

# Computational Analysis of Binomial Series

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**Abstract:** This paper discusses the computational analysis of binomial series. This idea can enable the scientific researchers to solve the real life problems.

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**Keywords:** computation, binomial coefficient, binomial series, combinatorics

## 1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series, a new idea stimulated his mind to create a combinatorial geometric series [1-9]. The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient  $V_n^r$ . In this article, binomial identities and multinomial theorem is provided using the binomial coefficients for combinatorial geometric series.

## 2. Combinatorial Geometric Series

The combinatorial geometric series [1-9] is derived from the multiple summations of geometric series[10-19]. The coefficient of each term in the combinatorial refers to the binomial coefficient

$$\sum_{i_1=0}^n \sum_{i_2=i_1}^n \sum_{i_3=i_2}^n \cdots \sum_{i_r=i_{r-1}}^n x^{i_r} = \sum_{i=0}^n V_i^r x^i \quad \& \quad V_n^r = \frac{(n+1)(n+2)(n+3) \cdots (n+r-1)(n+r)}{r!},$$

where  $n \geq 0, r \geq 1$  and  $n, r \in N = \{0, 1, 2, 3, \dots\}$ .

Here,  $\sum_{i=0}^n V_i^r x^i$  refers to the combinatorial geometric series and

$V_n^r$  is the binomial coefficient for combinatorial geometric series.

## 3. Computational Analysis of Binomial Series

We know that the general binomial series is  $\sum_{i=0}^n V_i^{n-i} x^i y^{n-i} = (x+y)^n$ .

**Theorem 3. 1:**  $\sum_{i=0}^n V_i^{n-i} x^i (1-x)^{n-i} = 1$ .

*Proof.* Let us substitute  $y = 1 - x$  in  $\sum_{i=0}^n V_i^{n-i} x^i y^{n-i} = (x+y)^n$ .

$$\text{Then } \sum_{i=0}^n V_i^{n-i} x^i (1-x)^{n-i} = (x+1-x)^n \Rightarrow \sum_{i=0}^n V_i^{n-i} x^i (1-x)^{n-i} = 1.$$

Hence, the theorem is proved.

$$\textbf{Theorem 3.2: } \sum_{i=0}^n V_i^{n-i} (1-x)^i x^{n-i} = 1.$$

$$\textit{Proof.} \text{ Let us substitute } y = 1-x \text{ in } \sum_{i=0}^n V_i^{n-i} y^i x^{n-i} = (y+x)^n.$$

$$\text{Then } \sum_{i=0}^n V_i^{n-i} (1-x)^i x^{n-i} = (1-x+x)^n \Rightarrow \sum_{i=0}^n V_i^{n-i} (1-x)^i x^{n-i} = 1.$$

Hence, the theorem is proved.

$$\textbf{Corollary 3.1: } \sum_{i=0}^n (-1)^{n-i} V_i^{n-i} 2^i = 1.$$

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#### 4. Conclusion

In this article, theorems and corollaries on binomial coefficients were constituted. This idea can enable the scientific researchers to solve the real world problems.

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