otimes e_i]$$

$$\mathcal{H} = \frac{1}{8} (e_i \otimes e_j + e_j \otimes e_i) \otimes (e_j \otimes e_k + e_k \otimes e_j) \otimes (e_k \otimes e_i + e_i \otimes e_k)$$

$$= \frac{1}{8} (a_{ij} + a_{ji}) \otimes (a_{jk} + a_{kj}) \otimes (a_{ki} + a_{ik})$$

$$= \frac{1}{8} \Big(a_{ij} \otimes a_{jk} \otimes a_{ki} + a_{ij} \otimes a_{jk} \otimes a_{ik}$$
$$+ a_{ij} \otimes a_{kj} \otimes a_{ki} + a_{ij} \otimes a_{kj} \otimes a_{ik}$$
$$+ a_{ji} \otimes a_{jk} \otimes a_{ki} + a_{ji} \otimes a_{jk} \otimes a_{ik}$$
$$+ a_{ji} \otimes a_{kj} \otimes a_{ki} + a_{ji} \otimes a_{kj} \otimes a_{ik} \Big)$$

$$= \frac{1}{8} \Big(ijjkki + ijjkik$$
$$+ ijkjki + ijkjik$$
$$+ jijkki + jijkik$$
$$+ jikjki + jikjik \Big)$$

$e_m \otimes e_n \otimes e_p \otimes e_q \otimes e_r \otimes e_s$

$mn|pq|rs$

$$= \frac{1}{8} \Big(\delta_{mi}\delta_{nj}\delta_{pj}\delta_{qk}\delta_{rk}\delta_{si} + \delta_{mi}\delta_{nj}\delta_{pj}\delta_{qk}\delta_{ri}\delta_{sk}$$
$$+ \delta_{mi}\delta_{nj}\delta_{pk}\delta_{qj}\delta_{rk}\delta_{si} + \delta_{mi}\delta_{nj}\delta_{pk}\delta_{qj}\delta_{ri}\delta_{sk}$$
$$+ \delta_{mj}\delta_{ni}\delta_{pj}\delta_{qk}\delta_{rk}\delta_{si} + \delta_{mj}\delta_{ni}\delta_{pj}\delta_{qk}\delta_{ri}\delta_{sk}$$
$$+ \delta_{mj}\delta_{ni}\delta_{pk}\delta_{qj}\delta_{rk}\delta_{si} + \delta_{mj}\delta_{ni}\delta_{pk}\delta_{qi}\delta_{ri}\delta_{sk} \Big)$$

$$= \frac{1}{8} \Big(\delta_{ms}\delta_{np}\delta_{qr} + \delta_{mr}\delta_{np}\delta_{qs} + \delta_{ms}\delta_{nq}\delta_{pr} + \delta_{mr}\delta_{nq}\delta_{ps}$$
$$+ \delta_{mp}\delta_{ns}\delta_{qr} + \delta_{mp}\delta_{nr}\delta_{qs} + \delta_{mq}\delta_{ns}\delta_{pr} + \delta_{mq}\delta_{nr}\delta_{ps} \Big)$$

implementierbar

$e_m \otimes e_n \otimes e_p \otimes e_q \otimes e_r \otimes e_s$ ✓