Enrico's Chart of Phase Noise and Two-Sample Variances

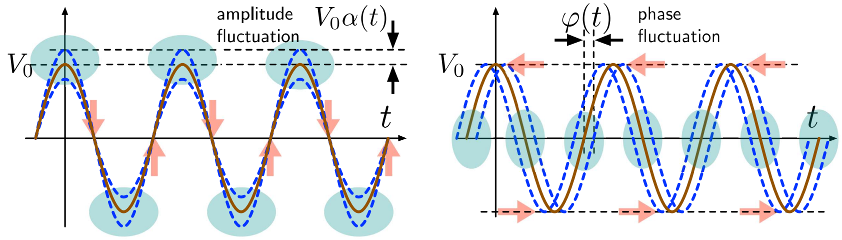
Enrico Rubiola – http://rubiola.org
European Frequency and Time Seminar – http://efts.eu
Oscillator Instability Measurement Platform http://oscillator-imp.com

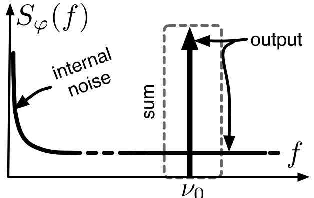
Download the Companion paper

Download the Enrico's Chart

Thanks to FIRST-TF https://first-tf.com

Clock signal $v(t) = V_0[1+\alpha(t)]\cos[2\pi\nu_0 t + \varphi(t)]$

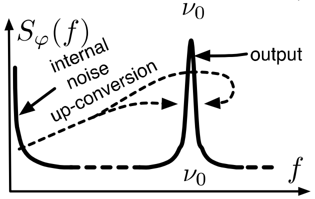




Additive Noise

RF noise added to the carrier

Statistically independent AM & PM



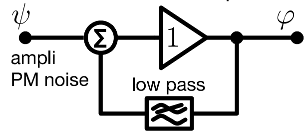
Parametric Noise

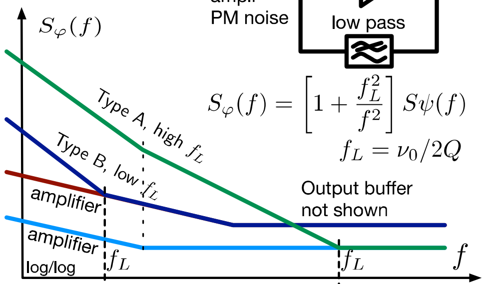
Near-dc noise modulates the carrier

AM & PM related and narrowband

Leeson effect

Oscillator's PM-noise equivalent





$$S_\varphi(f) = \left[1+\frac{f_L^2}{f^2}\right] S\psi(f)$$

$$f_L = \nu_0/2Q$$

Boldface notation

total = nominal + fluctuation		
$\boldsymbol{\varphi}(t) = 2\pi\nu_0 t + \varphi(t)$	phase	
$\boldsymbol{\nu}(t) = \nu_0 + (\Delta\nu)(t)$	frequency	
$\mathbf{x}(t) = t + \mathrm{x}(t)$	time	
$\mathbf{y}(t) = 1 + \mathrm{y}(t)$	fractional frequency	

Two-sample (Allan-like) variances

Definition

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\bar{y}_2 - \bar{y}_1\right]^2\right\}$$

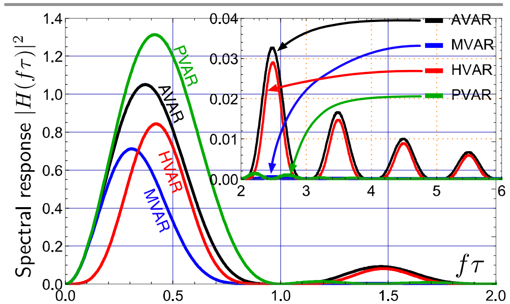
$y(t) \to \bar{y}$ averaged over τ
$\bar{y}_2$ and $\bar{y}_1$ are contiguous

Bare mean $\bar{y} \to$ Allan variance AVAR
Weighted averages $\to$ MVAR, PVAR, etc.

Evaluation

$$\sigma_y^2(\tau) = \frac{1}{2(M-1)}\sum_{k=1}^{M-1}\left[\bar{y}_{k+1} - \bar{y}_k\right]^2$$

M contiguous samples of $\bar{y}$



Phase noise spectrum

Definition

$S_\varphi(f)$ [rad^2/Hz] is the one-sided PSD ($f>0$) of $\varphi(t)$

$$S_\varphi(f) = 2\mathcal{F}\{\mathbb{E}\{\varphi(t)\varphi(t+\tau)\}\}, \quad f>0$$

Evaluation

$$S_\varphi(f) = \tfrac{2}{T}\left\langle \Phi_T(f)\,\Phi_T^*(f)\right\rangle_m$$

avg on m data, $\Phi_T(f)$ = DFT of $\varphi(t)$ truncated on T

Usage

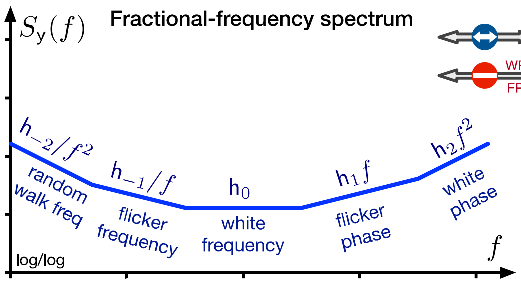
most often, 'phase noise' refers to $\mathscr{L}(f)$

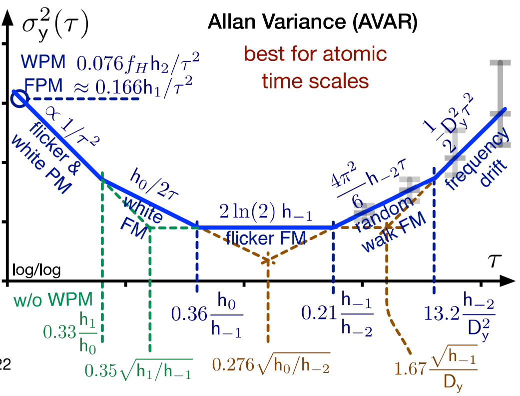
Only $10\log_{10}[\mathscr{L}(f)]$ is used, given in dBc/Hz

Definition: $\mathscr{L}(f) = \frac{1}{2}S_\varphi(f)$ [the unit c/Hz never used]

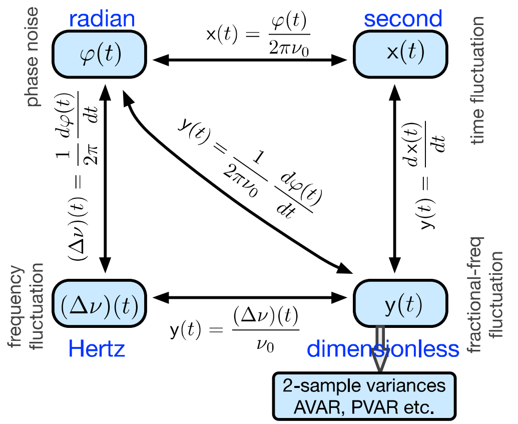
Literally, the unit 'c' is a squared angle, $\sqrt{\text{c}} = \sqrt{2}$ rad $\simeq 81°$

Frequency fluctuation PSD <—> Allan Variance

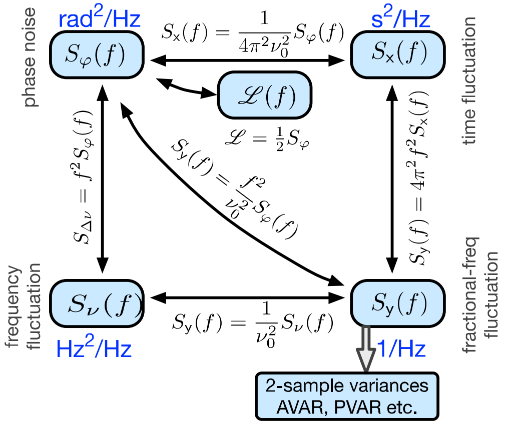




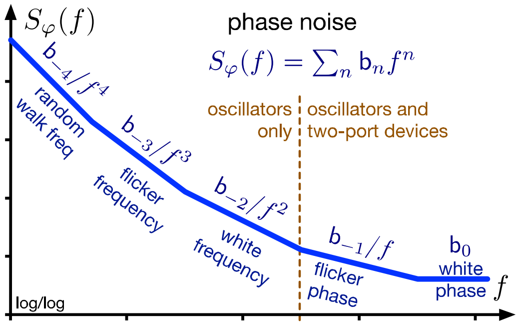
Time Domain

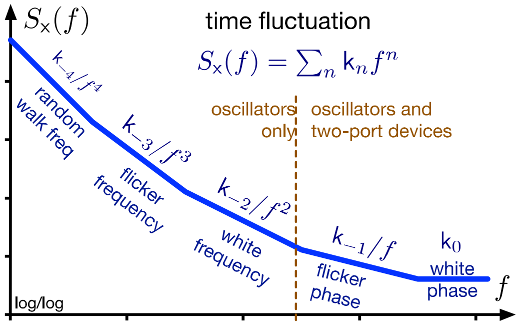


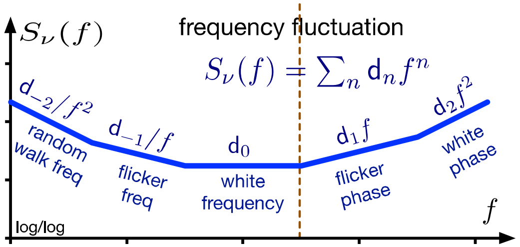
Frequency Domain

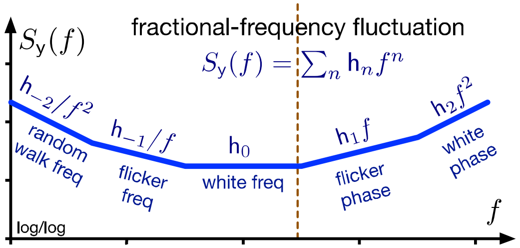


Spectra and Polynomial Law









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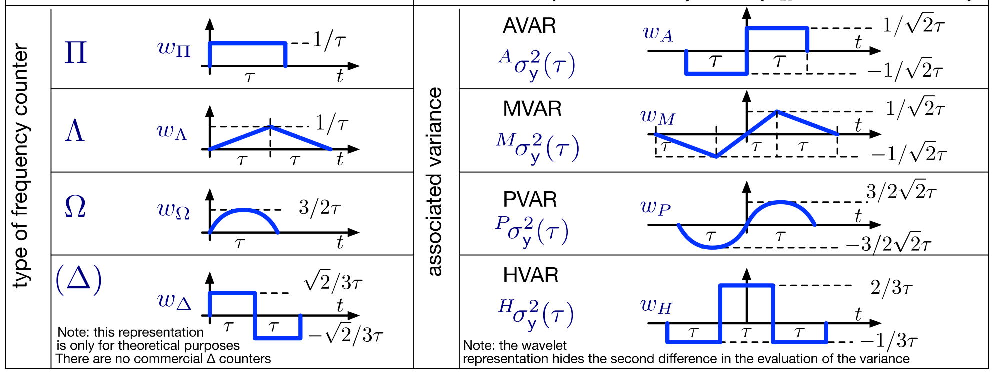
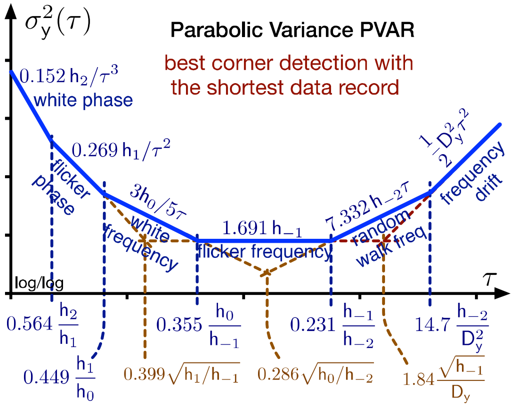
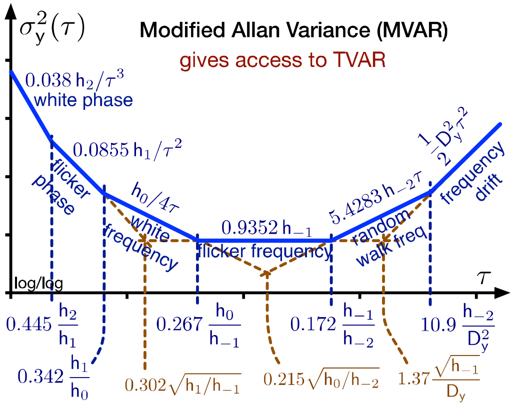
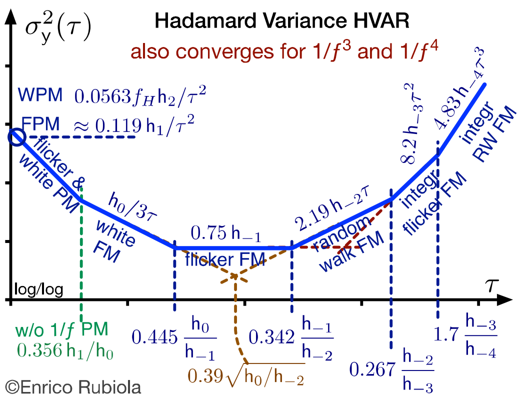
Frequency Counter	Wavelet Variance
$\bar{\mathsf{y}}(\tau) = \int_{\mathbb{R}} \mathsf{y}(t)\, w(t;\tau)\, dt$	$\sigma^2_{\mathsf{y}}(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\bar{\mathsf{y}}_2 - \bar{\mathsf{y}}_1\right]^2\right\} = \mathbb{E}\left\{\int_{\mathbb{R}}\left[\mathsf{y}(t)\, w(t;\tau)\right]^2 dt\right\}$

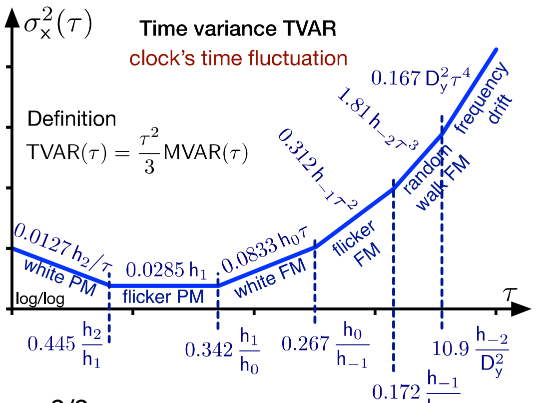


Other Two-Sample Variances









Spectra to Variances Conversion

noise type	$S_y(f)$	AVAR $^A\sigma^2_y(\tau)$	MVAR $^M\sigma^2_y(\tau)$	HVAR $^H\sigma^2_y(\tau)$	PVAR $^P\sigma^2_y(\tau)$	TVAR $^T\sigma^2_x(\tau)$
Blue PM	$h_3 f^3$	$\frac{3f_H^2}{8\pi^2}\frac{h_3}{\tau^2}$ $0.0380\, f_H h_3/\tau^2$	$\frac{10\gamma+\ln 48+10\ln(\pi f_H\tau)}{16\pi^4}\frac{h_3}{\tau^4}$ $\frac{[10\gamma+\ln 48+10\ln\pi]}{16\pi^4}=0.0135$	$\frac{5f_H^2}{18\pi^2}\frac{h_3}{\tau^2}$ $0.0281\, f_H h_3/\tau^2$	$\frac{9[\gamma+\ln(4\pi f_H\tau)]}{4\pi^4}\frac{h_3}{\tau^4}$ $\frac{9[\gamma+\ln(4\pi)]}{4\pi^4}=0.0718$	$\frac{10\gamma+\ln 48+10\ln(\pi f_H\tau)}{48\pi^4}\frac{h_3}{\tau^2}$ $\frac{10\gamma+\ln 48+10\ln\pi}{48\pi^2}=0.00451$
White PM	$h_2 f^2$	$\frac{3f_H}{4\pi^2}\frac{h_2}{\tau^2}$ $0.0760\, f_H h_2/\tau^2$	$\frac{3}{8\pi^2}\frac{h_2}{\tau^3}$ $0.0380\, h_2/\tau^3$	$\frac{5f_H}{9\pi^2}\frac{h_2}{\tau^2}$ $0.0563\, f_H h_2/\tau^2$	$\frac{3}{2\pi^2}\frac{h_2}{\tau^3}$ $0.152\, h_2/\tau^3$	$\frac{1}{8\pi^2}\frac{h_2}{\tau}$ $0.0127\, h_2/\tau$
Flicker PM	$h_1 f$	$\frac{3\gamma-\ln 2+3\ln(2\pi f_H\tau)}{4\pi^2}\frac{h_1}{\tau^2}$ $[3\gamma-\ln 2+3\ln 2\pi]/4\pi^2=0.166$	$\frac{(24\ln 2-9\ln 3)}{8\pi^2}\frac{h_1}{\tau^2}$ $0.0855\, h_1/\tau^2$	$\simeq\frac{5[\gamma+\ln(\sqrt[10]{48}\,\pi f_H\tau)]}{9\pi^2}\frac{h_1}{\tau^2}$ $5[\gamma+\ln(\sqrt[10]{48}\,\pi)]/9\pi^2=0.119$	$\frac{3[\ln(16)-1]}{2\pi^2}\frac{h_1}{\tau^2}$ $0.269\, h_1/\tau^2$	$\frac{(8\ln 2-3\ln 3)}{8\pi^2}h_1$ $0.0285\, h_1$
White FM	h_0	$\frac{1}{2}\frac{h_0}{\tau}$	$\frac{1}{4}\frac{h_0}{\tau}$	$\frac{1}{3}\frac{h_0}{\tau}$	$\frac{3}{5}\frac{h_0}{\tau}$	$\frac{1}{12}h_0\tau$
Flicker FM	$h_{-1}f^{-1}$	$2\ln(2)\, h_{-1}$ $1.39\, h_{-1}$	$\frac{27\ln 3-32\ln 2}{8}h_{-1}$ $0.935\, h_{-1}$	$\frac{8\ln 2-3\ln 3}{3}h_{-1}$ $0.750\, h_{-1}$	$\frac{2[7-\ln(16)]}{5}h_{-1}$ $1.69\, h_{-1}$	$\frac{27\ln 3-32\ln 2}{24}h_{-1}\tau^2$ $0.312\, h_{-1}\tau^2$
Random walk FM	$h_{-2}f^{-2}$	$\frac{2\pi^2}{3}h_{-2}\tau$ $6.58\, h_{-2}\tau$	$\frac{11\pi^2}{20}h_{-2}\tau$ $5.43\, h_{-2}\tau$	$\frac{2\pi^2}{9}h_{-2}\tau$ $2.19\, h_{-2}\tau$	$\frac{26\pi^2}{35}h_{-2}\tau$ $7.33\, h_{-2}\tau$	$\frac{11\pi^2}{60}h_{-2}\tau^3$ $1.81\, h_{-2}\tau^3$
Integrated flicker FM	$h_{-3}f^{-3}$	not convergent	not convergent	$\frac{\pi^2[27\ln 3-32\ln 2]}{9}h_{-3}\tau^2$ $8.205\, h_{-3}\tau^2$	not convergent	not convergent
Integrated RW FM	$h_{-4}f^{-4}$	not convergent	not convergent	$\frac{44\pi^2}{90}h_{-4}\tau^3$ $4.825\, h_{-4}\tau^3$	not convergent	not convergent
linear drift D_y		$\frac{1}{2}D_y^2\tau^2$	$\frac{1}{2}D_y^2\tau^2$	0	$\frac{1}{2}D_y^2\tau^2$	$\frac{1}{6}D_y^2\tau^4$
Spectral response $\lvert H(\theta)\rvert^2$, $\theta=\pi f\tau$		$\frac{2\sin^4\theta}{\theta^2}$	$\frac{2\sin^6\theta}{\theta^4}$	$\frac{16\sin^6\theta}{9\theta^2}$	$\frac{9\left[2\sin^2\theta-\theta\sin 2\theta\right]^2}{2\theta^6}$	$\frac{\tau^2}{3}\frac{2\sin^6\theta}{\theta^4}$

$^T\sigma^2_x(\tau) = \frac{\tau^2}{3}\,{}^M\sigma^2_y(\tau)$

$\gamma = 0.577$ is the Euler Mascheroni constant. Formulae hold for $\tau \gg f_H/2$ where appropriate, f_H = bandwidth (sharp cutoff filter). MVAR, PVAR and TVAR require $\tau \gg \tau_0$, where τ_0 = sampling interval.