Lorentz Invariance from Period-Wavelength Counting and Reflection/Refraction?

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Fermat's principle of least time may be utilized to predict reflection and refraction paths. This entails writing t in terms of distance(x)/speed of light / and varying x with dx/dt =0. It seems, however, that there may exist a different way of analysing reflection and refraction.
In the case of reflection and refraction, time and frequency do not change for a reflected ray or for the x-projection of a refracted ray. For the refracted ray, both speed of light and wavelength change, but it appears that Fermat's principle is equivalent to keeping the number of wavelengths and also periods in the x-projection of the incident and refracted rays the same.

From this idea, one may create two invariants namely Et+px and -Et+px where p=hbar/wavelength. The second may be varied to give E=pc and so is used. This invariant follows from counting wavelengths and periods. Thus if one has a rest frame and a second frame moving at a constant velocity v in the x direction, the counting of wavelengths and periods does not change even though E,t,x and p may change (i.e. people in the two frames have different values). Thus -Et+px should also be an invariant for constant moving frames. In other words, one obtains a Lorentz invariant (with the metric) from an invariant appearing in reflection/refraction. In principle, one could simply use -Et+px with E proportional to 1/T and p to 1/wavelength to argue that -Et+px is a scalar representing the number of periods in t and wavelengths in x. Given that this is a scalar, one could argue it should be a Lorentz invariant as well. In this note we try to argue that -Et+px is not only an invariant in special relativity, but applies to other physical examples such as reflection or refraction.

Fermat's Principle

Fermat's minimum time principle applied to light may be used to predict the outcome of a reflection or refraction. For reflection or refraction a fixed length L along the x axis (which is also the axis of the mirror or media interface) exists and a fixed y projection is used. The idea is then to have an incident ray hit the x axis at x and write an expression for time using distance/light velocity. This is a function of x so one may take dt/dx=0 and find the value of x. Given x and the y projection, one has the angle of incidence and reflection/refraction.

Fermat's Principle Result Based on Wavelength Considerations Alone

It is possible to obtain Fermat's minimum time results for reflection and refraction by wavelength counting arguments alone.

For reflection, consider an incident ray making an angle A with the normal and having a y axis projection of Y. This ray contains a certain number of wavelengths. By adjusting Y one may make this a whole number. Consider next a reflected ray making the same angle A. It has the same number of wavelengths. Next draw a circle using the reflected ray. The center is the point at which the incident ray hits the mirror. If the angle A is increased, the new reflected ray must

still create a y-axis projection of Y and so extends outside of the circle. Thus the number of wavelengths is increased in the reflected ray. If the angle A is decreased, the new ray with a projection of Y does not reach the circle so again there are fewer wavelengths. Thus having the incident and reflected rays have the same number of wavelengths is equivalent to finding a stationary solution of the number of wavelengths and this is equivalent to Fermat's principle.
Alternatively, one could use the x projection of the incident and reflected rays and argue that the number of periods T and wavelengths in each should be the same. This is equivalent to conservation of momentum along the x axis. A similar idea is applied to refraction, although in this case the wavelength changes in different media.

For refraction, consider having the same number of wavelengths and time periods in the x-projection incident ray as in the x-projection refracted ray. The speed of light in medium 1 is c/n1 and in medium 2 c/n2. To keep energy the same in both media one may use v=f wavelength where f=1/energy (up to a proportionality constant). Thus if v=c/ni then wavelength = wavelength (vacuum)/ ni as well. We denote wavelength(vacuum) by wv.

If one uses the same length L for the incident and refracted ray then the x projection of the incident ray is L sin(A) and of the refracted L sin(B) where A and B are incident and refracted angles measured from the y axis (normal). Then:

Lsin(A) = (c/n1) nT and Lsin(B) = (c/n2) nT where T is the period proportional to 1/E and n is the number of periods. This leads to Snell's law: sin(A)/sin(B) = n2/n1. ((1))

The number of wavelengths in the x projections are:
Lsin(A) / (wv/n1) and Lsin(B) / (wv/n2) ((2))

Using Snell's law gives: L n2 sin(B)/wv and Ln2 sin(B)/wv so the two are the same.
Given that L is the same for the incident and refracted rays, this means that the number of wavelengths and periods that fit into the two rays is not the same, only for the x-projections.

We note that the x-projection coincides with conservation of momentum in the x direction i.e. p(in) sin(A) = p(out) sin(B) where p= hbar/(wv/ni).

Creating an Invariant

Given that E and t are the same for both reflected and x projection of the refracted cases and that E=b/T where T is the period:

Et= b * the number of periods in the time t ((3))

This implies that there should be the same number of wavelengths in these same distances. Using p=b/(wv/n1) one needs a length of x= n(wv/n1) where wv is the vacuum wavelength so that px = b* the number of periods in t.

Thus one may create an invariant which is 0 i.e.

A= -Et + px ((4))

This has the added feature that dA=0 = -E dt + p dx yields dx/dt = E/p or E=pc which holds for light.

Lorentz Invariance

The invariant created in the above section basically represents n-n where n is both the number of periods and wavelengths. Given that this is linked directly to a number (scalar) (i.e. counting) which should be invariant in other cases, one may consider two frames, one at rest and the other moving. Let v be the velocity of the moving frame in the direction of x, for the sake of argument. For someone sitting in the rest frame the arguments of the above sections hold. Imagine that a viewer in the rest frame watches what happens in the moving frame. In such a case, p, E, x and t may not be the same values as observed in the moving frame. Let us call them p',E',x',t'. Given that -Et+px = n-n where is a countable value, this value should not change as viewed from one frame or another. Thus:

-E't'+p'x' = -Et + px ((5))

In other words the invariant created for the refraction/reflection scenario carries over into a moving frame scenario i.e into Lorentz invariants. This result does not depend on Fermat's principle.

One may note that an argument based on -Et+px representing numbers of periods and wavelengths could be introduced without considering reflection/refraction. One could then proceed to argue that -Et+px should be a scalar when considered in different frames moving along the x axis with different speeds. We, however, wish to show that -Et+px is not only an invariant for moving frames (Lorentz transformations), but appears in other physical examples such as reflection and refraction.

Conclusion

In conclusion, we argue that Fermat's principle may be replaced by counting periods and wavelengths in a special way for reflection/refraction i.e. using the x-projection i.e the direction in which momentum is conserved.This notion leads to Snell's law with no derivatives or minimum considerations as well as the angle of reflection equals the angle of incidence . Furthermore it leads to an invariant quantity namely -Et+px where E=b/T and p=b/(wv/ni) where wv is the wavelength in a vacuum, ni the index of refraction and b=hbar. This invariant is essentially n-n where n is the number of periods and also the number of wavelengths, in other words a countable number or scalar. We argue that if one considers two moving frames (with motion along the x direction), even if E,p,x, t do not have the same values as viewed from both frames, a countable (scalar) should have the same value so -Et+px should be an invariant, in other words a Loretnz invariant.

References

1. https://en.wikipedia.org/wiki/Fermat%27s_principle
2. https://en.wikipedia.org/wiki/Snell%27s_law