Transversal Isotropic

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Define engineering quantities

E_i Youngs modulus along direction i ..... $E_i = \dfrac{\sigma_{ii}}{\varepsilon_{ii}}$

G_{ij} Shear modulus in plane with normal i in direction j $\quad G_{ij} = \dfrac{\sigma_{ij}}{2\,\varepsilon_{ij}}$

ν_{ij} Poisson's ratio due to load applied in direction i and measured response in direction j $\quad \nu_{ij} = -\dfrac{\varepsilon_{jj}}{\varepsilon_{ii}}$

$i, j \in [x, y, z]$

Implications $\quad G_{ij} = G_{ji}$ because of symmetry of σ, ε

$\left(\dfrac{\nu_{12}}{E_1} = \dfrac{\nu_{21}}{E_2} \right.$ because of symmetry of compliance (see below)
note, that $\nu_{12} \neq \nu_{21}$! Distinguish cause and effect! $\left.\right)$

Orthotropy

$$\sigma^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & & & \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & & 0 & \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & & & \\ & & & \frac{1}{G_{23}} & & \\ & 0 & & & \frac{1}{G_{31}} & \\ & & & & & \frac{1}{G_{12}} \end{bmatrix}_{\text{Voigt}} \overset{\text{nicer}}{=} \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_1} & & 0 & \\ & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & & & \\ & & \frac{1}{E_3} & & & \\ & \text{sym} & & \frac{1}{G_{23}} & & \\ & & & & \frac{1}{G_{13}} & \\ & & & & & \frac{1}{G_{12}} \end{bmatrix}_{\text{Voigt}}$$

$\Rightarrow$ $\boxed{\text{9 parameters:}}$

E_1, E_2, E_3

$\nu_{12}, \nu_{13}, \nu_{23}$

G_{12}, G_{13}, G_{23}

unused (dependent):

$\nu_{21} = \dfrac{E_2}{E_1}\nu_{12}$

$\nu_{31} = \dfrac{E_3}{E_1}\nu_{13}$

$\nu_{32} = \dfrac{E_3}{E_2}\nu_{23}$

Bounds: $\quad E_m > 0 \quad m \in [1,2,3]$

$\qquad G_n > 0 \quad n \in [12, 13, 23]$

Transversal Isotropy

Special case of Orthotropy with

bindings:

$E_2 = E_3$

$G_{12} = G_{13}$

$\nu_{12} = \nu_{13}$

In plane of Isotropy [$\nu_{23} = \frac{E_2}{2 G_{23}} - 1$

$\Rightarrow$

Primary arguments:	Aliases
E_1	$E_{\parallel}$
E_2	E_3, $E_\perp$
G_{12}	G_{13}, $G_{\parallel}$, G_{21}, G_{31}
G_{23}	$G_\perp$, G_{32}, $G_{\text{in iso plane}}$
or ν_{23}	ν_{32} due to isotropy in plane
ν_{12}	ν_{13}

$\Rightarrow$ TransversalIsotropic = Orthotropic(

Prefered, primary args

$E_1 = E_1$

$E_2 = E_2$

$E_3 =$ "

$\nu_{12} = \nu_{12}$ or $\frac{E_1}{E_2} \nu_{21}$

$\nu_{13} =$ " "

$\nu_{23} = \nu_{23}$ or $\frac{E_2}{2 G_{23}} - 1$

$G_{12} = G_{12}$

$G_{13} =$ "

$G_{23} = \frac{E_2}{2(1+\nu_{23})}$ or G_{23}

)

Valid argument combinations:

Combinations of length 5 out of $\{E_1, E_2, G_{23}, G_{12}, \nu_{12}, \nu_{21}, \nu_{23}\}$

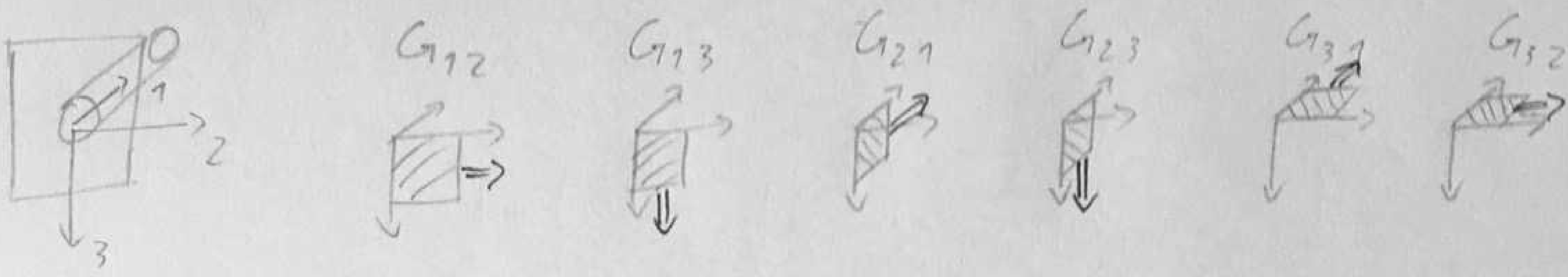
with $[E_1, E_2, G_{12}]$ in combinations

and G_{23} or ν_{23} in combinations

and ν_{12} or ν_{21} in combinations

$$\Rightarrow [E_1 \; E_2 \; G_{12}] + \left| \begin{matrix} [G_{23}, \nu_{12}] \\ [G_{23}, \nu_{21}] \\ [\nu_{12}, \nu_{23}] \\ [\nu_{21}, \nu_{23}] \end{matrix} \right\} \text{4 combinations + aliases}$$

Notes on Engineering constants



$G_{ij} = G_{ji}$ because σ, ε are symmetric