Multi-step optimization of the purchasing options of power retailers to feed their portfolios of consumers[☆]

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Abstract

The liberalization of the retail market of electricity increased the tariff choice of end-use consumers. Retailers compete in the retail market for customers, obtaining private portfolios of end-use consumers to manage. Retailers buy electricity at wholesale markets to feed their customers' demands. They can use spot, derivatives, and bilateral markets to acquire the energy they need. The increasing levels of variable renewable energy sources trading at spot markets, increase the price volatility of these markets. To hedge against the volatility of spot prices, retailers may negotiate standard physical or financial bilateral contracts at derivatives markets. Alternatively, they can also negotiate private bilateral contracts. This article addresses the optimization of the retailers purchasing options, to increase their risk-return ratio from electricity markets, and offer more competitive tariffs to consumers. Considering the risk attitude of retailers, they use a multi-step purchasing model composed of a multi-level risk-return optimization and a decision support system. The article presents an agent-based study considering a retailer with a portfolio of 312 real-world consumers. Risk-seeking and risk-neutral retailers obtained a return up to 38%, less than 7% of the optimal return. However, risk-neutral retailers are subject to four times higher risk in their returns than risk-seeking retailers. The results support the conclusion that wholesale markets of electricity are more favourable to risk-seeking retailers, considering their real returns.

Keywords: Agent-based simulation; Purchasing options of power retailers;

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Nomenclature

Acronyms and Abbreviations

Agg	aggregation
AggRes	Agg of residentials
AggSCom	Agg of small commercials
BMF	Buy monthly futures
BQF	Buy quarter futures
BRP	Balance Responsible Party
CH	Calinski-Harabasz
CVaR	conditional VaR
DAM	day-ahead market
DSS	decision support system
ETOMinR	Minimum return tariff
Ind	industrial
itp	start of the trading period
LCom	large commercial
lcp	last closing price
MIBEL	Iberian market of electricity
MTS	Multivariate Time Series
OMIE	the Spanish market operator
OMIP	the Portuguese market op.
POM	purchasing optimization model
SMF	Sell monthly futures
SQF	Sell quarter futures
VaR	value-at-risk
VREs	variable renewable energy sources

Symbols

$[w]$	option weight
$[Cov]$	covariance
$[\hat{r}]$	expected return
$[T_{\Delta t}]$	consumers tariffs
α	confidence level
$\hat{R}_o$	expected profit
λ	risk attitude
μ	expected return average
Π	optimal return
σ	standard deviation
I	investment
K_t	period discriminator
P_t	electricity price
$Pforward$	forward price
$Pfutures_t$	futures price
$Pspot_t$	spot price
$q_{j,h}$	consumer consumption
q_{max}	maximum quantity of energy
r	real return
r^*	cut-off return
R_o	profit
$T_{j,h}$	tariff
v	forecast weight

Subscripts

h	period of the tariff
i	time step
j	consumer number
l	lag
o	purchasing option
t	current period
y	previous year
Δh	number of tariff periods
Δi	number of time steps
Δt	period duration
J	number of consumers
L	number of lags
O	number of options
Y	number of previous years

Risk attitude; Risk-return optimization; Wholesale markets.

1. Introduction

The deregulation of the electricity supply industry has brought full competition to both wholesale and retail markets (see, e.g., [1, 2]).

A wholesale market is a market where competing generators offer their electricity output to demand-side players [3]. Market participants can trade electricity in three key (sub-)markets [4]: spot, derivatives, and non-organized bilateral markets. Spot markets include mainly day-ahead (DAM) and intraday or real-time markets, where participants can submit bids involving prices and quantities of electricity (as well as some other complex conditions) [5]. Derivatives markets allow players to sign standardized bilateral contracts to hedge against spot price volatility. They include forwards, futures, options, and swaps (or contracts for differences). Non-organized bilateral markets allow players to privately negotiate the terms and conditions of tailored bilateral contracts, typically covering the delivery of large amounts of energy over long periods (months to years). As Balance Responsible Parties (BRPs), the trades agreed by all players on these markets, lead to a programmed dispatch that they have to comply with, to avoid the payment of penalties. Unbalances between supply and demand may affect the security of the power system because of frequency deviations (normally there is a threshold of 1%), which are solved by the balancing reserves. These reserves are traded at the balancing markets, and their costs are paid by the BRPs that deviate from their dispatch schedules [6].

A retail market exists when customers can choose their suppliers from competing power retailers [7]. Retailers buy energy from wholesale markets and sell it to end-use consumers (end-users). They usually try to attract as many customers as possible, signing bilateral contracts with them, and thus defining non-optimal portfolios (in terms of risk and return). In other words, retailers usually pursue a "business as usual strategy", meaning that they offer high tariffs to clients, which are equal for customers with similar consumption patterns. Also, they often consider a high risk premium, making the energy part of the tariff (retail price) substantially larger than the (wholesale) spot price [8, 9]. The risk premium depends on their attitude towards risk. Generally speaking, three attitudes towards risk are often discussed in the literature: risk-averse, risk-neutral, and risk-seeking. The risk attitude is also known as risk preference or aversion. Risk-averse retailers tend to define stable portfolios and consider "small" risk premiums, allowing them to propose "reduced" tariffs to consumers. Retailers with more stable portfolios can obtain better outputs in both favourable and unfavourable scenarios [7]. Risk-neutral retailers mainly focus on the expected return of portfolios, meaning that they do not care much about the uncertainty (risk) of their decisions. Risk-seeking retailers are, to a certain extent, the opposite of risk-averse retailers. In this sense, they may obtain an advantage in retail competition, since high variations in the expected return typically lead to lower variations in risk [9]. In practice, retailers trade energy in spot markets, which are characterized by relevant price volatilities because of the increasing levels of variable renewable energy sources (VREs). VREs have near-zero marginal costs, and stochastic outputs, which together with demand uncertainty increase the uncertainty of the net-load, and the need to balance the power system, increasing the penalties paid by the unbalanced parties [6, 10]. Against this background, retailers also sign standard and private bilateral contracts to hedge against spot price volatility. Their main objective is to trade electricity and sign contracts so that the resulting mix can increase their return and lead to a reduced risk. Retailers may plan their portfolio of purchasing options to guarantee they obtain the electricity required by their customers and a reasonable profit.

This article addresses the risk-return optimization of the wholesale purchasing options of power retailers. The literature mainly focuses on the optimization of the wholesale selling options of power producers [11] and optimal retail tariffs [12]. Some pieces of work focus on models to maximize the profit of traders [13] and optimize the wholesale purchasing options of retailers [14–16]. Algarvio and Lopes [7] presented a literature review on their main features. But only a few pieces of work focus on both the market price and the demand uncertainty, considering real prices and consumers when optimizing the wholesale purchasing options of power retailers. For instance, Kettenun *et al.* [17] studied optimal portfolios of forward contracts by considering retailers with different risk attitudes, facing both price risk (associated with the DAM) and consumption risk (related to end-users). The authors concluded that risk-neutral retailers tend to be more concerned with price-related uncertainty, while risk-averse retailers tend to favour forward contracts to hedge against the DAM price volatility, thus being normally more concerned with the associated risk premium. They assumed, however, a strong correlation between the DAM price and the demand of retailers, which typically depends on the quantity of energy traded. Sun *et al.* [18] used the conditional value-at-risk (CVaR) in the risk assessment phase to maximize the profit of retailers. They select the best options of wholesale markets to buy electricity and feed their consumers, considering tariffs with time-of-use or real-time pricing rates. They concluded that the risk attitude of retailers affects the quantity of electricity acquired through different time horizons. While risk-averse retailers tend to trade in the long term, risk-seeking retailers prefer trading in the short term. They also concluded that real-time pricing rates may increase the retailer's profit when compared with time-of-use rates, but a set with both types of rates is the solution that maximizes the retailer's profit. Koltsakis and Dagoumas [19] considered companies that own power plants and portfolios of consumers, participating in both wholesale and retail markets. They presented an optimal clearing model of the wholesale market, considering a power exchange with an optimal dispatch of the power plants to feed the consumption needs of end-users. They concluded that economically viable companies should have a balanced share of energy in both wholesale and retail markets, a form of risk hedging. Otherwise, they are exposed to the volatility of the market prices. When they have an energy deficit, *i.e.* when the energy they need to feed their consumers is higher than the energy produced by their power plants, they benefit when marginal prices are small. On the contrary, companies with an energy excess benefit when marginal prices are high.

The majority of pieces of work that focus on the purchasing options of retailers only consider an optimization model that selects the best set of products and/or power plants used to feed a given demand. The proposed model upgrades the literature by considering a multi-step model that contains both a multi-level risk-return optimization of the purchasing options in the first step, filtered by a decision support system in the second step. The model deals with both price and consumption uncertainties. This model uses multiple time steps, which means that on each time step, each level of the optimization can be constrained by previously acquired options. Against this background, the purpose of this paper is twofold:

1. To present a multi-step model for optimizing the wholesale purchasing options of agent-based retailers to feed their portfolio of end-users, concerning: i) a multi-level risk-return optimization, ii) a decision support system, and iii) short term and long term forecast methodologies.
2. To test the new model by considering situations involving retailers with different risk attitudes and real-world consumers.

Thus, the work presented here refines and extends previous work on portfolio optimization [7, 9] and risk management [20–22].

The remainder of the paper is structured as follows. Section 2 presents an overview of wholesale competition in electricity markets. Section 3 describes the risk-return optimization model. Section 4 describes a simulation study to test the model in a real-world setting. Finally, concluding remarks are presented in section 5.

2. Wholesale competition in electricity markets

Typically, electricity retailers sign long-term fixed-price contracts with consumers in the retail market, buy energy in spot markets, and/or sign bilateral physical contracts with producers in the wholesale market. Spot prices are highly volatile and uncertain, which increases retailers' risks, and forwards contracts normally have higher prices because of the risk premium requested by their sellers, which decreases retailers' returns. Another form of risk hedging in the wholesale market is the acquisition of financial contracts (futures, options, swaps, etc.). However, the prices of financial contracts are normally higher than spot prices [5, 23].

2.1. Spot markets

Spot markets include mainly the DAM and, an intraday or real-time markets. The DAM clears to meet bid-in demand for an entire day, one day in advance. The pricing mechanism is founded on the marginal pricing theory. Considering the system/locational marginal pricing algorithm, generator companies compete to supply demand by submitting bids in the form of price and quantity pairs [5]. These bids are ranked in increasing order of price, leading to a supply curve. Similarly, retailers and possibly other demand-side participants submit offers to buy certain amounts of energy at specific prices. These purchase offers are ranked in order of decreasing price, leading to a demand curve. The market-clearing price is defined by the intersection of the supply curve with the cumulative demand curve. This price is determined periodically and applied to all generators uniformly, regardless of their bids or location, or is computed locationally in the case of a system/locational marginal pricing algorithm, respectively. Generators are instructed to produce the amount of energy corresponding to their accepted bids, and buyers are informed of the amount of energy that they are allowed to draw from the system.

Closer to real-time operation exist auction-based intraday or real-time markets, but also continuous intraday markets, where players can adjust their trading position. The continuous intraday market is cleared considering the pay-as-bid scheme, *i.e.* in case of opposite bids the system automatically clears them.

The spot prices volatility increases the risk of the retailers' portfolio. Retailers usually sign long-term contracts with consumers with a fixed price, periodic revision, and variable volumes. So, retailers may forecast the market prices to propose a price for the energy component of the tariff. Considering the markets prices volatility, in the computed tariff, retailers have to consider the expected return that they intend to have, plus a risk premium concerning the market risks. The goal of this work is to reduce the market premium by using risk mitigation measures, such as investment in forward contracts and financial contracts.

2.2. Forward and derivatives markets

Bilateral contracts are often negotiated in the forward/derivatives market. The transactions in the forward market are performed assuming their liquidation, *i.e.* the seller delivers the product and the buyer pays the product price on a future date. Whether physical or financial, a bilateral contract is typically negotiated weeks or months before its delivery and can include the following specifications: 1) starting date and time, 2) ending date and time, 3) price per hour (€/MWh), 4) variable megawatt (MW) quantity, and 5) range of hours when the contract is to be delivered. In a more general form, the quantity and price could be time-varying over the contract duration in forward markets. In the derivatives market can only be traded standard contracts, considering fixed quantities and restricted conditions [20]. Depending on the amount of time available and the quantities to be traded, buyers and sellers will resort to the following forms of bilateral trading [1]: customized long-term contracts, trading ´over the counter" and electronic trading.

Forwards contracts are contracts to purchase and sell a given amount and quality of an asset (financial or otherwise), in a specific future date, at a price set in the present, and negotiated bilaterally (outside the power exchange) [21]. After the trading date, the buyer is bound to pay the agreed price (strike price), and the seller is linked to the delivery of the asset under agreed conditions during the delivery period (between the delivery and the maturity date). These contracts may be subject to physical settlement (where the seller delivers the goods sold) or financial settlement (in which there is no physical delivery of the goods, but only a reckoning due to the market price of the asset on the settlement date). Unlike futures contracts, which are contracts traded multilaterally in a power exchange, and subject to a high degree of standardization, forward contracts are likely to be drawn freely according to the will of the parties. In the specific case of retailers, normally they are interested in the physical settlement of the contract. In the case of only financial settlement, during the delivery period, financial forwards are similar to financial futures. So, when the spot index rises above the strike price, buyers have to be compensated, otherwise are the sellers who are compensated. The difference is in the strike prices, while in forwards the strike prices are defined bilaterally in the trading date, in futures, they are defined in the power exchange at the beginning of the delivery period. Normally, the futures prices of electricity serve as a reference to the forward contracts. So, who sells forwards do not want to sell below the futures prices (normally they consider higher prices because in forward contracts they have practically all the risk) and buyers do not want to buy above the futures price (but as they have very few risks, they normally do it).

Customized forward contracts (fixed prices but variable quantity) can be more expensive (sellers can request a higher risk premium) than standardized forward contracts (fixed prices and quantities). Retailers should consider the futures index prices of electricity, and also their price forecasts for the same period. If financial markets have prices significantly higher than the expected spot prices, considering the margin retailers have in their contracts with consumers, they can adopt a selling position and sell financial contracts. However, this strategy may be used carefully, since it increases the risk of the portfolio. In this case, retailers may use financial contracts as profit-seeking measures. Otherwise, if the futures index is similar or below the forecast prices, retailers can adopt the previously mentioned strategy of buying physical forward contracts to mitigate risk, and in some cases get a higher profit than at spot markets.

Futures contracts are standardized contracts, reversible, buying and selling a given quantity and quality of an asset, at a future date, with a price fixed in the present, the trading price. The buyer is bound to pay the agreed price and the seller to deliver the asset under the agreed conditions. These contracts may also be subject to financial or physical settlement (the energy is acquired in the DAM). Unlike forward contracts, which can be negotiated outside the power exchange, and can be drawn based on the will of both parties, futures contracts are fully standardized, meaning that the price is the only variable allowed to be negotiated. During the trading period, futures contracts allow the parties to reverse their contracts, by doing an operation opposite to the initial, *i.e.* by either selling/buying contracts of the same quantities initially purchased/sold. Furthermore, during this period there will exist profits and losses due to daily cash settles (differences between the settlement prices). At the expiration date (the end of the trading period), ends the possibility of selling or buying the contract, is defined the last closing price (lcp), and starts the delivery period. During this period, there will be daily cash settles with values equal to the difference between the lcp and the spot reference price. Furthermore, in the case of physical futures, the seller will have to deliver the product (physically) to the buyer (see Figure 1).

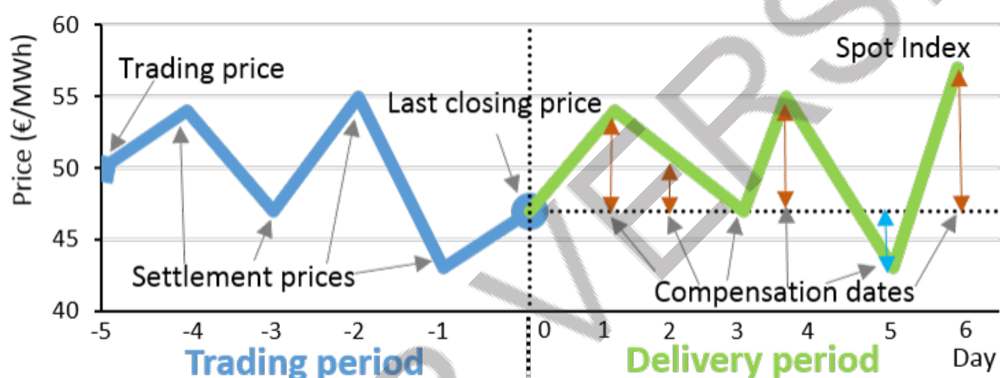

Figure 1: Characteristics of futures trading.

2.3. The Iberian market

The Iberian market (MIBEL [24]) involves a DAM, an intraday market based both on auctions with six sessions and continuous trading, managed by the Spanish electricity market operator (OMIE [25]), a derivatives market managed by the Portuguese electricity market operator (OMIP [26]), and a non-organized forward market. The clearing of the DAM and each intraday auction uses EUPHEMIA [27]. EUPHEMIA considers the system marginal pricing theory. It may consider simple and complex bids from both supply and demand sides, such as the physical constraints of the cross-zonal capacity [5]. By computing the price and quantity for each bidding zone, the algorithm also defines the day-ahead flows between bidding zones [23]. The derivatives market only considers standard forwards, futures, and options as physical and financial instruments, and swaps as financial instruments [20, 22].

3. Risk management and optimization of the electricity purchasing options

Retailer agents should select the best options to trade electricity–that is, they should select the markets (*e.g.*, the day-ahead market in conjunction with the derivatives market) and define the bids to submit to each market. To this end, they should consider a risk management process, which typically includes three key phases [28]: (i) risk assessment, (ii) risk characterization, and (iii) risk mitigation. In the risk assessment phase, retailers recognize the risk factors and identify the main deterministic and stochastic variables. Next, in the risk characterization phase, they make efforts to measure the risk using one or more methods, such as variance, correlation, regression, VaR, and CVaR. Finally, in the risk mitigation phase, they select the best set of products that allow them to reduce the risk.

3.1. Optimization of retailers' wholesale purchasing options of electricity

Considering the aforementioned risk management process, in the first phase, retailers face the following risk factors: market price volatility and consumption uncertainty (of customers in the portfolio). In the second phase, retailers consider the VaR to analyse how the previous risk factors affect the portfolios, which is given by the following expression:

$$VaR = -I \cdot r^* = I(\alpha\sigma\sqrt{\Delta t} - \mu\Delta t) \tag{1}$$

where:

(i) I is the investment made by a retailer;

(ii) r^* is the cut-off return;

(iii) α, σ and μ are the confidence level, the standard deviation and the average of the expected return, respectively;

(iv) Δt is the time period under consideration (duration of the contract).

In the third phase, retailers consider their risk attitude and the tariffs proposed to consumers, to obtain the point that optimizes their risk-return ratio, which represents a different share of purchasing options. The purchasing options optimization model (POM) plans the participation of retailers in wholesale markets. It has the dual objective of maximizing the return and minimizing the risk, as follows:

$$[w] = arg \max_{w_o \geq 0} \left([w]^T [\hat{r}] - \lambda [w]^T [Cov] [w] \right) \tag{2}$$

Subject to

$$\sum_{o=1}^{O} w_o = 1 \tag{3}$$

where:

(i) $[w]$ is the matrix containing the weight w_o of each purchasing option o in the portfolio;

(ii) $[\hat{r}]$ is the matrix of the expected return of each purchasing option o;

(iii) λ is the risk attitude/aversion of the retailer;

(iv) $[Cov]$ is the covariance matrix that relates the expected return between every purchased option o.

The quantification of the risk attitude of the retailer diverges in the literature. Generally, the literature considers that risk-neutral agents have a risk attitude equal to 0, which is the value considered in the proposed model. Bodie *et al.* [29] modelled the risk attitude as an aversion to risk that varies from 1 (highest risk-seeking) to 5 (highest risk-averse). Von Neumann and Morgenstern [30] modelled the risk attitude between -1 to 1, being 0 the risk-neutral. The proposed model was calibrated to consider risk attitudes as risk aversions higher or equal to 0. How closer to 0 higher is the risk-seeking of the agent, and how far from 0, higher is its risk-aversion (see [7, 9] for more details).

The expected return is computed by considering the difference between the tariffs offered to consumers and the expected cost of each market option. This parameter may involve some uncertainty concerning future prices of electricity, notably when retailers trade energy in day-ahead markets. However, in the case of retailers considering mainly forward contracts to buy electricity, the uncertainty is essentially related to the consumption of consumers (the uncertainty associated with the price is reduced). The covariance of the expected return of each market option is important to select "complementary" purchasing options to avoid a high variation (uncertainty) in the expected return of the entire portfolio.

The selection of "complementary" purchasing options allows retailers to hedge against potential unfavourable situations, *i.e.* when spot prices rise above retail tariffs if retailers bought a given quantity of futures contracts for a price below their tariffs, they guarantee a fixed profit, that consists in the price difference between their tariffs and futures prices. Against this background, from the point of view of retailers, buying energy at the DAM and through futures contracts are "complementary" options, which conduct to small values in the covariance of their returns. On the contrary, buying energy at the DAM and selling it through futures contracts are not "complementary" options, because they have similar outputs, a financial loss in case of a rise in spot prices, and profit in case of a fall in spot prices.

3.2. Trading strategy and forecast methods

The model depends on the tariffs proposed to each consumer. Such tariffs involve a fixed payment for power (contracted capacity fee) and a price per unit of electricity (variable fee). Both fees are divided into several parts, but the most important for the return of retailers is the energy part [9]. Accordingly, retailers may set an expected return tax ($\hat{r}$) for each consumer, the markup, considering the risk-free of deposits in global markets and the risk-premium, which depends on several factors, such as the risk associated with the market prices and the consumption.

This paper considers a flat rate, computed using the strategy "Equal tariff optimization strategy at a minimum return (ETOMinR)" adapted from [9]. This pricing strategy is not personalized, so it proposes the same tariff to all consumers (only the energy part of the variable term, the other parts are fixed and depend on rates defined by the regulator). To guarantee a minimum target return from all consumers, the retailer will compute a tariff to each consumer, and the maximum tariff between all computed tariffs is selected because it guarantees that the retailer receives its target minimum return:

$$T_{j,\Delta t} = \frac{(\hat{r}+1)\cdot\ \sum_{j=1}^{J}\sum_{t=1}^{T}\hat{q}_{j,t}\hat{I}_t}{\sum_{t=1}^{\Delta t}\hat{q}_{j,t}K_t} \tag{4}$$

$$[T_{\Delta t}] = \begin{bmatrix} T_{1,\Delta t} \\ \dots \\ T_{j,\Delta t} \end{bmatrix} \tag{5}$$

$$T_{\Delta t} = \max\,[T_{\Delta t}] \tag{6}$$

where:

(i) $T_{j,\Delta t}$ is the prices of each consumer j tariff, that guarantees the minimum expected return to the retailers during period Δt;

(ii) $\hat{r}$ is the retailer's expected return;

(iii) $\hat{I}_t$ is the retailer's expected investment (cost) with consumer j per period t;

(iv) $\hat{q}_{j,t}$ is the electricity expected consumption of consumer j in period t.

(v) K_t is the period discriminator, in case of existing several periods (*i.e.*, when the retailer wants to give more value to peak periods); otherwise, it is equal to one for all periods.

(vi) $[T_{\Delta t}]$ is a matrix containing each consumer j tariff, that guarantees the minimum expected return to retailers during period Δt;

(vii) $T_{\Delta t}$ is the minimum price(s) that retailers need to charge to all consumers to receive the expected return during period Δt;

Using the Calinski-Harabasz (CH) criterion is possible to obtain the optimal number of clusters of the real data, by considering the consumption profile of each consumer [31]. It computes the Euclidean distance between the clusters and compares it with the internal sum of squared errors for each cluster. Using the k-means clustering algorithm is possible to divide consumers by their consumption segment, identified by the CH criterion, and compute their segment typical load profile. This algorithm is a robust model that minimizes the distance between each point to the centre of its respective cluster. The CH criterion and the k-means algorithm have been adapted from [7]. To feed the model with a future prediction of the electricity consumption has been used a multivariate time series (MTS) forecast method, adapted from [7]. To compute the expected yearly futures prices and the expected hourly spot prices one year ahead, has been used a MTS forecast method adapted from [9]. Throughout the year, the expected hourly spot prices are used to compute the average spot price on each month and quarter, $\bar{P}spot_t$, which is used to compute the respective quarter and monthly expected spot prices of electricity, $\hat{P}spot_t$, using a forecast methodology based on tendency growth rates:

$$\hat{P}spot_t = Pspot_{t-1} e^{\frac{Pspot_{t-1} - \hat{P}spot_{t-1}}{\hat{P}spot_{t-1}}} e^{\frac{\bar{P}spot_t - Pspot_{t-1}}{Pspot_{t-1}}} \tag{7}$$

where:

(i) $Pspot_{t-1}$ is the spot price of electricity in the previous period $t-1$;

(ii) $\hat{P}spot_{t-1}$ is the expected spot price of electricity computed for the previous period $t-1$;

The forecast methodology considers both the error between the expected and real spot prices of electricity in the same period (first term), and the expected growth rate of the spot prices between periods (second term). A similar approach is used to compute the expected futures prices of electricity $\hat{P}futures_t$, considering the previous observed futures prices, $Pfutures_{t-l,y}$, at lag l, and previous year(s), y, observed futures prices in the same period, $Pfutures_{t-l,y-1}$, weighting, $v_{(t-l),y}$, the past data with L lags (previous periods):

$$\hat{P}futures_t = Pfutures_{t-1} \times \sum_{y=0}^{Y} \sum_{t=0}^{\Delta t} \sum_{l=0}^{L} e^{v_{t-l,y} \frac{Pfutures_{(t-k-1),y} - Pfutures_{(t-k),y}}{Pfutures_{(t-k),y}}} \tag{8}$$

$$\sum_{y=0}^{Y} \sum_{t=0}^{\Delta t} \sum_{k=0}^{K} v_{(t-k),y} = 1 \tag{9}$$

3.3. Expected return

Considering the type of market option, o, where the retailer will invest, the expected return, r_o, will vary, such as the volatility of the future expected profit, $\hat{R}_o$, and the expected investment required, $\hat{I}_o$. The expected return is equal to:

$$\hat{r}_o = \hat{R}_o / \hat{I}_o \tag{10}$$

The financial products only consider a backup budget (deposit) in case of financial losses, not a direct investment like spot markets and physical contracts. Retailers have portfolios of end-users to satisfy, so financial products are only considered together with the day-ahead market and/or physical contracts. Considering the models and behaviour of the day-ahead market and of each product of the derivatives market analysed in section 2, their formulation follows:

1. Buy at the day-ahead market:

$$\hat{R}_1 = \sum_{t=1}^{\Delta t} \sum_{j=1}^{J} \sum_{i_j=1}^{\Delta i_j} \sum_{h=h_{j,i_j}}^{\Delta h_{j,i_j}} \left(T_{j,i} - \hat{P}spot_{t,h} \right) \hat{q}_{j,t,h} \tag{11}$$

where:

(i) $T_{j,i}$ is the prices of the tariff charged for consumer j at period i;

(ii) $\hat{P}spot_{t,h}$ is the expected spot prices at hour h of day t;

(iii) $\hat{q}_{j,t,h}$ is the expected electricity consumption of consumer j at hour h of day t;

(iv) Δt is the duration of the contract, in days;

(v) J is the number of consumers;

(vi) H_j is the tariff discretization (number of periods that the tariff has) for customer j;

(vii) i_{j,h_j} is the initial time step for the period h_j of customer's j tariff;

(viii) $\Delta i_{j,h_j}$ is the number of time steps for the period h_j of customer's j tariff;

$$\hat{I}_1 = \sum_{t=1}^{\Delta t} \sum_{j=1}^{J} \sum_{i=1}^{I} \hat{P}spot_{t,h} \hat{q}_{j,t,h} \tag{12}$$

2. Buy physical forwards:

$$R_2 = \sum_{t=1}^{\Delta t} \sum_{j=1}^{J} \sum_{i_j=1}^{\Delta i_j} \sum_{h=h_{j,i_j}}^{\Delta h_{j,i_j}} \left(T_{j,i} - Pforward_{base} \right) q_{j,t,h,base} \tag{13}$$

where:

(i) $Pforward_{base}$ is the forward contract cost at the *base* period (typically, standardized contracts have three periods discretization: base for the whole day, peak only for peak periods, and weekends);

(ii) $q_{j,h,base}$ is the reserved quantity for customer j at hour h of day t at $base$ period.

In this contract, the contracted quantity for the base period, q_{base}, should not surpass the sum of the reserved quantity for every customer j, $q_{j,t,h,base}$.

$$q_{base} = \sum_{j=1}^{J} \sum_{i_j=1}^{\Delta i_j} \sum_{h=h_{j,i_j}}^{\Delta h_{j,i_j}} q_{j,t,h,base} \tag{14}$$

The remuneration from the standardized forward contracts is certain (not expected). However, in standardized forwards every contract has to have 1 MW of quantity (tick volume), so the contracted quantity can exceed or scarce the required quantity. In these cases, the excess or scarcity of energy should be traded in the DAM.

$$I_2 = \sum_{t=1}^{\Delta t} \sum_{h=h_{j,i_j}}^{\Delta h_{j,i_j}} Pforward_{base} q_{base} \tag{15}$$

3. Buy physical futures:

$$\hat{R}_3 = \hat{R}_{3,t=0} + \sum_{t=1}^{\Delta t} \sum_{j=1}^{J} \sum_{i_j=1}^{\Delta i_j} \sum_{h=h_{j,i_j}}^{\Delta h_{j,i_j}} \left(T_{j,i} - \hat{P}futures_{base} \right) q_{j,t,h,base} \tag{16}$$

where $\hat{R}_{3,t=0}$ is the cumulative financial profit/loss since the acquisition of the contract in the beginning of the trading period (itp), t_{itp}, at trading price $Pfutures_{itp,base}$. The strike price is the last closing price (lcp) of the trading period ($t = 0$).

$$\hat{R}_{3,t=0} = \sum_{t=itp}^{t=0} \left(\hat{P}futures_{t,base} - \hat{P}futures_{t-1,base} \right) q_{base} \tag{17}$$

$$\hat{I}_3 = (1 + I_{3,t=itp}) \sum_{t=1}^{\Delta t} \sum_{h=h_{j,i_j}}^{\Delta h_{j,i_j}} \hat{P}futures_{base} q_{base} \tag{18}$$

where $I_{3,t=itp}$ is the deposit required by the broker in the acquisition of the futures contracts in the itp, t_{itp}, in order to avoid credit risks, during the trading period (so in case of a 10% deposit, the $I_{3,t=itp}$ can be equal to 0.1).

4. Sell financial futures and buy at the day-ahead market:

$$\hat{R}_4 = \hat{R}_{4,t=0} + \sum_{t=1}^{\Delta t} \sum_{h=h_{j,i_j}}^{\Delta h_{j,i_j}} \left(\hat{P}futures_{base} - \hat{P}spot_{t,h} \right) q_{j,t,h,base} \tag{19}$$

$$\hat{R}_{4,t=0} = \sum_{t=itp}^{t=0} \left(\hat{P}futures_{t-1,base} - \hat{P}futures_{t,base} \right) q_{base} \tag{20}$$

The investment $\hat{I}_4$ is equal to $\hat{I}_1$ plus the deposit.

3.4. Decision Support System

Considering the portfolio of consumers, and the forward and futures prices of electricity, the retailer computes forecasts of the consumption and electricity prices, to feed the POM and the decision support system (DSS). Retailers need to satisfy the consumptions needs of their clients, so they must buy physical electricity. The POM indicates the best share of products to purchase energy according to the risk preference of the retailer. It is a multi-level optimization that on each optimization step, constrains the products' share according to the quantities of electricity already traded. So, the long-term optimization problem has two steps: (i) define the physical markets where to buy electricity and their share, then use (ii) risk mitigation or profit-seeking strategies to select the best set of financial markets to invest. Finally, in the short run and until each product's gate closure, retailers have to decide if they will acquire such purchasing options defined by the POM. So, they may use a DSS that indicates if they are making a good decision.

3.4.1. Long-run risk mitigation measures

In this case, physical or financial contracts are only used to mitigate the risks associated with the transaction of electricity, so any speculative measure is considered in this section. Thus, a buyer (or a retailer) selling financial contracts will not be considered, because although it can be possible to increase its return, it increases the risk of the buyer (profit-seeking strategies). The choice of a product considers a forecast of the electricity prices, $\hat{P}spot_t$, for period t. That expected price is compared with the futures prices of the electricity for the same period, $Pfutures_t$. Now, considering retailers in a buyer position, they behaviour to mitigate the price risk will be:

- $\hat{P}spot_t >> Pfutures_t$:

  Favourable: Buy physical forwards, because if it is expected that spot prices will be higher than the current futures prices, physical forward contracts guarantee that buyers buy the requested quantity of energy at a fixed price, lower than spot prices. Buy financial futures only close to the lcp, to avoid a rise in the futures prices that can be higher than the expected spot prices.

  Avoid: Buy futures far away from the delivery period, at the spot markets or sell contracts. In this case, buy (financial or physical) futures contracts is not a good option because with the expectation that spot prices will be higher, the futures index will tend to increase (short-run return). So, the lcp will be higher than the current futures price, which will decrease or mitigate a future return during the delivery period. Buy the electricity at the spot markets is also to avoid, since signing forward contracts at this time could be more advantageous, to mitigate against the rising of spot prices. Although buying electricity at spot markets can be considered a short-run decision, it has been prepared as a long-run issue.

- $\hat{P}spot_t \geq Pfutures_t$:

  Favourable: Buy physical forwards, considering the uncertainty that the spot prices are going to be higher than the forwards prices. Only buy futures if this futures price is the lcp. In some periods, buying electricity in spot markets can be more advantageous (short-run decision).

  Avoid: Buy futures, at spot markets or sell contracts.

- $\hat{P}spot_t \leq Pfutures_t$:

  Favourable: Buy at the spot markets and physical forwards. In this case, is preferable to buy the majority of the electricity using spot markets, and only part of it through forwards (risk mitigation). Financial futures (for risk-seeking agents) can be used as risk mitigation, in the case that the spot prices go above the lcp (strike price of the contracts). In this case, the agent has to be careful when buying futures contracts, because the tendency can be a decrease in the futures price of electricity, which can result in a loss.

Avoid: Buy financial futures far ahead from the trading period. If the futures prices tend to decrease, buying futures will result in a short-run loss during the trading period, which will be worse how lower spot prices are concerning the strike price of futures.

- $\hat{P}spot_t << Pfutures_t$:

  Favourable: Buy all the electricity using spot markets, and sell contracts. However, from the point of view of buyers, selling contracts can be seen as speculative, risk-taking, and profit-seeking measures (see section 3.4.3).

  Avoid: Buying futures and forward contracts can result in high cumulative losses for buyers.

As in the opposite side is the seller, the favourable and the avoidance situations are exactly the opposite concerning the buyer.

3.4.2. Short-run risk mitigation measures

Now, as these long-run mitigation measures are based on forecasts of the market prices, some short-run mitigation measures must be considered in case of happening the opposite situation:

- If $\hat{P}spot_t > Pfutures_t$ but $Pspot_t < Pfutures_t$:

  Favourable: If it is expected that this situation will occur before the trading period, sell the long-term position of the forwards and futures contracts. Otherwise, if it is temporary, sell short-term forwards and futures to mitigate the risks associated with that position. Buy the required electricity using spot markets, and if this situation is not temporary, buy short-run forwards in some periods.

  Avoid: Buying financial contracts will result in financial losses because of the decrease in spot prices.

- If $\hat{P}spot_t < Pfutures_t$ but $Pspot_t > Pfutures_t$:

  Favourable: Buy medium/short-run physical forwards if this situation is temporary/permanent, and financial forwards and futures as risk mitigation and profit-seeking measures.

Avoid: Buying the majority of the electricity at spot markets, since in this case it increases the risk and reduces the return.

The strategies used in this section should consider the long-term contracts of the portfolio, such as the medium/short-run bidding and contracting strategies and results.

3.4.3. Long-run profit-seeking measures

These strategies are favourable to be used by risk-seeking agents, since they have the main goal of increasing their return, but they also increase their risk:

- If $\hat{P}spot_t > Pfutures_t$:

  Use the favourable measures for long-run risk mitigation with an increase in the number of financial contracts to purchase.

- If $\hat{P}spot_t < Pfutures_t$:

  An agent that needs to buy electricity, can sell financial contracts. This strategy can substantially increase the return of the agent, but it is also risky. In the case of retailers, if spot prices start rising, the losses can also be substantially high in two ways: (i) the difference between the consumers' tariffs and spot prices, and (ii) the losses in the financial contracts.

The same strategies used for short-run risk mitigation can be used for short-run profit-seeking, but with an increase in the number of financial contracts.

3.4.4. DSS suggestions

The decision support system considers a sorting algorithm that orders all options with positive expected returns from descending order for a given period, not considering the spot market, $o > 1$. The DSS suggestions algorithm is presented in Algorithm 1.

For a specific period the retailer will only acquire the best derivatives options according to the POM+DSS models. In case of disagreements between the models, the retailer will only acquire the energy from spot markets.

Algorithm 1 DSS suggestions

procedure SORT($\hat{r}_o$)
  $aux \leftarrow 0$ ▷ Auxiliary variable
  while $and(\max \hat{r}_o > 0, o > 1)$ **do** ▷ Verifies if exists a positive return
    $opt(aux) \leftarrow index(\max \hat{r}_o)$ ▷ Gets the index of the best option
    $\hat{r}_{opt(aux)} \leftarrow -1$ ▷ Removes the best option
    $aux \leftarrow aux + 1$
  end while
end procedure
procedure SUGGEST(opt)
  $aux \leftarrow 0$
  while $aux < length(opt)$ **do** ▷ Verifies if exists any suggestion
    if $w_{opt(aux)} > 0$ **then** ▷ Verifies if the POM suggests that option
      Acquire the option $opt(aux)$
    end if
    $aux \leftarrow aux + 1$
  end while
  Do not acquire all the other derivatives options
end procedure

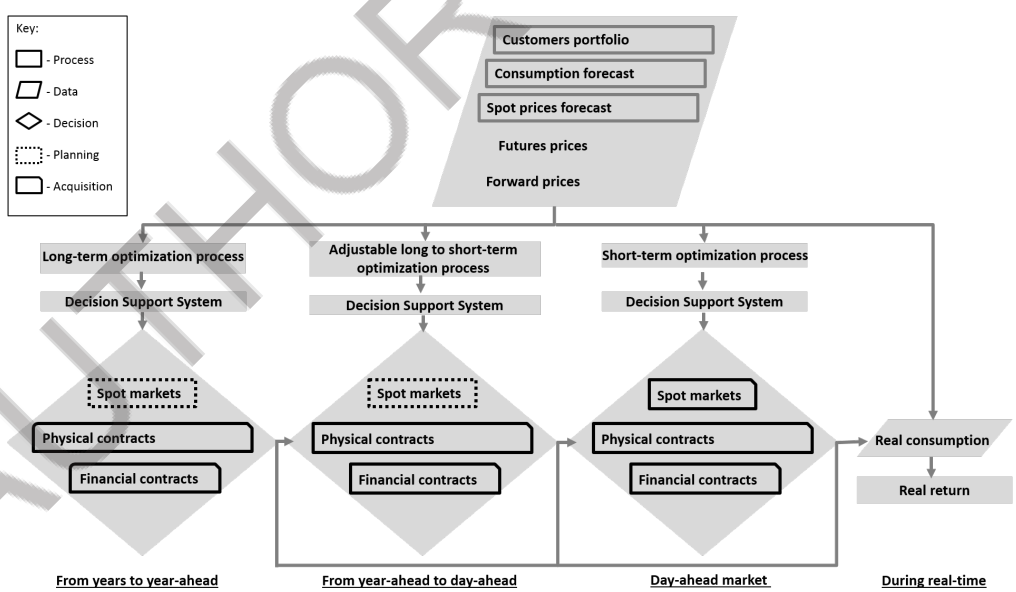

Figure 2: Multi-step optimization of the purchasing options.

3.5. Deterministic optimization

A deterministic optimization formulation can be used to validate the results of the POM+DSS models. It receives as input the observed prices and consumptions, and for each period, computes which is the wholesale option that maximizes the retailer's return, considering a maximum quantity of energy, q_{max}, that can be acquired per tariff period, h. The optimal return, Π, is computed considering Equations 10–20 using observed data instead of forecasts, as follows:

$$\Pi = \max \sum_{o=0}^{O} \sum_{t=1}^{\Delta t} \sum_{h=h_{j,i_j}}^{\Delta h_{j,i_j}} r_o \tag{21}$$

Subject to

$$q_h = q_{max} \tag{22}$$

The next subsection summarizes the entire model.

3.6. The multi-step optimization of the purchasing options model

Figure 2 illustrates the entire model. To use the model, retailers have to define their risk preference, target return, and target period. They should start by analysing the consumption uncertainty of their portfolios, such as market prices volatility, forecasting these variables for the target period (see Equations 7–9 and the used MTS methodologies in [7, 9]). Then they should select a pricing strategy that computes tariffs that guarantee their target return for the computed forecasts (see Equations 4–6 and pricing strategies in [7, 9, 12]). According to each product maturity, the model considers multiple time steps until the end of the selected target period [26]. In the first time step of the model, retailers plan the purchasing options during the target period using the first level of the POM formulation (see Equations 2 and 3). In this step, the model uses real prices and tariffs, such as forecast prices and consumptions to compute the expected returns of each option and their covariance matrices (see Equations 10–20). Considering the previous matrices and the risk preference of retailers, the POM computes the optimal quantities (weights) of each product and the respective expected return and VaR (see Equation 1). In the second step, the model uses the DSS to filter the results of the POM using Algorithm 1. If both the POM and the DSS suggest the acquisition of the same market products, retailers purchase them if they are close to the delivery period. Otherwise, retailers only plan their acquisitions. Then, during real-time is possible to compute the return of retailers considering the consumptions of their portfolios, such as the prices of the day-ahead market. In the second and subsequent times steps, each level of the POM is constrained by the weights of the previously acquired products. So, retailers have to consider those products independently of the DSS filter.

The next section presents a study to test the model.

4. Case study

This section presents a study involving a real price-taker retailer with a portfolio composed of 312 consumers from Portugal, corresponding roughly to 5% of the total demand of the country [32]. The studied period has a duration of 24 months: from January 1, 2012, to December 31, 2013.[1] Using the CH criterion and the k-means clustering algorithm is possible to divide the data set into five classes of consumption: aggregation (Agg) of residentials (AggRES), aggregation of small commercials (AggSCom), large commercials (LCom), industrials (Ind) and others (aggregation of different types of consumers). Each class contains the following consumers: Ind (10 consumers), LCom (11 consumers), AggSC (189 consumers), AggRes (71 consumers), and other (31 consumers). The retailer is modelled as a software agent–that is, computer systems capable of autonomous action and able to meet their design objectives.

The retailer agent is equipped with the purchasing options optimization model (POM), with the decision support system (DSS), and with the forecast methodologies described in section 3. It will plan its purchasing options in the Portuguese market in 2013. So, in the first step of the planning phase, it uses real data from MIBEL and consumers up to 2012 [25, 26]. Until 2013, the MIBEL derivatives market only allows trades of futures contracts, being the prices of such agreements used to simulate potential forward agreements [26]. Furthermore, the retailer uses the MTS forecast methodologies to compute the expected market prices and its portfolio's needs during 2013. It was expected a decrease in the day-ahead market prices of 4.83% and an increase in the portfolio needs of 0.36%. Equipped with this information and

[1]This data set can be found in an online repository in `https://archive.ics.uci.edu/ml/datasets/ElectricityLoadDiagrams20112014#`.

methodologies, and using the ETOMinR pricing strategy with a target expected return of 19%, the retailer computes a single tariff of 56.66 €/MWh to charge its consumers during 2013. The study involves three main parts. The first part is devoted to computing the optimized share of purchasing options of the retailer, considering only the POM, and both the POM and the DSS.

By the end of 2012, in the first stage, the retailer receives the real yearly, quarters, and first three months futures prices of the electricity in Portugal [26]. It uses real data to compute the expected behaviour of the spot index and futures prices during 2013. The purchasing options considered by the retailer are the following: day-ahead market, buy yearly physical forwards, buy physical quarter or monthly futures and sell financial quarter or monthly futures. To trade forwards contracts is considered the real prices of the yearly futures contracts since until this year there is not possible to trade standard forward agreements in Portugal. Armed with this information, and equipped with the optimization model, the retailer then determines the optimized share of purchasing options in the first stage. Furthermore, the retailer can use the DSS to filter the solutions obtained through the POM, following them in case of validation, or only purchasing in the DAM, otherwise. The second part presents the real return of the retailer considering different simulations involving its risk attitude, its participation only in the DAM, and the optimal deterministic considering perfect information, *i.e.* observed data instead of forecasts. Specifically, the retailer agent considers both the spot and futures market prices published by MIBEL and the consumption data of the 312 consumers. The retailer computes its "real" return considering the aforementioned information, the optimized purchasing options, and the tariff proposed to consumers obtained in the first part of the study. In the second part of the study, a systematic comparison is made between the expected return and the "real" return. The third part of the study compares the results of the proposed model with an open-access third-party model. Finally, some conclusions are drawn from the simulations.

4.1. Optimal purchasing offers of retailers

The retailer agent starts by planning its purchasing offers considering: i) the expected demand of its portfolio of consumers, ii) the real yearly, quarter, and the first three months futures prices, iii) the annual, quarter, and monthly expected spot prices, and iv) the forecast of the last nine months futures prices. Equipped with this information, in the first stage of the multi-level optimization is possible to use the DSS to identify the best decisions for the entire year, the first quarter, or the first month of 2013. Table 1 presents the main results of the DSS in the first step of the optimization.

Table 1: Monthly real and expected prices of the purchasing options in the first step of the optimization.

Month	Expected spot price (€/MWh)	Forward price (€/MWh)	Quarter futures price (€/MWh)	Monthly futures price (€/MWh)	Expected monthly futures price (€/MWh)	Decision Support System
January	51.95	52.87	52.90	53.77	–	Sell monthly or quarter futures
February	55.11	52.87	52.90	53.78	–	Buy monthly or sell quarter futures
March	49.49	52.87	52.90	52.26	–	Sell monthly or quarter futures
April	44.27	52.87	51.22	–	51.24	Sell monthly or quarter futures
May	44.19	52.87	51.22	–	47.64	Sell monthly or quarter futures
June	53.52	52.87	51.22	–	44.55	Buy monthly or sell quarter futures
July	50.35	52.87	53.86	–	43.31	Buy monthly or quarter futures
August	49.52	52.87	53.86	–	44.30	Buy monthly or quarter futures
September	48.38	52.87	53.86	–	45.26	Buy monthly or quarter futures
October	46.01	52.87	53.23	–	47.49	Sell quarter or monthly futures
November	42.46	52.87	53.23	–	51.92	Sell quarter or monthly futures
December	42.86	52.87	53.23	–	58.48	Sell quarter or monthly futures

Analysing Table 1 is possible to verify that the DSS proposes two options every month, the first is the best solution, but the second can also bring a positive return to the retailer. The DSS indicates that the best decision considers the selling of January futures together with the acquisition of energy at the DAM. A good decision considers selling the first quarter financial futures. The DSS indicates that the acquisition of yearly forward contracts will result in an economic loss since it is expected that spot prices are lower than the yearly forward price during 2013. Also, it is expected that buying futures contracts in the first month or quarter of the year will result in an economic loss. Then, feeding the optimization model with the previously mentioned information is possible to compute the planned share of each purchasing option in the first step, considering the risk attitude of the retailer (see Table 2). In this study, the high risk-seeking attitude (8) of the retailer is selected by considering the maximization of the expected return minus the VaR (first factor). The high risk-averse attitude (100) is selected by considering the maximization of the expected return/VaR ratio (second factor). The other risk attitudes stand between the aforementioned risk attitudes, decreasing the first factor values and increasing the second factor values while increasing the risk aversion.

Table 2: Retailer's planning share of purchasing options in the first step of the multi-level optimization model considering its risk attitude.

Risk Preference	Spot (%)	Buy yearly forward (%)	Buy quarter futures (%)	Sell quarter futures (%)	Buy monthly futures (%)	Sell monthly futures (%)
Neutral	0	0	0	0	0	100
High seeking	54.89	0	0	14.36	19.93	10.82
Moderate seeking	75.83	4.30	0	0	17.52	2.35
Small seeking	52.31	36.31	0	0	9.03	2.35
Small aversion	25.85	69.44	0	0	3.55	1.17
Moderate aversion	12.62	86.00	0	0	0.80	0.58
High aversion	9.97	89.31	0	0	0.25	0.46

Analysing Table 2 is possible to conclude that the optimization model is in line with the DSS for selling January futures for all risk attitudes, but the number of contracts differs. If the retailer uses the DSS to filter the POM output, it will pass the planned acquisition of yearly forwards to the spot market, otherwise, it will continue as planned with the POM (see Table 3).

Analysing Table 3 is possible to verify that during January, while a risk-neutral retailer will sell all available quantity through monthly financial futures, retailers with different risk attitudes decrease the number of contracts while increasing the risk-aversion. In the case of not using the DSS, the risk-attitude from moderate seeking to high aversion retailers also acquire

Table 3: Retailer's results for January 2013 considering its risk attitude.

Risk Preference	λ	Expected Return (%)	VaR (%)	Purchasing Options	Price (€/MWh)	Quantity (MW)
Neutral	0	51.39	53.49	Sell monthly futures	53.77	212
High seeking	8	27.81	12.34	Sell monthly futures	53.77	116
Moderate seeking	15	19.02	9.04	Buy yearly forwards	52.87	9
				Sell monthly futures	53.77	42
Small seeking	20	16.15	6.92	Buy yearly forwards	52.87	77
				Sell monthly futures	53.77	24
Small aversion	60	11.44	3.54	Buy yearly forwards	52.87	147
				Sell monthly futures	53.77	10
Moderate aversion	80	9.09	1.93	Buy yearly forwards	52.87	182
				Sell monthly futures	53.77	3
High aversion	100	8.62	1.63	Buy yearly forwards	52.87	189
				Sell monthly futures	53.77	3

yearly forward contracts, being the number of contracts proportional to the risk-aversion, *i.e.* how higher is the risk aversion (preference) higher will be the quantity of energy acquired through yearly forward contracts. In this case, the share of yearly forward contracts, and the weights of selling January futures and buying at the DAM are already defined, being included as weight constraints on the next step of the multi-level optimization by the end of January 2013. Otherwise, in the case of using the DSS, only the weight of selling January futures and buying at the DAM are included as weight constraints of the next step of the multi-level optimization. Table 4 presents the planning share in the last step of the multi-level optimization model considering both the POM and the DSS.

The last step of the optimization model ends by the end of September 2013 with the acquisition of the last quarter futures (BQF). Therefore, until the end of the year, the retailer only buys the required energy by its consumer's portfolio from the DAM. Analysing Table 4 is possible to verify that concerning Table 2, the main differences stand in the acquisition of quarter futures instead of monthly futures, and a higher share of spot markets instead of yearly forward contracts. Using the DSS, the planned share of forward contracts is passed to the spot market. Otherwise, it is considered the forward share of the first step, and the spot share is the difference between its share presented in this table and the forward share presented in Table 3.

Table 4: Retailer's planning share of purchasing options in the last step of the multi-level optimization model considering its risk attitude.

Risk Preference	Spot (%)	Buy yearly forward (%)	Buy quarter futures (%)	Sell quarter futures (%)	Buy monthly futures (%)	Sell monthly futures (%)
Neutral	0	0	25.00	0	0	75.00
High seeking	4.52	0	23.87	0	0	71.61
Moderate seeking	8.52	0	22.87	0	0	68.61
Small seeking	40.73	0	14.82	0	0	44.45
Small aversion	84.90	0	3.78	0	0	11.33
Moderate aversion	90.42	0	2.40	0	0	7.19
High aversion	93.73	0	1.57	0	0	4.70

Feeding the deterministic model with the real data from 2013 is possible to identify the best purchasing decisions. Table 5 presents the DSS suggestions until the last step of the optimization according to the expected spot prices and the real futures prices, and compares them with the optimal decisions that only consider observed data.

Table 5: Monthly real and expected prices of the purchasing options in the last step of the optimization.

Month	Expected spot price (€/MWh)	Real Spot price (€/MWh)	Quarter futures price (€/MWh)	Monthly futures price	DSS Suggestion	Optimal Decision
January	51.95	48.53	52.90	53.77	SMF	SQF
February	48.30	43.74	52.90	49.83	SMF	SQF
March	33.68	22.78	52.90	44.48	SMF	SQF
April	13.68	16.15	43.94	35.90	SMF	SMF
May	7.97	43.24	43.94	43.01	SMF	BMF
June	24.92	41.71	43.94	45.70	SMF	SMF
July	18.52	51.41	47.96	47.78	SMF	BMF
August	46.46	48.28	47.96	49.65	SMF	SMF
September	37.63	51.53	47.96	48.24	SMF	SMF
October	51.34	(51.64)	47.21	49.96	BQF	(BQF)
November	50.64	(46.41)	47.21	49.23	BQF	(BQF)
December	54.55	(64.55)	47.21	50.68	BQF	(BQF)

Analysing Table 5 is possible to conclude that the forecasts have substantial errors in May and July. These errors originate the failure of the

DSS suggestions, resulting in a small economic loss (0.23 €/MWh) in May and a substantial economic loss in July (3.63 €/MWh). In May and July, spot prices are higher than futures prices, so the retailer will have a financial loss by selling monthly futures (SMF) instead of buying them (BMF). In the first quarter of the year, the DSS indicates that will be preferable to sell monthly futures than to sell quarter futures (SQF), conducting to lower returns when compared with the optimal decision. The price of the energy in the day-ahead market was highly volatile in 2012 and 2013, meaning that the price risk is indeed a key risk faced by retailers [7]. The other key risk is the consumption uncertainty of their customers.

4.2. Return of the retailer according to its risk attitude

It is possible to compute the expected and "real" results of the retailer, considering scenarios with i) only the extreme risk attitudes of the retailer using the POM with and without the DSS, ii) trading only in spot markets, and iii) a deterministic optimization considering perfect information (see Table 6). The "real" returns of the other risk attitudes stand between the returns of the high-seeking and high-averse risk attitudes, and therefore are not interesting for discussion. For each scenario has been computed the return difference between the "real" and the expected return.

Table 6: Expected return in the first stage and real return considering the POM with and without the DSS, and the optimal deterministic result.

Simulation	Expected Return (%)	POM "Real" Return (%)	POM+DSS "Real" Return (%)	Return difference of the POM+DSS (%)
Neutral	51.39	37.37	37.37	-14.02
High seeking	27.81	31.90	36.90	9.09
High averse	8.62	4.25	23.69	15.07
Only Spot	19.21	–	23.50	4.29
Deterministic	–	–	43.44	–

Analysing Table 6 is possible to conclude that the maximum return that the retailer could obtain in 2013 was 43.44%, considering the proposed tariff, the prices of the purchasing options, spot prices, and consumers' consumption. It is a substantial value since the literature considers real-world markups around 20% [7, 33].

Considering the risk-neutral and the risk-seeking attitudes, their POM+DSS returns are close to the optimal value, which means that this model can be appropriated for real-world retailers. This model can be upgraded by considering more sophisticated forecast methodologies. If the retailer only considers the spot market it will obtain a return of 23.50%, which is in line with real-world markups [33]. The spot prices decreased concerning the expected values by 4.37%, increasing the return by 4.29%.

The use of the DSS increases the return of the retailer for all risk attitudes, except the risk-neutral. The DSS suggestions are in line with the POM results of a risk-neutral retailer, so they propose the same purchasing options. The use of the DSS is very important in the case of the high risk-averse retailer, since it avoids the acquisition of yearly forwards for high prices, increasing the return from 4.25% to 23.69%. Even considering that the consumers' tariff is higher than the yearly forwards price, the consumption of the portfolio is variable. So, on average, during the year the spot prices are lower than the yearly forwards price, which means that the retailer may have a loss when it has to sell its excess of energy at spot markets.

The real return of the retailer increases concerning the expected return in the first stage of the optimization, except the risk-neutral attitude because it decreases by -14.02%. The expected return of the risk-neutral is subject to a VaR higher than it. It was expected that spot prices of electricity were lower than futures prices during almost the whole year (see Table 1). So, as during half the year the spot prices are higher than futures prices, the return of selling financial monthly futures is lower than expected (see Table 5).

Figure 3 presents the main results of the study.

The worst-case return is the minimum expected return and reflects the subtraction of the VaR to the expected return. Analysing the figure is possible to verify that the VaR of the risk-neutral retailer is very high, even higher than the expected return, which means that its minimum expected return is negative. The real return of the high risk-seeking retailer is slightly below the one of the risk-neutral, but its VaR is more than four times lower than the one of the risk-neutral. The high risk-averse retailer has a small VaR, which means that its return is practically guaranteed. This conservative attitude towards risk leads to more stable returns but potentially lower than considering other risk attitudes. Potentially, risk-neutral retailers may obtain higher outputs but at very high risk. However, the final results are very dependent on the forecast accuracy, because, in the case of practically any forecast errors, risk-neutral retailers may obtain higher returns, *i.e.* returns close to the expected returns. Otherwise, in case of substantial forecast errors, risk-averse retailers may obtain returns closer to their expected returns when compared

Figure 3: Main results of the study.

with the other risk attitudes. Furthermore, in case of unfavourable scenarios, *i.e.* higher prices than expected, they may have higher returns than retailers with other risk attitudes. These results reflect the benefits of using optimization models that consider the dual objective of maximizing return and minimizing risk. Models that neglect the trading risk may bring high losses to retailers. Concluding, wholesale markets of electricity seem favourable to risk-seeking retailers because they are the ones that maximize their minimum expected return.

The next section compares the results of the presented model with ojAlgo, a third-party open-access library [34].

4.3. Comparing the results with ojAlgo outputs

This section presents and uses ojAlgo to benchmark the results of the proposed model [34]. It is one of the fastest Java libraries to solve linear algebra problems with large matrices [35]. It has a portfolio optimization model for the stock exchange that can be adapted to optimize the purchasing options of power retailers. Adapting its portfolio model makes it the closest open-access optimization model with the proposed POM. Considering the same optimization time scale of the proposed POM, the first time step considers the optimization of the purchasing options from January 2013 onwards for risk-neutral, high risk-seeking, and high risk-averse attitudes. The last time step considers the optimization of the purchasing options of December 2013.

Table 7 presents the initial and final purchasing options of the retailer with the aforementioned risk-attitudes.

Table 7: Retailer's planning share of purchasing options using ojAlgo, considering its risk attitude.

Risk Preference	Time step	Spot (%)	Buy yearly forwards (%)	Buy quarter futures (%)	Buy monthly futures (%)	Sell monthly futures (%)
Neutral	initial	0	0	0	0	100
	final	0	25.00	0	0	75.00
High seeking	initial	69.95	14.22	0	3.14	12.69
	final	65.86	14.22	0	2.48	17.45
High aversion	initial	4.21	95.31	0	0.48	0
	final	4.35	95.31	0	0.34	0

Analysing Tables 2, 4, and 7 is possible to conclude that a retailer with a risk-neutral attitude has the same results using both models. This occurs because the risk-neutral retailer maximizes its return, and as the input data is the same, both models suggest the same purchasing options. Concerning the other risk preferences, ojAlgo is more conservative by proposing higher weights to purchase forward contracts. Table 8 presents the expected and real results using the initial and final portfolios allocation proposed by ojAlgo, respectively.

Table 8: Expected return in the first stage and real return considering the final portfolio suggested by ojAlgo.

Risk Preference	Expected Return (%)	VaR (%)	OjAlgo "Real" Return (%)	Return difference (%)
Neutral	51.39	53.49	37.37	-14.02
High seeking	19.30	9.20	22.05	2.75
High averse	7.89	1.24	3.80	-4.09

By proposing the acquisition of higher quantities of forward contracts ojAlgo obtains lower returns when compared with the POM. The majority of the proposed models only consider an optimization model. Against this background, the obtained results validate the benefit of using a decision support system to filter the results of the optimization, increasing the retailer's return.

Concluding, wholesale markets seem favourable to risk-seeking retailers. Furthermore, they can also be favourable to risk-neutral retailers if equipped with forecast methodologies that provide expected prices and consumptions with small errors. Otherwise, the VaR that risk-neutral retailers face does not compensate their potential benefit concerning retailers with higher risk aversions. On the contrary, the retail market of electricity is favourable to risk-averse retailers, which means that having a stable portfolio in terms of consumption is an important step to plan the purchasing options in wholesale markets [7].

5. Conclusion and final remarks

The liberalization of the electricity sector led to retail competition for customers in retail markets. Retailers face the volatility of spot prices and the consumption uncertainty of their portfolios composed of end-users. Spot prices are highly volatile, mainly because of the stochastic nature of variable renewable energy sources (VREs), but also because of demand uncertainty. Increasing levels of VREs increase the market price risk of all market participants. Against this background, this paper proposed a risk management model, that may help retailers lead with both the volatility of spot prices and the consumption uncertainty.

This article presented a multi-step risk-return optimization of the purchasing options of retailers with different risk attitudes, considering: i) a multi-level optimization model of the purchasing options (POM), and ii) a decision support system (DSS). The POM considers the dual objective of maximizing return and minimizing risk. It indicates the optimal purchasing options of retailers according to their risk preference, considering historical and forecast data, while the DSS sorts the best options based on forecast results, indicating if retailers are making a good decision.

The model was tested in a real-world setting, involving a price-taker retailer with different risk attitudes, real data from the Iberian electricity market, and 312 consumers from Portugal. The scenario is positive from the point of view of the retailer since the real spot prices were lower than the predictions obtained with the forecast method. Specifically, the study involved an expected decrease of 4.83% in spot prices, when the real decrease was 9.2%. The forecast results analysed by the DSS only failed in two months, reducing the return of the retailer. Even so, results from the study prove that in the case of small forecast errors, the participation of retailers in wholesale markets is favourable to risk-neutral retailers. The use of the DSS together with the POM increases the retailer's return, considering all risk preferences except the risk-neutral. The real return of the retailer with risk-neutral and risk-seeking attitudes stands below the optimal deterministic return by less than 7%, which validates the proposed model. However, even considering this positive scenario, the real return of the risk-neutral retailer decreased by 14.02% concerning the expected return. The risk-neutral retailer is the only one that has a lower return than expected, proving the influence of the risk of its portfolio with a value-at-risk (VaR) of 53.29%, which is higher than its expected return, and more than four times higher than the VaR of the other risk preferences. The risk-seeking retailer obtains a slightly lower return with substantially less risk than the risk-neutral retailer, which means that risk-seeking retailers may have better results in both favourable and not favourable scenarios, and in case of high forecast errors. These results confirm the benefit of using a dual objective (risk-return) optimization model, instead of only maximizing the return. Indeed, price and quantity risks are key factors of retailers when planning their purchasing options.

In conclusion, while wholesale markets are favourable to risk-seeking retailers, the retail market of electricity is favourable to risk-averse retailers. Having stable portfolios in terms of consumption is important to plan the purchasing options of retailers. In case of unstable portfolios of end-users, retailers may acquire call or put options by paying a premium, hedging against consumption uncertainty. Wholesale markets can also be favourable to risk-neutral retailers, in scenarios where their forecast methodologies provide small errors. However, the risk of the purchasing options of risk-neutral retailers is very high, which means that generally, their potential benefit is not enough to cover their potential losses.

The presented study only considers price-taker retailers. Considering retailers with substantial market shares, there is a need to upgrade the model. For future work is going to be considered the strategic bidding process of retailers in wholesale markets, enabling to simulate the wholesale competition of price-maker retailers.

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