More than 80 Years of Experience with the Method of Characteristics - from Graphical Analyses to Advanced Driveability Analyses by Scripts.

Peter Middendorp

Allnamics Geotechnical & Pile Testing Experts, The Netherlands

ABSTRACT

Most are not aware that the first practical applications of stress wave simulations were based on the Method of Characteristics and were performed using graphical tools. Today these tools have been replaced by computers and the computer algorithm for the Method of Characteristics (MOC) is almost certainly the most commonly used algorithm for stress wave simulations, like driveability studies (both for impact hammers and vibratory hammers), signal matching (both to assess pile capacity and to perform pile shape analysis), the design of hammer parts, hammer cushions, and Rapid Load Testing simulations. This paper will be a sequel to the paper “Thirty Years of Experience with the wave equation solution based on the Method of Characteristics” that was presented at the Stress Wave Conference in Kuala Lumpur, 2004. As such it will provide an overview of the application of the method in foundation testing: from the early beginnings using the graphical methods to the current advanced driveability analyses. The overview will be illustrated by actual examples generated by the wave equation program AllWave that also demonstrate how the outcome of soil investigation testing is transformed into soil model parameters (incl. soil fatigue parameters) to ensure that the analysis generates reliable results.

Keywords: Method of Characteristics (MOC), Wave Equation Analysis (WEQ) program, driveability, hammer modelling, conversion soil investigation to soil model parameters, soil fatigue, impact driving, vibratory driving, Rapid Load Test (RLT) prediction, signal matching, automation, scripting, batching

1. INTRODUCTION

As early as the Bronze Age, houses on the banks of European rivers were built on piles. As a result, they were elevated and thus did not flood when the river levels were high. But in Europe it was the Romans who were the first to use piles as the foundation for bridge abutments. An impressive example is Caesar’s Rhine Bridge (Fig. 1, Hinz 2017, Middendorp et al. 2022), which was constructed in 10 days. His engineers had to answer the same questions that are posed today: what equipment is needed to drive a pile to a required penetration, how many piles are needed and how much penetration is required to obtain the necessary capacity to support the superstructure safely. Even today getting the answer to those questions is often a learning loop of trial and error. This learning starts with building up local experience, performing pile tests, the use of rules of thumb and, especially in the past, the use of pile driving formulas. The introduction of the wave equation theory (WEQ)

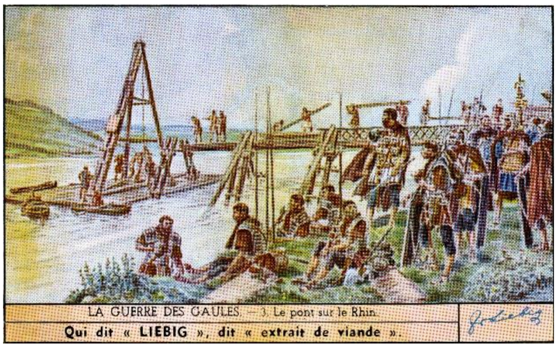


Fig 1. Artists impression of Caesar's Rhine Bridge. construction with floating pile driving rigs, 55 B.C.

gave a deeper understanding of the stress wave phenomena as a result of each blow as the pile was driven into the soil. Before computers became available, wave propagation, stresses, velocities and displacement could be quantified by means of graphical tools, and these tools were the basis for the present-day computer algorithm using the Method of Characteristics (MOC). This is almost certainly the most applied method for explaining stress wave phenomena by graphics (Fig.2) and the most commonly used algorithm for stress wave simulations, like drive-ability studies (both for impact hammers and vibratory hammers), signal matching (both to assess pile capacity and to perform pile shape analysis), the design of hammer parts, hammer cushions, and rapid load testing simulations

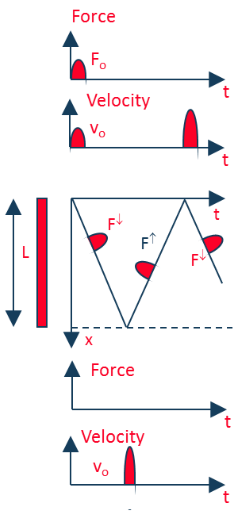


Fig. 2, Example of a travelling stress wave on for a free pile.

2. FROM ESTIMATE TO PREDICTION

There are several publications that give an overview of the history of stress wave applications and its contributors, incl. Fellenius (1996) and Hussein et al. (2004). This section will highlight some of the contributions mentioned in these publications, and also add some that according to this author deserve more attention

Capacity Estimates and Pile Driving Formulas

The first capacity estimate for a driven pile was by the French engineer A. Rondelet (1802) who tried to determine this empirically. According to him, the bearing capacity of a pile was simply a function of the pile cross-sectional area or approximately 30 kg/cm^2 (2.94 MPa). A drawback of this method was that this estimate was completely independent of the pile driving process and the quality of the soil. A completely different approach was to derive a formula to determine the load bearing capacity. According to Lintsen (1994), in the Netherlands the first pile driving formula was developed in 1821 based on the work of the Prussian hydraulic engineer Eytelwein (1808). He assumed that the bearing capacity of a pile was equal to the resistance that a pile experienced during pile driving. This resistance R_u could be found with the formula:

$$R_u = \frac{e_h . E_h}{n.s\left[1 + \frac{w}{W}\right]}$$

with E_h = Rated Energy, e_h = Efficiency, s = permanent set after a blow, w = weight pile, W = weight ram, n = safety factor

These types of formulas became very popular, because they seem intuitively correct. If the pile showed a considerably set after impact, the bearing capacity was small and if the load was increased by taking a heavier ram or an increased drop height, then a small set meant a large bearing capacity. However, even at that time it was clearly understood that the assumptions used for these formulas were rather doubtful, and therefore a safety factor "n" was introduced. Usually, a factor of 6 was used, which in and of itself is telling. Although pile driving formulas proved to be notoriously unreliable, they still cropped up everywhere and are even used to this very day. The basic misconceptions of pile driving formulas are the assumption of a rigid pile and thus neglecting stress waves, as well as ignoring the soil type influence and the action and energy transfer of the various components of a hammer. These misconceptions are extensively highlighted by Allin et al. (2015).

Stress Wave Phenomena

The first awareness of energy propagation in a row of impacting ivory balls (Fig.3) was probably by Mariotte (1717).

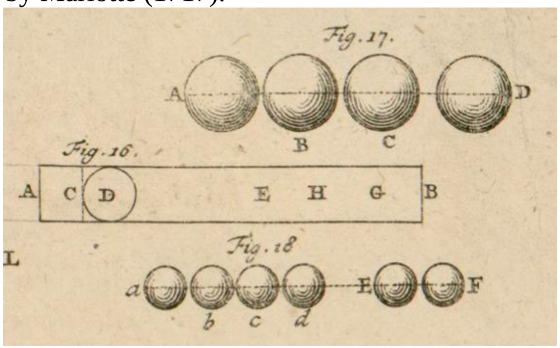


Fig.3. Mariotte's impacting balls experiment figures.

Note: The credits with the modern version of the executive toy went to Newton and it is still being described as "Newton's cradle" even while Mariotte was quoted in Newton's Principia (Cross, 2012).

Stress wave phenomena have been understood for a long time. d'Alembert (1747) discovered the wave equation and Saint Venant's (1867) found the wave equation solution for impacting rods, but for a long time it was not really applied to pile driving.. The first observation of stress waves in piles is by Isaacs (1931), who created an integration technique best described as a semi graphical one. He developed a mathematical model based on the successive transmission and reflection of waves, like the method of characteristics. A sample solution is given in Figure 4, in this case showing multiple impacts. Isaacs constructed a drafting machine to draw the solution, a diagram of which is shown in Figure 5.

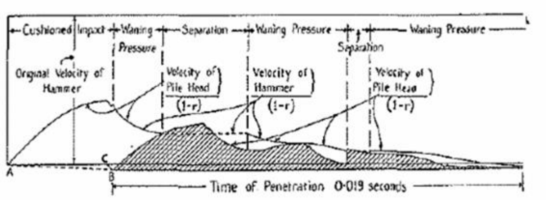


Fig. 4. Wave Diagram. Pile well cushioned, a light hammer, and medium Driving. Isaacs (1931)

Isaacs also gave a sample solution for a propagating wave for an “ideal uncushioned impact of a short hammer on a long pile” (Fig. 6). For comparison the author made a similar simulation with the MOC program AllWave. It is

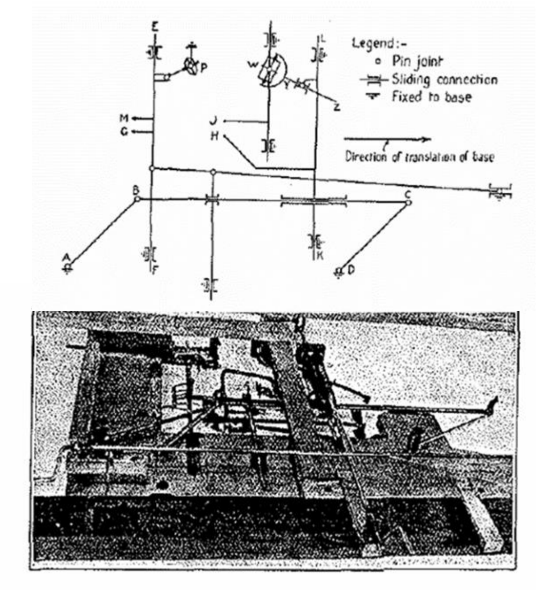


Fig. 5. Principle and wave equation drafting machine by Isaacs (1931)

impressive how Isaacs already obtained this result with limited means and it shows his deep understanding of stress wave phenomena in the ram and in the pile.

Graphical Tools

However, there were additional development in hydraulic and mechanical engineering based on graphical methods to predict wave propagation. Bergeron (1937) proposed a graphical method to predict the propagation of waves in water channels and piles (Fig.7).

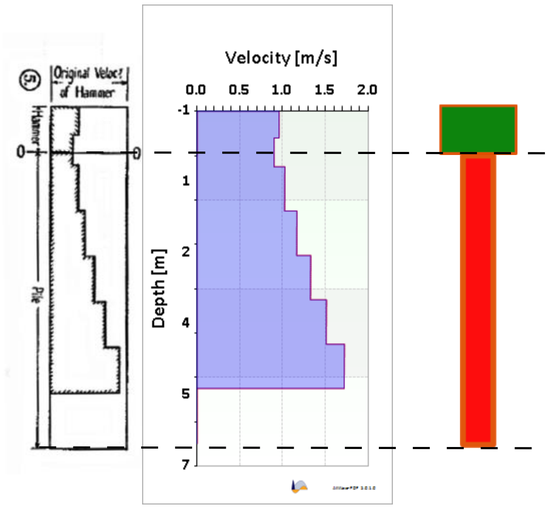


Fig. 6. Snapshot of a travelling wave according to Isaacs (1931) and AllWave (2022) for "Ideal uncushioned impact of a short hammer on a long pile."

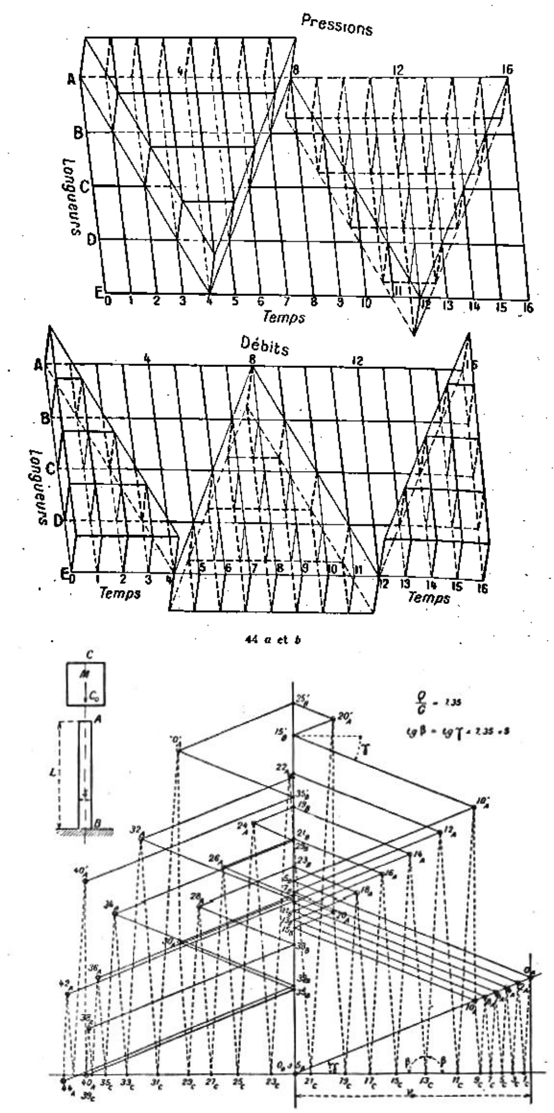


Fig 7. Hydraulics and mechanics graphical MOC analysis examples taken from Bergeron (1950)

In principle Bergeron already described how to incorporate phenomena like friction (sudden jumps) for the propagating of waves with discrete points along a pile:

In other cases the waves undergo a deformation arising from the homogeneous medium itself in proportion to the advance of the waves This modification is then continuous; ***but, one can sometimes conceive of it as being produced by sudden jumps at points in space, equidistant and sufficiently close****, and thus one constructs an approximation which falls on the preceding case where the limits would be the locations of the sudden jumps. In principle, a wave is than a physical phenomenon in motion; started from some point, it remains the same for an observer who travels with its velocity,* ***and this velocity is constant*** *between the limits of the homogeneous medium where the phenomenon has occurred.*

In mechanical engineering one was interested in the stresses generated by impacting rods, and similar graphical tools as described by Bergeron were used to study the subject. An excellent example is the work of De Juhasz (1949) with impressive graphics (Fig.8) of stress wave propagation for cases exceeding the elastic limit of the rod material.

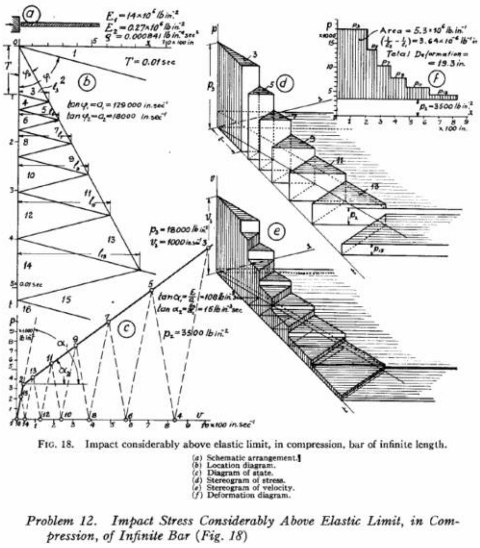


Fig. 8, Graphical MOC analysis by De Juhasz (1949)

The method of characteristics was used in the Netherlands for the prediction of the propagation of tidal waves, based on the work of Massau (1914) and Schonfeld (1951). In a Dutch publication that has now been translated by the SWC2022 organizing committee, De Josselin de Jong (1956) applied the MOC to pile driving and proposed a model for the toe resistance including porewater pressure phenomena (Fig.9).

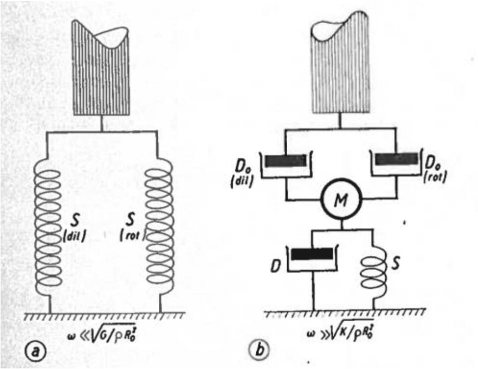


Fig. 9. Proposed toe resistance model by De Josselin de Jong (1956).

Fischer (1960) applied MOC with his grapho-dynamical method and performed extensive research on the influence of ram and anvil dimensions for rods penetration into soil. An example is presented in Figure 10.

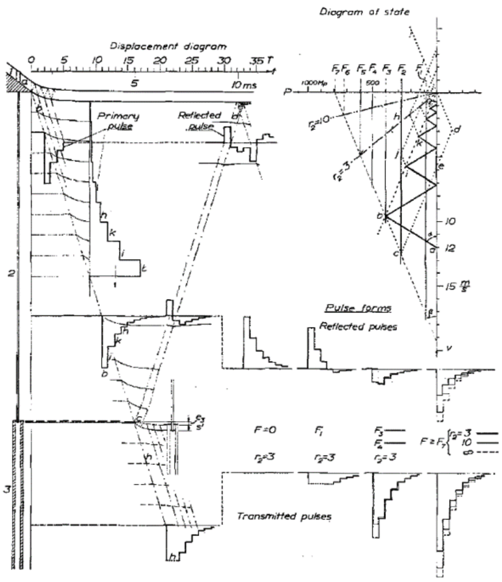


Fig. 10. Graphodynamical MOC analysis by Fischer (1960)

Computer Applications

While it had been long recognized that pile driving created travelling “stress waves”, solutions solving real issues were not available until the advent of the digital computer. A practical digital application for the wave equation was first developed and implemented on IBM computers by the mid 1950’s by. Smith (1960). He published his method, which was a finite difference solution using masses and springs to realistically model the various components using the engineering properties of the hammer, driving system components (helmet and cushions), elastic pile allowing stress wave propagation, and soils of various types having both static and dynamic behaviour.

Voitus van Hamme (1974) chose a different approach for the design and simulations for the Hydroblok, an advanced offshore hammer (Fig. 13). Starting with the analytical wave equation solution valid for the frictionless case and then adapting it in an obvious manner, he adopted an approach similar to that formulated by Bergeron. His method assumes that the continuous skin-friction can be replaced by a great number of concentrated frictional forces (Fig.11). In this way a simple straightforward method of calculation is obtained, which is well suited for practical applications:

When the friction is concentrated at a number of points, the parts of the pile between these points are not subjected to friction and the simple theory is valid for them.

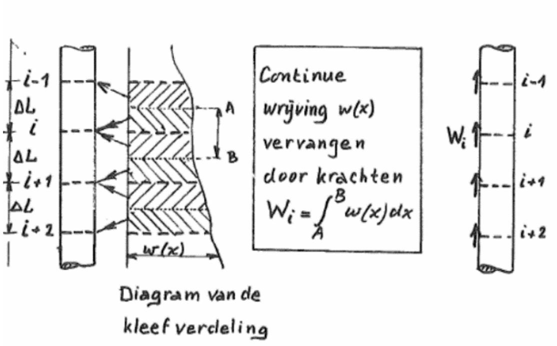


Fig.11. Sketch by Voitus van Hamme (1984) explaining friction discretization for MOC.

The discontinuities, which occur at the points where the friction is modeled, can be dealt with in a simple manner by equilibrium and continuity conditions. Analysis shows that the downward wave is reduced by half the amount of the frictional force, while the upward wave gains an equal amount. At the same time the wave theory remains valid: the initial ram conditions at impact, the impedance discontinuities in the pile and within the hammer, and a toe resistance soil model can be applied without restrictions. Anvils, helmet, followers and cushions can be dealt with, and the method can be applied for any hammer. Until 1976 there was not any computer program based on MOC available and Voitus van Hamme (1980) stated at the Numerical Methods in Offshore Piling conference:

It is, however, astonishing that none of the programs known to the writer is based on a solution of the wave equation with the exception of the HBG pile driving program PILEWAVE designed by the writer.

3. WAVE EQUATION ALGORITHMS

The author has a hydrodynamics background and his thesis (Middendorp, 1977) and his first publication (Middendorp, 1981) were based on the Long Wave Theory in channels, which is based on the MOC. While working for the research organization TNO and reading the work of Voitus van Hamme (1976) the author decided to use this straightforward and elegant MOC approach for the development of the wave equation program TNOWAVE (Middendorp et al. 1986) and formulated the following algorithm (Fig.12).

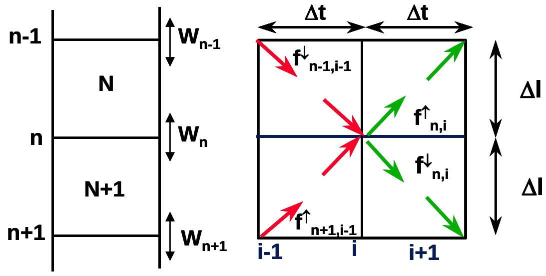


Fig. 12. Discrete levels and wave propagation for a time step

$$f^{\downarrow}_{n,i} = \left(\frac{Z_N - Z_{N+1}}{Z_N + Z_{N+1}}\right) f^{\uparrow} + \left(\frac{Z_{N+1}}{Z_N + Z_{N+1}}\right)\left(2 f^{\downarrow} - W_{n,i}\right)$$

$$f^{\uparrow}_{n,i} = \left(-\frac{Z_N - Z_{N+1}}{Z_N + Z_{N+1}}\right) f^{\downarrow} + \left(\frac{Z_N}{Z_N + Z_{N+1}}\right)\left(2 f^{\uparrow} + W_{n,i}\right)$$

$f^{\downarrow}$ = incident downward travelling wave at *n-1* and *i-1*,

$f^{\uparrow}$ = incident upward travelling wave at *n-1* and *i-1*,

$f^{\downarrow}_{n,i}$ = transmitted downward travelling force wave,

$f^{\uparrow}_{n,i}$ = transmitted upward travelling force wave,

n = discrete point or node number,

i = time step number,

Z_N = impedance of pile element N,

Z_{N+1} = impedance of pile element N+1,

N = pile element number.

W = soil interaction

$$W = W_u + W_v + W_a$$

Force: $f(n,i) = f^{\downarrow}_{n-1,i-1} + f^{\uparrow}_{n+1,i-1}$

Velocity: $v(n,i) = \dfrac{f^{\downarrow}_{n-1,i-1}}{Z_N} - \dfrac{f^{\uparrow}_{n+1,i-1}}{Z_{N+1}}$

Acceleration: $a(n,i) = (v(n,i) - v(n,i-1)) / \Delta t$

Displacement: $u(n,i) = \sum v(n,i).\Delta t$

Power: $P(n,i) = f(n,i).v(n,i)$

Energy: $E = \sum P(n,i)$

The algorithm was initially programmed by Cruys (1983). A more extensive description is given by Middendorp (2004)

Misconceptions

There are still misconceptions about the MOC, one of which can be found in Hussein et. Al. (2014). That publication discusses the lump mass approach by Smith and the MOC and the authors state:

Thus, two approaches developed for the analysis of a pile under impact: the more flexible lumped mass method of Smith and the Donnell-de Juhasz method of characteristics. The latter method is more exact for ideally elastic, continuous systems. However, it is more difficult to apply to the pile capacity problem due to the difficulty of including a realistic soil, hammer and driving system model.

The statement that MOC is more exact is true, but the limiting statements are not correct as proven by numerous publications (e.g. Fischer et al. 2015,.Kourelis, 2018, Buckley et al. 2021) and the wide application of the MOC based wave equation programs like TNOWAVE, ALLWAVE, CAPWAP (Rausche, 1983) and IMPACT (Randolph 2008). It should be noted that ALLWAVE can be considered as the successor of TNOWAVE (Middendorp, et al. 2012).

Method of Characteristics Advantages

Voitus van Hamme (1980) stated the following about the MOC approach: *It has important advantage over piledriving programs based on concentrated masses interconnected by springs. First, force and velocity are always calculated for the same points (the grid points, at intervals of the order of 1 ft), whereas with conventional programs the forces in the pile are calculated for the springs and the velocities for the concentrated masses. Secondly, phenomena which occur at places where no traction can be sustained (e.g., between a pile and an add-on) can be assessed accurately: the time when a gap occurs is found, and how the gap increases and eventually decreases until the parts come into contact again. Thirdly, the piledriving hammer, even a rather complicated hammer such as the Hydroblok (with a built-in gas buffer), and the pile cap, with cushions if these are used, can easily be incorporated in the system. Fourthly, this 'solution of the wave equation' theory not only leads to a simple computer program but also provides a much better understanding of what really happens during piledriving.*

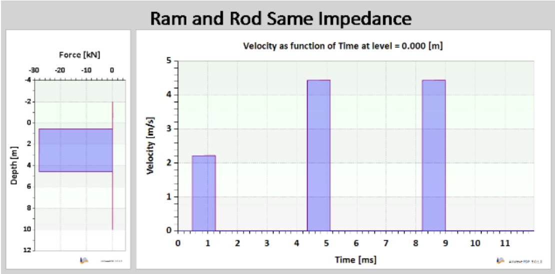
Rausche (1983) stated as MOC advantages: *High viscous damping forces do not lead to unstability, as the Smith model does. If soil segments are chosen at every third pile element and at a 6000 sps frequency, then the CAPWAP/C analysis is approximately 20% faster than CAPWAP. Further time savings can be obtained in cases with little or no skin friction over substantial pile portions (offshore). The response at time 2L/c is much more accurate than that of the lumped mass analysis, in particular on long piles. Thus model changes to avoid phase shifts are unnecessary.*

The conversion from the Smith approach to the MOC improved CAPWAP according to Horvath et al. (1988).

Randolph (2008) mentioned: *There are several advantages to the use of the characteristic solutions of the wave equation in pile driving analysis, rather than a finite difference or finite element approximation. The method has the simplicity of explicit time integration (avoiding the need to assemble and solve a global stiffness matrix for the pile) and yet is completely stable numerically. Wave propagation within the pile is modelled exactly, with only the soil resistance being lumped at nodes. The time increment is directly proportional to the length of the pile elements and will generally be rather larger than is necessary for accurate solution using finite element or finite difference approaches.*

Check on Algorithm Performance

A check on algorithm performance and/or programming can be done by comparing the outcome with theoretical solutions, such as for a ram with the same impedance as the pile (which should result in a rectangular pulse wave, Fig.13) or for a ram with a larger impedance than the pile (which should result in a step wise decreasing block shaped pulse. Fig.14).



Fig, 13. MOC Result for ram and pile having the same impedance. Pile length = 10m, Diameter = 20mm, Ram length = 2m, Ram Diameter = 20mm, Solid Steel.

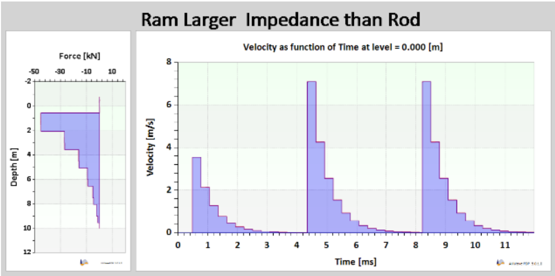


Fig. 14. MOC Result for a ram with a larger impedance than the pile. Pile length = 10m, Diameter=20mm, Ram length = 0.75m, Ram Diameter= 40mm, Solid Steel

4. IMPACT HAMMER MODELLING

`Because the MOC algorithm is stable, does not generate spurious reflections and agrees with theoretical solutions, it is an excellent tool to model impact hammer components (like rams, anvils cushions) and their interaction. Components and cushions may be of different materials, and cushioning by fluids with belonging stress wave propagation can be modelled as well. Also, the influence of the soil on transferred energy can be determined, which influence can indicate incorrectly that the hammer is less efficient. The first impact hammer modelling by the MOC was the Hydroblok (Fig.15, Janz et al. 1976), a rather complicated hammer with a built-in nitrogen gas buffer.

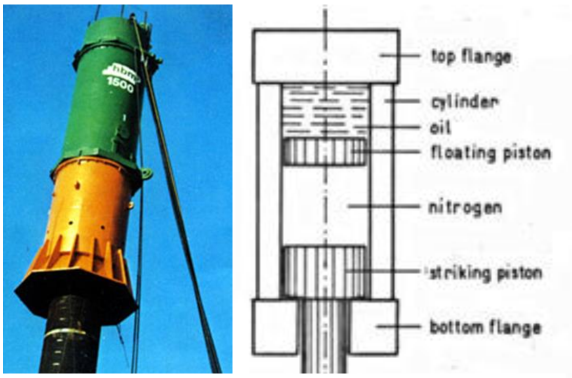


Fig. 15. A Hydroblok hammer and its principle (1976).

Nowadays a long list of hammer types (e.g., steam hammers, diesel hammers, hydraulic hammers, rapid load testing devices and vibratory hammers)

Fig. 16. IHC Pulse and Menck MNRU Noise reduction systems

have been modelled. Also advanced impact noise reduction systems like PULSE (IQIP) and MNRU (Menck) can be modelled. For these noise

Fig.17. Blue Piling water vessel hammer.

reduction systems are basically cushions to reduce high frequencies and to spread the impact load over a longer duration. With PULSE this is achieved by a pressurized water column, while MNRU applies mechanical spring elements. Examples of the effects of such noise reduction systems are given in Figure 16. Another option to obtain a softer impact is the Blue Piling Hammer (BPH) with a water vessel as the ram (Fig.17). .Figure 18 shows typical simulation results for the maximum stress in ram, anvil and pile as function of depth along pile and the impact force as function of time at the pile top (negative depth is along the ram). It should be noted, however, that the results for PULSE, MNRU and

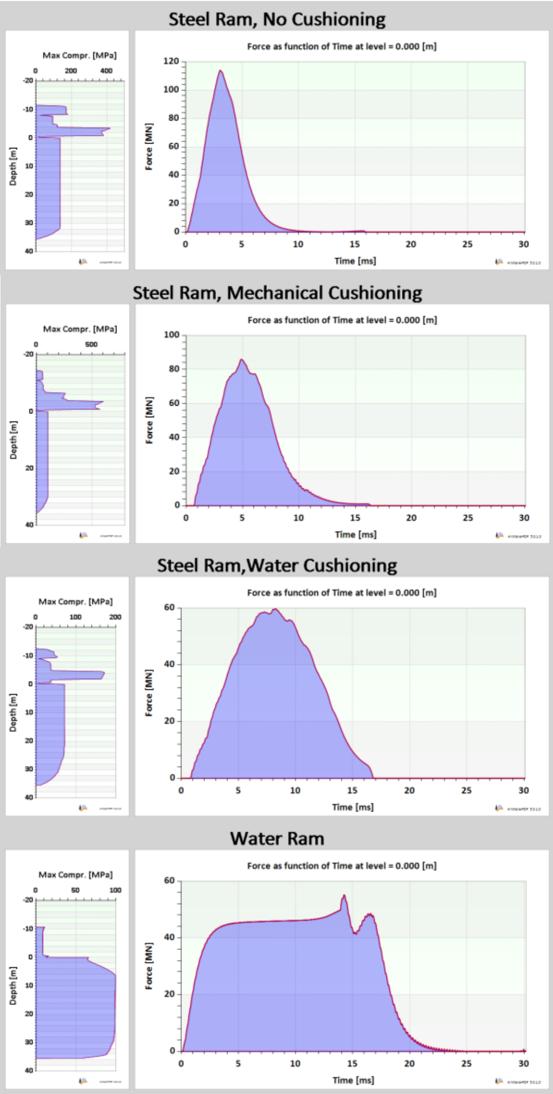


Fig. 18. Cushioning influence examples for, no cushioning, situation, MNRU, PULSE and a water ram

BPH are indicative and that the results are not based on the actual dimensions.

The MOC combines very well with finite element methods for detailed 2D or 3D stress studies of hammer components. For global modelling, in which flat wave propagation can be assumed in the components, the 1D MOC approach can be used for further analysis. Rapid Load Testing (RLT) is applying cushioning to the extreme, so that the impact results in a quasi-static situation in the pile and soil. Figure 19 represent a StatRapid (STR) device with cushioning system based on rubber springs. MOC simulations are used to predict maximum load values and load durations for a particular test (accurately modelling the drop height and mass, the springs, the pile and the soil) as well as for checking the validity of the UPM method (Fig. 20, 21, Middendorp 2019).

Fig. 19. StatRapid Load Testing Device and rubber cushioning system.

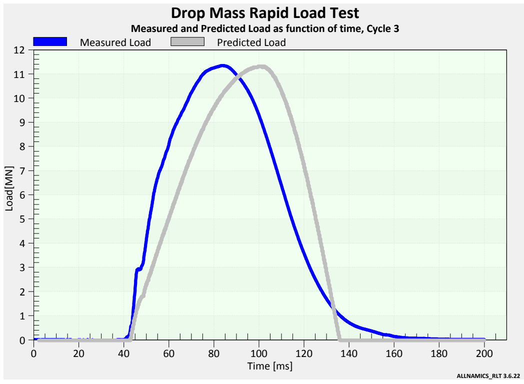


Fig. 20. Predicted and measured RLT Pile top Loads

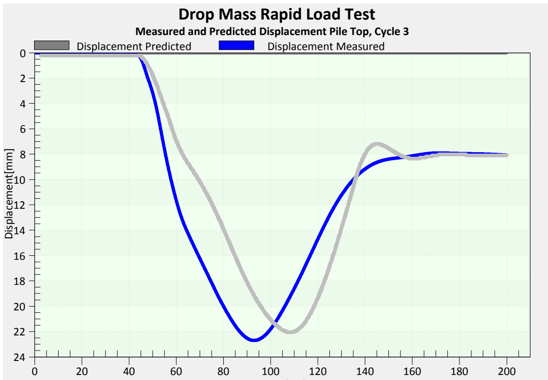


Fig. 21 Predicted and measured pile top displacements.

5. SIGNAL MATCHING

Signal matching is the technique whereby model parameters are modified such that a good agreement is obtained between measured and simulated signals. In the foundation industry it is used to obtain realistic soil model parameters or to fine tune hammer component model parameters. In case of the former, the derived static parts of the soil model are then used to the static load displacement behaviour of a pile. This signal matching technique was first applied by Goble et all (1980). The signal matching Kalman Filtering technique applied in TNOWAVE and AllWave (Courage et al. 1992) is these days also used for machine learning applications. To the author's knowledge, all signal matching programs available in the market are based on the MOC (CAPWAP, TNOWAVE, ALLWAVE_DLT, IMPACT).

6. FROM SOIL INVESTIGATION TO STATIC AND DYNAMIC SOIL PARAMETERS

An important step with applying MOC is the introduction of the soil resistance W at specific pile toe penetrations, which can be split up in displacement dependent resistance W(u) (spring), velocity dependent resistances W(v) (damper) and acceleration dependent resistances W(a) (added mass). A flow chart how the parameters for these soil resistance models are derived is given in Figure 22. The derivation of soil model parameters starts normally with soil investigation results, e.g., CPT data. Using the measured cone resistance q_c and sleeve friction f_s, the soil behavior type can be determined according to Robertson (2010) and correlations are available to estimate soil density and other soil properties. Some of these parameters (like yield stress and stiffness parameters like quake or dynamic shear modulus) are then introduced into the W(u) resistance spring models.

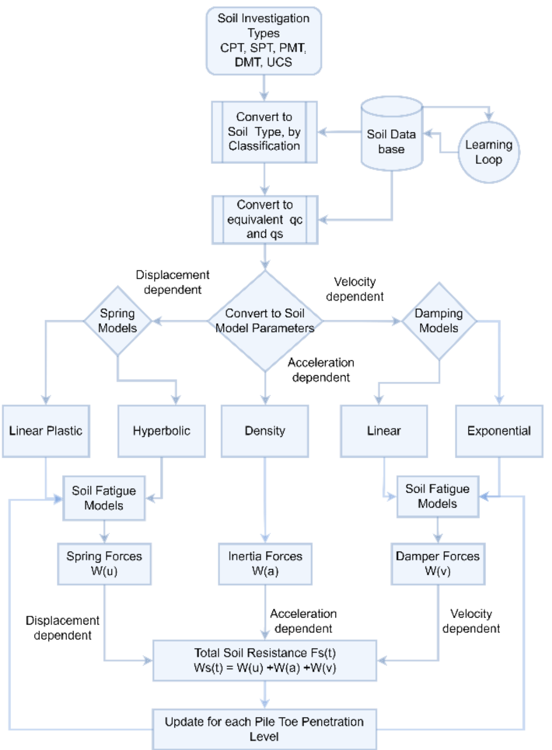


Fig. 22. Flow diagram representing derivation of soil parameters.

The Figures 23 and 24 represents such CPT data presentations and soil type derivations. Multiple CPT file formats like GEF and AGS can be processed and a soil interpretation type can be selected. A similar approach is used for other soil investigation methods based on empirical correlation factors and published empirical correlation data like from Verbrugge et al. (2016).

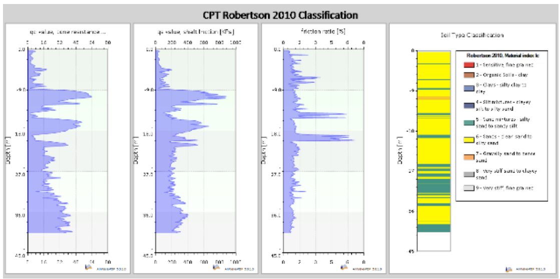


Fig. 23. Robertson CPT data interpretation example

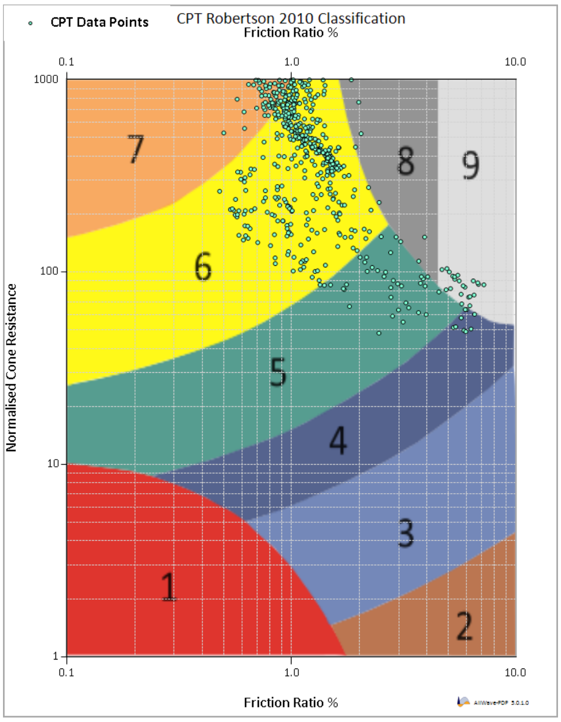


Fig.. 24.. Robertson CPT Soil classification example from GEF file format.

7. SOIL RESISTANCE MODELLING

The most applied static resistance model is the linear plastic behaviour, based on quake values (uq1, uq2) and yield values (Fy1, Fy2). However, a hyperbolic loading cycle approach is closer to the real cyclic soil loading behaviour. (Fig.25).

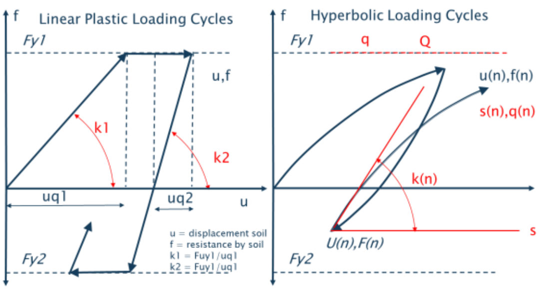


Fig. 25. Static soil resistance models

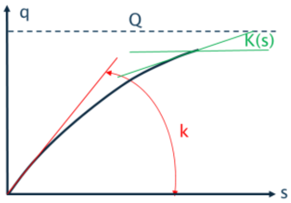


Fig. 26. Hyperbola definition

There are several hyperbolic model approaches for cyclic loading (e.g., Kondner (1963)), but a convenient one to implement in MOC is based on the hyperbola formula from Kee (1970) (Fig.26).

$$q = \frac{s}{(bs + a)}$$

$$a = 1/k$$

$$b = 1/Q$$

$$K(s) = \frac{dq}{ds} = \frac{a}{(a + bs)^2}$$

with s = displacement, q = load, k= initial spring stiffness, Q= asymptote hyperbola.

Q is related to the soil yield strength Fy1 for loading and Fy2 for unloading. The initial spring stiffness is related to the shear modulus. The Figures 25 and 26 represent vibratory driving simulation result with elasto-plastic modelling and hyperbolic modelling.

8. SOIL FATIGUE MODELLING

Pile driving introduces repeating loading cycles for each soil layer level along the pile shaft, resulting in soil fatigue with soil strength reduction. For impact driving the fatigue pattern changes with each hammer blow as loading cycle histories are added at each soil level. At the end of driving, the soil near ground level or seabed will have experienced the maximum, number of loading cycles, while near the toe there have been only a few loading cycles. To consider this change of fatigue pattern during driving, the soil strength models at the elevation must be updated for each pile toe penetration level, as was also stated by Schneider et al. (2010).

"*to accurately assess the effects of friction fatigue during drivability studies, a separate wave equation analysis would need to be performed for each tip depth* ".

This procedure is considered (Fischer et al. 2015). Several methods are available to introduce the soil fatigue behaviour into the MOC. As an example, Figure 27 represent the initial soil strength and the soil fatigue strength with the reduction factor pattern halfway final penetration and at end of driving. This pattern is updated for each penetration level.

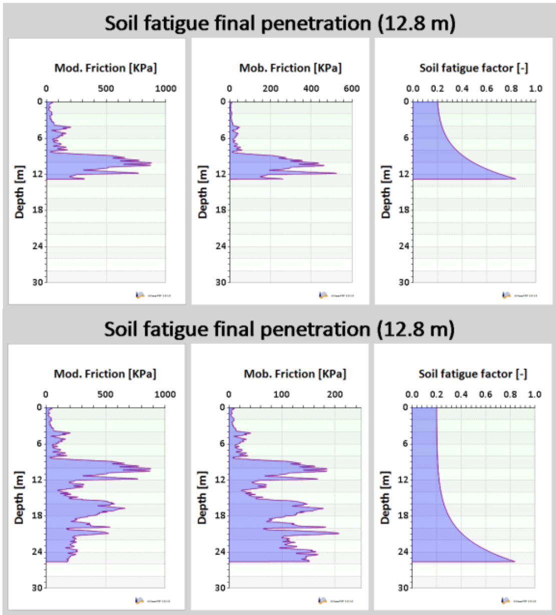


Fig. 27. Initial soil strength and fatigue reduced soil strength with fatigue reduction factor half way final penetration and at final penetration.

There are various fatigue models published in the literature (e.g., Toolan & Fox (1977), Stevens (1982), Alm & Amre (1998), Alm & Hamre (2001)), but in the author's opinion the model presented by Fischer et al. 2015 should be considered as it emphasizes mixed fatigue modelling per soil layer designating each soil layer with a different fatigue model. Also, an interesting development is also the Unified CPT-based method (Lehane et al. 2020). The method is developed for estimating the axial capacity of driven piles in sand and considers the aging (set up) of the soil after driving, which is the recovery from soil fatigue. Alternatively, the recovery model can also be transferred to soil fatigue modelling

9. VIBRATORY DRIVING

Like with impact driving, it was initially assumed that with vibratory driving the pile was vibrating as a rigid body and that maximum soil resistances at each level were acting at the same moment. However, this assumption is only valid for short piles (e.g., sheet piles). For longer piles a wave equation approach is required to correctly model the shift in strain state along the pile. With this approach the model will reflect that the particles along the pile shaft are vibrating with different and varying amplitudes and are in different phases. This was also clearly shown in the extensive work of O'Neill et al. (1989). To behave like a rigid body, it was required that the driving frequency was chosen to be equal or less than 10% of the natural frequency of the full-sized pile as a freely vibrating rod, expressed as:

$$f_d \leq 0.1 f_n = 0.1\, c_b/2L$$

Or

$$L \leq c_b\,/20 f_n$$

with f_d = driving frequency [Hz], f_n = longitudinal natural frequency of a free slender bar/pile [Hz], c_b = pile stress wave velocity [m/s], L = length of the pile [m].

For steel piles and a typical vibratory driving frequency of 23Hz this means that the rigid body assumption is only valid for pile with a length smaller than:

$$L \leq \frac{5172}{400} \approx 11\ [m]$$

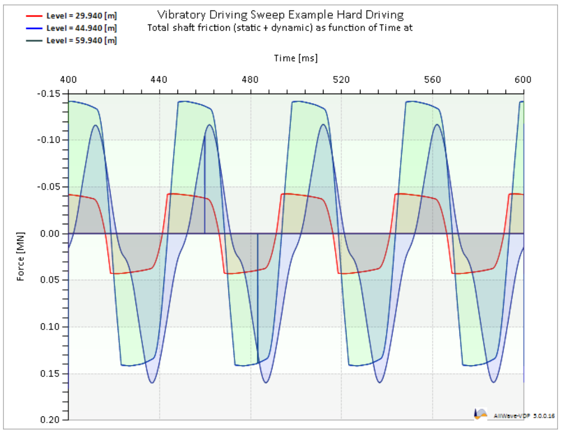


Fig. 28. Snapshot of a VDP simulation based on an elastoplastic soil resistance model.

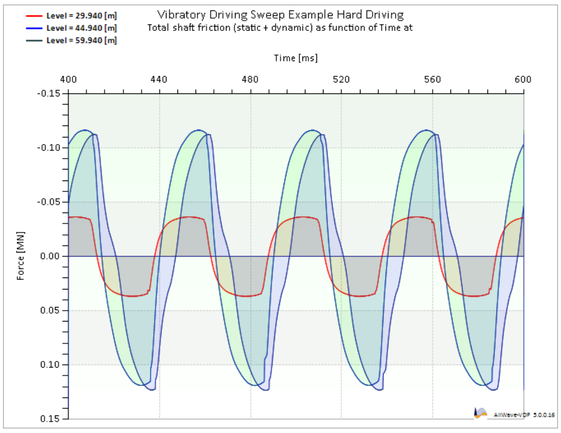


Fig. 29. Snapshot of a VDP simulation based on an hyperbolic soil resistance model.

and that for longer piles the maximum soil resistances along the pile do not occur at the same moment during vibratory driving.

Figures 28 and 29 represent a snapshot of an AllWave-VDP simulation result for a 72 m long steel pipe pile, with a diameter of 3.5m, wall thicknesses varying from 55mm to 75mm and a total weight of about 400 tons. The graphs represent the shaft friction at three different levels from the pile top during driving at 20Hz and a penetration of 50m.

It can be clearly seen that that the friction force acts out of phase at each level and cannot be combined to a simple friction force acting on a rigid body pile. Therefore, the rigid body models (e.g., those presented by Whenham (2012) and Massarsch (2020)) are only applicable for relatively short piles.

Soil fatigue modelling is mainly based on the Beta method (Jonker 1987). With AllWave-VDP the driveability is determined by the penetration speed at the pile toe penetration level. A penetration speed approaching zero is considered refusal. The penetration speed is determined from the displacement of the pile toe during a vibratory driving sweep for multiple cycles. The figures 29 and 30 represent results for easy driving and close to refusal respectively. The penetration speed is determined at the end of the trend line through the displacement signal. By subtracting the trendline from the displacement signal the displacement amplitude is obtained. The vibration sweeps are performed at several penetration intervals normally each 0.25m to 0.5m.

The first MOC based publications for vibratory driving of offshore piles was by Jonker et al. (1988) and Middendorp et al. (1988). Another

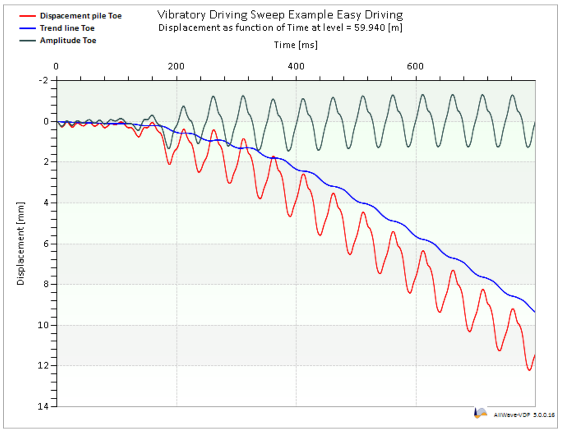


Fig. 30. Pile displacement for easy driving with trend line for penetration speed determination

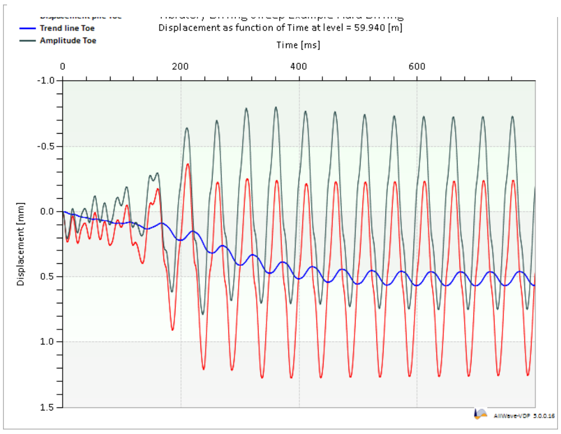


Fig. 31. Pile displacement for hard driving with trend line for penetration speed determination

paper that must be mentioned is the publication by Jonker in 1987, where he described the Beta method for soil fatigue modelling for vibratory driving.

In 2010 the companies APE, CAPE-Holland, APE-China and Allnamics teamed up to convince the Chinese contractor First Harbor Marine Group China that a massive multi-unit vibrohammer could be used to drive 49 m long, 22 m diameter steel caisson pipe piles weighing 600 tonnes each 25 m into the bed of the South China Sea, where the soil consists of silty clay, clay and sand with SPT N-values ranging from 8 to 40. (Fig.32). The driveability studies were performed by AllWave-VDP, which showed the feasibility of driving these gigantic piles into the bottom of the sea to the required depth. (Middendorp et al. 2012).

Following the successful vibratory driving at the Hong Kong-Zhuhai-Macau Bridge project the contractor Seaway Heavy Lifting opted for the use of a vibratory hammer for the installation of the monopiles for the Riffgat project. (de Neef at al. 2013). By choosing this innovative way of pile installation they could adhere to the strict

Fig. 32. Vibratory driving of a 22m diameter caisson for the Hong Kong-Zhuhai-Macau Bridge project

environmental rules, which apply in Germany, and keep the environmental impact due to noise and vibrations within acceptable limits. Traditional piling techniques using conventional hydraulic hammers on the other hand would have resulted in noise levels that cause major damage to marine life. Other advantage of vibratory driving is that the pile can be repositioned easily when the initial installation angle is out of tolerance (as was experienced during project execution), Furthermore vibratory driving results in lower stress levels reducing induced pile material fatigue.

In the view of the author, vibratory driving and impact driving are complimentary, and as a minimum vibratory driving can be used for stabbing the piles. If final penetration levels cannot be achieved by vibratory driving, the impact hammer can finish the job. This approach, i.e., performing vibratory driving predictions with MOC and the installation of monopiles by vibratory hammers, are common practice in the design and construction of offshore wind energy farms nowadays.

10. DRIVEABILITY PREDICTIONS WITH SCRIPTING AND ARTIFICIAL INTELLIGENCE

The current development of offshore wind energy farms may require the installation of hundreds of piles. To perform the design in an efficient way driveability studies must be automated, much like many other processes are automated. This requires that the MOC simulation programs allow the option to be operated externally by scripting methods. In this case scripting refers to a program or a sequence of instructions that is carried out by another program rather than by the computer processor. To the best of the author's knowledge, only AllWave piledriving simulation software can be run in this manner, but it seems only a matter of time before the industry will demand this as a standard feature and it may well be common practice when the next Stress Wave conference is hosted.

The next logical development is the merger of driveability prediction programs with Artificial Intelligence to make full use of all the data that has been collected in the 80 years that the Method of Characteristics has been used for pile driving simulation and modelling.

11. CONCLUSIONS

The author is impressed with the ingenuity of our forefathers to solve and apply the WEQ solution without the assistance of computers

The MOC graphics method is widely used to explain and understand stress wave propagation phenomena.

The MOC algorithm has many advantages over the finite difference method (lumped masses and springs).

The MOC is used extensively in many stress wave equation applications for impact driving, vibratory driving, rapid load testing, signal matching and integrity testing.

The MOC is used for hammer components design like rams and noise reduction systems.

Soil fatigue behaviour during impact driving can be obtained by updating the soil model parameters for each pile toe penetration depth.

The use of scripting will be the next logical development for driveability studies so the process can be more automated.

Artificial Intelligence and MOC will increasingly be merged in driveability prediction programs.

12. REFERENCES

Allin, R., Likins, G., Honeycutt, J. (2015) Pile driving formulas revisited. *Proceedings of the International Foundations Congress and Equipment Expo 2015*, San Antonio, Texas. Eds. M. Iskander et al.

Alm, T., Hamre., L (1998) *Soil Model for Driveability Predictions*. Offshore Technology Conference. OTC 8835, 13.

Alm, T., Hamre., L (2001) *Soil Model for Pile Driveability Predictions based on CPT Interpretations*. Proceedings of the 15th International Conference on Soil Mechanics and Geotechnical Engineering, 2, Istanbul, Turkey, 1297-1302.

Bergeron, L., (1937), "Méthode Graphique Générale de Calcul des Propagation d'Ondes Planes" (General Graphical Method of Calculating the Propagation of Plane Waves), *Bulletin de la Société des Ingénieurs Civils de France*, Paris, July 1937, 93 pp., with 43 figures.

Bergeron, L., (1950), Du coup de bélier en hydraulique au coup de foudre en électricité, méthode graphique générale, Dunod, Paris

Buckley, R.M., Kontoe, S., Jardine, R.J., Barbosa, P., Schroeder, F. C. (2021). Pile driveability in low-to medium-density chalk. *Canadian Geotechnical Journal*,*58*(5), 650-665

Courage, W.M.G, Bielefeld, M.W., (1992). TNOWAVE automatic signal matching. Application of Stress-Wave Theory to Piles. F.B.J. Barends (ed.) 1992 Balkema, Rotterdam, ISBN 9054100826.

Cross, R., (2012)Edme Mariotte and Newton's Cradle, The Physics Teacher, Vol. 50, April 2012, Australia

Cruys, G.W., (1983), Poging tot Bepaling van het gedrag van de grond uit het lastzakkingsdiagram (Dutch), (A Trial to Determine Soil Behaviour from a Load Displacement Diagram), *TNO-Report BI-83-74.* TNO, The Netherlands

De Josselin De Jong, G. (1956), "Wat gebeurt er in de grond tijdens het heien (Dutch)" (What happens in the soil during pile driving) *De Ingenieur, No. 25,* Breda, The Netherlands,

De Josselin De Jong, G. (1956, 2022), What happens in the soil during pile driving, *English Translated*

Version, 11th International Stress Wave Conference, Rotterdam, The Netherlands,. 2022

De Juhasz, K.J., (1949), "Graphical Analysis of Impact of Bars stressed above the Elastic Range", J.*Franklin* Inst , 248,. E141

Eytelwein. J.A., (1808), *Handbuch der Statik fester Körper (German),* Berlin.

Fellenius, B.H., (1996), Reflections on Pile Dynamics, *Keynote paper to the 5th International Conference on the Application of Stress-Wave Theory to Piles*, Orlando, Florida

Fischer, H.C., (1960), On longitudinal impact IV,.*Appl. sci. Res., Section A, Volume 9*,

Fischer, J., Middendorp, P., Tara, D., Verbeek, G.E.H., (2015) Soil Fatigue Analysis for Pile Driving Simulations Using an Impact Hammer, *Conference: 40th Annual Conference on Deep Foundations*, At: Oakland, CA, USA, Volume: AM-2015

Goble, G. G., Rausche, F., (1980) Pile drivability predictions by Capwap. Numerical methods in offshore piling. January 1980, 29-36

Hinz, D., (2017), Caesars Rheinübergang (German), *Historisches Institut, Alte Geschichte* Universität zu Köln

Horvath. H.G., and Killeavy, M., (1988) "Variation of "CAPWAP Results With Blows Selected for Analysis. *Proceedings of the Third International Conference on the Application of Stress- Wave Theory to Piles*. Ottawa, Ontario, 25-27 May. pp.735-748

Hussein. M H., and Goble. GG, (2004). A Brief History of the Application of Stress-Wave Theory to Piles, Current Practices and Future Trends in Deep Foundations. *Geotechnical Special Publication No. 115*., American Society of Civil Engineers: Rest on. VA

Isaacs, D. V, (1931), Reinforced Concrete Pile Formula. *Inst. Aust. Eng. Jour., Vol. 12*.

Jansz, J.W., Voitus van Hamme, G.E.J.S.L., Gerritse, A., Bomer, H. (1976), Controlled Pile Driving Above and Under Water with A Hydraulic Hammer., *OTC 2477. Offshore Technology Conference.*

Jonker, G., (1987), Vibratory Pile Driving Hammers for Oil Installation and Soil Improvement Projects., *Proc. of Nineteenth Annual Offshore Technology Conf., Dallas, Texas*, OTC 5422, pp. 549-560, 1987

Jonker, G., Middendorp, P., (1988), Subsea installations using vibratory piling hammers, *20th OTC*, Houston, Texas.

Kee, Ch. F., (1970) Estimation of the Ultimate Load of Piles from tests not carried to failure. *Proc. 2. Southeast Asian Conf. Soil Eng*. Singapur, pp. 81-90

Kondner, R. L., (1963). "Hyperbolic Stress-Strain Response; Cohesive Soils." *Journal of the Soil Mechanics and Foundations Division,* ASCE. VoL 89, No. SMI, pp. 115-143, Jan.

Kourelis, I., (2018), Pile driveability analyses using driving records for the Horns Rev III offshore windfarm in the Danish North Sea, Imperial College London, Faculty of Engineering, Department of Civil and Environmental Engineering.

Lehane, B. M., Liu, Z., Bittar, E., Nadim, F., Lacasse, S., Jardine, R., Carotenuto, P., Rattley, M., Gavin, K., & More Authors (2020). A New 'Unified' CPT-Based Axial Pile Capacity Design Method for Driven Piles in Sand. In Z. Westgate (Ed.), *Proceedings Fourth International Symposium on Frontiers in Offshore Geotechnics (pp. 462-477). [3457]*

Le Rond D'Alembert (1747). Recherches sur la courbe que forme une corde tenduë mise en vibration (Researches on the curve that a tense cord forms [when] set into vibration), *Histoire de l'académie royale des sciences et belles lettres de Berlin*, vol. 3, pages 214-219.

Lintsen, H.W., (1994), History of technology in the Netherlands. The emergence of a modern society 1800-1890. Part V. Technology, Profession and Practice (in Dutch). *Walburg Press*, Zutphen

Mariotte, E., (1717), Œuvres de Mariotte. Réédition : *J. Peyroux, Bordeaux, 2001. Texte en ligne*

Mayne, P.W. (2016). Evaluating effective stress parameters and undrained shear strength of soft-firm clays from CPT and DMT. *Australian Geomechanics Journal* 51 (4): 27-55

Massarsch, K. R., Wersäll, C.L. Fellenius, B, (2020). Vibratory driving of piles and sheet piles - State of practice. *Proceedings of the Institution of Civil Engineers* - Geotechnical Engineering. 1-43. 10.1680/jgeen.20.00127.

Massau. J., (1904), Memoire sur l'integration graphique des equations aux derivees partielles, (Note on the graphical integration of partial differential equations). *Annales de l'Association des ingenieurs sortis des ecoles speciales de Gand*, 23 (1900), p. 95–214 ; (3e s´er.) 1 (1902), p. 135–226, 393–434 ; 2 (1903), p. 383–436 ; 3 (1904), p. 65–147.

Middendorp, P., (1981), Fender Forces from Berthing Ships, *The Dock & Harbour Authority*, Vol. LXII, No. 731.

Middendorp, P., van Weele, A.F, (1986), *Application of the characteristic stress wave method in offshore practice*. Proceedings 3rd International Conference on Numerical methods in Offshore Pilling, Nantes, France. Supplement; 6-18.

Middendorp, P. and Jonker, G. (1988). Prediction of Vibratory Hammer Performance by Stress Wave Analysis. *Proceedings of the 3th International Conference on Application of Stresswave Theory to Piles*, Ottawa.

Middendorp, P. (2004), 'Thirty years of experience with the wave equation solution based on the method of characteristics", 7th *Int. Conf. on the Application of Stress Wave Theory to Piles*, Kuala Lumpur. Malaysia.

Middendorp, P. and Verbeek, G. (2012). At the cutting edge of pile foundations and pile testing. *Proceedings of the 9th International Conference on Testing and Design Methods for Deep Foundations*, Kanazawa, Japan, pp. 931-937.

Middendorp, P., Bielefeld, M. W., Bakker, J., (2019) Rapid Load Testing Prediction Models, *Stress Wave Theory and Testing Methods for Deep Foundations: 10th International Conference, ASTM*, STP1611, USA

Middendorp, P., Verbeek, G., Bielefeld, M. W., (2022) Post Driveability Study for Caesar's Rhine Bridge 55 B.C., *11th International Stress Wave Conference, Rotterdam, The Netherlands*

Neef de, L., Middendorp, P. and Bakker, J. (2013) Installation of monopiles by vibrohammers for the Riffgat Project. *Proceedings of the Institute for Soil Mechanics and Foundation Engineering Technische Universität Braunschweig*, Germany,14 pages

O'Neill, M.W., Vipulanandan, C., (1989), Laboratory evaluation of piles installed with vibratory drivers, National Cooperative Highway Research Program, Report No. 316, *National Research Council Washington* DC, Vol 1, 1-51

Rausche, F., (1983), CAPWAP Analysis Using the Characteristics Approach, *PDA Users Day*, Philadelphia, USA.

Randolph M. F. (2008). *IMPACT - Dynamic analysis of pile driving*, Manual.

Robertson, P.K., (2010), Soil Behaviour Type from CPT: an update. *Proceedings of the 2nd International Symposium on Cone Penetration Testing, CPT'10*. Huntington Beach, CA.

Rondelet (1802), *L'Art de Batir*, Paris

Saint-Venant. B. de, (1867), Memoire sur le doc longitudinal de deux barres elastiques, *Journal de Mathematique*, 2, ser XII, pp 237-376.

Schneider, J. A., Harmon, I. A., 2010. Analyzing Drivability of Open Ended Piles in Very Dense Sands, *DFI Journal, The Journal of the Deep Foundations Institute*, Volume 4, Issue 1, pp. 32-44.

Schonfeld, J.C., (1951), Voortplanting van getijden en soortgelijke golven (Dutch) Propagation of tides and similar waves. *Thesis University of Delft*, The Netherlands

Stevens, R.S., Wiltsie, E.A., Turton, T.H. (1982) Evaluating Pile Driveability for Hard Clay, Very Dense Sand, and Rock. *14th Annual Offshore Technology Conference in Houston*, Texas, May 3-6, OTC 4205.

Toolan, F.E. and Fox, D.A. (1977) Geotechnical Planning of Piles Foundations for Offshore Platforms. *Proc. Instn. Civ. Engrs*, Part 1, 62, May, 221-244.

Verbrugge, J.; Schroeder, C, (2018), Geotechnical Correlations for Soils and Rocks, *Wiley-ISTE (2018) ISBN 10: 1786302799 / ISBN 13: 9781786302793*

Voitus van Hamme, G.E.J.S.L., Jansz J.W. Bomer H.,and Arentsen, D., (1974), Hydroblok and Improved Pile Driving Analysis. *De Ingenieur*, Vol 86, no 8, pp 344-352, The Netherlands.

Voitus van Hamme, G.E.J.S.L., (1980), Institution of Civil Engineers, *Numerical Methods in Offshore Piling. ICE*, London, pp. 171-173.

Voitus van Hamme, G.E.J.S.L., (1984), De Theorie van het Heien (The Theory of Pile Driving). *Natuurkundige Voordrachten, Nieuwe Reeks No.63, Vis-Druk Alphen aan den Rijn, 1984*

Warrington D. C. (2013) Persistent Issues in the Wave Equation for Piling for Both Forward and

Inverse Methods. *University of Tennessee at Chattanooga*, USA

Whenham,V., Holeyman, A., (2012), Load transfers during vibratory driving. *Geotechnical and Geological Engineering* 30(5):1119–1135.