<u>Voigt notation</u>

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$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} \\ & \sigma_{22} & \sigma_{23} \\ \text{sym} & & \sigma_{33} \end{pmatrix}$$

$$\sigma^{v} = \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} \sigma^{v}_{1} \\ \sigma^{v}_{2} \\ \sigma^{v}_{3} \\ \sigma^{v}_{4} \\ \sigma^{v}_{5} \\ \sigma^{v}_{6} \end{pmatrix} \qquad \varepsilon^{v} = \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{pmatrix} = \begin{pmatrix} \varepsilon^{v}_{1} \\ \varepsilon^{v}_{2} \\ \varepsilon^{v}_{3} \\ \varepsilon^{v}_{4} \\ \varepsilon^{v}_{5} \\ \varepsilon^{v}_{6} \end{pmatrix}$$

$$\sigma^{v}_{\alpha} = C^{v}_{\alpha\beta}\, \varepsilon_{\beta} \qquad \alpha, \beta \in [1,2,3,4,5,6]$$

$$\sigma^{v} = \left(\begin{array}{ccc|ccc} C^{v}_{11} & C^{v}_{12} & C^{v}_{13} & C^{v}_{14} & C^{v}_{15} & C^{v}_{16} \\ C^{v}_{21} & C^{v}_{22} & & & \cdots & \\ C^{v}_{31} & & C^{v}_{33}\ \textcircled{1} & \textcircled{2} & & \\ \hline C^{v}_{41} & & \textcircled{3} & \textcircled{4}\ C^{v}_{44} & & \\ C^{v}_{51} & \vdots & & & C^{v}_{55} & \\ C^{v}_{61} & & & & & C^{v}_{66} \end{array}\right) \varepsilon^{v}$$

<u>Mandel:</u>

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\sigma_{23} \\ \sqrt{2}\sigma_{13} \\ \sqrt{2}\sigma_{12} \end{pmatrix} = \left(\begin{array}{ccc|ccc} C^{m}_{11} & C^{m}_{12} & C^{m}_{13} & C^{m}_{14} & C^{m}_{15} & C^{m}_{16} \\ \vdots & & & & & \\ \hline \vdots & & & & & \\ C^{m}_{61} & & & & & C^{m}_{66} \end{array}\right) \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \sqrt{2}\varepsilon_{23} \\ \sqrt{2}\varepsilon_{13} \\ \sqrt{2}\varepsilon_{12} \end{pmatrix}$$

$$= \left(\begin{array}{cc|cc} C_{1111} & & \sqrt{2}\,C_{1123} & \\ & \boxed{1} & \boxed{\sqrt{2}} & \\ \hline \sqrt{2}\,C_{2311} & \boxed{\sqrt{2}} & \boxed{2} & \\ & & & 2\,C_{1212} \end{array}\right)$$

Voigt | $\boxed{\text{stiffness}}$ | Mandel

1) $\sigma_{11} = C^{V}_{11}\ \varepsilon_{11}$ $\qquad\qquad$ $\sigma_{11} = C^{M}_{11}\ \varepsilon_{11}$

$$\Rightarrow C^{V}_{11} = C^{M}_{11} = C_{1111}$$

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2) $\sigma_{11} = C^{V}_{14}\ 2\,\varepsilon_{23}$ $\qquad\qquad$ $\sigma_{11} = C^{M}_{14}\ \sqrt{2}\,\varepsilon_{23}$

$$\Rightarrow 2\,C^{V}_{14} = \sqrt{2}\,C^{M}_{14} \qquad \Rightarrow C^{V}_{14} = \frac{\sqrt{2}}{2}\,C^{M}_{14} = \frac{1}{\sqrt{2}}\,C^{M}_{14} = \frac{\sqrt{2}}{\sqrt{2}}\,C_{1123}$$

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3) $\sigma_{23} = C^{V}_{41}\ \varepsilon_{11}$ $\qquad\qquad$ $\sqrt{2}\,\sigma_{23} = C^{M}_{41}\ \varepsilon_{11}$

$$\Rightarrow C^{V}_{41} = \frac{1}{\sqrt{2}}\,C^{M}_{41} = \cancel{\frac{\sqrt{2}}{\sqrt{2}}}\,C_{2311}$$

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4) $\sigma_{23} = C^{V}_{44}\ 2\,\varepsilon_{23}$ $\qquad\qquad$ $\sqrt{2}\,\sigma_{23} = C^{M}_{44}\ \sqrt{2}\,\varepsilon_{23}$

$$\Rightarrow 2\,C^{V}_{44} = C^{M}_{44} \qquad \Rightarrow C^{V}_{44} = \frac{1}{2}\,C^{M}_{44} = \cancel{\frac{2}{2}}\,C_{1212}$$

$$\mathbb{C}^{V} = \left(\begin{array}{c|c} 1 & \frac{1}{\sqrt{2}} \\ \hline \frac{1}{\sqrt{2}} & \frac{1}{2} \end{array}\right)_{C^{M}}$$

Factors which have to be applied to components C^{M} in specified quadrants to get component C^{V}

$$\Rightarrow \mathbb{C}^{M} = \left(\begin{array}{c|c} 1 & \sqrt{2} \\ \hline \sqrt{2} & 2 \end{array}\right)_{C^{V}}$$

Compliance

Mandel: $\mathbb{S}^M = \left(\mathbb{C}^M\right)^{-1}$

Voigt:

$$\varepsilon^V = \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{pmatrix} = \left(\begin{array}{c|c} S^V_{11} \quad S^V_{12} & \\ \quad\quad (1) & (2) \\ \hline \quad\quad (3) & (4) \\ & \end{array}\right) \sigma^V$$

Voigt | Model

1) $\varepsilon_{11} = S^V_{11}\, \sigma_{11}$ $\qquad$ $\varepsilon_{11} = S^M_{11}\, \sigma_{11}$

$$\Rightarrow S^V_{11} = S^M_{11}$$

2) $\varepsilon_{11} = S^V_{14}\, \sigma_{23}$ $\qquad$ $\varepsilon_{11} = S^M_{14}\, \sqrt{2}\, \sigma_{23}$

$$\Rightarrow S^V_{14} = \sqrt{2}\, S^M_{14}$$

3) $2\varepsilon_{23} = S^V_{41}\, \sigma_{11}$ $\qquad$ $\sqrt{2}\, \varepsilon_{23} = S^M_{41}\, \sigma_{11}$

$$\Rightarrow \frac{S^V_{41}}{2} = \frac{S^M_{41}}{\sqrt{2}} \Rightarrow S^V_{41} = \frac{2}{\sqrt{2}} S^M_{41} = \sqrt{2}\, S^M_{41}$$

4) $2\varepsilon_{23} = S^V_{44}\, \sigma_{23}$ $\qquad$ $\sqrt{2}\, \varepsilon_{23} = S^M_{44}\, \sqrt{2}\, \sigma_{23}$

$$\Rightarrow \frac{S^V_{44}}{2} = S^M_{44} \Rightarrow S^V_{44} = 2\, S^M_{44}$$

$$\mathbb{S}^V = \left(\begin{array}{c|c} 1 & \sqrt{2} \\ \hline \sqrt{2} & 2 \end{array}\right)_{\mathbb{S}^M}$$

$$\mathbb{S}^M = \left(\begin{array}{c|c} 1 & \frac{1}{\sqrt{2}} \\ \hline \frac{1}{\sqrt{2}} & \frac{1}{2} \end{array}\right)_{\mathbb{S}^V}$$

~~to_model~~

~~stess~~
~~strain~~
~~stiffness~~
~~compliance~~

~~A2_to_model~~

~~t4_to_model~~

~~v2_to_~~

What should be implemented?

Extend Converter by to_voigt

to model		to_tensor	to_voigt
2 * shape_t		1 * shape_m	
4 * shape_t		2 * shape_m	
stess	Warning	stess	stess:
strain		strain	2 * shape_t
stiffness		stiffness	1 * shape_m
compliance		compliance	strain:

missuse by passing voigt to it without flag

ϟ: model passed with voigt flag

what simple converter limited to voigt_to_model

voigt_to_model (item, voigtType)	model_to_voigt (item, voigtType)
stess	stess
strain	strain
stiffness	stiffness
compliance	compliance

Stress

$$\sigma^M = \begin{pmatrix} - \\ - \\ - \\ \sqrt{2}\,\sigma_{23} \\ \sqrt{2}\,\sigma_{13} \\ \sqrt{2}\,\sigma_{12} \end{pmatrix}, \quad \sigma^V = \begin{pmatrix} - \\ - \\ - \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix}$$

$$\sqrt{2}\,\sigma_{23} = \sigma_4^M \qquad \sigma_{23} = \sigma_4^V$$

compare
$$\Rightarrow \quad \sigma_4^V = \frac{1}{\sqrt{2}}\,\sigma_4^M \quad \Rightarrow \quad \sigma^V = \left(\begin{array}{c} 1 \\ \hline \frac{1}{\sqrt{2}} \end{array}\right)_{\sigma^M}$$

$$\sigma^M = \left(\begin{array}{c} 1 \\ \hline \sqrt{2} \end{array}\right)_{\sigma^V}$$

Strain

$$\varepsilon^M = \begin{pmatrix} - \\ - \\ - \\ \sqrt{2}\,\varepsilon_{23} \\ \sqrt{2}\,\varepsilon_{13} \\ \sqrt{2}\,\varepsilon_{12} \end{pmatrix} \qquad \varepsilon^V = \begin{pmatrix} - \\ - \\ - \\ 2\,\varepsilon_{23} \\ 2\,\varepsilon_{13} \\ 2\,\varepsilon_{12} \end{pmatrix}$$

$$\varepsilon_4^M = \sqrt{2}\,\varepsilon_{23} \qquad \varepsilon_4^V = 2\,\varepsilon_{23}$$

compare
$$\Rightarrow \quad \frac{1}{\sqrt{2}}\,\varepsilon_4^M = \frac{1}{2}\,\varepsilon_4^V$$

$$\Rightarrow \quad \varepsilon_4^V = \sqrt{2}\,\varepsilon_4^M \quad \Rightarrow \quad \varepsilon^V = \left(\begin{array}{c} 1 \\ \hline \sqrt{2} \end{array}\right)_{\varepsilon^M}$$

$$\varepsilon^M = \left(\begin{array}{c} 1 \\ \hline \frac{1}{\sqrt{2}} \end{array}\right)_{\varepsilon^V}$$

Mandel_to_Voigt

stress:

voigt[shear] = mandel[shear] $\cdot \frac{1}{\sqrt{2}}$

strain:

" = " $\cdot \sqrt{2}$

stiffness:

voigt[q2] = mandel[q2] $\frac{1}{\sqrt{2}}$
q3 q3 $\frac{1}{\sqrt{2}}$
" q4 " q4 $\frac{1}{2}$

compliance:

voigt[q2] = mandel [q2] $\cdot \sqrt{2}$

" q3 " q3 $\cdot \sqrt{2}$

q4 q4 $\cdot$ 2

shear = np.s_[3 : 6]

q1 = np.s_[0:3, 0:3]

q2 = np.s_[0:3, 3:6]

q3 = " [3:6, 0:3]

q4 = " [3:6, 3:6]

Voigt_to_Mandel

stress:

mandel[shear] = voigt[shear] $\cdot \sqrt{2}$

strain

" = " $\cdot \frac{1}{\sqrt{2}}$

stiffness

mandel[q2] = voigt[q2] $\cdot \sqrt{2}$
" q3 " q3 $\sqrt{2}$
" q4 = " q4 2

compliance

mandel[q2] = voigt[q2] $\frac{1}{\sqrt{2}}$

q3 = q3 $\frac{1}{\sqrt{2}}$

q4 = q4 $\frac{1}{2}$