

Binomial Coefficients of Combinatorial Geometric Series: System of Natural Numbers

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Abstract: This paper presents the binomial coefficients of combinatorial geometric series as a system of natural numbers. The coefficient for each term in combinatorial geometric series refers to a binomial coefficient. This idea can enable the scientific researchers to solve the real life problems.

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1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series, a new idea was stimulated his mind to create a combinatorial geometric series [1-9]. The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r . In this article, we show that the binomial coefficients in combinatorial geometric are a system of natural numbers.

2. Combinatorial Geometric Series

The combinatorial geometric series [1-9] is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial refers to the binomial coefficient V_n^r .

$$\sum_{i_1=0}^n \sum_{i_2=i_1}^n \sum_{i_3=i_2}^n \cdots \sum_{i_r=i_{r-1}}^n x^{i_r} = \sum_{i=0}^n V_i^r x^i \text{ \& } V_n^r = \frac{(n+1)(n+2)(n+3) \cdots (n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1$ & $n, r \in N$. Here, $N = \{1, 2, 3, 4, 5, \dots\}$ is a system of natural numbers.

$\sum_{i=0}^n V_i^r x^i$ refers to the combinatorial geometric series;

V_n^r is the binomial coefficient of combinatorial geometric series and

$V_n^r = \frac{(n+r)!}{n! r!}$ is a positive integer (say, k), i.e. $(n+r)! = k \times n! \times r!$.

Theorem 2.1: $\{V_n^r \mid n \geq 0, r \geq 1 \text{ \& } n, r \in N\}$ is a system of natural numbers, i.e. binomial coefficients of combinatorial geometric series are a system of natural numbers.

Proof. Let us show that the binomial coefficients refer to a system of natural numbers.

Here, $V_0^1 = 1$; $V_1^1 = 2$; $V_2^1 = 3$; $V_3^1 = 4$; $V_4^1 = 5$; $V_5^1 = 6$; \dots

$\therefore N = \{V_0^1, V_1^1, V_2^1, V_3^1, V_4^1, V_5^1, \dots\}$ is a system of natural numbers.

Note that addition and multiplication of any binomial coefficients of combinatorial geometric series are binomial coefficients which belong to a system of natural numbers.

$W = \{0, V_0^1, V_1^1, V_2^1, V_3^1, V_4^1, V_4^1, \dots\}$ is a system of whole numbers .

$Z = \{\dots, -V_2^1, -V_1^1, -V_0^1, 0, V_0^1, V_1^1, V_2^1, \dots\}$ is a system of integers.

If $x = 1$ in the combinatorial geometric series, then $\sum_{i=0}^n V_i^r x^i = \sum_{i=0}^n V_i^r$,
i. e. $V_0^r + V_1^r + V_2^r + V_3^r + \dots + V_{n-1}^r + V_n^r = V_n^{r+1} \Rightarrow V_{n-1}^{r+1} + V_n^r = V_n^{r+1}$.

Note that $V_n^r + V_{n-1}^{r+1} = V_n^{r+1} \Rightarrow V_n^{r-1} + V_{n-1}^{r-1} + V_{n-1}^{r+1} = V_n^{r+1}$,
 $(\because V_n^r = V_n^{r-1} + V_{n-1}^{r-1}$, which is the sum of partitions of V_n^r).

$\sum_{i=0}^n 2V_i^r = 2V_n^{r+1}$; $\sum_{i=0}^n 3V_i^r = 3V_n^{r+1}$; $\sum_{i=0}^n 3V_i^r = 3V_n^{r+1}$; \dots ; $\sum_{i=0}^n nV_i^r = nV_n^{r+1}$,
for $n = 1, 2, 3, 4, 5, \dots$

If r is either a real or complex number, then $\sum_{i=0}^n rV_i^r = rV_n^{r+1}$.

Let $aV_m^n = \alpha$; $aV_p^q = \beta$ and $aV_r^s = \gamma$. Then $aV_m^n + aV_p^q = aV_r^s \Rightarrow \alpha + \beta = \gamma$.
Here, α, β, γ are either real or complex numbers.

$\sum_{i=0}^n rV_i^r x^i = \sum_{i=0}^n \mu_i x^i$, ($rV_i^r = \mu_i$ for $i = 0, 1, 2, 3, \dots$),

where μ_i , ($i = 0, 1, 2, 3, \dots$), are real or complex numbers and x is a real or complex variable.

3. Conclusion

In this article, the binomial coefficients of combinatorial geometric series were showed as natural numbers. This new idea can enable the scientific researchers for research and development further.

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