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A New Perspective of Neutrosophic Hyperconnected Spaces

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Abstract. The focus of this article is to introduce a new class of sets namely neutrosophic semi j-open and neutrosophic semi j-closed sets in neutrosophic topological space. Using this, we present the new spaces neutrosophic hyperconnected and neutrosophic semi j-hyperconnected. Also we explore the characteristics of neutrosophic semi j-open sets, neutrosophic semi j-closed set, neutrosophic hyperconnectedness. Finally we examine the properties of neutrosophic semi j-hyperconnectedness with some existing sets.

Keywords: Neutrosophic semi j-open, Neutrosophic semi j-closed, Neutrosophic hyperconnected, Neutrosophic semi j-hyperconnectedness.

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1. **Introduction**

In 1965, L.Zadeh introduced the concept of fuzzy sets and fuzzy logic. It is an important concept in handling uncertainity in real life where each element has a membership functions [22]. In 1986, Attanassov proposed the concept of intuitionistic fuzzy sets, which is a generalization of fuzzy sets [10]. Intuitionstic fuzzy sets are characterized by the membership function and non-membership function with each element, whereas in real life we need to handle the incompleteness and indeterminancy. In this context, Smarandache applied neutrosophic set theory to solve real world practical problems. Smarandache's neutrosophic set theory focused on medical, engineering fields, social science etc. [5, 7], Neutrosophic sets are characterized by membership, indeterminancy and non-membership functions [8, 21].

In 2012, Salama and Alblowi defined neutrosophic topological space by using neutrosophic sets [14]. Further researchers have carried out to investigate the various properties of neutrosophic sets in different fields [2–4, 6].In 1970, Steen and Seebach [20] introduced the notion of hyperconnectedness in topological spaces. Several researchers examined the properties of hyperconnectedness in general topology [1, 11, 12, 15, 17, 18]. Jayasree chakraborty, Baby bhattacharya and Arnab paul defined fuzzy hyperconnectedness in fuzzy topological space [9].

Recently Sasikala.D and Deepa.M introduced semi j-hyperconnected space using semi j-open sets [16]. This paper communicates the role of hyperconnectedness in the field of neutrosophic topological spaces. We ideate a new class of sets called neutrosophic semi j-open set and neutrosophic semi j-closed set exercised with theorems and appropriate examples. Also we proposed the novel space namely neutrosophic semi j-hyperconnected space by neutrosophic semi j-open sets and analyses the essential characteristics of this space.

Throughout this paper neutrosophic topological space $[\mathcal{X}, \tau]$ is simply denoted by $\mathcal{X}$.

2. Preliminaries

Definition 2.1. [13] Let $\mathcal{X}$ be a non empty set. A neutrosophic set P is an object having the form $P = \{< x, \lambda_P(x), \mu_P(x), \nu_P(x) >: x \in \mathcal{X}\}$ where $\lambda_P(x), \mu_P(x)$ and $\nu_P(x)$ represents the degree of membership function, the degree of indeterminancy and the degree of non membership function respectively of each element $x \in \mathcal{X}$ to the set P. It is simply denoted by $P =< \lambda_P(x), \mu_P(x), \nu_P(x) >$.

Definition 2.2. [14] Let $P =< \lambda_P(x), \mu_P(x), \nu_P(x) >$ be a neutrosophic set on $\mathcal{X}$, then the complement of the set P can be defined by the following three kinds as

$(i) C[P] = \{x, 1 - \lambda_P(x), 1 - \mu_P(x), 1 - \nu_P(x) >: x \in \mathcal{X}\}$.

$(ii) C[P] = \{x, \nu_P(x), \mu_P(x), \lambda_P(x) >: x \in \mathcal{X}\}$.

$(iii) C[P] = \{x, \nu_P(x), 1 - \mu_P(x), \lambda_P(x) >: x \in \mathcal{X}\}$.

Proposition 2.3. *[14] For any neutrosophic set P, the following conditions hold:*

(i) $0_N \subseteq P, 0_N \subseteq 0_N$.

(ii) $P \subseteq 1_N, 1_N \subseteq 1_N$.

Definition 2.4. [14] Let τ be a collection of all neutrosophic subsets on $\mathcal{X}$. Then τ is called a neutrosophic topology on $\mathcal{X}$ if the following conditions hold.

(i) $0_N, 1_N \in \tau$.

(ii) union of any numbers of neutrosophic sets in τ also belongs to τ.

(iii) intersection of any two of neutrosophic sets in τ also belongs to τ.

Then the pair $(\mathcal{X}, \tau)$ is called neutrosophic topological space. A neutrosophic set Q is neutrosophic closed if and only if complement of Q is neutrosophic open.

Definition 2.5. [13] Let $\mathcal{X}$ be neutrosophic topological space and $P =< \lambda_P(x), \mu_P(x), \nu_P(x) >$ be a neutrosophic set in $\mathcal{X}$. Then the neutrosophic closure and neutrosophic interior of P are defined by

$Ncl[P] = \cap\{M : M$ is a neutrosophic closed set in $\mathcal{X}$ and $P \subseteq M\}$.

$Nint[P] = \cup\{N : N$ is a neutrosophic open set in $\mathcal{X}$ and $N \subseteq P\}$.

It follows that $Ncl[P]$ is neutrosophic closed set and $Nint[P]$ is a neutrosophic open set in

$\mathcal{X}$.

(i) P is neutrosophic open set if and only if $P = Nint[P]$.

(ii) P is neutrosophic closed set if and only if $P = Ncl[P]$.

Proposition 2.6. *[13] For any neutrosophic set P in $\mathcal{X}$ we have*

(i) $Ncl(C[P]) = C(Nint[P])$.

(ii) $Nint(C[P]) = C(Ncl[P])$.

Definition 2.7. [21] A neutrosophic set P in a neutrosophic topological space $\mathcal{X}$ is called

(i) Neutrosophic semiopen set if $P \subseteq Ncl[Nint[P]]$.

(ii) Neutrosophic preopen set if $P \subseteq Nint[Ncl[P]]$.

(iii) Neutrosophic regular open set if $P = Nint[Ncl[P]]$.

(iv) Neutrosophic j-open set if $P \subseteq Nint[Npcl[P]]$.

Definition 2.8. [4] A neutrosophic subset P in a neutrosophic topological space $\mathcal{X}$ is called

(i) neutrosophic dense if $Ncl[P] = 1_N$.

(ii) neutrosophic nowhere dense if $Nint[Ncl[P]] = 0_N$.

Proposition 2.9. *[13] Let $\mathcal{X}$ be a neutrosophic topological space and P, Q be two neutrosophic subsets in $\mathcal{X}$. Then the following conditions hold:*

(i) $Nint[P] \subseteq P$.

(ii) $P \subseteq Ncl[P]$.

(iii) $P \subseteq Q \implies Nint[P] \subseteq Nint[Q]$.

(iv) $P \subseteq Q \implies Ncl[P] \subseteq Ncl[Q]$.

(v) $Nint[Nint[P]] = Nint[P]$.

(vi) $Ncl[Ncl[P]] = Ncl[P]$.

(vii) $Nint[P \cap Q] = Nint[P] \cap Nint[Q]$.

(viii) $Ncl[P \cup Q] = Ncl[P] \cup Ncl[Q]$.

(ix) $Nint[0_N] = 0_N$.

(x) $Nint[1_N] = 1_N$.

(xi) $Ncl[0_N] = 0_N$.

(xii) $Ncl[1_N] = 1_N$.

(xiii) $P \subseteq Q \implies C[Q] \subseteq C[P]$.

(xiv) $Ncl[P \cap Q] \subseteq Ncl[P] \cap Ncl[Q]$.

(xv) $Nint[P \cup Q] \supseteq Nint[P] \cup Nint[Q]$.

Definition 2.10. [19] A topological space $\mathcal{X}$ is said to be hyperconnected if every non empty open subset of $\mathcal{X}$ is dense in $\mathcal{X}$.

3. Neutrosophic semi j-open sets

In this part, we define a new set namely neutrosophic semi j-open set in neutrosophic topological spaces. Also some of its basic properties are discussed.

Definition 3.1. Let P be a neutrosophic subset of a neutrosophic topological space $\mathcal{X}$. Then P is said to be neutrosophic semi j-open set of $\mathcal{X}$ if and only if $P \subseteq Ncl[Nint[Npcl[P]]]$.

Example 3.2. Let $\mathcal{X} = \{s, t, r\}$ and the neutrosophic subsets P, Q, R and S in $\mathcal{X}$ as follows,
$P = \{< s, 0.4, 0.3, 0.8 >, < t, 0.5, 0.2, 0.6 >, < r, 0.4, 0.2, 0.6 >; s, t, r \in \mathcal{X}\}$,
$Q = \{< s, 0.3, 0.4, 0.5 >, < t, 0.6, 0.4, 0.6 >, < r, 0.3, 0.4, 0.6 >; s, t, r \in \mathcal{X}\}$,
$R = \{< s, 0.4, 0.4, 0.5 >, < t, 0.6, 0.4, 0.6 >, < r, 0.4, 0.4, 0.6 >; s, t, r \in \mathcal{X}\}$,
$S = \{< s, 0.3, 0.3, 0.8 >, < t, 0.5, 0.2, 0.6 >, < r, 0.3, 0.2, 0.6 >; s, t, r \in \mathcal{X}\}$.
Then $\tau = \{0_N, P, Q, R, S, 1_N\}$ is a neutrosophic topological space $\mathcal{X}$.
Let $E = \{< s, 0.4, 0.4, 0.5 >, < t, 0.5, 0.4, 0.7 >, < r, 0.4, 0.4, 0.7 >; s, t, r \in \mathcal{X}\}$ be a neutrosophic subset in $\mathcal{X}$, then $Ncl[Nint[Npcl[E]]] = \{< s, 0.5, 0.6, 0.5 >, < t, 0.6, 0.6, 0.6 >, < r, 0.6, 0.6, 0.4 >; s, t, r \in \mathcal{X}\}$ Therefore $E \subseteq Ncl[Nint[Npcl[E]]]$. Hence E is a neutrosophic semi j-open set.

Theorem 3.3. *Let $\{P_\alpha : \alpha \in \Delta\}$ be a collection of neutrosophic semi j-open sets in neutrosophic topological space $\mathcal{X}$. Then $\bigcup_{\alpha\in\Delta} P_\alpha$ is also neutrosophic semi j-open in $\mathcal{X}$.*

Proof. Since P_α is neutrosophic semi j-open set in $\mathcal{X}$. Then $P_\alpha \subseteq Ncl[Nint[Npcl[P_\alpha]]]$. $\bigcup_{\alpha\in\Delta} P_\alpha \subseteq \bigcup_{\alpha\in\Delta} Ncl[Nint[Npcl[P_\alpha]]] \subseteq Ncl[Nint[Npcl[\bigcup_{\alpha\in\Delta} P_\alpha]]]$. Hence $\bigcup_{\alpha\in\Delta} P_\alpha$ is also neutrosophic semi j-open set in $\mathcal{X}$. □

Remark 3.4. The intersection of any two neutrosophic semi j-open sets of neutrosophic topological space $\mathcal{X}$ need not be a neutrosophic semi j-open set as verified by the following example.

Example 3.5. Let $\mathcal{X} = \{s, t\}$ and the neutrosophic subsets P, Q, R and S in $\mathcal{X}$ as follows,
$P = \{< s, 0.2, 0.1, 0.8 >, < t, 0.3, 0.1, 0.4 >; s, t \in \mathcal{X}\}$,
$Q = \{< s, 0.1, 0.2, 0.5 >, < t, 0.4, 0.3, 0.4 >; s, t \in \mathcal{X}\}$,
$R = \{< s, 0.2, 0.2, 0.5 >, < t, 0.4, 0.3, 0.4 >; s, t \in \mathcal{X}\}$,
$S = \{< s, 0.1, 0.1, 0.8 >, < t, 0.3, 0.1, 0.4 >; s, t \in \mathcal{X}\}$.
Then $\tau = \{0_N, P, Q, R, S, 1_N\}$ is a neutrosophic topological space $\mathcal{X}$.
Let $E = \{< s, 0.9, 0.1, 0.6 >, < t, 0.3, 0.2, 0.6 >; s, t \in \mathcal{X}\}$ and
$F = \{< s, 0.6, 0.7, 0.3 >, < t, 0.5, 0.7, 0.4 >; s, t \in \mathcal{X}\}$ be the neutrosophic subsets in $\mathcal{X}$. Then $Ncl[Nint[Npcl[E]]] = 1_N$ and $Ncl[Nint[Npcl[F]]] = 1_N$. This implies $E \subseteq Ncl[Nint[Npcl[E]]]$ and $F \subseteq Ncl[Nint[Npcl[F]]]$. Here $E \cap F = \{< s, 0.6, 0.1, 0.6 >, <$

$t, 0.3, 0.2, 0.6 >; s, t \in \mathcal{X}\}$.Therefore E and F are neutrosophic semi j-open sets and $E \cap F$ is not neutrosophic semi j-open set in $\mathcal{X}$.

Theorem 3.6. *In a neutrosophic topological space $\mathcal{X}$, let P be a neutrosophic semi j-open set and $P \subseteq Q \subseteq Ncl[P]$. Then Q is also a neutrosophic semi j-open set in $\mathcal{X}$.*

Proof. Since P is neutrosophic semi j-open in $\mathcal{X}$. Then $P \subseteq Ncl[Nint[Npcl[P]]]$. $Ncl[P] \subseteq Ncl[Ncl[Nint[[Npcl[P]]]]$. Using proposition 2.9, $Ncl[P] \subseteq Ncl[Nint[Npcl[P]]]$. By hypothesis $P \subseteq Q \subseteq Ncl[P]$, then $Q \subseteq Ncl[Nint[Npcl[P]]]$. We have $P \subseteq Q$, therefore $Ncl[Nint[Npcl[P]]] \subseteq Ncl[Nint[Npcl[Q]]]$, which implies $Q \subseteq Ncl[Nint[Npcl[Q]]]$. Hence Q is a neutrosophic semi j-open set in $\mathcal{X}$. □

Theorem 3.7. *In a neutrosophic topological space $\mathcal{X}$, every neutrosophic j-open set is neutrosophic semi j-open.*

Proof. Let P be a neutrosophic j-open set in $\mathcal{X}$. Then $P \subseteq Nint[Npcl[P]]$, $Ncl[P] \subseteq Ncl[Nint[Npcl[P]]]$. We know that $P \subseteq Ncl[P]$. Therefore $P \subseteq Ncl[Nint[Npcl[P]]]$. Hence P is a neutrosophic semi j-open set in $\mathcal{X}$. □

Remark 3.8. Converse of the above theorem need not be true as shown in the following example.

Example 3.9. Let $\mathcal{X} = \{s, t\}$ and the neutrosophic subsets P and Q in $\mathcal{X}$ as follows,
$P = \{< s, 0.2, 0.2, 0.5 >, < t, 0.4, 0.3, 0.4 >; s, t \in \mathcal{X}\}$,
$Q = \{< s, 0.1, 0.1, 0.8 >, < t, 0.3, 0.1, 0.4 >; s, t \in \mathcal{X}\}$.
Then $\tau = \{0_N, P, Q, 1_N\}$ is a neutrosophic topological space on $\mathcal{X}$.
Let $R = \{< s, 0.2, 0.1, 0.6 >, < t, 0.4, 0.4, 0.5 >; s, t \in \mathcal{X}\}$. $Nint[Npcl[R]] = P$, this implies $R \nsubseteq P$. and $Ncl[Nint[Npcl[R]]] = P^C$. Therefore $R \subseteq P^C$. Hence R is neutrosophic semi j-open but not neutrosophic j-open.

Theorem 3.10. *Every neutrosophic open sets in $\mathcal{X}$ is neutrosophic semi j-open.*

Proof. Let P be a neutrosophic open sets in $\mathcal{X}$. Then $P = Nint[P]$. We know that $Nint[P] \subseteq P \subseteq Ncl[P]$ and $Npcl[P] \subseteq Ncl[P]$. This implies $P \subseteq Npcl[P] \subseteq Ncl[P]$. Using proposition 2.9,
$\implies Nint[P] \subseteq Nint[Npcl[P]] \subseteq Nint[Ncl[P]]$
$\implies Ncl[Nint[P]] \subseteq Ncl[Nint[Npcl[P]]] \subseteq Ncl[Nint[Npcl[P]]]$
$\implies Ncl[P] \subseteq Ncl[Nint[Npcl[P]]]$
$\implies P \subseteq Ncl[Nint[Npcl[P]]]$.
Hence P is neutrospohic semi j-open. □

Remark 3.11. Converse of the above theorem need not be true as shown in the following example.

Example 3.12. Consider $\mathcal{X} = \{s\}$ and the neutrosophic subsets P and Q as follows
$P = \{< s, 0.4, 0.5, 0.3 >; s \in \mathcal{X}\}$,
$Q = \{< s, 0.1, 0.5, 0.5 >; s \in \mathcal{X}\}$.
Then $\tau = \{0_N, P, Q, 1_N\}$ is a neutrosophic topological space $\mathcal{X}$.
Here $R = \{< s, 0.3, 0.6, 0.5 >; s \in \mathcal{X}\}$ is neutrosophic semi j-open but not neutrosophic open.

4. **Neutrosophic semi j-closed sets**

Definition 4.1. A neutrosophic subset S of a neutrosophic topological space $\mathcal{X}$ is said to be neutrosophic semi j-closed set if and only if $Nint[Ncl[Npint[S]]] \subseteq S$.

Example 4.2. Let $\mathcal{X} = \{s_1, s_2, s_3\}$ and the neutrosophic subsets Q_1, Q_2 and Q_3 as follows
$Q_1 = \{< s_1, 0.6, 0.5, 0.6 >, < s_2, 0.7, 0.4, 0.4 >, < s_3, 0.6, 0.4, 0.4 >; s_1, s_2, s_3 \in \mathcal{X}\}$,
$Q_2 = \{< s_1, 0.7, 0.6, 0.3 >, < s_2, 0.8, 0.6, 0.4 >, < s_3, 0.6, 0.6, 0.4 >; s_1, s_2, s_3 \in \mathcal{X}\}$,
$Q_3 = \{< s_1, 0.6, 0.5, 0.4 >, < s_2, 0.7, 0.5, 0.4 >, < s_3, 0.6, 0.5, 0.4 >; s_1, s_2, s_3 \in \mathcal{X}\}$.
Then $\tau = \{0_N, Q_1, Q_2, Q_3, 1_N\}$ is a neutrosophic topological space $\mathcal{X}$. Put $F = \{< s_1, 0.5, 0.4, 0.5 >, < s_2, 0.6, 0.5, 0.3 >, < s_3, 0.4, 0.4, 0.3 >; s_1, s_2, s_3 \in \mathcal{X}\}$. Then $Nint[Ncl[Npint[F]]] \subseteq F$. Therefore F is a neutrosophic semi j-closed set in $\mathcal{X}$.

Theorem 4.3. *Take S be a neutrosophic subset of $\mathcal{X}$, then S is neutrosophic semi j-closed if and only if $C(S)$ is neutrosophic semi j-open.*

Proof. Assume S is neutrosophic semi j-closed set in $\mathcal{X}$. Then $Nint[Ncl[Npint[S]]] \subseteq S$, taking compliments on both sides, we obtain $C[S] \subseteq C[Nint[Ncl[Npint[S]]]] = Ncl[Nint[Npcl[C[S]]]]$ using proposition 2.6. Hence $C[S]$ is neutrosophic semi j-open set in $\mathcal{X}$. Conversely assume $C[S]$ is a neutrosophic semi j-open set in $\mathcal{X}$. Then $C[S] \subseteq Ncl[Nint[Npcl[C[S]]]]$. We obtain $C[Ncl[Nint[Npcl[C[S]]]]] \subseteq C[C[S]]$ by taking compliments on both sides. This implies $Nint[Ncl[Npint[S]]] \subseteq S$. Hence S is a neutrosophic semi j-closed set in $\mathcal{X}$. □

Theorem 4.4. *Let $\{S_\alpha : \alpha \in \Delta\}$ be a family of neutrosophic semi j-closed set in $\mathcal{X}$. Then arbitrary intersection of neutrosophic semi j-closed sets is also neutrosophic semi j-closed.*

Proof. Let $\{S_\alpha : \alpha \in \Delta\}$ be a family of neutrosophic semi j-closed sets in $\mathcal{X}$ and $P_\alpha = \{S_\alpha\}^c$. Then $\{S_\alpha : \alpha \in \Delta\}$ is a family of neutrosophic semi j-open sets in $\mathcal{X}$. Using theorem 3.3 $\bigcup_{\alpha \in \Delta} P_\alpha$ is neutrosophic semi j-open. Then $\{\bigcup_{\alpha \in \Delta} P_\alpha\}^C$ is neutrosophic semi j-closed which implies $\bigcap_{\alpha \in \Delta} {P_\alpha}^c$ is neutrosophic semi j-closed. Hence $\bigcap_{\alpha \in \Delta} S_\alpha$ is neutrosophic semi j-closed. □

Theorem 4.5. *In a neutrosophic topological space $\mathcal{X}$, every neutrosophic j-closed set is also neutrosophic semi j-closed.*

Proof. Let S be a neutrosophic j-closed set in $\mathcal{X}$. Then $Ncl[Npint[S]] \subseteq S$. $Nint[Ncl[Npint[S]]] \subseteq Nint[S]$. We know that $Nint[S] \subseteq S$, therefore $Nint[Ncl[Npint[S]]] \subseteq S$. Hence S is neutrosophic semi j-closed. □

Remark 4.6. Converse of the above theorem need not be true, as verified by the following example.

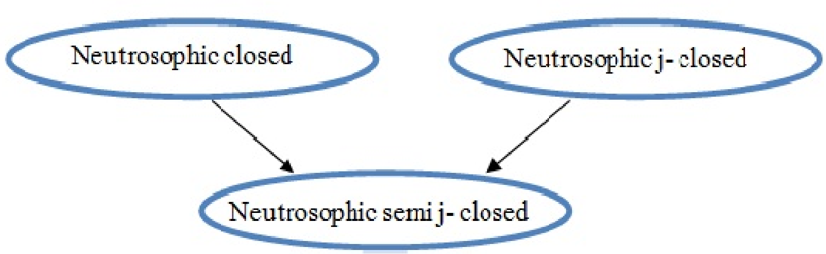
Example 4.7. Let $\mathcal{X} = \{t_1, t_2, t_3\}$ and the neutrosophic subsets Q_1, Q_2 as follows,
$Q_1 = \{< t_1, 0.2, 0.5, 0.4 >, < t_2, 0.2, 0.4, 0.5 >, < t_3, 0.1, 0.0, 0.5 >; t_1, t_2, t_3 \in \mathcal{X}\}$,
$Q_2 = \{< t_1, 0.3, 0.4, 0.5 >, < t_2, 0.4, 0.3, 0.2 >, < t_3, 0.2, 0.3, 0.4 >; t_1, t_2, t_3 \in \mathcal{X}\}$.
Put $\tau = \{0_N, Q_1, Q_2, Q_1 \cup Q_2, 1_N\}$. Let $G = \{< t_1, 0.4, 0.5, 0.4 >, < t_2, 0.3, 0.4, 0.2 >, < t_3, 0.3, 0.4, 0.5 >; t_1, t_2, t_3 \in \mathcal{X}\}$. Then G is a neutrosophic semi j-closed but not neutrosophic j-closed. Since $Nint[Ncl[Npint[G]]] = Q_1 \subseteq G$, but $Ncl[Npint[G]] = [Q_1 \cup Q_2]^c \nsubseteq G$.

Theorem 4.8. *In a neutrosophic topological space $\mathcal{X}$, every neutrosophic closed set is neutrosophic semi j-closed.*

Proof. Let S be neutrosophic closed set in $\mathcal{X}$. Then $S = Ncl[S]$. We know that $Nint[S] \subseteq Npint[S]$. $Ncl[Nint[S]] \subseteq Ncl[Npint[S]] \subseteq Ncl[S]$. It follows that $Nint[Ncl[Nint[S]]] \subseteq Nint[Ncl[Npint[S]]]$. Hence S is a semi j-closed in $\mathcal{X}$. □

Remark 4.9. The converse of the above theorem may not be true, as shown by the following example. In example 4.7, $Ncl[G] \neq G$. This implies G is neutrosophic semi j-closed set but not neutrosophic closed set.

From the above results, we have the following indications: But the converse of the above



indications need not be true as shown by 4.7 and 4.9.

5. Neutrosophic hyperconnected space

Definition 5.1. A neutrosophic topological space $\mathcal{X}$, is said to be neutrosophic hyperconnected if for every non empty neutrosophic open subsets of $\mathcal{X}$ is neutrosophic dense in $\mathcal{X}$.

Example 5.2. Consider $\mathcal{X} = \{s_1, s_2\}$ with $\tau = \{0_N, 1_N, P_1, P_2, P_3, P_4\}$ where
$P_1 = \{< s_1, 0.2, 0.4, 0.3 >, < s_2, 0.5, 0.1, 0.4 >, s_1, s_2 \in \mathcal{X}\}$,
$P_2 = \{< s_1, 0.1, 0.5, 0.6 >, < s_2, 0.4, 0.2, 0 >, s_1, s_2 \in \mathcal{X}\}$,
$P_3 = \{< s_1, 0.2, 0.5, 0.3 >, < s_2, 0.5, 0.2, 0 >, s_1, s_2 \in \mathcal{X}\}$,
$P_4 = \{< s_1, 0.1, 0.4, 0.6 >, < s_2, 0.4, 0.1, 0.4 >, s_1, s_2 \in \mathcal{X}\}$.
Here every non empty neutrosophic open sets $P_1, P_2, P_3, P_4, 1_N$ are neutrosophic dense in $\mathcal{X}$. ie.,
$Ncl[P_1] = 1_N$,
$Ncl[P_2] = 1_N$,
$Ncl(P_3) = 1_N$,
$Ncl(P_4) = 1_N$,
$Ncl(1_N) = 1_N$.
Therefore $\mathcal{X}$ is neutrosophic hyperconnected space.

Definition 5.3. A neutrosophic topological space $\mathcal{X}$ is called as neutrosophic extremely disconnected if the neutrosophic closure of each neutrosophic open set is neutrosophic open in $\mathcal{X}$.

Theorem 5.4. *In a neutrosophic topological space $\mathcal{X}$, every neutrosophic hyperconnected space is neutrosophic extremely disconnected.*

Proof. Let us take $\mathcal{X}$ be neutrosophic hyperconnected. Then for any neutrosophic open set P, $Ncl[P] = 1_N$. This implies that $Ncl[P]$ is neutrosophic open. Therefore $\mathcal{X}$ is neutrosophic extremely disconnected. □

Remark 5.5. The following example shows that the converse of the above theorem need not be true.

Example 5.6. Let $\mathcal{X} = \{s\}$ with $\tau = \{0_N, P_1, P_2, P_3, P_4, 1_N\}$ where
$P_1 = \{< s, 0.5, 0.3, 0.2 >; s \in \mathcal{X}\}$,
$P_2 = \{< s, 0.2, 0.3, 0.5 >; s \in \mathcal{X}\}$,
$P_3 = \{< s, 0.3, 0.3, 0.5 >; s \in \mathcal{X}\}$,
$P_4 = \{< s, 0.5, 0.3, 0.5 >; s \in \mathcal{X}\}$.
Here $Ncl[P_1] = \{< s, 0.5, 0.3, 0.2 >; s \in \mathcal{X}\}$,
$Ncl[P_2] = \{< s, 0.2, 0.3, 0.5 >; s \in \mathcal{X}\}$,

$Ncl(P_3) = \{< s, 0.5, 0.3, 0.5 >; s \in \mathcal{X}\}$,

$Ncl(P_4) = \{< s, 0.5, 0.3, 0.5 >; s \in \mathcal{X}\}$.

This example shows that $[\mathcal{X}, \tau]$ is neutrosophic extremely disconnected. Since $Ncl[P_1]$, $Ncl[P_2]$, $Ncl(P_3)$ and $Ncl(P_4)$ are neutrosophic open but not neutrosophic dense . Therefore, $\mathcal{X}$ is not neutrosophic hyperconnected.

Theorem 5.7. *In a neutrosophic topological space $\mathcal{X}$, the following properties are equivalent.*
(a) $\mathcal{X}$ is neutrosophic hyperconnected.
(b) In $\mathcal{X}$, the only neutrosophic regular open sets are 0_N and 1_N.

Proof. (a) $\implies$ (b)

Let $\mathcal{X}$ be a neutrosophic hyperconnected space. If P is a non-empty neutrosophic regular open set, then by the definition $P = Nint[Ncl[P]]$. This implies that $[Nint[Ncl[P]]]^c = [1_N - Nint[Ncl[P]]] = Ncl[1_N - Ncl[P]] = Ncl(P^c) = P^c \neq 1_N$. Since $P \neq 0_N$. This is a contradiction to the assumption. Hence the only neutrosophic regular open sets in $\mathcal{X}$ are 0_N and 1_N.

(b) $\implies$ (a)

Assume that 0_N and 1_N are the only neutrosophic regular open subsets in $\mathcal{X}$. Suppose that $\mathcal{X}$ is not neutrosophic hyperconnected. Then there exist a non empty neutrosophic open subset P of $\mathcal{X}$ such that $Ncl[P] \neq 1_N$. This implies $Ncl[Nint[P]] \neq 1_N$. Therefore, we have $Ncl[Nint[P]] = 0_N$. This gives $Ncl[P] = 0_N$. Since $P \neq 0_N$. It contradicts our assumption that $\mathcal{X}$ is not neutrosophic hyperconnected . Hence $\mathcal{X}$ is neutrosophic hyperconnected space.

□

Theorem 5.8. *A neutrosophic topological space $\mathcal{X}$ is neutrosophic hyperconnected if and only if for every neutrosophic subset of $\mathcal{X}$ is either neutrosophic dense or neutrosophic nowhere dense.*

Proof. Suppose $\mathcal{X}$ be a neutrosophic hyperconnected space and let P be any neutrosophic subset of $\mathcal{X}$ such that $P \subseteq 1_N$. Assume P is not neutrosophic nowhere dense. Then $Ncl[1_N - Ncl[P]] = 1_N - [Nint[Ncl[P]]] \neq 1_N$. Since $Nint[Ncl[P]] \neq 0_N$. This implies that $Ncl[Nint[Ncl[P]]] = 1_N$. Since $Ncl[Nint[Ncl[P]]] = 1_N \subseteq Ncl[P]$. Therefore, $Ncl[P] = 1_N$. Hence P is neutrosophic dense set.

For the converse part, let P_1 be any non empty neutrosophic open set in $\mathcal{X}$, then $P_1 \subset Nint[Ncl[P_1]]$. This implies that P_1 is not neutrosophic nowhere dense set. By the hypothesis, P_1 is neutrosophic dense set. □

Proposition 5.9. *If $\mathcal{X}$ be a neutrosophic hyperconnected space, then the intersection of any two neutrosophic semi open sets is also neutrosophic semi open.*

Proof. Let P_1 and P_2 be the two non empty neutrosophic semi open sets in a neutrosophic hyperconnected space $\mathcal{X}$. Then, we have $P_1 \subseteq Ncl[Nint[P_1]]$ and $P_2 \subseteq Ncl[Nint[P_2]]$. It follows that, $Ncl[P_1] = Ncl[Nint[P_1]] = 1_N$ and $Ncl[P_2] = Ncl[Nint[P_2]]] = 1_N$, where P_1 and P_2 are two non empty neutrosophic semi open sets in a neutrosophic hyperconnected space. Also we have $P_1 \wedge P_2 \neq 0_N$. Therefore, $Ncl[Nint[P_1 \wedge P_2]] = Ncl[Nint[P_1]] \wedge Ncl[Nint[P_2]] = 1_N$. This implies $P_1 \wedge P_2 \subseteq Ncl[Nint[P_1]] \wedge Ncl[Nint[P_2]] = Ncl[Nint[P_1 \wedge P_2]]$. Hence $P_1 \wedge P_2$ is neutrosophic semi open set. □

6. Neutrosophic semi j-hyperconnected spaces

Definition 6.1. A neutrosophic subset P of $\mathcal{X}$ is said to be neutrosophic semi j-interior of P if the union of all neutrosophic semi j-open sets of $\mathcal{X}$ contained in P. It is denoted by $Nint_{sj}[P]$.

A neutrosophic subset Q of $\mathcal{X}$ is said to be neutrosophic semi j-closure of Q if the intersection of all neutrosophic semi j-closed sets of $\mathcal{X}$ containing Q. It is denoted by $Ncl_{sj}[Q]$.

Example 6.2. Consider $\mathcal{X} = \{s_1, s_2, s_3\}$ and the neutrosophic subsets S_1, S_2, S_3 in $\mathcal{X}$ as follows,

$S_1 = \{< s_1, 0.3, 0.4, 0.3 >, < s_2, 0.6, 0.2, 0.4 >, < s_3, 0.5, 0.2, 0.3 >; s_1, s_2, s_3 \in \mathcal{X}\}$,

$S_2 = \{< s_1, 0.2, 0.6, 0.5 >, < s_2, 0.4, 0.2, 0.3 >, < s_3, 0.2, 0.3, 0.1 >; s_1, s_2, s_3 \in \mathcal{X}\}$,

$S_3 = \{< s_1, 0.3, 0.6, 0.3 >, < s_2, 0.6, 0.2, 0.3 >, < s_3, 0.5, 0.3, 0.1 >; s_1, s_2, s_3 \in \mathcal{X}\}$.

Take $\tau = \{0_N, S_1, S_2, S_3, 1_N\}$. For this 0_N, 1_N, S_1, S_2, $S_1 \cup S_2$, $S_1 \cup S_3$, $S_2 \cup S_3$ are the neutrosophic semi j-open sets and 0_N, 1_N, S_1^c, S_2^c, $(S_1 \cup S_2)^c$, $(S_1 \cup S_3)^c$, $(S_2 \cup S_3)^c$ are the neutrosophic semi j-closed sets. Put $T = \{< s_1, 0.5, 0.6, 0.2 >, < s_2, 0.7, 0.3, 0.3 >, < s_3, 0.6, 0.4, 0.2; s_1, s_2, s_3\mathcal{X} >\}$ is a neutrosophic subset in $\mathcal{X}$. Then we have $Nint_{sj}(T) = S_1$ and $Ncl_{sj}(T) = 1_N$.

Definition 6.3. A neutrosophic topological space $\mathcal{X}$ is said to be neutrosophic semi j-hyperconnected space if for each nonempty neutrosophic semi j-open subset A of $\mathcal{X}$ is neutrosophic semi j-dense in $\mathcal{X}$. ie., $Ncl_{sj}(A) = 1_N$ for every A in $\mathcal{X}$.

Example 6.4. Let $\mathcal{X} = \{s_1, s_2, s_3\}$ and the neutrosophic subsets P_1, P_2, P_3 in $\mathcal{X}$ as follows,

$P_1 = \{< s_1, 0.1, 0.3, 0.2 >, < s_2, 0.4, 0.1, 0.3 >, < s_3, 0.3, 0.1, 0.2 >; s_1, s_2, s_3 \in \mathcal{X}\}$,

$P_2 = \{< s_1, 0.1, 0.4, 0.5 >, < s_2, 0.3, 0.1, 0.0 >, < s_3, 0.2, 0.0, 0.1 >; s_1, s_2, s_3 \in \mathcal{X}\}$,

$P_3 = \{< s_1, 0.2, 0.4, 0.2 >, < s_2, 0.4, 0.1, 0.0 >, < s_3, 0.3, 0.1, 0.0 >; s_1, s_2, s_3 \in \mathcal{X}\}$.

Put $\tau = \{0_N, P_1, 1_N\}$. Then the collection of neutrosophic semi j-open sets are 0_N, 1_N, $P_1 \cup P_2$ and $P_1 \cup P_3$ i.e.,$P_1 \subseteq Ncl[Nint[Npcl[P_1]]]$, $P_1 \cup P_2 \subseteq Ncl[Nint[Npcl[P_1 \cup P_2]]]$ and

$P_1 \cup P_3 \subseteq Ncl[Nint[Npcl[P_1 \cup P_2]]]$. Here every non empty neutrosophic semi j-open sets are neutrosophic semi j-dense in $\mathcal{X}$. i.e., $Ncl_{sj}[P_1] = 1_N$, $Ncl_{sj}[P_1 \cup P_2] = 1_N$, $Ncl_{sj}[P_1 \cup P_3] = 1_N$ and $Ncl_{sj}[1_N]$. Therefore a neutrosophic topological space $\tau = \{0_N, P_1, 1_N\}$ is neutrosophic semi j-hyperconnected space.

Theorem 6.5. *In a neutrosophic topological space, every neutrosophic hyperconnected space is neutrosophic semi j-hyperconnected.*

Proof. Let $\mathcal{X}$ be a neutrosophic hyperconnected space and P be a neutrosophic open subset of $\mathcal{X}$. Then $Ncl[P] = 1_N$. This implies that $Nint[Ncl[P]] = 1_N$. Therefore P is neutrosophic preopen. P^C is neutrosophic preclosed. Since every neutrosophic open set is neutrosophic preopen and its complement is neutrosophic preclosed. It follows that $Npcl[P] = Ncl[P] = 1_N$ which implies $Ncl[Nint[Npcl[P]]] = 1_N$. Therefore P is neutrosophic semi j-open $\implies$ $Ncl_{sj}[P] = 1_N$ for any neutrosophic open set in $\mathcal{X}$. Hence $\mathcal{X}$ is neutrosophic semi j-hyperconnected space. □

Definition 6.6. Let $\mathcal{X}$ be a neutrosophic topological space and P be a neutrosophic semi j-open set of $\mathcal{X}$. Then

(a) P is said to be neutrosophic semi j-regular open set if and only if $P = Nint_{sj}[Ncl_{sj}[P]]$.

(b) P is said to be neutrosophic semi j-regular closed set if and only if $Ncl_{sj}[Nint_{sj}[P]] = P$.

Theorem 6.7. *Let $\mathcal{X}$ be a neutrosophic topological space, then each of the following statements are equivalent.*

(a) *$\mathcal{X}$ is neutrosophic semi j-hyperconnected.*

(b) *$\mathcal{X}$ has no two proper neutrosophic semi j-regular open or proper semi j-regular closed subset.*

(c) *Let P and Q be the proper disjoint neutrospohic semi j-open subsets in $\mathcal{X}$, then $\mathcal{X}$ does not have P and Q such that $Ncl_{sj}[P] \cup Q = P \cup Ncl_{sj}[Q] = 1_N$.*

(d) *$\mathcal{X}$ has no proper semi j-closed subset S and T such that $\mathcal{X} = S \cup T$ and $Nint_{sj} \cap T = S \cap Nint_{sj}(T) = 0_N$.*

Proof. (a) $\implies$ (b) Let $0_N \neq P$ be neutrosophic semi j-regular open subset in $\mathcal{X}$. Then $P = Nint_{sj}[Ncl_{sj}[P]]$. Since $\mathcal{X}$ is a neutrosophic semi j-hyperconnected space. then $Ncl_{sj}[P] = 1_N$. This implies $P = 1_N$. Clearly P is not a proper neutrosophic semi j-regular open subset of $\mathcal{X}$. Similarly $\mathcal{X}$ cannot have a proper neutrosophic semi j-regular closed subset.

(b) $\implies$ (c) Suppose P and Q are the neutrosophic subsets in $\mathcal{X}$ and $P \cap Q = 0_N$ such that

$Ncl_{sj}[P] \cup Q = P \cup Ncl_{sj}[Q] = 1_N$. This implies $0_N \neq Ncl_{sj}[P]$ is the neutrosophic semi j-regular closed set in $\mathcal{X}$. Since $P \cap Q = 0_N$ and $Ncl_{sj}[P] \cap Q = 0_N \implies Ncl_{sj}[P] \neq 1_N$ which implies $\mathcal{X}$ has a proper neutrosophic semi j-regular closed subset P. This contradicts (b).

(c) $\implies$ (d) Suppose, there exist two proper neutrosophic semi j-closed subset $0_N \neq S$ and $0_N \neq T$ in $\mathcal{X}$ such that $\mathcal{X} = S \cup T$, $Nint_{sj}(S) \cap T = S \cap Nint_{sj}(T) = 0_N$. Then $P = 1_N - S$, $Q = 1_N - T$ are the two non-empty neutrosophic semi j-open sets. Then $Ncl_{sj}[P] \cup Q = Ncl_{sj}(1_N - S) \cup Q = [1_N - Nint_{sj}(S)] \cup Q = 1_N. \implies Ncl_{sj}[P] \cup Q = P \cup Ncl_{sj}[Q] = 1_N$ which contradicts (c).

(d) $\implies$ (a) Suppose there exist a proper neutrosophic semi j-open set $0_N \neq P$ of $\mathcal{X}$ such that $Ncl_{sj}[P] \neq 1_N$. Then $Nint_{sj}[Ncl_{sj}[P]] \neq 1_N$. Take $S = Ncl_{sj}[P]$ and $T = 1_N - Nint_{sj}[Ncl_{sj}[P]]$. This implies $S \cup T = Ncl_{sj}[P] \cup [1_N - Nint_{sj}[Ncl_{sj}[P]]] = Ncl_{sj}[P] \cup Ncl_{sj}[1_N - Ncl_{sj}[P]] \implies Ncl_{sj}[P] \cup Ncl_{sj}[C(S)] \implies S \cup C(S) = 1_N$. Since S is neutrosophic semi j-closed set. Then $Nint_{sj}[Ncl_{sj}[P]] \cap [1_N - Nint_{sj}[Ncl_{sj}[P]]] = 0_N$. $\implies Ncl_{sj}[P] \cap Nint_{sj}[1_N - Nint_{sj}[Ncl_{sj}[P]]] = S \cap Nint_{sj} Ncl_{sj}[1_N - Ncl[P]] = S \cap Nint_{sj} Ncl_{sj}\ [C(S)] = S \cap C(S) = 0_N$. Since $C(S)$ is neutrosophic semi j-open. Thus $\mathcal{X}$ has two proper neutrosophic semi j-closed sets S and T such that $\mathcal{X} = S \cup T$ and $Nint_{sj} S \cap T = S \cap Nint_{sj} C[T] = 0_N$. This is a contradiction to (d). □

Theorem 6.8. *In a neutrosophic semi j-hyperconnected space $\mathcal{X}$. Let $0_N \neq P$ and $0_N \neq Q$ be the two neutrosophic semi j-open subsets in $\mathcal{X}$, then $P \cap Q$ is also non-empty.*

Proof. Suppose $P \cap Q = 0_N$, for any $0_N \neq P$ and $0_N \neq Q$ neutrosophic semi j-open sets in $\mathcal{X}$. Then $Ncl_{sj}[P] \cap Q = 0_N$. This implies P is not neutrosophic semi j-dense. We have P is neutrosophic semi j-open then $P \subseteq Ncl[Nint[Npcl[P]]]$ and P is not neutrosophic semi j-dense which is a contradiction to our assumption that $P \cap Q = 0_N$. Hence $P \cap Q \neq 0_N$. □

Theorem 6.9. *In a neutrosophic semi j-hyperconnected space, intersection of any two neutrosophic semi j-open sets are neutrosophic semi j-open.*

Proof. Let $0_N \neq P$, $0_N \neq Q$ be the two neutrosophic semi j-open sets in a neutrosophic semi j-hyperconnected space $\mathcal{X}$. Then $P \subseteq Ncl[Nint[Npcl[P]]]$ and $Q \subseteq Ncl[Nint[Npcl[P]]]$. We have $Ncl_{sj}[P] = 1_N$ and $Ncl_{sj} = 1_N$. This implies $Ncl[Nint[Npcl[P]]] = Ncl[Nint[Npcl[Q]]] = 1_N$, also we have $P \cap Q \neq 0_N$ using proposition 2.9. It follows that $P \cap Q \subseteq Ncl[Nint[Npcl[P]]] \cap Ncl[Nint[Npcl[Q]]] = Ncl[Nint[Npcl[P \cap Q]]] = 1_N$. Therefore $P \cap Q \subseteq Ncl[Nint[Npcl[P \cap Q]]] = 1_N$. Hence $P \cap Q$ is also neutrosophic semi j-open. □

7. Conclusion

The characteristics of neutrosophic semi j-open sets, neutrosophic semi j-closed sets, neutrosophic hyperconnectedness and neutrosophic semi j-hyperconnectedness are discussed in this paper. Nowadays neutrosophic sets have began to play a vital role by helping in the analysis of real life situations. In future, neutrosophic hyperconnected spaces will assist in determining solutions in each situations where indeterminancy occurs as the main crisis.

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