CAUSATION, 18(4): 5–88

DOI:10.5281/zenodo.7079056
Received: September 14, 2022
Accepted: September 14, 2022
Published: September 14, 2022
Deutsche Nationalbibliothek Frankfurt

Time and gravitational field

Research article

Ilija Barukčić[1]

[1] Internist, Horandstrasse, 26441 Jever, Germany

* **Correspondence:** E-Mail: Barukcic@t-online.de;
Tel: +49-4466-333; Fax: +49-4466-333.

Abstract

Background:

Our investigations of what exists might seduce us to have little to say about an objective reality, in which nothing exists.

Methods:

The usual rules of tensor algebra have been used.

Results:

Time is equivalent to gravitational field. The four basic field of nature are geometrized. The structure of the fourth basic field of nature is identified.

Conclusion:

Theoretically, the beginning of our world out of an empty negative appears to be possible.

Keywords: Energy; Time; Space; Cause; Effect; Causal relationship k; Causality; Causation

1. Introduction

I have already tried to provide essential answers about the fundamental relationship between energy, time and space in numerous publications of mine. I come back to this topic again, because my former presentation of this subject does not satisfy more. However, there are further theoretical approaches to proof this relationship from another different point of view. While following the time-honoured principle of going **from the known to the unknown**, a new focus on widely discussed notions like energy, time and space might widen our view.

Energy,

pure energy as such, existing independently and outside of human mind an consciousness, objectively and real is an energy without any further determination, an energy which is in its own self equal only to itself. In point of fact, the other side of pure energy is that pure energy is also not unequal with respect to another. Pure energy has no difference within itself and pure energy has no difference outwardly. If anything concrete or any determination or content could be identified in pure energy as distinct, or if pure energy were posited by such a determination as distinct from an other, pure energy would thereby fail to hold fast to its purity. In last consequence, pure energy is equally pure emptiness and at the end indeterminateness as such. Investigations into the nature of time and discussions of various issues related to time have had an important role in science as early as a very long time ago.

Time,

pure time, is similar to pure energy just simple equality with itself, complete emptiness, complete absence of any determination and content, a negation which is equally devoid of any reference. Pure time is the lack of all distinction within itself. No wonder that pure time is the same determination or rather the absence of any determination, and thus altogether the same as what pure energy is. As outlined in view words before, pure energy and pure time are the same. However, it is necessary to consider that neither energy nor time, but rather that energy has passed over into time and time has passes over into energy. However and besides of all, it is important to note that pure energy and pure time are at the end not without any distinction. It is more likely that pure energy and pure time are not the same. Pure energy and pure time are absolutely distinct even if equally unseparated and inseparable. Each of both, each of pure energy and pure time, immediately vanishes into its own opposite.

Space,

is this movement of the immediate vanishing of pure energy into pure time or of the one into its own other and vice versa. However, such an understanding of the relationship between energy and time as stated before is not without deeper issues. In contrast to such a view and following **the first law of thermodynamics** (see Clausius, 1867, du Châtelet, 1740) energy can be transformed from one energy to another, but can be neither destroyed nor created. However, time itself is not energy, it is the other of energy. Under conditions where energy passes over into time or time into energy the impression solidifies that the first law of thermodynamics is violated. Nonetheless, space as the unity and the struggle between energy and time is a movement in which these two, pure energy and pure time, are distinguished too. However, it would be necessary to consider that it is this distinction which immediately dissolved itself too. Authors customary oppose time to energy. However, energy as an already determined and self-organised entity distinguishes itself from another energy. In other words, the time which is opposed to energy is also the time of a certain or concrete energy, a determinate time. Here, time should be viewed in its simplicity as pure time. Pure time is non-energy and as such deemed to oppose pure energy. In point of fact, in pure time as non-energy there is contained the reference to pure energy too. In other words, we have reason to suppose that non-energy is both, pure time and equally its own negation, its own other, pure energy. At the end all, pure time and pure energy summarized in one, determines space.

2. Material and methods

Scientific knowledge and objective reality are more than only interrelated. It cannot be repeated often enough that objective reality or processes of objective reality is the foundation of any scientific knowledge. In point of fact, seen by light, grey is never merely simply grey. In general, human experience teaches us that a high mountain can be conquered by different paths.

2.1. Material

In general, it is appropriate to ensure as much as possible a broader consideration of a research question and to take into account the different facets and viewpoints of an issue investigated in order to reach a goal.

2.2. Methods

Definitions should help us to provide and assure a systematic approach to a scientific issue. It also goes without the need of further saying that a definition as such need to be logically consistent and correct.

2.2.1. Basic definitions

Definition 2.1 **(Energy).**

Let E denote energy which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$E = M \times c^2 \tag{1}$$

where M is the matter and c is the speed of the light in vacuum.

Definition 2.2 **(Matter).**

Let M denote matter which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. In our understanding of the matter we follow Einstein's explanations very closely.

> "... '**Materie**'bezeichnet ... nicht nur
>
> die **'Materie'im üblichen Sinne**, sondern auch das **elektromagnetische Feld**. "
>
> (Einstein, 1916, p. 802/803)

In broken English, '**matter** denotes ... not only **matter in the ordinary sense**, but also the **electromagnetic field**. 'It is worth noting that the equivalence of matter (M) and energy (E) lies at the core of today's physics and has been described by Einstein as follows:

"Gibt ein Körper die Energie L in Form von Strahlung ab, so verkleinert sich seine Masse um L/V^2 ... Die Masse eines Körpers ist ein Maß für dessen Energieinhalt "

(see also Einstein, 1905b, p. 641)

In general it is

$$M \equiv \frac{E}{c^2} \tag{2}$$

(see also Einstein, 1905b, p. 641)

where M denotes the matter(see also Tolman, 1912) and c is the speed of the light in vacuum. In other words, Einstein is demanding the equivalence of matter and energy as the most important upshot of his special theory of relativity.

"Eines der wichtigsten Resultate der Relativitätstheorie ist die Erkenntnis,

daß jegliche Energie E eine ihr proportionale Trägheit (E/c²) besitzt"

(see also Einstein, 1912, p. 1062)

Definition 2.3 (**Anti energy**).

Let $\underline{E}$ denote non-energy or anti energy, the other of energy, the complementary of energy, the opposite of energy which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$\underline{E} = S - E \tag{3}$$

Definition 2.4 (**Time**).

Let t denote time, the other of anti-time, the complementary of anti - time, the opposite of anti-time which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. Let $\underline{t}$ denote anti time. It is

$$t = S - \underline{t} \tag{4}$$

Definition 2.5 (**Anti time**).

Let $\underline{t}$ denote non-time or anti-time, the other of time, the complementary of time, the opposite of time which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$\underline{t} = S - t \tag{5}$$

Definition 2.6 (**Gravitational field**).

Let g denote the gravitational field. The gravitational field g is quite often defined by the gravitational potential. Nonetheless, it is necessary to distinguish the gravitational field and the gravitational potential, both are not identical. Even if it is a little questionable to refer so often to Einstein's position, as long as the same is logically sound, it is also very difficult to simply ignore the same. Although it is much too often overlooked today, let us again refer to Einstein's understanding of the relationship between matter and gravitational field. Einstein defined the gravitational field **ex negativo** as follows.

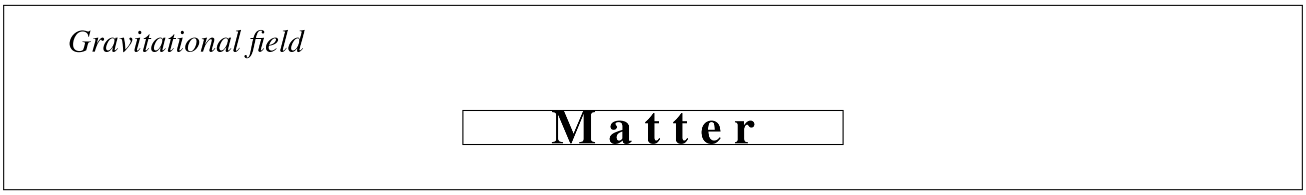
"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß

alles außer dem Gravitationsfeld als '**Materie**'**bezeichnet** wird, also nicht nur

die 'Materie'im üblichen Sinne, sondern auch das elektromagnetische Feld. "

(Einstein, 1916, p. 802/803)

Again, Einstein's position translated into English: 'We distinguish in the following between 'gravitational field'and 'matter', in the sense that everything except the gravitational field is regarded as 'matter', that is not only 'matter'in the ordinary sense, but also the electromagnetic field.'The following and only symbolic figure might illustrate this relationship in more detail.



Mathematically, we express this relationship as follows:

$$g = U - M \tag{6}$$

Definition 2.7 (**Space**).

Let S denote the space which is which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. We assume that energy and time are determining space. It is

$$S = E + t \tag{7}$$

In the further progress of the research it should be possible to demonstrate beyond any reasonable doubt that

$$S - t = E \tag{8}$$

and that the most general formulation of the Einstein field equations could be

$$\left(S \times g_{\mu\nu}\right) - \left(t \times g_{\mu\nu}\right) = \left(E \times g_{\mu\nu}\right) \tag{9}$$

Definition 2.8 (U).

Let U denote the unity and the struggle between matter and gravitational field which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$U = \frac{S}{c^2} \tag{10}$$

Definition 2.9 (Four basic fields of nature).

We define the four basic fields of nature (Barukčić, 2016b,c, 2020a,c,d, 2021) as $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$. Exemplarily, covariant tensors are used. The tensors can also be formulated in a mixed or contravariant from without any loss of information. The table 1 will provide us with an overview of the general definition of the relationships between these four basic (Barukčić, 2016b,c, 2021) fields of nature under conditions of the general theory of relativity where $R_{\mu\nu} = a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}$ is

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Table 1. The four basic fields of nature

the Ricci tensor, $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of electromagnetic field, $G_{\mu\nu}$ is Einstein's curvature tensor, $\underline{G}_{\mu\nu}$ is the "**anti tensor**" (Barukčić, 2016c) of Einstein's curvature tensor, $E_{\mu\nu}$ is the stress-energy tensor of energy, $\underline{E}_{\mu\nu}$ is the tensor of non-energy, the anti-tensor of the stress-energy tensor of energy. It is

$$a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu} \tag{11}$$

or

$$b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu} - a_{\mu\nu} \tag{12}$$

Furthermore, it is (Barukčić, 2016b,c, 2020a,c,d, 2021)

$$
\begin{aligned}
a_{\mu\nu} + b_{\mu\nu} &\equiv \frac{8 \times \pi \times \gamma}{c^4} \times T_{\mu\nu} \\
&\equiv G_{\mu\nu} + \Lambda \times g_{\mu\nu} \\
&\equiv \frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \times g_{\mu\nu} \\
&\equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \\
&\equiv E \times g_{\mu\nu} \\
&\equiv E_{\mu\nu}
\end{aligned}
\tag{13}
$$

and

$$
\begin{aligned}
a_{\mu\nu} + c_{\mu\nu} &\equiv G_{\mu\nu} \\
&\equiv R_{\mu\nu} - \frac{R}{2} \times g_{\mu\nu} \\
&\equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}
\end{aligned}
\tag{14}
$$

It was possible to provide evidence (Barukčić, 2016b,c, 2020a,c,d, 2021) that

$$
\begin{aligned}
c_{\mu\nu} + d_{\mu\nu} &\equiv \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \\
&\equiv \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \\
&\equiv \underline{E} \times g_{\mu\nu} \\
&\equiv \underline{E}_{\mu\nu}
\end{aligned}
\tag{15}
$$

and that (Barukčić, 2016b,c, 2020a,c,d, 2021)

$$
\begin{aligned}
b_{\mu\nu} + d_{\mu\nu} &\equiv E_{\mu\nu} - a_{\mu\nu} + \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - c_{\mu\nu} \\
&\equiv E_{\mu\nu} + \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - a_{\mu\nu} - c_{\mu\nu} \\
&\equiv E_{\mu\nu} + \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - G_{\mu\nu} \\
&\equiv \frac{R}{2} \times g_{\mu\nu} + E_{\mu\nu} - \Lambda \times g_{\mu\nu} - G_{\mu\nu} \\
&\equiv \frac{R}{2} \times g_{\mu\nu}
\end{aligned}
\tag{16}
$$

Definition 2.10 (The Einstein field equations). *The Einstein field equations (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) describe the relationship between the presence of matter (represented by the stress-energy tensor* $\left(\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu}\right)$ *in a given region of spacetime and the curvature in that region by the equation*

$$R_{\mu\nu}-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)\equiv\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv E_{\mu\nu} \tag{17}$$

(*Einstein, 1916, 1917*)

*where $R_{\mu\nu}$ is the Ricci tensor (Ricci-Curbastro and Levi-Civita, 1900) of 'Einstein's general theory of relativity' (Einstein, 1916), R is the Ricci scalar, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold, Λ is the Einstein's cosmological (Barukčić, 2015a, Einstein, 1917) constant, $\underline{\Lambda}$ is the "**anti cosmological constant**" (Barukčić, 2015a), $g_{\mu\nu}$ is the metric tensor of Einstein's general theory of relativity, $G_{\mu\nu}$ is Einstein's curvature tensor, $\underline{G}_{\mu\nu}$ is the "**anti tensor**" (Barukčić, 2016c) of Einstein's curvature tensor, $E_{\mu\nu}$ is the stress-energy tensor of energy, $\underline{E}_{\mu\nu}$ is the tensor of non-energy, the anti-tensor of the stress-energy tensor of energy, $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field, c is the speed of the light in vacuum, γ is Newton's gravitational "**constant**" (Barukčić, 2015a,b, 2016a,c), π is Archimedes constant pi.*

Table 2 may provide a more detailed and preliminary overview of the definitions (Barukčić, 2016b,c) before.

		Curvature YES	Curvature NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}\equiv(c_{\mu\nu}+\Lambda\times g_{\mu\nu})$	$\frac{8\times\pi\times\gamma\times T}{c^4\times D}\times g_{\mu\nu}\equiv\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu}$
	NO	$c_{\mu\nu}\equiv(b_{\mu\nu}-\Lambda\times g_{\mu\nu})$	$d_{\mu\nu}\equiv(\frac{R}{2}\times g_{\mu\nu}-b_{\mu\nu})$	$\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu}$
		$G_{\mu\nu}\equiv\left(\frac{R}{D}-\frac{R}{2}\right)\times g_{\mu\nu}$	$\frac{R}{2}\times g_{\mu\nu}$	$R_{\mu\nu}\equiv\frac{R}{D}\times g_{\mu\nu}$

Table 2. Four basic fields of nature and Einstein's field euqations.

2.3. Tensor Algebra

Geometry can be traced back to the first trials of systematic logical thinking of humans. Still, the nature of the relation between the definitions, axioms, theorems, and proofs in a system of geometry and objective reality has to be considered in detail. Tensors are one mathematical approach to geometry. The tensor (see also Voigt, 1898, p. 20) calculus has been developed in some greater detail by Ricci-Curbastro (see Ricci-Curbastro and Levi-Civita, 1900) and his student Levi-Civita on the basis of earlier work of authors Riemann, Christoffel, Bianchi and others. Especially, Einstein's general theory of relativity is expressed by the mathematical technology of tensors.

2.3.1. Tensor addition

Definition 2.11 (Tensor addition).

The sum of two second rank co-variant tensors has the properties of associativity and commutativity and is defined as

$$\begin{aligned} C_{\mu\nu} &\equiv A_{\mu\nu} + B_{\mu\nu} \\ &\equiv B_{\mu\nu} + A_{\mu\nu} \end{aligned} \tag{18}$$

The sum of two second rank contra-variant tensors has the properties of associativity and commutativity and is defined as

$$\begin{aligned} C^{\mu\nu} &\equiv A^{\mu\nu} + B^{\mu\nu} \\ &\equiv B^{\mu\nu} + A^{\mu\nu} \end{aligned} \tag{19}$$

The sum of two second rank mixed tensors has the properties of associativity and commutativity and is defined as

$$\begin{aligned} C_{\mu}{}^{\nu} &\equiv A_{\mu}{}^{\nu} + B_{\mu}{}^{\nu} \\ &\equiv B_{\mu}{}^{\nu} + A_{\mu}{}^{\nu} \end{aligned} \tag{20}$$

2.3.2. Anti tensor I

Definition 2.12 (Anti tensor I).

Let $a_{\mu\nu}$ denote a co-variant (lower index) second-rank tensor. Let $b_{\mu\nu}$, $c_{\mu\nu}$ et cetera denote other co-variant second-rank tensors. Let $E_{\mu\nu}$ denote the sum of these co-variant second-rank tensors. Let the relationship $a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + ... \equiv E_{\mu\nu}$ be given. A co-variant second-rank anti tensor (see also Barukčić, 2020d) of a tensor $a_{\mu\nu}$ denoted in general as $\underline{a}_{\mu\nu}$ is defined

$$\begin{aligned} \underline{a}_{\mu\nu} &\equiv E_{\mu\nu} - a_{\mu\nu} \\ &\equiv b_{\mu\nu} + c_{\mu\nu} + ... \end{aligned} \tag{21}$$

2.3.3. Anti tensor II

Definition 2.13 (Anti tensor II).

Let $a^{\mu\nu}$ denote a contra-variant (upper index) second-rank tensor. Let $b^{\mu\nu}$, $c^{\mu\nu}$ et cetera denote other contra-variant (upper index) second-rank tensors. Let $E^{\mu\nu}$ denote the sum of these contra-variant (upper index) second-rank tensors. Let the relationship $a^{\mu\nu} + b^{\mu\nu} + c^{\mu\nu} + ... \equiv E^{\mu\nu}$ be given. A co-variant second-rank anti tensor of a tensor $a^{\mu\nu}$ denoted in general as $\underline{a}^{\mu\nu}$ is defined

$$\begin{aligned} \underline{a}^{\mu\nu} &\equiv E^{\mu\nu} - a^{\mu\nu} \\ &\equiv b^{\mu\nu} + c^{\mu\nu} + \ldots \end{aligned} \tag{22}$$

2.3.4. Anti tensor III

Definition 2.14 (Anti tensor III).

Let $a_{\mu}{}^{\nu}$ denote a mixed second-rank tensor. Let $b_{\mu}{}^{\nu}$, $c_{\mu}{}^{\nu}$ et cetera denote other mixed second-rank tensors. Let $E_{\mu}{}^{\nu}$ denote the sum of these mixed second-rank tensors. Let the relationship $a_{\mu}{}^{\nu} + b_{\mu}{}^{\nu} + c_{\mu}{}^{\nu} + ... \equiv E_{\mu}{}^{\nu}$ be given. A mixed second-rank anti tensor of a tensor $a_{\mu}{}^{\nu}$ denoted in general as $\underline{a}_{\mu}{}^{\nu}$ is defined

$$\begin{aligned} \underline{a}_{\mu}{}^{\nu} &\equiv E_{\mu}{}^{\nu} - a_{\mu}{}^{\nu} \\ &\equiv b_{\mu}{}^{\nu} + c_{\mu}{}^{\nu} + \ldots \end{aligned} \tag{23}$$

2.3.5. Tensor subtraction

Definition 2.15 (Tensor subtraction).

The subtraction of two second rank co-variant tensors is defined as

$$C_{\mu\nu} \equiv A_{\mu\nu} - B_{\mu\nu} \tag{24}$$

The subtraction of two second rank contra-variant tensors is defined as

$$C^{\mu\nu} \equiv A^{\mu\nu} - B^{\mu\nu} \tag{25}$$

The subtraction of two second rank mixed tensors is defined as

$$C_{\mu}{}^{\nu} \equiv A_{\mu}{}^{\nu} - B_{\mu}{}^{\nu} \tag{26}$$

2.3.6. Symmetric and anti symmetric tensors

Definition 2.16 (**Symmetric and anti symmetric tensors**).

Symmetric tensors of rank 2 may represent many physical properties objective reality. A co-variant second-rank tensor $a_{\mu\nu}$ is symmetric if

$$a_{\mu\nu} \equiv a_{\nu\mu} \tag{27}$$

However, there are circumstances, where a tensor is anti-symmetric. A co-variant second-rank tensor $a_{\mu\nu}$ is anti-symmetric if

$$a_{\mu\nu} \equiv -a_{\nu\mu} \tag{28}$$

Thus far, there are circumstances were an anti-tensor is identical with an anti-symmetrical tensor.

$$a_{\mu\nu} \equiv E_{\mu\nu} - b_{\mu\nu} + ... \equiv E_{\mu\nu} - \underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{29}$$

Under conditions where $E_{\mu\nu} = 0$, an anti-tensor is identical with an anti-symmetrical tensor or it is

$$-\underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{30}$$

However, an anti-tensor is not identical with an anti-symmetrical tensor as such.

Definition 2.17 (**Multiplication of tensors**). *Let g_{kl} or $g_{\mu\nu}$ denote a 2-index metric tensors. Let $g_{kl\mu\nu}$ denote a 4-index metric tensors. Let $g_{kl\mu\nu\,...}$ denote a n-th index metric tensor. The n-index metric tensor $g_{kl\mu\nu\,...}$ itself is a covariant symmetric tensor and equally an example of a tensor field. If we pause for a moment today and rely on Einstein's "Die Grundlage der allgemeinen Relativitätstheorie " (see Einstein, 1916, p. 784), it is*

$$g_{kl\mu\nu} \equiv g_{kl} g_{\mu\nu} \tag{31}$$

and in the case of n-th rank order

$$g_{kl\mu\nu\,...} \equiv g_{kl} g_{\mu\nu ...} \tag{32}$$

The mixed and contra-variant cases are similar. Riemann defined the distance between two neighbouring points more or less by a quadratic differential form. The geometry based on the positive definite Riemannian metric tensor is called the Riemannian geometry. However, tensor calculus as a generalization of classical linear algebra should assure that formulae are invariant under coordinate transformations and that the same are independent of any kind of the rank order of the metric tensor chosen. Albert Einstein (see Einstein, 1916) presented some rules of tensor algebra in his important publication issued in the year 1916.

$$T_{\mathrm{a\,b\,c}} \equiv A_{\mathrm{a\,b}} B_{\mathrm{c}} \tag{33}$$

(see Einstein, 1916, p. 784)

Furthermore, it is

$$T^{\mathrm{a\,b\,c\,d}} \equiv A^{\mathrm{a\,b}} B^{\mathrm{c\,d}} \tag{34}$$

(see Einstein, 1916, p. 784)

and equally

$$T_{\ c\ \ d}^{\ a\ \ b} \equiv A^{\mathrm{a\,b}} B_{\mathrm{c\,d}} \tag{35}$$

(see Einstein, 1916, p. 784)

A covariant tensor of the second rank type is defined as

$$T_{\mathrm{c\,d}} \equiv A_{\mathrm{c}} B_{\mathrm{d}} \tag{36}$$

(see Einstein, 1916, p. 782)

A contravariant tensor of the second rank type is defined as

$$T^{\mathrm{c\,d}} \equiv A^{\mathrm{c}} B^{\mathrm{d}} \tag{37}$$

(see Einstein, 1916, p. 782)

A mixed tensor of the second rank type is defined by Einstein as follows.

$$T_{\ c}^{\ \ \ d} \equiv A_{\mathrm{c}} B^{\mathrm{d}} \tag{38}$$

(see Einstein, 1916, p. 783)

A scalar F, or a tensor of zero rank, is given by the relationship

$$F \equiv F_{\ b}^{\ b} \equiv F_{\ a\ \ b}^{\ a\ \ b} \equiv F_{\mathrm{a\,b}} F^{\mathrm{a\,b}} \tag{39}$$

(see Einstein, 1916, p. 785)

This relationship (see equation 39, p. 17) is of importance for the fundamental invariants of the electromagnetic field too. The covariant and contravariant products of two rank 2 tensors give the same value and result in a scalar. In general, scalar products are operations on two tensors of the same rank that yield a scalar.

2.3.7. The metric tensor $g_{\mu\nu}$ and the inverse metric tensor $g^{\mu\nu}$

General relativity is a theory of the geometrical properties of space-time to, while the metric tensor $g_{\mu\nu}$ itself is of fundamental importance for general relativity. The metric tensor $g_{\mu\nu}$ is something like the generalization of the Pythagorean theorem. Thus far, it does not appear to be necessary to restrict the validity of the Pythagorean theorem only to certain situations. The question is justified why the Riemannian geometry should be oppressed by the quadratic restriction. In this context, **Finsler geometry**, named after Paul Finsler (1894 - 1970) who studied it in his doctoral thesis (see Finsler, 1918) in 1918, appears to be a kind of metric generalization of Riemannian geometry without the quadratic restriction and justifies the attempt to systematize and to extend the possibilities of general relativity.

Definition 2.18 (**Kronecker delta**).

The Kronecker delta (see Zehfuss, 1858), a notation invented by Leopold Kronecker (1823-1891) in 1868 (see Kronecker, 1868) appears in many areas of physics, mathematics, and engineering and is defined as

$$g_{\mu\rho} \times g^{\nu\rho} \equiv g_{\mu}{}^{\nu} \equiv \delta_{\mu}{}^{\nu} \tag{40}$$

Technically, the Kronecker delta itself is a mixed second-rank tensor.

Definition 2.19 (**The metric tensor $g_{\mu\nu}$ and the inverse metric tensor $g^{\mu\nu}$**).

The distance between any two points in a given space can be described geometrically by a generalized Pythagorean theorem, the metric tensor $g_{\mu\nu}$. Sharing Einstein's point of view, it is in general

$$g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\nu}{}^{\nu} \equiv D \tag{41}$$

where D might denote the number of space-time dimensions. The quantity

$$\delta_{\mathrm{i}}{}^{\mathrm{i}} \equiv \delta_{1}{}^{1} + \delta_{2}{}^{2} + ... + \delta_{\mathrm{D}}{}^{\mathrm{D}} \equiv D \tag{42}$$

is an invariant. In other words, an index which is repeated inside an expression means summation over the repeated index (Einstein summation convention). Vectors and scalars are invariant under coordinate transformations. In point of fact, Einstein field equations (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) were initially formulated by Einstein himself in the context of a four-dimensional theory even though Einstein field equations need not to break down under conditions of D space-time dimensions (see Stephani, 2003). Nonetheless, based on Einstein's statement (Einstein, 1916, p. 796), one gets (see also Einstein, 1923b, p. 91)

$$g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\nu}{}^{\nu} \equiv D \equiv +4 \tag{43}$$

or

$$\frac{1}{g_{\mu\nu} \times g^{\mu\nu}} \equiv \frac{1}{4} \tag{44}$$

where $g^{\mu\nu}$ is the matrix inverse of the metric tensor $g_{\mu\nu}$. The inverse metric tensor or the metric tensor, which is always symmetric, allow tensors to be transformed into each other and are used to lower and raise indices. Einstein's point of view is that

> "... in the general theory of relativity ... must be ... the tensor $g_{\mu\nu}$ of the gravitational potential"
> (Einstein, 1923b, p. 88)

Definition 2.20 (The metric tensor $g_{\mu\nu}$ decomposed). *The fundamental difference between the metric tensors of the four basic fields of nature, denoted as $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$, finds its complete expression in equation 45 as*

$$_{a}g_{\mu\nu} + {}_{b}g_{\mu\nu} + {}_{c}g_{\mu\nu} + {}_{d}g_{\mu\nu} \equiv g_{\mu\nu} \tag{45}$$

where $_{a}g_{\mu\nu}$ is the metric tensor of the ordinary force, $_{b}g_{\mu\nu}$ is the metric tensor of electromagnetism, $_{c}g_{\mu\nu}$ is the metric tensor of gravitational field, $_{d}g_{\mu\nu}$ is the metric tensor of gravitational waves and $g_{\mu\nu}$ is the metric tensor of Einstein's general theory of relativity. We distinguish here between the four basic field of nature, as follows. Details are illustrated by table 3.

Table 3. The metric field decomposed

		Curvature		
		YES	NO	
Momentum	YES	$({}_{a}g_{\mu\nu})$	$({}_{b}g_{\mu\nu})$	$({}_{E}g_{\mu\nu})$
	NO	$({}_{d}g_{\mu\nu})$	$({}_{d}g_{\mu\nu})$	$({}_{\underline{E}}g_{\mu\nu})$
		$({}_{G}g_{\mu\nu})$	$({}_{\underline{G}}g_{\mu\nu})$	$(g_{\mu\nu})$

In this publication, let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the covariant second rank tensors of the four basic fields of nature were $a_{\mu\nu} \equiv a \times g_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu} \equiv b \times g_{\mu\nu}$ et cetera.

Definition 2.21 (The metric tensor $_{gw}g_{\mu\nu}$ of gravitational waves). *Let $g_{\mu\nu}$ denote the metric tensor of Einstein's general theory of relativity. Let $_{gw}g_{\mu\nu}$ denote the metric tensor of* ***gravitational waves*** *of Einstein's general theory of relativity. Let $_{\underline{gw}}g_{\mu\nu}$ denote the metric tensor of* ***anti-gravitational waves*** *of Einstein's general theory of relativity. In general, we define*

$$_{\underline{E}}g_{\mu\nu} \equiv {}_{\underline{gw}}g_{\mu\nu} + {}_{gw}g_{\mu\nu} \tag{46}$$

Definition 2.22 (The metric tensor $\eta_{\mu\nu}$ of special relativity). *There is a fundamental difference between Special and General Relativity regarding the metric tensor. Let $\eta_{\mu\nu}$ denote the metric tensor of Einstein's special theory of relativity. In general, depending upon circumstances, it is $\eta_{\mu\nu} = \left\{\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{matrix}\right\}$ (see Einstein, 1916, p. 778). Let $\underline{\eta}_{\mu\nu}$ denote* ***the anti-metric tensor of Ein-***

stein's special theory of relativity. *Let $g_{\mu\nu}$ denote the metric tensor of Einstein's general theory of relativity. In general, it is (see equation 21)*

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \underline{\eta}_{\mu\nu} \tag{47}$$

There are circumstances where ${}_{d}g_{\mu\nu} \equiv {}_{gw}g_{\mu\nu} \equiv \underline{\eta}_{\mu\nu}$. The n-th index relationship follows (see equation 21) as

$$g_{kl\mu\nu\ldots} \equiv \eta_{kl\mu\nu\ldots} + \underline{\eta}_{kl\mu\nu\ldots} \tag{48}$$

Definition 2.23 (Index raising). *According to Einstein (see also Einstein, 1916, p. 790), it is*

$$F_{\mu\nu} \equiv g_{\mu\alpha} g_{\nu\beta} F^{\alpha\beta} \tag{49}$$

and equally

$$F^{\mu\nu} \equiv g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta} \tag{50}$$

In other (Kay, 1988) words (see Einstein, 1916, p. 790), an order-2 tensor, twice multiplied by the contra-variant metric tensor and contracted (Einstein, 1916, p. 785) in different indices, raises each index. It is

$$F^{\left(\begin{smallmatrix} 1 & 3 \\ \mu & c \end{smallmatrix}\right)} \equiv g^{\left(\begin{smallmatrix} 1 & 2 \\ \mu & \nu \end{smallmatrix}\right)} \times g^{\left(\begin{smallmatrix} 3 & 4 \\ c & d \end{smallmatrix}\right)} \times F_{\left(\begin{smallmatrix} \nu & d \\ 2 & 4 \end{smallmatrix}\right)} \tag{51}$$

or more professionally

$$F^{\mu\ c} \equiv g^{\mu\nu} \times g^{cd} \times F_{\nu\ d} \tag{52}$$

Following Einstein, it is $g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\mu}{}^{\mu}$ (Einstein, 1916, p. 796). Furthermore, in conjunction with another view of Einstein (see Einstein, 1916, p. 785), it is

$$F \equiv F_{\mu\nu}{}^{\mu\nu} \equiv F_{\mu\nu} \times F^{\mu\nu} \tag{53}$$

2.4. Extended tensor algebra

In the following, for the sake of better understanding, we consider tensors of order two. As is known, the components of a tensor of order two can be displayed in 4 × 4 matrix form.

2.4.1. Zero tensor

Definition 2.24 (Zero tensor).

The second-rank co-variant zero tensor is defined as

$$0_{\mu\nu} \equiv \underbrace{\begin{pmatrix} 0_{00} & 0_{01} & 0_{02} & 0_{03} \\ 0_{10} & 0_{11} & 0_{12} & 0_{13} \\ 0_{20} & 0_{21} & 0_{22} & 0_{23} \\ 0_{30} & 0_{31} & 0_{32} & 0_{33} \end{pmatrix}}_{0_{\mu\nu} \text{ tensor}} \tag{54}$$

This definition is also valid for contra-variant or mixed tensors too.

2.4.2. The negation of one

Definition 2.25 (The negation of one).

The negation of one, denoted as $\neg(1)$, is defined by division as

$$\neg(1) = \frac{0}{1} \tag{55}$$

In general, it is

$$\neg(1) \times 1 = +1 - 1 = \frac{0}{1} \times 1 = \frac{1}{1} \times 0 = 0 \tag{56}$$

The negation of one, denoted as $\neg$, is defined by subtraction as

$$\neg = 1 - \tag{57}$$

In general, it is

$$\neg 1 = 1 - 1 = 0 \tag{58}$$

2.4.3. Unity tensor

Definition 2.26 (Unity tensor).

The second-rank co-variant unity tensor is defined as

$$1_{\mu\nu} \equiv \underbrace{\begin{pmatrix} 1_{00} & 1_{01} & 1_{02} & 1_{03} \\ 1_{10} & 1_{11} & 1_{12} & 1_{13} \\ 1_{20} & 1_{21} & 1_{22} & 1_{23} \\ 1_{30} & 1_{31} & 1_{32} & 1_{33} \end{pmatrix}}_{1_{\mu\nu}\text{ tensor}} \tag{59}$$

This definition is also valid for contra-variant or mixed tensors too.

2.4.4. The negation of zero

Definition 2.27 (The negation of zero).

The negation of zero, denoted as $\neg(0)$, is defined by division as

$$\neg(0) = \underline{0} = \frac{1}{0} \tag{60}$$

In general, it is

$$\neg(0) \times 0 = \underline{0} \times 0 = \frac{1}{0} \times 0 = \frac{0}{0} = 1 \tag{61}$$

The negation of zero, denoted as $\neg(0)$ or as $\underline{0}$, is defined by subtraction as

$$\neg = 1 - \tag{62}$$

In general, it is

$$\neg 0 = \underline{0} = 1 - 0 = 1 \tag{63}$$

2.4.5. The tensor of the number 2

Definition 2.28 (The tensor of the number 2).

The second-rank co-variant tensor of the number 2 is defined as

$$2_{\mu\nu} \equiv \underbrace{\begin{pmatrix} 2_{00} & 2_{01} & 2_{02} & 2_{03} \\ 2_{10} & 2_{11} & 2_{12} & 2_{13} \\ 2_{20} & 2_{21} & 2_{22} & 2_{23} \\ 2_{30} & 2_{31} & 2_{32} & 2_{33} \end{pmatrix}}_{2_{\mu\nu}\text{ tensor}} \tag{64}$$

This definition is also valid for contra-variant or mixed tensors an other numbers too. Whether it makes sense to define numbers or scalars et cetera in the form of a tensor is worth being discussed. However, such an approach has various advantages too.

2.4.6. Speed of the light tensor

Definition 2.29 (Speed of the light tensor).

Scientists and thinkers have been fascinated by the speed of light since ever. Aristotle (384-322 BCE) himself has been of the opinion that the speed of light was infinite. Let c denote the speed of the light in vacuum. The second-rank co-variant tensor of speed of the light is defined as

$$c_{\mu\nu} \equiv \underbrace{\begin{pmatrix} c_{00} & c_{01} & c_{02} & c_{03} \\ c_{10} & c_{11} & c_{12} & c_{13} \\ c_{20} & c_{21} & c_{22} & c_{23} \\ c_{30} & c_{31} & c_{32} & c_{33} \end{pmatrix}}_{\text{c}_{\mu\nu}\text{ tensor}} \tag{65}$$

2.4.7. Archimedes' constant tensor

Definition 2.30 (Archimedes' constant tensor).

The second-rank co-variant tensor of the Archimedes of Syracuse (c. 287 – c. 212 B. C. E.) constant π is defined as

$$\pi_{\mu\nu} \equiv \underbrace{\begin{pmatrix} \pi_{00} & \pi_{01} & \pi_{02} & \pi_{03} \\ \pi_{10} & \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{20} & \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{30} & \pi_{31} & \pi_{32} & \pi_{33} \end{pmatrix}}_{\pi_{\mu\nu}\text{ tensor}} \tag{66}$$

This definition is also valid for contra-variant or mixed tensors too.

2.4.8. Newton's constant tensor

Definition 2.31 (Newton's constant tensor).

The second-rank co-variant tensor of the Newton's constant is defined, as

$$\gamma_{\mu\nu} \equiv \underbrace{\begin{pmatrix} \gamma_{00} & \gamma_{01} & \gamma_{02} & \gamma_{03} \\ \gamma_{10} & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{20} & \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{30} & \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}}_{\gamma_{\mu\nu}\text{ tensor}} \tag{67}$$

This definition is also valid for contra-variant or mixed tensors too.

2.4.9. Planck's constant tensor

Definition 2.32 (Planck's constant tensor).

Plato (424/423 – 348/347 BCE), a Greek philosopher born in Athens, defined a circle as follows

> "Rund ist doch das, dessen Enden überall gleich weit von der Mitte entfernt sind? "
>
> (see also Plato, 1910, p. 26)

Max Karl Ernst Ludwig Planck (1858-1947) quantized the energy ${}_{R}E_{t}$ as

$$
{}_{R}E_{t} \equiv n \times h \times {}_{R}f_{t} \tag{68}
$$

where h is Planck's constant (Planck, 1901), ${}_{R}f_{t}$ is the frequency and n is an integer number. In the following, Paul Adrien Maurice Dirac (1902-1984) defined the so-called Dirac's constant $\hbar$ (Dirac, 1926) as

$$
\begin{aligned}
h &\equiv 2 \times \pi \times \hbar \\
&\equiv \pi \times (2 \times \hbar) \\
&\equiv \pi \times s
\end{aligned} \tag{69}
$$

Figure 1 might illustrate these basic relationships.

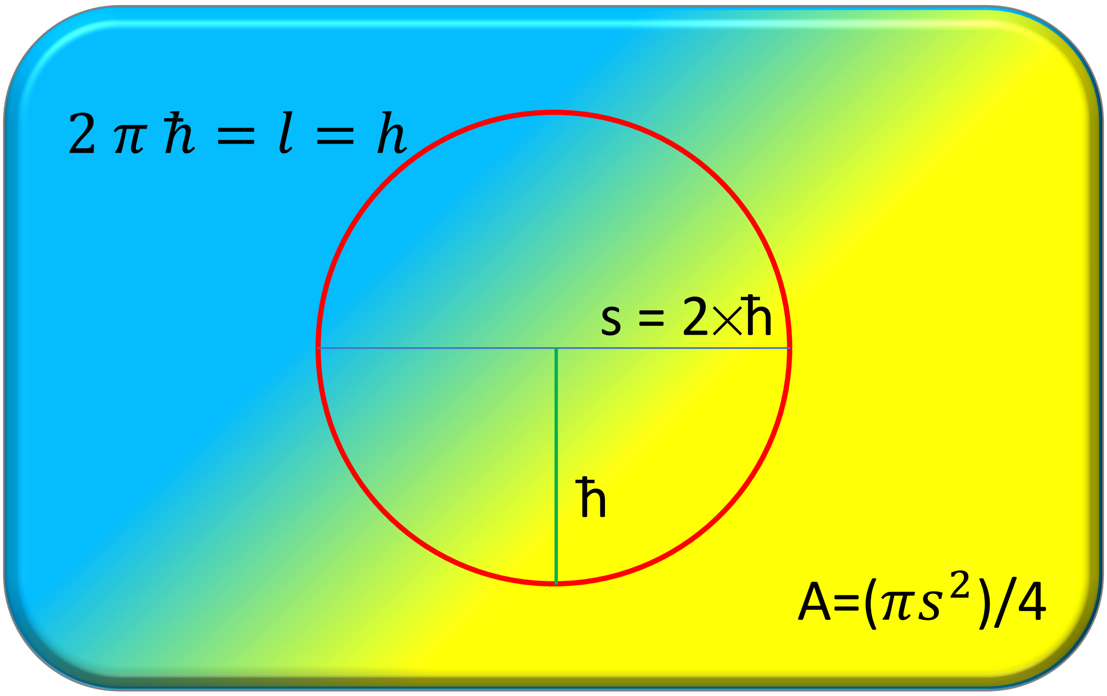


Figure 1. Planck's constant h, quantum loop and string theory.

A few thoughts - which are necessarily first thoughts - might consider circumstances where h can be regarded as a loop, denoted as l, of quantum loop theory, while s is treated as a string of string theory. Under these conditions, it is

$$l \equiv \pi \times s \tag{70}$$

or

$$\pi \equiv \frac{l}{s} \tag{71}$$

Equation 71 implies due to our experience that π can hardly be treated as a constant. In this context, the second-rank co-variant tensor of Planck's constant h (Planck, 1901) is defined, as

$$h_{\mu\nu} \equiv \underbrace{\begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ h_{30} & h_{31} & h_{32} & h_{33} \end{pmatrix}}_{\text{h}_{\mu\nu}\text{ tensor}} \tag{72}$$

This definition is also valid for contra-variant or mixed tensors too.

2.4.10. Dirac's constant tensor

Definition 2.33 (Dirac's constant tensor).

The second-rank co-variant tensor of Dirac's constant $\hbar$ is defined as

$$\hbar_{\mu\nu} \equiv \underbrace{\begin{pmatrix} \hbar_{00} & \hbar_{01} & \hbar_{02} & \hbar_{03} \\ \hbar_{10} & \hbar_{11} & \hbar_{12} & \hbar_{13} \\ \hbar_{20} & \hbar_{21} & \hbar_{22} & \hbar_{23} \\ \hbar_{30} & \hbar_{31} & \hbar_{32} & \hbar_{33} \end{pmatrix}}_{\hbar_{\mu\nu}\text{ tensor}} \tag{73}$$

This definition is also valid for contra-variant or mixed tensors too.

2.4.11. The commutative multiplication of tensors

Definition 2.34 (The commutative multiplication of tensors).

Multiplication is something which is equivalent to a repeated addition. Addition itself has the properties of associativity and commutativity. The question is justified whether there might exist something like a commutative multiplication of tensors. Let $U_{\mu\nu}$ denote a second-rank tensor. Let $W_{\mu\nu}$ denote another second-rank tensor. The commutative multiplication of two second-rank tensors is defined as an entry wise multiplication of both tensors. It is,

$$U_{\mu\nu} \cap W_{\mu\nu} \equiv X_{\mu\nu} \tag{74}$$

where the sign $\cap$ denotes a commutative multiplication of tensors of the same rank. The commutative multiplication of two tensors of the same rank is commutative, associative and distributive.

Example.

Example of an entrywise multiplication of two tensors of the same rank.

$$\begin{aligned}&\underbrace{\begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} \\ u_{10} & u_{11} & u_{12} & u_{13} \\ u_{20} & u_{21} & u_{22} & u_{23} \\ u_{30} & u_{31} & u_{32} & u_{33} \end{pmatrix}}_{\mathrm{U}_{\mu\nu}\ \text{tensor}} \cap \underbrace{\begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03} \\ w_{10} & w_{11} & w_{12} & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \\ w_{30} & w_{31} & w_{32} & w_{33} \end{pmatrix}}_{\mathrm{W}_{\mu\nu}\ \text{tensor}} \\ &= \underbrace{\begin{pmatrix} (u_{00}\times w_{00}) & (u_{01}\times w_{01}) & (u_{02}\times w_{02}) & (u_{03}\times w_{03}) \\ (u_{10}\times w_{10}) & (u_{11}\times w_{11}) & (u_{12}\times w_{12}) & (u_{13}\times w_{13}) \\ (u_{20}\times w_{20}) & (u_{21}\times w_{21}) & (u_{22}\times w_{22}) & (u_{23}\times w_{23}) \\ (u_{30}\times w_{30}) & (u_{31}\times w_{31}) & (u_{32}\times w_{32}) & (u_{33}\times w_{33}) \end{pmatrix}}_{x_{\mu\nu}}\end{aligned} \tag{75}$$

Jacques Salomon Hadamard (1865-1963), a French mathematician, defined a similar operation of two matrices of the same dimension $i \times j$ (see also Hadamard, 1893) which is commutative, associative and distributive. The Hadamard product (also known as the Issai Schur (see also Schur, 1911, p. 14) (1875 – 1941) product (see also Davis, 1962) or the point wise product) is of use for a commutative matrix multiplication and is defined something as

$$(u \circ w)_{\mathrm{ij}} \equiv u_{\mathrm{ij}} w_{\mathrm{ij}} \tag{76}$$

where the sign $\circ$ denotes an entry wise matrix multiplication.

2.4.12. The tensor double dot product on the closest indices

Definition 2.35 (The tensor double dot product on the closest indices).

Two tensors can be contracted over the first two indices of the second tensor or over the last two indices of the first tensor (double contraction). As is known, a double dot product between two tensors of orders m and n will result in a tensor of order (m + n - 4). Let $u_{\mu\nu}$ and $w_{\mu\nu}$ denote two second-rank tensors. Let : denote the contraction of two tensors $u_{\mu\nu}$ and $w_{\mu\nu}$ on the closest indices, then

$$u : w = u_{\mu\nu} w_{\nu\mu} \tag{77}$$

2.4.13. The tensor double dot product on the non-closest indices

Definition 2.36 (The tensor double dot product on the non-closest indices).

Let $u_{\mu\nu}$ and $w_{\mu\nu}$ denote two second-rank tensors. Let $\underline{:}$ denote the contraction of two tensors $u_{\mu\nu}$ and $w_{\mu\nu}$ on the non-closest indices, then

$$u\underline{:}w = u_{\mu\nu}w_{\mu\nu} \tag{78}$$

Especially under conditions where both second-rank tensors are symmetric, both definitions of the tensor double dot product coincide but not necessarily in general.

2.4.14. The division of tensors

Definition 2.37 (The division of tensors).

Division is something which is related to multiplication. Let $a_{\mu\nu}$ denote a second-rank tensor. Let $b_{\mu\nu}$ denote another second-rank tensor. Let $U_{\mu\nu}$ denote another second-rank co-variant tensor. In general, let it be that

$$a_{\mu\nu} + b_{\mu\nu} \equiv U_{\mu\nu} \tag{79}$$

The probability of a tensor $a_{\mu\nu}$, denoted as $p(a_{\mu\nu})$, is calculated entry wise as follows.

$$p(a_{\mu\nu}) \equiv \begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} / \begin{pmatrix} U_{00} & U_{01} & U_{02} & U_{03} \\ U_{10} & U_{11} & U_{12} & U_{13} \\ U_{20} & U_{21} & U_{22} & U_{23} \\ U_{30} & U_{31} & U_{32} & U_{33} \end{pmatrix} \equiv \begin{pmatrix} \frac{a_{00}}{U_{00}} & \frac{a_{01}}{U_{01}} & \frac{a_{02}}{U_{02}} & \frac{a_{03}}{U_{03}} \\ \frac{a_{10}}{U_{10}} & \frac{a_{11}}{U_{11}} & \frac{a_{12}}{U_{12}} & \frac{a_{13}}{U_{13}} \\ \frac{a_{20}}{U_{20}} & \frac{a_{21}}{U_{21}} & \frac{a_{22}}{U_{22}} & \frac{a_{23}}{U_{23}} \\ \frac{a_{30}}{U_{30}} & \frac{a_{31}}{U_{31}} & \frac{a_{32}}{U_{32}} & \frac{a_{33}}{U_{33}} \end{pmatrix} \tag{80}$$

2.4.15. The exponentiation of a tensor to the power n

Definition 2.38 (The exponentiation of a tensor to the power n)**.**

A second-rank co-variant tensor to the power n, denoted by ${}^{n}U_{\mu\nu}$, is determined by the fact that every single component of such a tensor is multiplied by itself n-times. In general, it is

$$
{}^{\mathrm{n}}U_{\mu\nu} = \underbrace{\begin{pmatrix}
\underbrace{(u_{00} \times u_{00} \times \ldots)}_{n-times} & \underbrace{(u_{01} \times u_{01} \times \ldots)}_{n-times} & \underbrace{(u_{02} \times u_{02} \times \ldots)}_{n-times} & \underbrace{(u_{03} \times u_{03} \times \ldots)}_{n-times} \\
\underbrace{(u_{10} \times u_{10} \times \ldots)}_{n-times} & \underbrace{(u_{11} \times u_{11} \times \ldots)}_{n-times} & \underbrace{(u_{12} \times u_{12} \times \ldots)}_{n-times} & \underbrace{(u_{13} \times u_{13} \times \ldots)}_{n-times} \\
\underbrace{(u_{20} \times u_{20} \times \ldots)}_{n-times} & \underbrace{(u_{21} \times u_{21} \times \ldots)}_{n-times} & \underbrace{(u_{22} \times u_{22} \times \ldots)}_{n-times} & \underbrace{(u_{23} \times u_{23} \times \ldots)}_{n-times} \\
\underbrace{(u_{30} \times u_{30} \times \ldots)}_{n-times} & \underbrace{(u_{31} \times u_{31} \times \ldots)}_{n-times} & \underbrace{(u_{32} \times u_{32} \times \ldots)}_{n-times} & \underbrace{(u_{33} \times u_{33} \times \ldots)}_{n-times}
\end{pmatrix}}_{{}^{\mathrm{n}}U_{\mu\nu}}
= \underbrace{\begin{pmatrix}
(u_{00})^{\mathrm{n}} & (u_{01})^{\mathrm{n}} & (u_{02})^{\mathrm{n}} & (u_{03})^{\mathrm{n}} \\
(u_{10})^{\mathrm{n}} & (u_{11})^{\mathrm{n}} & (u_{12})^{\mathrm{n}} & (u_{13})^{\mathrm{n}} \\
(u_{20})^{\mathrm{n}} & (u_{21})^{\mathrm{n}} & (u_{22})^{\mathrm{n}} & (u_{23})^{\mathrm{n}} \\
(u_{30})^{\mathrm{n}} & (u_{31})^{\mathrm{n}} & (u_{32})^{\mathrm{n}} & (u_{33})^{\mathrm{n}}
\end{pmatrix}}_{{}^{\mathrm{n}}U_{\mu\nu}} \tag{81}
$$

This definition is also valid for contra-variant or mixed tensors too.

2.4.16. The exponentiation of a tensor to the power 1/n

Definition 2.39 (The exponentiation of a tensor to the power 1/n)**.**

A second-rank co-variant tensor to the power n, denoted by ${}^{\mathrm{n}}\mathrm{U}_{\mu\nu}$, is determined by the fact that every single component of such a tensor is multiplied by itself (1/n)-times. In general, it is

$$
{}^{1/\mathrm{n}}U_{\mu\nu} = \underbrace{\begin{pmatrix}
(u_{00})^{1/\mathrm{n}} & (u_{01})^{1/\mathrm{n}} & (u_{02})^{1/\mathrm{n}} & (u_{03})^{1/\mathrm{n}} \\
(u_{10})^{1/\mathrm{n}} & (u_{11})^{1/\mathrm{n}} & (u_{12})^{1/\mathrm{n}} & (u_{13})^{1/\mathrm{n}} \\
(u_{20})^{1/\mathrm{n}} & (u_{21})^{1/\mathrm{n}} & (u_{22})^{1/\mathrm{n}} & (u_{23})^{1/\mathrm{n}} \\
(u_{30})^{1/\mathrm{n}} & (u_{31})^{1/\mathrm{n}} & (u_{32})^{1/\mathrm{n}} & (u_{33})^{1/\mathrm{n}}
\end{pmatrix}}_{{}^{1/\mathrm{n}}U_{\mu\nu}} \tag{82}
$$

This definition is also valid for contra-variant or mixed tensors too.

2.4.17. The expectation value of a co-variant second rank tensor

Let $\mathrm{E}({}_{\mathrm{R}}\mathrm{U}_{\mu\nu})$ denote the expectation value of a co-variant second rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$. Let $\mathrm{p}({}_{\mathrm{R}}\mathrm{U}_{\mu\nu})$ denote the probability of a tensor ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$. In general, we define

$$
E\left({}_{\mathrm{R}}U_{\mu\nu}\right) \equiv p\left({}_{\mathrm{R}}U_{\mu\nu}\right) \cap {}_{\mathrm{R}}U_{\mu\nu} \tag{83}
$$

and equally

$$^{2}E\left({}_{\mathrm{R}}U_{\mu\nu}\right) \equiv E\left({}_{\mathrm{R}}U_{\mu\nu}\right) \cap E\left({}_{\mathrm{R}}U_{\mu\nu}\right) \equiv p\left({}_{\mathrm{R}}U_{\mu\nu}\right) \cap p\left({}_{\mathrm{R}}U_{\mu\nu}\right) \cap {}_{\mathrm{R}}U_{\mu\nu} \cap {}_{\mathrm{R}}U_{\mu\nu} \tag{84}$$

Let $E({}_{\mathrm{R}}U_{kl\mu\nu\ldots})$ denote the expectation value of a co-variant n-index rank tensor ${}_{\mathrm{R}}U_{kl\mu\nu\ldots}$. Let $p({}_{\mathrm{R}}U_{kl\mu\nu\ldots})$ denote the probability of a co-variant n-index rank tensor ${}_{\mathrm{R}}U_{kl\mu\nu\ldots}$. In general, we define expectation value of a co-variant n-index rank tensor

$$E\left({}_{\mathrm{R}}U_{kl\mu\nu\ldots}\right) \equiv p\left({}_{\mathrm{R}}U_{kl\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}U_{kl\mu\nu\ldots} \tag{85}$$

It is equally true that

$$^{2}E\left({}_{\mathrm{R}}U_{kl\mu\nu\ldots}\right) \equiv E\left({}_{\mathrm{R}}U_{kl\mu\nu\ldots}\right) \cap E\left({}_{\mathrm{R}}U_{kl\mu\nu\ldots}\right) \equiv p\left({}_{\mathrm{R}}U_{kl\mu\nu\ldots}\right) \cap p\left({}_{\mathrm{R}}U_{kl\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}U_{kl\mu\nu\ldots} \cap {}_{\mathrm{R}}U_{kl\mu\nu\ldots} \tag{86}$$

2.4.18. The expectation value of a second rank anti tensor

Let $E({}_{\mathrm{R}}\underline{U}_{\mu\nu})$ denote the expectation value of the covariant second rank anti tensor ${}_{\mathrm{R}}\underline{U}_{\mu\nu}$. Let $p({}_{\mathrm{R}}\underline{U}_{\mu\nu})$ denote the probability of an anti tensor ${}_{\mathrm{R}}\underline{U}_{\mu\nu}$. In general, we define

$$\begin{aligned} E\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) &\equiv p\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) \cap U_{\mu\nu} \\ &\equiv \left(1_{\mu\nu} - p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\right) \cap {}_{\mathrm{R}}U_{\mu\nu} \end{aligned} \tag{87}$$

Euclid's theorem is a fundamental statement of geometry and has been proved by Euclid in his famous work Elements. According to Euclid's theorem, it is

$${}_{\mathrm{R}}U_{\mu\nu} \equiv E\left({}_{\mathrm{R}}U_{\mu\nu}\right) + E\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) \tag{88}$$

Theorem 1. *It is*

$${}_{R}U_{\mu\nu} \equiv E\left({}_{R}U_{\mu\nu}\right) + E\left({}_{R}\underline{U}_{\mu\nu}\right) \tag{89}$$

Proof. According to Euclid's theorem, it is

$${}_{\mathrm{R}}U_{\mathrm{t}} \equiv E\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) + E\left({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}\right) \tag{90}$$

Multiply ${}_{\mathrm{R}}U_{t}$ by the metric tensor $g_{\mu\nu}$ or just define

$${}_{\mathrm{R}}U_{\mathrm{t}} = {}_{\mathrm{R}}U_{\mu\nu} \tag{91}$$

Then the conclusion is true that

$${}_{\mathrm{R}}U_{\mu\nu} \equiv E\left({}_{\mathrm{R}}U_{\mu\nu}\right) + E\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) \tag{92}$$

□

♡

2.4.19. The expectation value of a second rank tensor raised to rower 2

Let $E({}^{2}{}_{R}U_{\mu\nu})$ denote the expectation value of the covariant second rank tensor ${}_{R}U_{\mu\nu}$ raised to the power 2. Let $p({}_{R}U_{\mu\nu})$ denote the probability of a tensor ${}_{R}U_{\mu\nu}$. In general, we define

$$\begin{aligned} E\left({}^{2}{}_{R}U_{\mu\nu}\right) &\equiv p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \\ &\equiv p\left({}_{R}U_{\mu\nu}\right) \cap \left({}^{2}{}_{R}U_{\mu\nu}\right) \end{aligned} \tag{93}$$

Let ${}^{2}E({}_{R}U_{kl\mu\nu\ldots})$ denote the expectation value of a co-variant n-index rank tensor ${}^{2}{}_{R}U_{kl\mu\nu\ldots}$. Let $p({}_{R}U_{kl\mu\nu\ldots})$ denote the probability of a co-variant n-index rank tensor ${}_{R}U_{kl\mu\nu\ldots}$. In general, we define the expectation value of a co-variant n-index rank tensor raised to rower 2 as

$${}^{2}E\left({}_{R}U_{kl\mu\nu\ldots}\right) \equiv p\left({}_{R}U_{kl\mu\nu\ldots}\right) \cap {}_{R}U_{kl\mu\nu\ldots} \cap {}_{R}U_{kl\mu\nu\ldots} \tag{94}$$

2.4.20. The variance of a tensor

Definition 2.40 (The variance of a tensor).

Let ${}_{R}U_{\mu\nu}$ denote a second-rank co-variant tensor. Let $p({}_{R}U_{\mu\nu})$ denote the probability of a tensor ${}_{R}U_{\mu\nu}$. The variance of a tensor ${}_{R}U_{\mu\nu}$, denoted as ${}^{2}\sigma\left({}_{R}U_{\mu\nu}\right)$, is defined as

$$\begin{aligned} {}^{2}\sigma\left({}_{R}U_{\mu\nu}\right) &\equiv E\left({}^{2}{}_{R}U_{\mu\nu}\right) - {}^{2}\left(E\left({}_{R}U_{\mu\nu}\right)\right) \\ &\equiv \left(p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu}\right) - \left(p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu} \cap p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu}\right) \\ &\equiv {}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \cap p\left({}_{R}U_{\mu\nu}\right) \cap \left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right) \end{aligned} \tag{95}$$

From equation 95 follows that

$${}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \equiv \frac{{}^{2}\sigma\left({}_{R}U_{\mu\nu}\right)}{p\left({}_{R}U_{\mu\nu}\right) \cap \left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right)} \tag{96}$$

and that

$${}_{R}U_{\mu\nu} \equiv \frac{\sigma\left({}_{R}U_{\mu\nu}\right)}{{}^{1/2}\left(p\left({}_{R}U_{\mu\nu}\right) \cap \left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right)\right)} \tag{97}$$

The standard deviation of a second-rank tensor, denoted as $\sigma\left({}_{R}U_{\mu\nu}\right)$, would follow as

$$\begin{aligned} \sigma\left({}_{R}U_{\mu\nu}\right) &\equiv {}^{1/2}\left({}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \cap p\left({}_{R}U_{\mu\nu}\right) \cap \left(\left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right)\right)\right) \\ &\equiv \sqrt[2]{\left({}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \cap p\left({}_{R}U_{\mu\nu}\right) \cap \left(\left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right)\right)\right)} \end{aligned} \tag{98}$$

Let ${}_{R}U_{kl\mu\nu\ldots}$ denote a co-variant n-index rank tensor. Let $p({}_{R}U_{klkl\mu\nu\ldots\ldots})$ denote the probability of a co-variant n-index rank tensor ${}_{R}U_{kl\mu\nu\ldots}$. The variance of a co-variant n-index rank tensor ${}_{R}U_{kl\mu\nu\ldots}$, denoted as ${}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)$, is defined as

$$\begin{aligned}
&{}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv E\left({}^{2}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)-{}^{2}\left(E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\\
&\equiv\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)-\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)
\end{aligned}\tag{99}$$

From equation 99 follows that

$${}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\equiv\frac{{}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)}\tag{100}$$

and that

$${}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\equiv\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{{}^{1/2}\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)}\tag{101}$$

The standard deviation of a second-rank tensor, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)$, would follow as

$$\begin{aligned}
&\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv{}^{1/2}\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)\right)\\
&\equiv\sqrt[2]{\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)\right)}
\end{aligned}\tag{102}$$

2.4.21. The co-variance of two tensors

Definition 2.41 (The co-variance of two tensors).

Let ${}_{\mathrm{R}}U_{\mu\nu}$ denote a second-rank co-variant tensor. Let $p({}_{\mathrm{R}}U_{\mu\nu})$ denote the probability of a tensor ${}_{\mathrm{R}}U_{\mu\nu}$. According to equation 80, the probability of a tensor ${}_{\mathrm{R}}U_{\mu\nu}$ is defined as $p({}_{\mathrm{R}}U_{\mu\nu})$. Let ${}_{\mathrm{R}}W_{\mu\nu}$ denote a second-rank co-variant tensor. Let $p({}_{\mathrm{R}}W_{\mu\nu})$ denote the probability of a tensor ${}_{\mathrm{R}}W_{\mu\nu}$ (see equation 80). Let $p({}_{\mathrm{R}}U_{\mu\nu},\ {}_{\mathrm{R}}W_{\mu\nu})$ denote the probability of a joint tensor of the two tensors ${}_{\mathrm{R}}U_{\mu\nu}$ and ${}_{\mathrm{R}}W_{\mu\nu}$.The co-variance of the two tensors ${}_{\mathrm{R}}U_{\mu\nu}$ and ${}_{\mathrm{R}}W_{\mu\nu}$, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mu\nu\ldots},{}_{\mathrm{R}}W_{\mu\nu\ldots}\right)$, is defined as

$$\begin{aligned}
&\sigma\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)\\
&\equiv E\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)-\left(E\left({}_{\mathrm{R}}U_{\mu\nu}\right)\times E\left({}_{\mathrm{R}}W_{\mu\nu}\right)\right)\\
&\equiv\left(p\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)\cap{}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}W_{\mu\nu}\right)\\
&-\left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap{}_{\mathrm{R}}U_{\mu\nu}\cap p\left({}_{\mathrm{R}}W_{\mu\nu}\right)\cap{}_{\mathrm{R}}W_{\mu\nu}\right)\\
&\equiv{}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}W_{\mu\nu}\cap\left(p\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)-\left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\times p\left({}_{\mathrm{R}}W_{\mu\nu}\right)\right)\right)
\end{aligned}\tag{103}$$

From equation 103 follows that

$$ {}_{\mathrm{R}}U_{\mu\nu} \cap {}_{\mathrm{R}}W_{\mu\nu} \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mu\nu}, {}_{\mathrm{R}}W_{\mu\nu}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mu\nu}, {}_{\mathrm{R}}W_{\mu\nu}\right) - \left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right) \times p\left({}_{\mathrm{R}}W_{\mu\nu}\right)\right)\right)} \tag{104} $$

Let ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ denote a co-variant n-index rank tensor. Furthermore, let $\mathrm{p}({}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots})$ denote the probability of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$. According to equation 80, the probability of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ is defined as $\mathrm{p}({}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots})$. Let ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$ denote a co-variant n-index rank tensor. Let $\mathrm{p}({}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots})$ denote the probability of this co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$ (see equation 80). Let $\mathrm{p}({}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots})$ denote the probability of a joint tensor of the two co-variant n-index rank tensors ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$.The co-variance of the two co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)$, is defined as

$$ \begin{aligned} &\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\ \equiv &E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \times E\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right) \\ \equiv &\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\ - &\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\ \equiv &{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots} \cap \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \times p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right) \end{aligned} \tag{105} $$

From equation 105 follows that

$$ {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots} \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \times p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)} \tag{106} $$

Definition 2.42 (**Energy $_{R}E_{t}$ and Matter $_{R}M_{t}$**).

"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß **alles außer dem Gravitationsfeld** als '**Materie**'bezeichnet wird, also nicht nur die **'Materie'im üblichen Sinne**, sondern auch das **elektromagnetische Feld**. "

(Einstein, 1916, p. 802/803)

Firstly. Everything but the gravitational field is matter, there is no third between matter and gravitational field, a third is not given, **tertium non datur**. Secondly. Matter, from the point of view of a stationary observer R, includes not only matter in the ordinary sense, but the electromagnetic field as well (Einstein, 1916, p. 802/803). Finally, one consequential relationship is necessary to mention. "Da Masse und Energie nach den Ergebnissen der speziellen Relativitätstheorie das Gleiche sind und die Energie formal durch den symmetrischen Energietensor (Tμv) beschrieben wird, so besagt dies, daß das G-Geld [gravitational field, author] durch den Energietensor der Materie bedingt und bestimmt ist "(Einstein, 1918). Matter or energy is the cause of the gravitational field. However, is this relationship valid vice versa to?

Definition 2.43 (**Time $_{R}t_{t}$ and gravitational field $_{R}g_{t}$**).

The fundamental relationship between gravitational field $_{R}g_{t}$ from the point of view of the stationary observer R and time $_{R}t_{t}$ from the point of view of the same stationary observer R is determined(Barukčić, 2011, 2013, 2016c,d) by the equation

$$_{R}g_{t} \equiv \frac{_{R}t_{t}}{c^{2}} \tag{107}$$

and from the point of view of a co-moving observer 0 by the equation

$$_{0}g_{t} \equiv \frac{_{0}t_{t}}{c^{2}} \tag{108}$$

Next we define(Barukčić, 2011, 2016d) the following mathematical identities related to time, to which a concrete physical meaning would have to be attached in the following of further development.

$$_{W}t_{t} \equiv v \times c \times {_{R}g_{t}} \tag{109}$$

In general, it is

$$_{W}t_{t}^{2} \equiv \left(v \times c \times {_{R}g_{t}}\right)^{2} \equiv {_{R}t_{t}}^{2} - {_{0}t_{t}}^{2} \tag{110}$$

and

$$_{W}g_{t} \equiv \frac{_{W}t_{t}}{c^{2}} \tag{111}$$

As such (see equation 110), it is a logical step to consider that

$$ {}_{R}g_{t} \equiv {}_{0}g_{t} + {}_{W}g_{t} \tag{112} $$

I should like to take this opportunity to express once again the possibility that ${}_{W}g_{t}$ itself might represent something similar to the gravitational waves. Let the mathematical identity ${}_{K}t_{t}$ be defined as follows.

$$ {}_{K}t_{t} \equiv \frac{{}_{W}t_{t} \times {}_{W}t_{t}}{{}_{R}t_{t}} \equiv \frac{{}_{W}t_{t}}{{}_{R}t_{t}} \times {}_{W}t_{t} \equiv \frac{(v \times c \times {}_{R}g_{t})^{2}}{c^{2} \times {}_{R}g_{t}} \equiv v^{2} \times {}_{R}g_{t} \tag{113} $$

The notion ${}_{K}t_{t}$ might indicate the time as determined by the relativistic kinetic energy ${}_{K}E_{t}$. Let the mathematical identity ${}_{P}t_{t}$ be defined as follows.

$$ {}_{P}t_{t} \equiv \frac{{}_{0}t_{t} \times {}_{0}t_{t}}{{}_{R}t_{t}} \equiv \frac{{}_{0}t_{t}}{{}_{R}t_{t}} \times {}_{0}t_{t} \equiv \left(\sqrt{1 - \frac{v^{2}}{c^{2}}}\right) \times {}_{0}t_{t} \tag{114} $$

The notion ${}_{P}t_{t}$ might indicate the time as determined by the relativistic potential energy ${}_{P}E_{t}$. In general, it is necessary to consider that,

$$ {}_{R}t_{t} \equiv {}_{P}t_{t} + {}_{K}t_{t} \tag{115} $$

Furthermore, the following identities are defined.

$$ {}_{K}g_{t} \equiv \frac{{}_{K}t_{t}}{c^{2}} \tag{116} $$

$$ {}_{P}g_{t} \equiv \frac{{}_{P}t_{t}}{c^{2}} \tag{117} $$

The identity ${}_{Kred}t_{t}$ is defined as

$$ {}_{Kred}t_{t} \equiv v \times {}_{R}g_{t} \tag{118} $$

Definition 2.44 (Space ${}_{R}S_{t}$).

We define the general relationship

$$ {}_{R}S_{t} \equiv {}_{0}S_{t} + {}_{0}\underline{S}_{t} \equiv {}_{R}U_{t} \times c^{2} \tag{119} $$

In case, that there are not justified reasons to doubt the correctness of Einstein's demand that all but matter is a gravitational field(Einstein, 1916, p. 802/803), we define

$$ {}_{R}U_{t} \equiv {}_{R}M_{t} + {}_{R}g_{t} \equiv \frac{{}_{R}S_{t}}{c^{2}} \tag{120} $$

where ${}_{R}U_{t}$ is the mathematical identity of matter ${}_{R}M_{t}$ and gravitational field ${}_{R}g_{t}$, ${}_{R}S_{t}$ is something like space and c is the speed of the light in vacuum. The following figure might illustrate this basic relationship from another point of view.

We multiply equation 120 by the term $\left(\sqrt{1-\frac{v^2}{c^2}}\right)$ where v is the relative velocity between a co-moving observer 0 and a stationary observer R. It is

$$\left({}_{\mathrm{R}}U_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right)\right) \equiv \left({}_{\mathrm{R}}M_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right)\right) + \left({}_{\mathrm{R}}g_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right)\right) \tag{121}$$

We define ${}_{0}\mathrm{U}_{\mathrm{t}}$ as

$$ {}_{0}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \tag{122}$$

According to Einstein, the rest-mass ${}_{0}\mathrm{m}_{\mathrm{t}}$ is given as

$$ {}_{0}m_{\mathrm{t}} \equiv {}_{\mathrm{R}}M_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \tag{123}$$

We define ${}_{0}\mathrm{g}_{\mathrm{t}}$ as

$$ {}_{0}g_{\mathrm{t}} \equiv {}_{\mathrm{R}}g_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \tag{124}$$

Equation 121 as seen from the point of view of a co-moving observer 0 becomes

$$ {}_{0}U_{\mathrm{t}} \equiv {}_{0}m_{\mathrm{t}} + {}_{0}g_{\mathrm{t}} \tag{125}$$

where ${}_{0}\mathrm{m}_{\mathrm{t}}$ indicates the rest mass as determined by the co-moving observer, ${}_{0}\mathrm{g}_{\mathrm{t}}$ is the gravitational field as determined by the co-moving observer and ${}_{0}\mathrm{U}_{\mathrm{t}}$ is the unity and the 'struggle' of both.

2.6. *Einstein's general theory or relativity*

Definition 2.45 (The Einstein field equations). *The Einstein field equations (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) describe the relationship between the presence of matter (represented by the stress-energy tensor* $\left(\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu}\right)$ *in a given region of spacetime and the curvature in that region by the equation*

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv E_{\mu\nu} \tag{126}$$

*where $R_{\mu\nu}$ is the Ricci tensor (Ricci-Curbastro and Levi-Civita, 1900) of 'Einstein's general theory of relativity' (Einstein, 1916), R is the Ricci scalar, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold, Λ is the Einstein's cosmological (Barukčić, 2015a, Einstein, 1917) constant, $\underline{\Lambda}$ is the "**anti cosmological constant**" (Barukčić, 2015a), $g_{\mu\nu}$ is the metric tensor of Einstein's general theory of relativity, $G_{\mu\nu}$ is Einstein's curvature tensor, $\underline{G}_{\mu\nu}$ is the "**anti tensor**" (Barukčić, 2016c) of Einstein's curvature tensor, $E_{\mu\nu}$ is the stress-energy tensor of energy, $\underline{E}_{\mu\nu}$ is the tensor of non-energy, the anti-tensor of the stress-energy tensor of energy, $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field, c is the speed of the light in vacuum, γ is Newton's gravitational "**constant**" (Barukčić, 2015a,b, 2016a,c), π is Archimedes constant pi.*

Einstein's field equations are defined in space-time dimensions (see Málek, 2012, p. 31) other than 3+1 too. Table 4 may provide a more detailed and preliminary overview of the definitions (Barukčić, 2016b,c) before.

		Curvature YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu} \equiv (c_{\mu\nu} + \Lambda\times g_{\mu\nu})$	$\frac{8\times\pi\times\gamma\times T}{c^4\times D}\times g_{\mu\nu} \equiv \left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv (b_{\mu\nu} - \Lambda\times g_{\mu\nu})$	$d_{\mu\nu} \equiv (\frac{R}{2}\times g_{\mu\nu} - b_{\mu\nu})$	$\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{D}-\frac{R}{2}\right)\times g_{\mu\nu}$	$\frac{R}{2}\times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D}\times g_{\mu\nu}$

Table 4. Four basic fields of nature and the Einstein field equations.

From Einstein's specific point of view, two wings are necessary to get to the core of the relationship between matter and gravitational field, just as two wings are essential for a bird that conquers the air.

We are quite privileged to consider in detail that

$$\underbrace{\left(\frac{R}{D}\times g_{\mu\nu}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)}_{the\ left-hand\ side}\equiv\underbrace{\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{\mu\nu}}_{the\ right-hand\ side} \tag{127}$$

while $R_{\mu\nu}\equiv a_{\mu\nu}+b_{\mu\nu}+c_{\mu\nu}+d_{\mu\nu}$ and the

> "... one wing ... is made of fine marble (**left side of the equation**) ...
>
> the other wing ... is built of low-grade wood (**right side of equation**).
>
> The phenomenological representation of matter is, in fact, only a crude substitute for a representation which would do justice to all known properties of matter. "
>
> (Einstein, 1936, p. 370)

Taken together, the n[th] index, D-dimensional Einstein's gravitational field equations (Barukčić, 2020d) follow as

$$\underbrace{\left(\frac{R}{D}\times g_{\mu\nu\pi\rho\ldots}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu\pi\rho\ldots}\right)+\left(\Lambda\times g_{\mu\nu\pi\rho\ldots}\right)}_{(local)\ space-time\ curvature}\equiv\underbrace{\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{\mu\nu\pi\rho\ldots}}_{(local)\ energy\ and\ momentum} \tag{128}$$

In general, the metric field (responsible for gravitational-inertial properties of bodies) on the left-hand side of Einstein's field equations, is completely determined by a tensorial but non-geometrical phenomenological representation of matter on the right-hand side. Einstein himself had a very differentiated view of these two sides of his field equations. In point of fact, the left part of the Einstein field equations (the Einstein tensor) is taken by Einstein as fine marble because of its geometrical nature, whereas the right side of the equations is lacking similar geometric significance and was degraded by Einstein himself to low-grade wood, the need for geometrical unification follows at least from such an asymmetrical state of affairs. An incorporation of electromagnetism and of other fields into spacetime geometry is desirable. In point of fact, a striving toward unification and simplification of the premises and of Einstein's general theory of relativity as a whole is necessary.

> "The mind striving after unification of the theory cannot be satisfied that two fields should exist which, by their nature, are quite independent. A mathematically **unified field theory** is sought in which **the gravitational field and the electromagnetic field are interpreted only as different components or manifestations of the same uniform field** ... The gravitational theory ... should be generalized so that it includes the laws of the electromagnetic field. "
>
> (Einstein, 1923a, p. 489)

Definition 2.46 (**The stress-energy tensor of ordinary matter $a_{\mu\nu}$**). *Howard Georgi and Sheldon Glashow (Georgi and Glashow, 1974) proposed in 1974 the first Grand Unified Theory (Buras et al., 1978). Grand Unified Theory (GUT) models predict the unification of the electromagnetic, the weak, and the strong forces into a single force. However, it appears to be more appropriate to unify the strong nuclear force and the weak nuclear force into an ordinary force. The matter as associated with an ordinary force can be calculated very precisely. Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity,* ***the stress-energy tensor of ordinary matter $a_{\mu\nu}$*** *which* ***is expected to unify the strong nuclear force and the weak nuclear force into an ordinary force*** *is defined / derived / determined as*

$$\begin{aligned} a_{\mu\nu} &\equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) - b_{\mu\nu} \\ &\equiv G_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) - b_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left(R \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) + d_{\mu\nu} \\ &\equiv (E - b) \times g_{\mu\nu} \\ &\equiv (G - c) \times g_{\mu\nu} \\ &\equiv a \times g_{\mu\nu} \end{aligned} \tag{129}$$

or

$$\begin{aligned} a_{\mu\nu} \equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) - \\ \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \end{aligned} \tag{130}$$

From our present point of view we can expect that there are conditions where

$$\begin{aligned} a_{\mu\nu} &\equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) - b_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) - \left(\frac{(4 + D) \times F_1}{4 \times \pi \times 4 \times D}\right)\right) \times g_{\mu\nu} \end{aligned} \tag{131}$$

where F_1 is Lorenz invariant.

Definition 2.47 (**The 4-index D dimensional a $_{kl\mu\nu}$**). *The 4-index D dimensional $a_{kl\mu\nu}$ is defined as:*

$$\begin{aligned} a_{kl\mu\nu} &\equiv (E - b) \times g_{kl\mu\nu} \\ &\equiv (G - c) \times g_{kl\mu\nu} \\ &\equiv a \times g_{kl\mu\nu} \end{aligned} \tag{132}$$

Definition 2.48 (**The n-index D dimensional a $_{kl\mu\nu\ldots}$**). *The n-index D dimensional $a_{kl\mu\nu\ldots}$ is defined as:*

$$\begin{aligned} a_{kl\mu\nu\ldots} &\equiv (E - b) \times g_{kl\mu\nu\ldots} \\ &\equiv (G - c) \times g_{kl\mu\nu\ldots} \\ &\equiv a \times g_{kl\mu\nu\ldots} \end{aligned} \tag{133}$$

Definition 2.49 (**Ricci scalar R**). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the Ricci scalar curvature R as the trace of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric is determined at each point in space-time by lamda Λ and anti-lamda (Barukčić, 2015a) $\underline{\Lambda}$ as*

$$R \equiv g^{\mu\nu} \times R_{\mu\nu} \equiv (\Lambda) + (\underline{\Lambda}) \equiv D \times S \tag{134}$$

where D is proved as the number of space-time dimension and $S \equiv \left(\frac{R}{D}\right)$. *A Ricci scalar curvature R which is positive at a certain point indicates that the volume of a small ball about the point has smaller volume than a ball of the same radius in Euclidean space. In other words, the density of space varies. In contrast to this, a Ricci scalar curvature R which is negative at a certain point indicates that the volume of a small ball is larger than it would be in Euclidean space. In general, it is (see Barukčić, 2015a)*

$$R \times g_{\mu\nu} \equiv \left(\Lambda \times g_{\mu\nu}\right) + \left(\underline{\Lambda} \times g_{\mu\nu}\right) \tag{135}$$

or

$$R \equiv (\Lambda) + (\underline{\Lambda}) \tag{136}$$

The cosmological constant can also be written algebraically as part of the stress–energy tensor, a second order tensor as the source of gravity (energy density).

Definition 2.50 (**Ricci tensor $\mathbf{R}_{\mu\nu}$**). *The Ricci tensor $R_{\mu\nu}$ is a geometric object which has been developed by Gregorio Ricci-Curbastro (1853 – 1925) (Ricci-Curbastro and Levi-Civita, 1900) and is able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. In this publication, let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the covariant second rank tensors of the four basic fields of nature were $a_{\mu\nu} \equiv {}_{f}a^2 \times g_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu} \equiv {}_{f}b^2 \times g_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field, $c_{\mu\nu} \equiv c^2 \times g_{\mu\nu}$ is the tensor of the gravitational field and $d_{\mu\nu} \equiv {}_{f}d^2 \times g_{\mu\nu}$ is the tensor of gravitational waves. The Ricci tensor $R_{\mu\nu}$ of 'Einstein's general theory of relativity' (Einstein, 1916) is determined by the stress-energy tensor* $\left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right)$ *and the anti stress-energy tensor* $\left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right)\right)$ *as*

$$\begin{aligned}
R_{\mu\nu} &\equiv \underbrace{\left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right)}_{stress-energy\ tensor} + \underbrace{\left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right)\right)}_{anti\ stress-energy\ tensor} \\
&\equiv \qquad a_{\mu\nu} + b_{\mu\nu} \qquad\qquad + c_{\mu\nu} + d_{\mu\nu} \\
&\equiv (S) \times g_{\mu\nu} \\
&\equiv \left(\frac{R}{D}\right) \times g_{\mu\nu}
\end{aligned} \tag{137}$$

while S might denote a scalar.

Definition 2.51 (**Laue's scalar T**). *Max von Laue (1879-1960) proposed the meanwhile so called Laue scalar (Laue, 1911) (criticised by Einstein (Einstein and Grossmann, 1913)) as the contraction of* *the the stress–energy momentum tensor $T_{\mu\nu}$ denoted as T and written without subscripts or arguments. Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, it is*

$$T \equiv g^{\mu\nu} \times T_{\mu\nu} \tag{138}$$

Taken Einstein seriously, $T_{\mu\nu}$ "denotes the co-variant energy tensor of matter" (see Einstein, 1923b, p. 88). In other words, "Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense." (see Einstein, 1923b, p. 93)

Definition 2.52 (The scalar E). *In general, we define the scalar E as*

$$\begin{aligned} E \equiv {}_{d}E_{t}{}^{2} &\equiv \left(\frac{8 \times \pi \times \gamma}{c^4 \times D}\right) \times T \\ &\equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \\ &\equiv \left(\frac{2 \times \pi \times 4 \times \gamma \times T}{c^4 \times D}\right) \\ &\equiv \left(\frac{h \times 4 \times \gamma \times T}{\hbar \times c^4 \times D}\right) \\ &\equiv \left(\frac{R}{D}\right) - \left(\frac{R}{2}\right) + \Lambda \end{aligned} \tag{139}$$

*where D is the space-time dimension, where c denote the speed of the light in vacuum, γ denote Newton's gravitational "**constant**" (Barukčić, 2015a,b, 2016a,c), π is the number pi and T denote Laue's scalar. The scalar E might correspond even to the total energy density squared of a (relativistic or quantum) system, and has the potential as such to bridge the gap between relativity theory and quantum mechanics under circumstances where the same is related or even identical with the Hamiltonian operator (squared).*

Definition 2.53 (Stress-energy and momentum tensor $E_{\mu\nu}$). *"Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense." (see also Einstein, 1923b, p. 93) The tensor of stress-energy-momentum denoted as $E_{\mu\nu}$ is determined in detail as follows.*

$$
\begin{aligned}
E_{\mu\nu} &\equiv a_{\mu\nu} + b_{\mu\nu} \\
&\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\
&\equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \\
&\equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \\
&\equiv \left(S - \left(\frac{R}{2}\right) + \Lambda\right) \times g_{\mu\nu} \\
&\equiv (G + \Lambda) \times g_{\mu\nu} \\
&\equiv G_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) \\
&\equiv R_{\mu\nu} - \underline{E}_{\mu\nu} \\
&\equiv E \times g_{\mu\nu}
\end{aligned}
\tag{140}
$$

while E might denote the scalar of, even something like 'energy density'. According to Einstein, it is necessary to consider that

"... a tensor, $T_{\mu\nu}$, of the second rank ... includes in itself

the energy density of the electromagnetic field

and of

ponderable matter;

we shall denote this in the following as the 'energy tensor of matter'"

(Einstein, 1923b, pp. 87/88)

Definition 2.54 (The scalar G). *In general, we define the scalar G (Barukčić, 2020b) as*

$$
\begin{aligned}
G \equiv {}_{d}G_{t}^{2} &\equiv \left(\left(\frac{R}{D}\right) - \frac{R}{2}\right) \\
&\equiv \left(E + {}_{R}t_{t} - \frac{R}{2}\right) \\
&\equiv \left(E + \left(\frac{R}{2} - \Lambda\right) - \frac{R}{2}\right) \\
&\equiv E - \Lambda
\end{aligned}
\tag{141}
$$

Definition 2.55 (**Einstein's curvature tensor $G_{\mu\nu}$**). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of curvature denoted by $G_{\mu\nu}$ is defined/derived/determined (see Barukčić, 2020b) as follows:*

$$
\begin{aligned}
G_{\mu\nu} &\equiv R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \\
&\equiv \left(\frac{R}{D} \right) \times g_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \\
&\equiv \left(\left(\frac{R}{D} \right) - \frac{R}{2} \right) \times g_{\mu\nu} \\
&\equiv a_{\mu\nu} + c_{\mu\nu} \\
&\equiv G \times g_{\mu\nu} \\
&\equiv \left(\frac{R}{D} \right) \times {}_{G}g_{\mu\nu}
\end{aligned}
\tag{142}
$$

Definition 2.56 (**The scalar $\underline{G}$**). *In general, we define the scalar $\underline{G}$ (see Barukčić, 2020b) as*

$$
\begin{aligned}
\underline{G} \equiv {}_{d}\underline{G}_{t}{}^{2} &\equiv \left(\left(\frac{R}{D} \right) - G \right) \\
&\equiv \left(\frac{R}{2} \right)
\end{aligned}
\tag{143}
$$

Definition 2.57 (**The scalar $\underline{E}$**). *In general, we define the scalar $\underline{E}$ as (see Barukčić, 2020b)*

$$
\begin{aligned}
\underline{E} \equiv {}_{d}\underline{E}_{t}{}^{2} &\equiv \left(\left(\frac{R}{D} \right) - E \right) \\
&\equiv \left(\frac{R}{2} - \Lambda \right)
\end{aligned}
\tag{144}
$$

Remark 2.1. *In the following of research, it is appropriate to prove the relationship between (1/X) and the complex conjugate of the wave function Ψ^{*} or the identity (1/X)$\equiv \Psi^{*}$.*

Definition 2.58 (**The anti Einstein's curvature tensor or the tensor of non-curvature $\underline{G}_{\mu\nu}$**). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of non-curvature is defined/derived/determined (Barukčić, 2020b) as follows:*

$$
\begin{aligned}
\underline{G}_{\mu\nu} &\equiv R_{\mu\nu} - G_{\mu\nu} \\
&\equiv R_{\mu\nu} - \left(R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \right) \\
&\equiv \left(\frac{R}{2} \right) \times g_{\mu\nu} \\
&\equiv b_{\mu\nu} + d_{\mu\nu} \\
&\equiv \underline{G} \times g_{\mu\nu}
\end{aligned}
\tag{145}
$$

Definition 2.59 (The 4-index D dimensional stress-energy and momentum tensor $E_{kl\mu\nu}$). *The 4-index D dimensional stress-energy-momentum tenosr denoted as $E_{kl\mu\nu}$ is determined in detail as*

$$\begin{aligned} E_{kl\mu\nu} &\equiv \left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{kl\mu\nu} \\ &\equiv R_{kl\mu\nu}-\left(\left(\frac{R}{2}\right)\times g_{kl\mu\nu}\right)+\left(\Lambda\times g_{kl\mu\nu}\right) \\ &\equiv G_{kl\mu\nu}+\left(\Lambda\times g_{kl\mu\nu}\right) \\ &\equiv R_{kl\mu\nu}-\underline{E}_{kl\mu\nu} \\ &\equiv a_{kl\mu\nu}+b_{kl\mu\nu} \\ &\equiv H\times g_{kl\mu\nu}\equiv H_{kl\mu\nu} \\ &\equiv E\times g_{kl\mu\nu} \end{aligned} \tag{146}$$

Definition 2.60 (The n-index D dimensional stress-energy and momentum tensor $E_{kl\mu\nu\ldots}$). *The n-index D dimensional stress-energy-momentum tenosr denoted as $E_{kl\mu\nu\ldots}$ is determined in detail as*

$$\begin{aligned} E_{kl\mu\nu\ldots} &\equiv \left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{kl\mu\nu\ldots} \\ &\equiv R_{kl\mu\nu\ldots}-\left(\left(\frac{R}{2}\right)\times g_{kl\mu\nu\ldots}\right)+\left(\Lambda\times g_{kl\mu\nu\ldots}\right) \\ &\equiv G_{kl\mu\nu\ldots}+\left(\Lambda\times g_{kl\mu\nu\ldots}\right) \\ &\equiv R_{kl\mu\nu\ldots}-\underline{E}_{kl\mu\nu\ldots} \\ &\equiv a_{kl\mu\nu\ldots}+b_{kl\mu\nu\ldots} \\ &\equiv H\times g_{kl\mu\nu\ldots}\equiv H_{kl\mu\nu\ldots} \\ &\equiv E\times g_{kl\mu\nu\ldots} \end{aligned} \tag{147}$$

Definition 2.61 (The tensor of non-energy $\underline{E}_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of non-energy or the anti tensor of the stress energy tensor is defined/derived/determined as follows:*

$$\begin{aligned} \underline{E}_{\mu\nu} &\equiv R_{\mu\nu}-\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu} \\ &\equiv\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)-\left(\Lambda\times g_{\mu\nu}\right) \\ &\equiv\left(\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu}\right) \\ &\equiv c_{\mu\nu}+d_{\mu\nu} \\ &\equiv\Psi\times g_{\mu\nu}\equiv\Psi_{\mu\nu} \\ &\equiv\underline{E}\times g_{\mu\nu} \end{aligned} \tag{148}$$

Definition 2.62 (The 4-index D dimensional tensor of non-energy $\underline{E}_{kl\mu\nu}$). *The 4-index D dimensional tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) of non-energy $\underline{E}_{kl\mu\nu}$ is defined as follows:*

$$
\begin{aligned}
\underline{E}_{kl\mu\nu} &\equiv \left(\frac{R}{D} \times g_{kl\mu\nu}\right) - \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{kl\mu\nu}\right)\\
&\equiv \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right) - \left(\Lambda \times g_{kl\mu\nu}\right)\\
&\equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{kl\mu\nu}\right)\\
&\equiv c_{kl\mu\nu} + d_{kl\mu\nu}\\
&\equiv \Psi \times g_{kl\mu\nu} \equiv \Psi_{kl\mu\nu}\\
&\equiv \underline{E} \times g_{kl\mu\nu}
\end{aligned} \tag{149}
$$

Definition 2.63 (The n-th index D dimensional tensor of non-energy $\underline{E}_{kl\mu\nu\ldots}$). *The n-th index D dimensional tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) of non-energy $\underline{E}_{kl\mu\nu\ldots}$ is defined as follows:*

$$
\begin{aligned}
\underline{E}_{kl\mu\nu\ldots} &\equiv \left(\frac{R}{D} \times g_{kl\mu\nu\ldots}\right) - \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{kl\mu\nu\ldots}\right)\\
&\equiv \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right) - \left(\Lambda \times g_{kl\mu\nu\ldots}\right)\\
&\equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{kl\mu\nu\ldots}\right)\\
&\equiv c_{kl\mu\nu\ldots} + d_{kl\mu\nu\ldots}\\
&\equiv \Psi \times g_{kl\mu\nu\ldots} \equiv \Psi_{kl\mu\nu\ldots}\\
&\equiv \underline{E} \times g_{kl\mu\nu\ldots}
\end{aligned} \tag{150}
$$

Definition 2.64 (The 4-index D dimensional Einstein's curvature tensor $G_{kl\mu\nu}$). *The Riemann tensor $R_{kl\mu\nu}$ does not appear explicitly in Einstein's gravitational field equations. Therefore, the question is justified whether Einstein's equation of gravitation are really the most general equations. Frèdèric Moulin proposed in the year 2017 a kind of a generalized 4-index gravitational field equation which contains the Riemann curvature tensor linearly (Moulin, 2017). Moulin himself ascribed an energy-momentum to the gravitational field itself (Moulin, 2017, p. 5/8) which is not without problems. Besides of all, it is known that the Riemann curvature tensor of general relativity $R_{kl\mu\nu}$ can be split into different ways, including the Weyl conformal tensor $C_{kl\mu\nu}$ and the anti-Weyl conformal tensor $\underline{C}_{kl\mu\nu}$ or in other words the parts which involve only the Ricci tensor $R_{\mu\nu}$ the curvature scalar R. Because of these properties $\left(R_{kl\mu\nu} \equiv C_{kl\mu\nu} + \underline{C}_{kl\mu\nu}\right)$ it is possible to reformulate the famous Einstein equation. The 4-index D dimensional Einstein's curvature tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) denoted by $G_{kl\mu\nu}$ is defined (see Barukčić, 2020b) as follows:*

$$
\begin{aligned}
G_{kl\mu\nu} &\equiv R_{kl\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right) \\
&\equiv \left(\frac{R}{D}\right) \times g_{kl\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right) \\
&\equiv \left(\left(\frac{R}{D}\right) - \frac{R}{2}\right) \times g_{kl\mu\nu} \\
&\equiv a_{kl\mu\nu} + c_{kl\mu\nu} \\
&\equiv G \times g_{kl\mu\nu}
\end{aligned} \tag{151}
$$

Definition 2.65 (The n-index D dimensional Einstein's curvature tensor $\mathbf{G}_{\mathbf{kl}\mu\nu\ldots}$). *The n-index D dimensional Einstein's curvature tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) denoted by $G_{kl\mu\nu\ldots}$ is defined (see Barukčić, 2020b) as follows:*

$$
\begin{aligned}
G_{kl\mu\nu\ldots} &\equiv R_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right) \\
&\equiv \left(\frac{R}{D}\right) \times g_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right) \\
&\equiv \left(\left(\frac{R}{D}\right) - \frac{R}{2}\right) \times g_{kl\mu\nu\ldots} \\
&\equiv a_{kl\mu\nu\ldots} + c_{kl\mu\nu\ldots} \\
&\equiv G \times g_{kl\mu\nu\ldots}
\end{aligned} \tag{152}
$$

Definition 2.66 (The 4-index D dimensional anti Einstein's curvature tensor or the tensor or non-curvature $\underline{\mathbf{G}}_{\mathbf{kl}\mu\nu}$). *The 4-index D dimensional anti Einstein's curvature tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) or the tensor of non-curvature denoted as $\underline{G}_{kl\mu\nu}$ is defined/derived/determined (Barukčić, 2020b) as follows:*

$$
\begin{aligned}
\underline{G}_{kl\mu\nu} &\equiv R_{kl\mu\nu} - G_{kl\mu\nu} \\
&\equiv R_{kl\mu\nu} - \left(R_{kl\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right)\right) \\
&\equiv \left(\frac{R}{2}\right) \times g_{kl\mu\nu} \\
&\equiv b_{kl\mu\nu} + d_{kl\mu\nu} \\
&\equiv \underline{G} \times g_{kl\mu\nu}
\end{aligned} \tag{153}
$$

Definition 2.67 (The n-index D dimensional anti Einstein's curvature tensor or the tensor of non-curvature $\underline{\mathbf{G}}_{\mathbf{kl}\mu\nu\ldots}$). *The n-index D dimensional anti Einstein's curvature tensor or the tensor of non-curvature denoted as $\underline{G}_{kl\mu\nu\ldots}$ is defined/derived/determined (Barukčić, 2020b) as follows:*

$$
\begin{aligned}
\underline{G}_{kl\mu\nu\ldots} &\equiv R_{kl\mu\nu\ldots} - G_{kl\mu\nu\ldots} \\
&\equiv R_{kl\mu\nu\ldots} - \left(R_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2} \right) \times g_{kl\mu\nu\ldots} \right) \right) \\
&\equiv \left(\frac{R}{2} \right) \times g_{kl\mu\nu\ldots} \\
&\equiv b_{kl\mu\nu\ldots} + d_{kl\mu\nu\ldots} \\
&\equiv \underline{G} \times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{154}
$$

Definition 2.68 (The first quadratic Lorentz invariant $\mathbf{F_1}$). *The inner product of Faraday's electromagnetic field strength tensor yields a Lorentz invariant. The Lorentz invariant does not change from one frame of reference to another. The first quadratic Lorentz invariant, denoted as F_1 is determined as*

$$
F_1 \equiv F_{kl} \times F^{kl} \tag{155}
$$

The electromagnetic field tensor $\mathrm{F_{kl}}$ has two Lorentz invariant quantities. One of the two fundamental Lorentz invariant quantities of the electromagnetic field (Escobar and Urrutia, 2014) is known be $F_{\mathrm{kl}} \times F^{\mathrm{kl}} = 2 \times \left(B^2 - E^2\right)$ where E denotes the electric E and B the magnetic field in the taken frame of reference.

Definition 2.69 (The second quadratic Lorentz invariant $\mathbf{F_2}$). *The second quadratic Lorentz invariant, denoted as F_2 is determined as*

$$
F_2 \equiv \varepsilon^{klmn} \times F_{kl} \times F_{mn} \tag{156}
$$

Definition 2.70 (The tensor $\mathbf{b}_{\mu\nu}$). *The co-variant Minkowski's stress-energy tensor of the electromagnetic field, in this context denoted by $b_{\mu\nu}$, is of order two and its components can be displayed by a 4 × 4 matrix too. The trace of energy-momentum tensor of the electromagnetic field is known to be null. Under conditions of Einstein's general theory of relativity (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932), the tensor $b_{\mu\nu}$ denotes the trace-less, symmetric stress-energy tensor for source-free electromagnetic field is defined in cgs-Gaussian units (**depending upon metric signature**) as*

$$
b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu}{}^{c} \right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de} \right) \right) \right) \tag{157}
$$

(see Lehmkuhl, 2011, p. 13) and equally as

$$
b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd} \right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de} \right) \right) \right) \tag{158}
$$

(see Hughston and Tod, 1990, p. 38) [1] *. The co-variant Minkowski's stress-energy tensor of the electromagnetic field is expressed under conditions of D = 4 space-time dimensions more compactly in a coordinate-independent (theorem 3.1, equation 80 Barukčić, 2020b, p. 157) form as*

$$\begin{aligned} b_{\mu\nu} &\equiv \left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\, c}\times F_{\nu\, d}\times g^{cd}\right)+\left(\frac{1}{4}\times g_{\mu\nu}\times F_{de}\times F^{de}\right)\right)\right) \\ &\equiv \left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\, c}\times F^{\mu\, c}\right)+\left(\frac{F_{I}}{4}\right)\right)\right)\times g_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{D}\right)-a-c-d\right)\times g_{\mu\nu} \\ &\equiv (E-a)\times g_{\mu\nu} \\ &\equiv b\times g_{\mu\nu} \end{aligned} \tag{159}$$

where F_{de} is called the (traceless) Faraday/electromagnetic/field strength tensor.

Definition 2.71 (The 4-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu}$). *The 4-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu}$ is defined as:*

$$\begin{aligned} b_{kl\mu\nu} &\equiv \left(\left(\frac{R}{D}\right)-a-c-d\right)\times g_{kl\mu\nu} \\ &\equiv (E-a)\times g_{kl\mu\nu} \\ &\equiv b\times g_{kl\mu\nu} \end{aligned} \tag{160}$$

Definition 2.72 (The n-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu\,\ldots}$). *The n-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu\,\ldots}$ is defined as:*

$$\begin{aligned} b_{kl\mu\nu\,\ldots} &\equiv \left(\left(\frac{R}{D}\right)-a-c-d\right)\times g_{kl\mu\nu\,\ldots} \\ &\equiv (E-a)\times g_{kl\mu\nu\,\ldots} \\ &\equiv b\times g_{kl\mu\nu\,\ldots} \end{aligned} \tag{161}$$

Definition 2.73 (The tensor $c_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of non-momentum and curvature is defined/derived/determined (Barukčić, 2020b) as follows:*

$$\begin{aligned} c_{\mu\nu} &\equiv b_{\mu\nu}-\left(\Lambda\times g_{\mu\nu}\right) \\ &\equiv (G-a)\times g_{\mu\nu} \\ &\equiv \left(\frac{R}{2}-\Lambda-d\right)\times g_{\mu\nu} \\ &\equiv (b-\Lambda)\times g_{\mu\nu} \\ &\equiv c\times g_{\mu\nu} \end{aligned} \tag{162}$$

[1] L. P. Hughston and K. P. Tod. An introduction to general relativity. Cambridge University Press, Cambridge ; New York, 1990. ISBN 978-0-521-32705-3.

Definition 2.74 (**The 4-index D dimensional tensor c** $_{\mathbf{kl}\mu\nu}$). *The 4-index D dimensional* $c_{kl\mu\nu}$ *is defined as:*

$$
\begin{aligned}
c_{kl\mu\nu} &\equiv (G-a)\times g_{kl\mu\nu}\\
&\equiv \left(\frac{R}{2}-\Lambda-d\right)\times g_{kl\mu\nu}\\
&\equiv (b-\Lambda)\times g_{kl\mu\nu}\\
&\equiv c\times g_{kl\mu\nu}
\end{aligned}
\tag{163}
$$

Definition 2.75 (**The n-index D dimensional tensor c** $_{\mathbf{kl}\mu\nu\ldots}$). *The n-index D dimensional* $c_{kl\mu\nu\ldots}$ *is defined as:*

$$
\begin{aligned}
c_{kl\mu\nu\ldots} &\equiv (G-a)\times g_{kl\mu\nu\ldots}\\
&\equiv \left(\frac{R}{2}-\Lambda-d\right)\times g_{kl\mu\nu\ldots}\\
&\equiv (b-\Lambda)\times g_{kl\mu\nu\ldots}\\
&\equiv c\times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{164}
$$

Definition 2.76 (**The tensor of neither curvature nor momentum d** $_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of neither curvature nor momentum is defined/derived/determined (Barukčić, 2020b) as follows:*

$$
\begin{aligned}
d_{\mu\nu} &\equiv \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)-b_{\mu\nu}\\
&\equiv \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)-\left(\Lambda\times g_{\mu\nu}\right)-c_{\mu\nu}\\
&\equiv \left(\frac{\left(\left(\frac{R}{D}\right)\times D\right)}{2}-b\right)\times g_{\mu\nu}\\
&\equiv \left(\frac{\left(\left(\frac{R}{D}\right)\times D\right)}{2}-\Lambda-c\right)\times g_{\mu\nu}\\
&\equiv \frac{R}{D}\times {}_{gw}g_{\mu\nu}\\
&\equiv d\times g_{\mu\nu}
\end{aligned}
\tag{165}
$$

There may exist circumstances where this tensor might indicate something like the density of gravitational waves. In detail, it is

$$
d_{\mu\nu}\equiv\frac{R}{D}\times {}_{gw}g_{\mu\nu}\equiv\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)-\left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\,c}\times F_{\nu\,d}\times g^{cd}\right)+\left(\frac{1}{4}\times g_{\mu\nu}\times F_{de}\times F^{de}\right)\right)\right)
\tag{166}
$$

Under these circumstances, the metric tensor of the gravitational waves ${}_{gw}g_{\mu\nu}$ *would follow as*

$$ {}_{d}g_{\mu\nu} \equiv {}_{gw}g_{\mu\nu} \equiv \frac{D}{R} \times \left(\left(\left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) - \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd} \right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de} \right) \right) \right) \right) \right) \tag{167} $$

The cosmic microwave background (CMBR) radiation (Penzias and Wilson, 1965) is an electromagnetic radiation which is part of the tensor $b_{\mu\nu}$.

Definition 2.77 (**The 4-index D dimensional d** $_{\mathbf{kl}\mu\nu}$). *The 4-index D dimensional* $d_{kl\mu\nu}$ *is defined as:*

$$ \begin{aligned} d_{kl\mu\nu} &\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - b \right) \times g_{kl\mu\nu} \\ &\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - \Lambda - c \right) \times g_{kl\mu\nu} \\ &\equiv d \times g_{kl\mu\nu} \end{aligned} \tag{168} $$

Definition 2.78 (**The n-index D dimensional d** $_{\mathbf{kl}\mu\nu\ldots}$). *The n-index D dimensional* $d_{kl\mu\nu\ldots}$ *is defined as:*

$$ \begin{aligned} d_{kl\mu\nu\ldots} &\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - b \right) \times g_{kl\mu\nu\ldots} \\ &\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - \Lambda - c \right) \times g_{kl\mu\nu\ldots} \\ &\equiv d \times g_{kl\mu\nu\ldots} \end{aligned} \tag{169} $$

2.7. *Axioms*

Whether science needs new and obviously generally valid statements (axioms) which are able to assure the truth of theorems proved from them may remain an unanswered question. In order to be accepted, a new axiom candidate (see Easwaran, 2008) should be at least as simple as possible and logically consistent to enable advances in our knowledge of nature. The importance of axioms is particularly emphasized by Albert Einstein. "**Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden**." (see Einstein, 1919, p. 17). In general, lex identitatis, lex contradictionis and lex negationis have the potential to denote the most simple, the most general and the most far-reaching axioms of science, the foundation of our today's and of our future scientific inquiry.

2.7.1. Principium identitatis (Axiom I)

Principium identitatis or **lex identitatis** or axiom I, is closely related to central problems of metaphysics, epistemology and of science as such. It turns out that it is more than rightful to assume that

$$+1 \equiv +1 \tag{170}$$

is true, otherwise there is every good reason to suppose that nothing can be discovered at all.

Identity as the epitome of a self-identical or of self-reference is at the same time different from difference, identity is free from difference, identity is not difference, identity is at the same time the other of itself, identity is non-identity. Identity as simple equality with itself is determined by a non-being, by a non-being of its own other, by a non-being of difference, identity is different from difference. Identity is in its very own nature different and is in its own self the opposite of itself (symmetry). It is equally

$$-1 \equiv -1 \tag{171}$$

In general, +1 and -1 are distinguished, however these distinct are related to one and the same 1. Identity as a vanishing of otherness, therefore, is this distinguishedness in one relation. It is

$$0 \equiv +1 - 1 \equiv 0 \times 1 \equiv 0 \tag{172}$$

Identity, as the unity of something and its own other is in its own self a separation from difference, and as a moment of separation might pass over into an equivalence relation which itself is reflexive, symmetric and transitive. Nonetheless, backed by thousands of years of often bitter human experience, the scientific development has taught us all that human knowledge is relative too. Even if experiments and other suitable proofs are of help to encourage us more and more in our belief of the correctness of a theory, it is difficult to prove the correctness of a theorem or of a theory et cetera once and for all. The challenge for all the science is the need to comply with Einstein's position: **"Niemals aber kann die Wahrheit einer Theorie erwiesen werden. Denn niemals weiß man, daß auch in Zukunft eine Erfahrung bekannt werden wird, die Ihren Folgerungen widerspricht..."** (Einstein, 1919).

Albert Einstein's position translated into English: 'But the truth of a theory can never be proven. For one never knows if future experience will contradict its conclusion; and furthermore, there are always other conceptual systems imaginable which might coordinate the very same facts.'Our human experience tells us that everything in life is more or less transitory, and that nothing lasts. As a result of our knowledge and experience, several scientific theories have a glorious past to look back on, but all the glory of such scientific theories might remain in the past if scientist don't continue to innovate. In a word, theories can be refuted by time.

"No amount of experimentation can ever prove me right;
a single experiment can prove me wrong."

(Albert Einstein according to: Robertson, 1998, p. 114)

In the light of the foregoing, it is clear that appropriate axioms and conclusions derived from the same are a main logical foundation of any 'theory'.

"**Grundgesetz (Axiome)** und **Folgerungen** zusammen bilden das was man **eine 'Theorie'** nennt. "

(Einstein, 1919)

However, another point is worth being considered again. One single experiment can be enough to refute a whole theory. Albert Einstein's (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a 'theory'* has still to get round. However, an axiom as a free creation of the human mind which precedes all science should be like all other axioms, as simple as possible and as self-evident as possible. Historically, the earliest documented use of **the law of identity** can be found in Plato's dialogue Theaetetus (185a) as "... each of the two is different from the other and the same as itself "[2] . However, Aristotle (384–322 B.C.E.), Plato's pupil and equally one of the greatest philosophers of all time, elaborated on the law of identity too. In Metaphysica, Aristotle wrote:

"... all things ... have some unity and identity. "

(see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica, Chapter IV, 999a, 25-30, p. 66)

[2]Plato's dialogue Theaetetus (185a), p. 104.

In Prior Analytics, [3] , [4] Aristotle, a tutor of Alexander, the thirteen-year-old son of Philip, the king of Macedon, is writing: “When A applies to the whole of B and of C, and is other predicated of nothing else, and B also applies to all C, A and B must be convertible. For since A is stated only of B and C, and B is predicated both of itself and of C, it is evident that B will also be stated of all subjects of which A is stated, except A itself. ”[5] , [6] For the sake of completeness, it should be noted at the outset that Aristotle himself preferred **the law of contradiction** and **the law of excluded middle** as examples of fundamental axioms. Nonetheless, it is worth noting that **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and equally the most simple axiom of science but has been treated by Aristotle in an inadequate manner without having any clear and determined meaning for Aristotle himself. Nonetheless, something which is really just itself is equally different from everything else. In point of fact, is such an equivalence (Degen, 1741) which everything has to itself inherent or must the same be constructed by human mind and consciousness. Can and how can something be **identical with itself** (Förster and Melamed, 2012, Hegel, Georg Wilhelm Friedrich, 1812a, Koch, 1999, Newstadt, 2015) and in the same respect different from itself. An increasingly popular view on identity is the one advocated by Gottfried Wilhelm Leibniz (1646-1716):

“Chaque chose est ce qu’elle est. Et dans autant d’exemples qu’on voudra
A est A,
B est B. ”
(Leibniz, 1765, p. 327)

or **A = A**, **B = B** or **+1 = +1**. In other words, a thing is what it is (Leibniz, 1765, p. 327). Leibniz’ **principium identitatis indiscernibilium** (p.i.i.), the principle of the indistinguishable, occupies a central position in Leibniz’ logic and metaphysics and was formulated by Leibniz himself in different ways in different passages (1663, 1686, 1704, 1715/16). All in all, Leibniz writes:

“C’est
le principe des indiscernables,
en vertu duquel
il ne saurait exister dans la nature deux êtres identiques.
...
Il n’y a point deux individus indiscernables. ”
(see Leibniz, Gottfried Wilhelm, 1886, p. 45)

Exactly in complete compliance with Leibniz, Johann Gottlieb Fichte (1762 - 1814) elaborates on this subject as follows:

[3] Aristotle, Prior Analytics, Book II, Part 22, 68a
[4] Kenneth T. Barnes. Aristotle on Identity and Its Problems. Phronesis. Vol. 22, No. 1 (1977), pp. 48-62 (15 pages)
[5] Aristotle, Prior Analytics, Book II, Part 22, 68a, p. 511.
[6] Ivo Thomas. On a passage of Aristotle. Notre Dame J. Formal Logic 15(2): 347-348 (April 1974). DOI: 10.1305/ndjfl/1093891315

“Each thing is what it is ;
it has those realities which are posited when it is posited,
(A = A.) ”
(Fichte, 1889)

Georg Wilhelm Friedrich Hegel (1770 – 1831) himself objected the Law of Identity by claiming that “A = A is ... an empty tautology. ”(see Hegel, Georg Wilhelm Friedrich, 1991, p. 413) provided an example of his own mechanical understanding of the Law of Identity. “the empty tautology: nothing is nothing; ... from nothing only nothing becomes ... nothing remains nothing. ”(see Hegel, Georg Wilhelm Friedrich, 1991, p. 84). Nonetheless, Hegel preferred to reformulate an own version of Leibniz principium identitatis indiscernibilium in his own way by writing that “All things are different, or: there are no two things like each other. ”(see Hegel, Georg Wilhelm Friedrich, 1991, p. 422). Much of the debate about identity is still a matter of controversy. This issue has attracted the attention of many authors and has been discussed by Hegel too. In this context, it is worth to consider Hegel’s radical position on identity.

“The other expression of the law of identity: A cannot at the same time be A and not-A, has a negative form; it is called
the law of contradiction. ”
(Hegel, Georg Wilhelm Friedrich, 1991, p. 416)

We may, usefully (see Barukčić, 2019), state Russell’s position with respect to the identity law as mentioned in his book ‘The problems of philosophy ’ (see Russell, 1912). In particular, according to Russell,

“...principles have been singled out by tradition under the name of ‘Laws of Thought.’ They are as follows:
(1) **The law of identity**: ‘Whatever is,is.
(2)**The law of contradiction**: ‘Nothing can both be and not be.’
(3) The law of excluded middle: ‘Everything must either be or not be.’
These three laws are samples of self-evident logical principles, but are not really more fundamental or more self-evident than various other similar principles: for instance, the one we considered just now, which states that what follows from a true premise is true. The name ‘laws of thought’ is also misleading, for what is important is not the fact that we think in accordance with these laws, but the fact that **things behave in accordance with them**; ”

(see Russell, 1912, p. 113)

Russell’s critique, that we tend too much to focus only on the formal aspects of the ‘Laws of Thoughts’ with the consequence that “... we thing in accordance with these laws” (see Russell, 1912, p. 113) is justified. Judged solely in terms of this aspect, it is of course necessary to think in accordance with the 'Laws of Thoughts'. But this is not the only aspect of the 'Laws of Thoughts'. The other and may be much more important aspect of these 'Laws of Thoughts'is the fact that quantum mechanical objects or that "... things behave in accordance with them" (see Russell, 1912, p. 113).

2.7.2. Principium contradictionis (Axiom II)

Principium contradictionis or **lex contradictionis** [7], [8], [9] or axiom II, the other of lex identitatis, the negative of lex identitatis, the opposite of lex identitatis, a complementary of lex identitatis, can be expressed mathematically as

$$+0 \equiv 0 \times 1 \equiv +1 \tag{173}$$

In addition to the above, from the point of view of mathematics, axiom II (equation 173) is equally the most simple mathematical expression and formulation of a contradiction. However, there is too much practical and theoretical evidence that a lot of 'secured'mathematical knowledge and rules differ too generously from real world processes, and the question may be asked whether mathematical truths can be treated as absolute truths at all. Many of the basic principle of today's mathematics allow every single author defining the real world events and processes et cetera in a way as everyone likes it for himself. Consequentially, a resulting dogmatic epistemological subjectivism and at the end agnosticism too, after all, is one of the reasons why we should rightly heed the following words of wisdom of Albert Einstein.

"I don't believe in mathematics."

(Albert Einstein cited according to Brian, 1996, p. 76)

In the long term, however, the above attitude of mathematics is not sustainable. History has taught us time and time again that objective reality has the potential to correct wrong human thinking slowly but surely, and many more than this. Objective reality has demonstrably corrected wrong human thinking again and again in the past.

[7] Horn, Laurence R., "Contradiction", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2018/entries/contradiction/.

[8] Barukčić I. Aristotle's law of contradiction and Einstein's special theory of relativity. Journal of Drug Delivery and Therapeutics (JDDT). 15Mar.2019;9(2):125-43. https://jddtonline.info/index.php/jddt/article/view/2389

[9] Barukčić, Ilija. (2020, December 28). The contradiction is exsiting objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

Despite all the adversities, it is necessary and crucial to consider that a self-identical as the opposite of itself is no longer only self-identity but a difference of itself from itself within itself. In other words, "All things are different, or: there are no two things like each other ... is, in fact, opposed to the law of identity ..."(see Hegel, Georg Wilhelm Friedrich, 1991, p. 422) Each on its own and without any respect to the other is distinctive within itself and from itself and not only from another. As the opposite of its own something, is no longer only self-identity, but also a negation of itself out of itself and therefore a difference of itself from itself within itself. In other words, in opposition, a self-identical is able to return into simple unity with itself, with the consequence that even as a self-identical the same self-identical is inherently self-contradictory. A question of fundamental theoretical importance is, however, why should something be itself and at the same time the other of itself, the opposite of itself, not itself? Is something like this even possible at all and if so, why and how? These and similar questions have occupied many thinkers, including Hegel.

"Something is therefore
alive only in so far as it contains contradiction within it,
and moreover is this power to
hold and endure the contradiction within it. "

(see Hegel, Georg Wilhelm Friedrich, 1991, p. 440)

However, as directed against identity, contradiction itself is also at the same time a source of self-changes of a self-identical out of itself.

"... contradiction
is the root of all movement and vitality;
it is only in so far as something has a contradiction within it
that it moves, has an urge and activity. "

(see Hegel, Georg Wilhelm Friedrich, 1991, p. 439)

The further advance of science will throw any contribution to scientific progress of each of us back into scientific insignificance, as long as principium contradictionis is not given enough and the right attention. **The contradiction [10] is existing objectively and real and is the heartbeat of every self-identical.** We have reason to be delighted by the fact that very different aspects of principium contradictionis have been examined since centuries from different angles by various authors. According to Aristotle, principium contradictionis applies to everything that is, it is the first and the firmest of all principles of philosophy.

[10] Barukčić, Ilija. (2020, December 28). The contradiction is existing objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

"... the same ... cannot at the same time belong and not belong to the same
... in the same respect ... This, then, is
the most certain of all principles "

(see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaph., IV, 3, 1005b, 16–22)

Principium contradictionis or axiom II has many facets. As long as we follow Leibniz in this regard, we should consider that "**Le principe de contradiction est en general ...** "(Leibniz, 1765, p. 327). Scientist inevitably have false beliefs and make mistakes. In order to prevent scientific results from falling into logical inconsistency or logical absurdity, it is necessary to posses among other the methodological possibility to start a reasoning with a (logical) contradiction too. However and in contrast to the way of reasoning with inconsistent premises as proposed by para-consistent (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) and other logic, in the absence of technical and other errors of reasoning, the contradiction itself need to be preserved. In other words, **from a contradiction does not anything follows but the contradiction itself** while the theoretical question is indeed justified "What is so Bad about Contradictions? " (Priest, 1998). Historically, **the principle of (deductive) explosion** (Carnielli and Marcos, 2001, Priest, 1998, Priest et al., 1989), coined by 12th-century French philosopher William of Soissons, demand us to accept that anything, including its own negation, can be proven or can be inferred from a contradiction. In short, according to **ex falso sequitur quodlibet**, a (logical) contradiction implies anything. Respecting the principle of explosion, the existence of a contradiction (or the existence of logical inconsistency) in a scientific theorem, rule et cetera is disastrous. However, the historical development of science shows that scientist inevitably revise the theories, false positions and claims are identified once and again, and we all make different kind of mistakes. In order to avert disproportionately great damage to science and to prevent reducing science into pure subjective belief, a negation of the principle of explosion is required. Nonetheless, a justified negation of the **ex contradictione quodlibet principle** (Carnielli and Marcos, 2001) does not imply the correctness of para consistent logic (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) as such as advocated especially by the Peruvian philosopher Francisco Miró Quesada (Quesada, 1977) and other (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989). In general, scientific theories appear to progress from lower and simpler to higher and more complex levels. However, high level theories cannot be taken for granted because high level theories are grounded on a lot of assumptions, definitions and other procedures and may rest upon too much erroneous stuff even if still not identified. Therefore, it should be considered to check at lower at simpler levels like with like.

2.7.2.1. Zero power zero

Theorem 2. *In general, it is*

$$+0^{+2} \equiv +0 \tag{174}$$

is false.

Proof by direct proof. The premise

$$+0 \equiv +1 \tag{175}$$

is false. In the following, any rearrangement of the premise which is free of (technical) errors, need to end up at a contradiction. In other words, the contradiction will be preserved. We obtain

$$+0 \times +0 \equiv +1 \times +0 \tag{176}$$

Equation 176 becomes

$$+0^{+2} \equiv +0 \tag{177}$$

□

2.7.2.2. Zero divided by zero

Theorem 3. *In general,*

$$\frac{1}{0} \equiv \frac{0}{0} \tag{178}$$

is false.

Proof by direct proof. If the premise

$$+1 \equiv +0 \tag{179}$$

is false, then the relationship

$$\frac{1}{0} \equiv \frac{0}{0} \tag{180}$$

is also false.

□

2.7.3. Principium negationis (Axiom III)

Lex negationis or axiom III, is often mismatched with simple opposition. However, from the point of view of philosophy and other sciences, identity, contradiction, negation and similar notions are equally mathematical descriptions of the most simple laws of objective reality. What sort of natural process is negation at the end? Mathematically, we define principium negationis or lex negationis or axiom III as

$$\text{Negation}(0) \times 0 \equiv \neg(0) \times 0 \equiv +1 \tag{181}$$

where $\neg$ denotes (logical (Boole, 1854) or natural) negation (Ayer, 1952, Förster and Melamed, 2012, Hedwig, 1980, Heinemann, Fritz H., 1943, Horn, 1989, Koch, 1999, Kunen, 1987, Newstadt, 2015, Royce, 1917, Speranza and Horn, 2010, Wedin, 1990b). In this context, there is some evidence that

$$\text{Negation}(1) \times 1 \equiv \neg(1) \times 1 = 0 \tag{182}$$

Logically, it follows that

$$\text{Negation}(1) \equiv 0 \tag{183}$$

In the following we assume that axiom I is universal. Under this assumption, the following theorem follows inevitably.

Theorem 4 (Zero divided by zero)**.** *According to classical logic, it is*

$$\frac{0}{0} \equiv 1 \tag{184}$$

Proof by direct proof. The premise

$$1 \equiv 1 \tag{185}$$

is true. It follows that

$$\begin{aligned} 0 &\equiv 0 \\ &\equiv 0 \times 1 \end{aligned} \tag{186}$$

In the following, we rearrange the premise (see equation 181, p. 59). We obtain

$$0 \times (\text{Negation}(0) \times 0) \equiv 0 \tag{187}$$

Equation 187 changes slightly (see equation 182, p. 59). It is

$$(\text{Negation}(1) \times 1) \times (\text{Negation}(0) \times 0) \equiv 0 \tag{188}$$

Equation 188 demands that

$$(\text{Negation}(1)) \times (\text{Negation}(0)) \times 0 \equiv 0 \tag{189}$$

Equation 189 is logically possible (see equation 172, p. 51) only if

$$(\text{Negation}(1)\ ) \times (\text{Negation}(0)\ ) \equiv 1 \tag{190}$$

(see theorem 2, equation 174) whatever the meaning of Negation(1) or of Negation(0) might be, equation 190 demands that

$$\text{Negation}(0)\ \equiv \frac{1}{\text{Negation}(1)} \tag{191}$$

and that

$$\text{Negation}(1) \equiv \frac{1}{\text{Negation}(0)} \tag{192}$$

Equation 191 simplifies as (see equation 183, p. 59)

$$\begin{aligned} \text{Negation}(0)\ &\equiv \frac{+1}{\text{Negation}(1)} \\ &\equiv \frac{+1}{+0} \end{aligned} \tag{193}$$

It follows that

$$\neg(0) \times 0 \equiv \frac{1}{0} \times 0 \equiv \frac{0}{0} \equiv 1 \tag{194}$$

To bring it to the point. Classical logic, assumed as generally valid, demands that

$$\frac{0}{0} \equiv 1 \tag{195}$$

□

Concepts like identity, difference, negation, opposition et cetera engaged the attention of scholars at least over the last twenty-three centuries (see also Horn, 1989, Speranza and Horn, 2010). As long as we first and foremost follow Josiah Royce, negatio or negation "is one of the simplest and most fundamental relations known to the human mind. For the study of logic, no more important and fruitful relation is known." (see also Royce, 1917, p. 265) But, do we really know what, for sure, what negation is? Based on what we know about negation, Aristotle (see also Wedin, 1990a) has been one of the first to present a theory of negation, which can be found in discontinuous chunks in his works the Metaphysics, the Categories, De Interpretatione, and the Prior Analytics (see also Horn, 1989, p. 1). Negation (see also Newstadt, 2015) as a fundamental philosophical concept found its own very special melting point especially in Hegel's dialectic and is more than just a formal logical process or operation which converts true to false or false to true. Negation as such is a natural process too and equally '**an engine of changes of objective reality** " (see also Barukčić, 2019). However, it remains an open question to establish a generally accepted link between this fundamental philosophical concept and an adequate counterpart in physics, mathematics and mathematical statistics et cetera. Especially the relationship between creation and conservation or creatio ex nihilio (see also Donnelly, 1970, Ehrhardt, 1950, Ford, 1983), determination and negation (see also Ayer, 1952, Hedwig, 1980, Heinemann, Fritz H., 1943, Kunen, 1987) has been discussed in science since ancient (see also Horn, 1989, Speranza and Horn, 2010) times too. Why and how does an event occur or why and how is an event created (creation), why and how does an event maintain its own existence over time (conservation)? The development of the notion of negation leads from Aristotle to Meister Eckhart (see also Eckhart, 1986) von Hochheim (1260-1328), commonly known as Meister Eckhart (see also Tsopurashvili, 2012) or Eckehart, to Spinoza (1632 – 1677), to Immanuel Kant (1724-1804) and finally to Georg Wilhelm Friedrich Hegel (1770-1831) and other authors too. One point is worth being noted, even if it does not come as a surprise, it was especially Benedict de Spinoza (1632 – 1677) as one of the philosophical founding fathers of the Age of Enlightenment who addressed the relationship between determination and negation in his lost letter of June 2, 1674 to his friend Jarig Jelles (see also Förster and Melamed, 2012) by the discovery of his fundamental insight that " **determinatio negatio est**" (see also Spinoza, 1674, p. 634). Hegel went even so far as to extended the slogan raised by Spinoza into to "Omnis determinatio est negatio" (see also Hegel, Georg Wilhelm Friedrich, 1812b, 2010, p. 87). Finally, it did not take too long, and the notion of negation entered the world of mathematics and mathematical logic at least with Boole's (see also Boole, 1854) publication in the year 1854. "Let us, for simplicity of conception, give to the symbol x the particular interpretation of men, then 1 - x will represent the class of 'not-men'." (see also Boole, 1854, p. 49). Finally, the philosophical notion negation found its own way into physics by the contributions of authors like Woldemar Voigt (see Voigt, 1887), George Francis FitzGerald (see FitzGerald, 1889), Hendrik Antoon Lorentz (see Lorentz, 1892, 1899), Joseph Larmor (see Larmor, 1897), Jules Henri Poincaré (see Poincaré, 1905) and Albert Einstein (see Einstein, 1905a) by contributions to the physical notion "Lorentz factor".

3. Results

3.1. Theorem. Matter and gravitational field

The fundamental relationship between matter and the gravitational field has been defined by Einstein as follows.

> "Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß
>
> **alles außer dem Gravitationsfeld** als '**Materie**'bezeichnet wird, also nicht nur
>
> die **'Materie'im üblichen Sinne**, sondern auch das **elektromagnetische Feld**. "
>
> (Einstein, 1916, p. 802/803)

Einstein's position translated into English. 'In the following we distinguish between 'gravitational field'and 'matter', in the sense that everything else but the gravitational field is termed as 'matter', i.e. not only 'matter'in the ordinary sense, but also the electromagnetic field. '

Theorem 5 (Matter and gravitational field)**.**

$$\underline{E} = g \times c^2 \tag{196}$$

Proof by direct proof. It is

$$1 = 1 \tag{197}$$

or

$$U = U \tag{198}$$

Rearranging equation 198, it is

$$U - M + M = U + 0 \tag{199}$$

or

$$g + M = U \tag{200}$$

Normalising the relationship between matter and gravitational field, it is

$$\frac{g}{U} + \frac{M}{U} = \frac{U}{U} = +1 \tag{201}$$

Rearranging equation 201 it is

$$\frac{g \times c^2}{U \times c^2} + \frac{M \times c^2}{U \times c^2} = +1 \tag{202}$$

or

$$\frac{g \times c^2}{S} + \frac{M \times c^2}{S} = +1 \tag{203}$$

and

$$\frac{g \times c^2}{S} + \frac{E}{S} = +1 \tag{204}$$

or

$$\frac{g \times c^2}{S} = +1 - \frac{E}{S} = \frac{\underline{E}}{S} \tag{205}$$

At the end, it is

$$g \times c^2 = \underline{E} \tag{206}$$

□

Remark 3.1. *Objective reality is not only determined by energy, there is also something other than energy, there is the complementary of energy, there is not energy or anti-energy. The other of energy, denoted as* $\underline{E}$*, the complementary of energy, the opposite of energy et cetera is identified for sure (see equation 206) as*

$$\underline{E} = g \times c^2 \tag{207}$$

However, which other meaning may we attribute to this relationship, can there be a more profound meaning of $\underline{E}$ *at all? In general, it is (see equation 3 and equation 4)*

$$E + \underline{E} = t + \underline{t} = S \tag{208}$$

Energy is given by the equation

$$E = t + \underline{t} - \underline{E} = S - \underline{E} \tag{209}$$

We add time to energy. It is

$$E + t = t + t + \underline{t} - \underline{E} = S + t - \underline{E} \tag{210}$$

Epistemologically it can not be denied that there can be circumstances where $+t = \underline{E}$ *with the consequence that* $+t - \underline{E} = 0$*. Under these circumstances, we can conclude that*

$$E + t = S + t - \underline{E} = S + 0 = S \tag{211}$$

We define energy in this way as all but time (ex negativo). In other words, there is no third between energy and time, tertium non datur. At the end, it is

$$E + t = S \tag{212}$$

There are conditions where it follows in a logically consistent way that

$$t = \underline{E} = g \times c^2 \tag{213}$$

Unfortunately, we have not presented a clear proof here at this passage that equation 213 is generally valid. However, under preliminary aspects we are inclined to consider that ***everything but time is energy****.*

3.2. Theorem. The relationship between the entity S and the dimension of space-time D

In general, the Ricci tensor $R_{\mu\nu}$ represents how a volume of space in a curved space-time differs from a volume of space in Euclidean space. Usually, the Ricci tensor $R_{\mu\nu}$ is defined in terms of mathematical objects called Christoffel symbols. The Christoffel symbols themselves are defined in terms of the metric tensor $g_{\mu\nu}$. At this location we would like to work out a proposal how to simplify the form of the Ricci tensor.

Theorem 6 (The relationship between the entity S and the dimension of space-time D)**.** *In general, the entity S is given by*

$$S \equiv \left(\frac{R}{D}\right) \tag{214}$$

Proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{215}$$

is true, **then** the conclusion

$$S \equiv \left(\frac{R}{D}\right) \tag{216}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{217}$$

is true. Multiplying this premise by the Ricci tensor it is

$$R_{\mu\nu} \equiv R_{\mu\nu} \tag{218}$$

We assume at this point, that there is an entity S which in combination with the metric tensor $g_{\mu\nu}$ describes the Ricci tensor $R_{\mu\nu}$ completely. It **should be** generally valid that

$$\boxed{R_{\mu\nu} \equiv S \times g_{\mu\nu} \qquad (219)}$$

This assumption leads to straightforward mathematical consequences. Rearranging equation before, it is

$$R_{\mu\nu} \times g^{\mu\nu} \equiv S \times g_{\mu\nu} \times g^{\mu\nu} \tag{220}$$

or in accordance to eq. 2.49

$$R \equiv S \times g_{\mu\nu} \times g^{\mu\nu} \tag{221}$$

In general, it is (see definition 2.19, equation 41)

$$R \equiv S \times D \tag{222}$$

The entity S is depending on the number of space-time dimensions D and follows as

$$S \equiv \left(\frac{R}{D}\right) \tag{223}$$

In other words, our conclusion is true.

□

Under conditions of D=1 space-time dimension, it is

$$R \equiv S \times D \equiv S \times 1 \equiv S \tag{224}$$

3.3. Theorem. The scalar form of Ricci tensor $R_{\mu\nu}$

Theorem 7 (The scalar form of Ricci tensor $R_{\mu\nu}$)**.** *The scalar form of Ricci tensor $R_{\mu\nu}$ is given as*

$$R_{\mu\nu} = \left(\frac{R}{D}\right) \times g_{\mu\nu} = S \times g_{\mu\nu} \tag{225}$$

Proof by direct proof. It is (see equation 223, p. 65)

$$S \equiv \left(\frac{R}{D}\right) \tag{226}$$

We multiply equation 226 by the metric tensor $g_{\mu\nu}$.An equivalent formulation of the Ricci tensor $R_{\mu\nu}$ in terms of a Scalar S is given by the equation

$$R_{\mu\nu} = S \times g_{\mu\nu} \equiv \left(\frac{R}{D}\right) \times g_{\mu\nu} \tag{227}$$

□

3.4. The geometrical structure of the four basic fields of nature

Time and again, we were able to identify the four fundamental fields of nature (Barukčić, 2016b,c, 2020b,c,d,d, 2021) as $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$. At this point, we would like to visualize these matters once again in our mind's eye (see table 5, p. 66).

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Table 5. Einstein field equations and the four basic fields of nature

As previously outlined elsewhere, the relationship between the four fundamental fields of nature $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ and the Ricci tensor $R_{\mu\nu}$ is given by the equation

$$a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu} \tag{228}$$

Nonetheless, even if the aforementioned is logically very plausible, the concrete structure and a detailed geometrical description of the four fundamental fields of nature remains quite doubtful in spite of many attempts of geometrization (Barukčić, 2016b,c, 2020b,c,d,d, 2021) of the same. At this stage we would like to approach this issue from a different viewpoint in order to possibly get closer to the solution of this problem.

3.4.1. Theorem. The geometrical structure of the basic field of nature $c_{\mu\nu}$

Theorem 8 (The geometrical structure of the basic field of nature $c_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $c_{\mu\nu}$ is given as*

$$c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{229}$$

Proof by direct proof. Einstein's tensor $G_{\mu\nu}$ (Barukčić, 2016b,c, 2020b,c,d,d, 2021) has been derived (but not defined) as

$$G_{\mu\nu} = a_{\mu\nu} + c_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2}\right) \times g_{\mu\nu} = \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \tag{230}$$

The validity of this tensor equation remains even under conditions under which the stress-energy tensor of the ordinary matter disappears or $a_{\mu\nu} = 0$. Under these conditions, it is

$$c_{\mu\nu} = 0 + c_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2}\right) \times g_{\mu\nu} = \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \tag{231}$$

Similarly, under the conditions of 1 space-time dimension, we must take the validity of this tensor equation as given. Under these conditions follows that

$$c_{\mu\nu} = \left(\frac{R}{1} - \frac{R}{2}\right) \times g_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{232}$$

The geometrical form of the fundamental field of nature $c_{\mu\nu}$ is given as

$$\boxed{c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu}} \tag{233}$$

□

3.4.2. Theorem. The geometrical structure of the basic field of nature $a_{\mu\nu}$

Theorem 9 (The geometrical structure of the basic field of nature $a_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $a_{\mu\nu}$ (stress-energy tensor of ordinary matter) is given as*

$$a_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) \tag{234}$$

Proof by direct proof. Einstein's tensor $G_{\mu\nu}$ (Barukčić, 2016b,c, 2020b,c,d,d, 2021) has been derived (but not defined) as

$$G_{\mu\nu} = a_{\mu\nu} + c_{\mu\nu} = \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \tag{235}$$

The stress-energy tensor of the ordinary matter, denoted as $a_{\mu\nu}$, is given as

$$a_{\mu\nu} = \left(\left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}\right) - c_{\mu\nu} \tag{236}$$

The tensor $c_{\mu\nu}$ has been determined as $c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu}$ (see equation 232, p. 67). Equation 236 becomes

$$a_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{237}$$

The geometrical form of the stress-energy tensor of the ordinary matter, denoted as $a_{\mu\nu}$, is given as

$$\boxed{a_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right)} \tag{238}$$

□

3.4.3. Theorem. The geometrical structure of the basic field of nature $d_{\mu\nu}$

Theorem 10 (The geometrical structure of the basic field of nature $d_{\mu\nu}$). *The geometrical structure of the basic field of nature $d_{\mu\nu}$ is given as*

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{239}$$

Proof by direct proof. In the meantime (Barukčić, 2016b,c, 2020b,c,d,d, 2021) it could be established (and not defined) that

$$c_{\mu\nu} + d_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \tag{240}$$

The tensor $c_{\mu\nu}$ has been determined as $c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu}$ (see equation 232, p. 67). Equation 240 becomes

$$\left(\frac{R}{2} \times g_{\mu\nu}\right) + d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) + \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{241}$$

or

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) + \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{242}$$

The geometrical structure of the basic field of nature $d_{\mu\nu}$ is given as

$$\boxed{d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right)} \tag{243}$$

□

3.4.4. Theorem. The geometrical structure of the basic field of nature $d_{\mu\nu}$

Theorem 11 (The geometrical structure of the basic field of nature $d_{\mu\nu}$). *The geometrical structure of the basic field of nature $d_{\mu\nu}$ is given as*

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{244}$$

Proof by direct proof. Axiom 1 or +1 = +1 is valid. Based on this axiom, we obtain

$$c_{\mu\nu} = c_{\mu\nu} \tag{245}$$

or (see equation 14, p. 12 and equation 15, p. 12)

$$G_{\mu\nu} - a_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{246}$$

We rearrange equation 246. It is

$$R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) - a_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{247}$$

Based on equation 225, p. 65, equation 247 changes slightly. We obtain

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) - a_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{248}$$

Equation 248 is generally valid. Rearranging equation 248, it is

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) - a_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{249}$$

or

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) - a_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{250}$$

The unrestricted validity of the previous equation (see equation 250) is also given if the tensor of ordinary matter $a_{\mu\nu}$ vanishes or if $\mathbf{a}_{\mu\nu} = \mathbf{0}$. We obtain

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{251}$$

The unrestricted validity of the previous equation (see equation251) is also given under conditions of **D = 1** space-time dimension. We obtain

$$\left(\frac{R}{1} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{252}$$

or

$$0 = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{253}$$

The geometrical structure of the basic field of nature $d_{\mu\nu}$ is given as

$$\boxed{d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right)} \tag{254}$$

□

Remark 3.2. *The geometric structure of the field $d_{\mu\nu}$ has been determined as $d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right)$. However, this raises at once several fundamental and far-reaching questions. Under the most different aspects, the Einstein cosmological constant Λ, usually represented by the Greek letter Λ (Lambda), is viewed as equivalent to the 'mass 'of empty space (which itself can be either positive or negative), and manny times associated with 'vacuum energy' (see also Huterer and Turner, 1999, Zwicky, 1933). In particular, as it may and will be in the end, the basic field of nature $d_{\mu\nu}$ appears to be an underlying background field that exists in space throughout the entire Universe.*

3.4.5. Theorem. The geometrical structure of the basic field of nature $b_{\mu\nu}$

Theorem 12 (The geometrical structure of the basic field of nature $b_{\mu\nu}$). *The geometrical structure of the basic field of nature $b_{\mu\nu}$ is given as*

$$b_{\mu\nu} = (b) \times g_{\mu\nu} = \left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \tag{255}$$

Proof by direct proof. Here we would like to reiterate once again that the following relationship has been established (and not defined) (Barukčić, 2016b,c, 2020b,c,d, 2021) . It is

$$b_{\mu\nu} + d_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{256}$$

The tensor $d_{\mu\nu}$ has been determined as $d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right)$ (see equation 243, p. 69). Equation 256 changes slightly. It is

$$b_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right) = \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{257}$$

The geometrical structure of the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ is given as

$$\boxed{b_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right)} \tag{258}$$

□

Remark 3.3. *Under certain circumstances, the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ is geometrized (Kalinowski, 1988) as $b_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right)$. Einstein's general theory of relativity is one small stepping stone towards a unified field theory which should be able to integrate somehow both gravitational and electromagnetic fields into a single hyper-field. As long as we are allowed to agree with Tonnelat's position, a unified field theory is "... a theory joining the gravitational and the electromagnetic field into one single hyperfield whose equations represent the conditions imposed on the geometrical structure of the universe." (see Tonnelat et al., 1955, p. 5) The geometrization of the fundamental fields of nature that has now been accomplished can be helpful in this view.* ***The geometrized hyper-field for electromagnetism and gravitation*** *is given as*

$$c_{\mu\nu} + b_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) = \left(R \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \tag{259}$$

There seem to exist conditions where ***the tensor of pure non-locality*** *is given by the equation*

$$b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = \left(R \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) = \left(R \times g_{\mu\nu}\right) \tag{260}$$

3.5.1. Objective reality without ordinary matter

Electrovacuum solution (electrovacuum) is one of the known an exact solution of the Einstein field equation. The stress-energy momentum tensor (see equation 2.53) is defined as

$$E_{\mu\nu} \equiv a_{\mu\nu} + b_{\mu\nu} \tag{261}$$

Under conditions where objective reality is determined by a vanishing tensor of ordinary matter ($a_{\mu\nu} = 0$) we obtain

$$E_{\mu\nu} \equiv (a_{\mu\nu} = 0) + b_{\mu\nu} \tag{262}$$

or

$$b_{\mu\nu} \equiv E_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \tag{263}$$

In other words, under these conditions, all stress energy and momentum is included in the stress energy tensor of the electromagnetic field. Nonetheless, a vanishing tenors of ordinary matter does not imply a vanishing of Einstein's tensor. The conditions outlined before do not imply that Einstein's tensor ($G_{\mu\nu}$) has to vanish too. Table 6 is providing us an overview of these relationships.

		Curvature YES	NO	
Momentum	YES	$a_{\mu\nu} = 0$	$b_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$	$\frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}$	$d_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu}$ - $(\frac{R}{D} - R) \times g_{\mu\nu}$	$\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu}$ - $R \times g_{\mu\nu} + R \times g_{\mu\nu}$

Table 6. Objective reality without ordinary matter.

It is important to emphasise here that objective relativity in which no ordinary matter is given ($a_{\mu\nu}$ = 0) is at the same time also a world in which momentum excludes curvature and vice versa. Curvature excludes momentum. But at the same time it is also a world which is not dead and not without any changes but a world full of life. We have to be theoretically prepared that this world might be the world of pure non-locality.

3.5.2. Objective reality under conditions of D=1 dimension

Under conditions of D = 1 dimension, the Einstein (Barukčić, 2016b,c, 2020b,c,d,d, 2021, Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) field equations (see equation 127) defined as

$$\underbrace{\left(\frac{R}{D}\times g_{\mu\nu}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)}_{the\ left-hand\ side}\equiv\underbrace{\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{\mu\nu}}_{the\ right-hand\ side} \tag{264}$$

becomes

$$\left(\frac{R}{1}\times g_{\mu\nu}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)\equiv\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^4\times 1}\right)\times g_{\mu\nu} \tag{265}$$

or

$$+\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)\equiv\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^4\times 1}\right)\times g_{\mu\nu} \tag{266}$$

Under these conditions, the Ricci tensor becomes

$$R_{\mu\nu}=\left(\frac{R}{D}\times g_{\mu\nu}\right)=R\times g_{\mu\nu} \tag{267}$$

but not $R_{\mu\nu} = 0$. Under conditions of D=1 space-time dimension, Einstein's tensor becomes

$$G_{\mu\nu}=\left(\frac{R}{1}\times g_{\mu\nu}\right)-\left(\frac{R}{2}\times g_{\mu\nu}\right)=\left(R\times g_{\mu\nu}\right)-\left(\frac{R}{2}\times g_{\mu\nu}\right)=\left(\frac{R}{2}\times g_{\mu\nu}\right) \tag{268}$$

Under conditions of D=1 space-time dimension and in contrast to a vacuum solution of general relativity neither the Einstein tensor vanishes nor the stress–energy tensor vanishes. Under conditions of D=1 space-time dimension the tensor $d_{\mu\nu}$ becomes

$$d_{\mu\nu}\equiv-\Lambda\times g_{\mu\nu}-(\frac{R}{D}-R)\times g_{\mu\nu})\equiv-\Lambda\times g_{\mu\nu}-(\frac{R}{1}-R)\times g_{\mu\nu})\equiv-\Lambda\times g_{\mu\nu}-(0)\equiv-\Lambda\times g_{\mu\nu} \tag{269}$$

From this relationship follows that

$$c_{\mu\nu}\equiv\frac{R}{2}\times g_{\mu\nu}-\Lambda\times g_{\mu\nu}-d_{\mu\nu}\equiv\frac{R}{2}\times g_{\mu\nu}-\Lambda\times g_{\mu\nu}-(-\Lambda\times g_{\mu\nu})\equiv\frac{R}{2}\times g_{\mu\nu} \tag{270}$$

In the last consequence we obtain the following picture (see table 7).

In general relativity, a vacuum region of objective reality is understood as a region whose Einstein tensor $G_{\mu\nu}$ vanishes. The Einstein tensor vanishes if

$$G_{\mu\nu}=R_{\mu\nu}-\left(\frac{R}{2}\right)\times g_{\mu\nu}=0 \tag{271}$$

which is especially the case under conditions of D = 2 space-time dimension. In general, vacuum solutions of the Einstein fields equations are distinct from the electrovacuum solutions (electromagnetic

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu} = 0$	$b_{\mu\nu} \equiv \left(\frac{R}{2}+\Lambda\right) \times g_{\mu\nu}$	$\left(\frac{R}{2}+\Lambda\right) \times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv \left(\frac{R}{2}\right) \times g_{\mu\nu}$	$d_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu}$	$\left(\frac{R}{2}-\Lambda\right) \times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{2}\right) \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv R \times g_{\mu\nu}$

Table 7. Objective reality under conditions of D=1 space-time dimension.

field, gravitational field) and are also distinct from the lambdavacuum solutions. In lambdavacuum solutions of the Einstein fields equations the only term in the stress–energy tensor is the cosmological constant term.

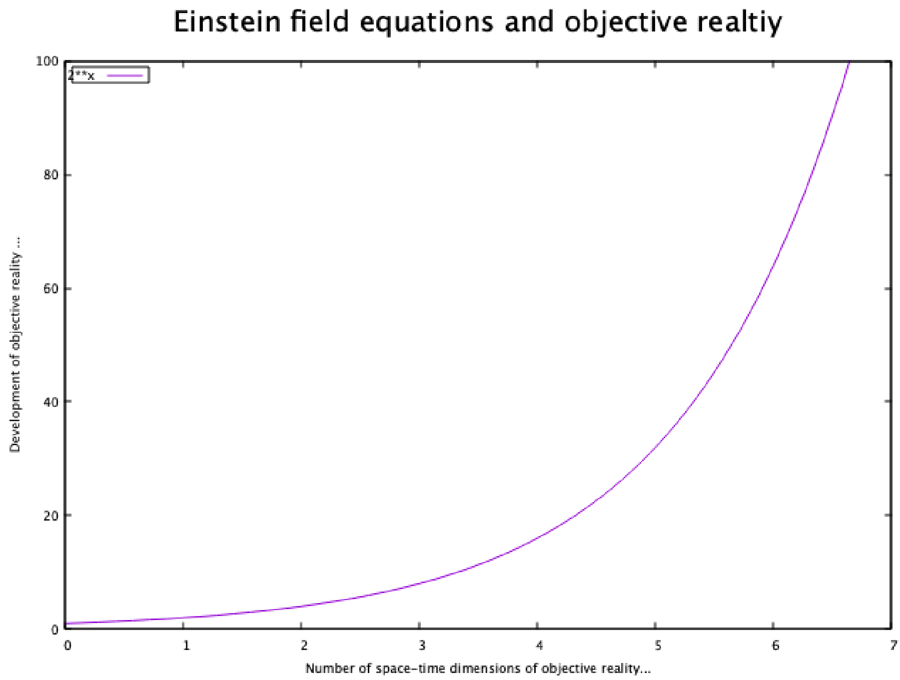


Figure 2. The evolution of objective reality

Under the previous and other conditions, one more point should be noted. The constancy of the speed of the light c in vacuum is something relative but not something absolute. Einstein is writing:

"Dagegen bin ich der Ansicht, daß das Prinzip der Konstanz der Lichtgeschwindigkeit sich nur insoweit aufrecht erhalten läßt, als man sich auf raum - zeitliche Gebiete von **konstantem Gravitationspotential** beschränkt. Hier liegt nach meiner Meinung die Grenze der Gültigkeit ... des Prinzips der **Konstanz der Lichtgeschwindigkeit** und damit unserer heutigen Relativitätstheorie. "

(see also Einstein, 1912, p. 1062)

Translated into English. 'On the other hand I am of the opinion that the principle of the constancy of the speed of light can be maintained only in so far as one restricts oneself to spatio-temporal areas of constant gravitational potential. Here lies in my opinion the limit of the validity... of the principle of the constancy of the speed of light and with it of our today's theory of relativity.'

3.6. Measurements of space-time dimensions

One aspect of the self-organization of the objective reality seems to be also the transition into an objective reality higher dimension. Whether this process is irreversible may be an open question for the present. However, the question may very well be asked whether our objective reality is already part of another, higher dimensional objective reality? Moreover, it would be desirable if this can be measured or verified experimentally somehow. Measurements of Laue's scalar (i. e. by wave-length et cetera) can be of help. It general, under conditions of 4 space-time dimensions, it is

$$\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times 4}\right)\times g_{\mu\nu}\right)=\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)\times g_{\mu\nu}\right) \tag{272}$$

or

$$D\times\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times 4}\right)\times g_{\mu\nu}\right)=\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4}\right)\times g_{\mu\nu}\right) \tag{273}$$

Simplifying, it is

$$D=\frac{\text{D Space time dimension}}{\text{4 Space time dimension}}=\frac{\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4}\right)\times g_{\mu\nu}\right)}{\left(\left(\frac{2\times \pi\times \gamma\times T}{c^4}\right)\times g_{\mu\nu}\right)}=\frac{\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4}\right)\right)}{\left(\left(\frac{2\times \pi\times \gamma\times T}{c^4}\right)\right)} \tag{274}$$

The determination of the dimension space-time by an measurement is given as

$$D=\frac{\text{D Space time dimension}}{\text{4 Space time dimension}}=\frac{\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4}\right)\right)}{\left(\left(\frac{2\times \pi\times \gamma\times T}{c^4}\right)\right)}=\frac{4\times \pi\times \gamma\times T_{\text{D Dimension}}}{\pi\times \gamma\times T_{\text{4 dimension}}} \tag{275}$$

4. Discussion

Today, a lot of cosmologists and theoretical physicists endorse the view that our universe was born about 13.7 billion years ago in a massive expansion, the so called big bang. The notion 'big bang '(see also Lemaître, 1931a,b) itself has been coined on 28 March 1949 by Fred Hoyle(see also Kragh, 2013) during his talk on the British Broadcasting Corporation (BBC). However, objective reality itself is our 'ultimate'truthmaker and can teach us very much about the beginning of our world.

The beginning of our world, as the foundation on which everything other is built, appears to be determined by laws of nature which are worth to be examined from a higher point of view before anything else. At the beginning of our world, another end is probably running on empty and we have nothing else but the beginning itself. However, it remains to be seen what and how such a beginning could be.

With what should the beginning of our world be made, what is there before us? Is it possible at all for our world to begin, it doesn't matter either our world is or it is not. In so far as our world is, our world is not just beginning, the world is already. In so far as our world is not, why should this world begin, how could this world begin? Thus, if no presupposition is to be made then the only determination of the beginning of our world as such is at the end to be the beginning of our world. In the same line, a beginning of our world may not presuppose anything. In point of fact, is there something like an absolute beginning at all, is there something which has been existed prior to the beginning of our world? Do we have to consider whether a preliminary labour need to be carried out before the beginning of our world? We should not let up at this point until the beginning of our world has been firmly established.

A reader who is concerned with the origin or the beginning of our world will have to consider at least the possibility of a creation or of a beginning of our world out of nothing, a **creatio ex nihilio** (see also Aquinas, 1964), however nothing itself might be defined. In this context, it is necessary to point out, that **nothing**, as an absence of something, **exists**. It is nothing, but it is given too. Has this world had a beginning (in nothing, in time, in ...) or the either way? In particular, has this world had no beginning in time, in nothing et cetera with the consequence that it always has existed and it always will exist. Are nothingness and the beginning of our world independent of each other (see table 8)? In point of fact, are questions like these beyond any human experience? In particular, is the question of our world's beginning more a matter of faith than of demonstration or science?

Table 8. Without nothingness no beginning of world?

		Beginning Of World		
		YES	NO	
Nothingness	YES	1	1	1
	NO	0	1	1
		1	1	1

The relationship without nothingness no beginning of our world is logically equivalent with the relationship if no absence of nothingness then no absence of the beginning of our world. However, this logical necessity need not imply that a beginning of our world is successful too. Their may have existed a lot of 'trials'until the beginning of our world was successful. However, what is nothingness, what is the structure of nothingness, where does it itself comes from? May all this not also be a little different and more likely the following way: if nothingness then beginning of our world?

Table 9. If nothingness, then beginning of world?

		Beginning Of World		
		YES	NO	
Nothingness	YES	1	0	1
	NO	1	1	1
		1	1	1

Again, does nothingness exist and what are the properties of nothingness? In order not to expose ourselves to the danger to favour a one-sided point of view (creatio ex nihilio), it is appropriate to consider whether the determination of the beginning of our world is comparable to a coin with two sides - **a beginning of our world out of itself which is necessary for itself** and equally **a beginning of our world which is a condition for its own further self-organised and self-determined development**. (see also Barukčić, 2007) It is hardly surprising, therefore, that in the first view of the nature of the beginning of our world, the beginning of our world out of itself which is necessary for itself appears to be something what is absolutely simple, that is, something what is the most general. In other words, it is very likely that that the beginning of our world cannot be made with anything containing a concrete relation within itself or anything concrete because such a concrete something need not to begin, such a concrete something is already existing. As a logical consequence, it difficult to consider that concrete something itself has been that from which the movement of our world started because the determinations contained in something concrete have already developed somehow. Thus, the developed and concrete something would exist before it started to exist. Consequently, anything which is in its own self a first and an other too implies equally that it has developed somehow, an advance from one to another has already been made. A concrete one has become somehow the concrete one that it is, some progress has already been made. In so far, that which constitutes the beginning of our world, **the beginning itself**, is to be taken as something simple and unfilled. If that which forms the beginning of our world would be something determined within itself, then this something that is determined within itself need likewise to be something otherwise concrete which the beginning of our world cannot be.

To address the question of emptiness, nothingness et cetera again and from a non-mathematical point of view, even after a removal of everything still remains something which is not constituted or determined by anything concrete, objective reality determined by neither curvature nor momentum, the emptiness as such, **the empty negative**, the infinitely flat, whatever this may be, what ever its properties. Therefore, in emptiness simply as such, in the empty negative which is necessary for itself, the beginning of our world can be found. The insight, that in the empty negative the beginning of our world can be found, is itself so simple that a beginning of our world as such out of the empty negative requires further introduction. How can there be any beginning of our world from the point of view of the general theory of relativity in nothingness, in the emptiness, in an empty negative?

$$d_{\mu\nu} = \left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) \quad (276)$$

Especially under conditions of D=2 space-time dimension it is

$$d_{\mu\nu} = \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times 2}\right) \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) \quad (277)$$

or

$$d_{\mu\nu} = -\left(\left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) \quad (278)$$

However, we have to demonstrate a certain amount of courage and to venture out from safe theoretical cover in order to ask ourselves at this point whether we are forbidden to ask the question about the theoretical possibility of the existence of an objective reality below D=2 space-time dimensions? What could such a bizarre objective reality look like in detail? In the end, even if we acknowledge with regard to this matter for the present to each his own point of view, the emptiness in which the beginning of our world can be found need to be an emptiness in which an advance from one to another has yet not been made, it is an abstract and not determinate emptiness. However, such an empty negative, the emptiness as such, is equally a self-related negativity, it is the negative of itself in its own self, it has a relation to the other of itself and is suffering and thirsty for the other of itself. From another point of view, only in what is simple there is nothing more than the pure beginning, only in such a simple, in emptiness as such, no advance yet has been made from one to an other. It might be reasonably assumed that **the beginning of our world began with the beginning itself**. Unfortunately, it appears to be that there is little to say in this respect since there were no eye-witnesses and there is no direct evidence in this regard. But even if epistemic self-doubt is not all the time so evidently justified, an important alternative which remains is the task of fact-finding as we **descend from the known to the unknown**.

Yet it remains central and helpful to consider that it is very difficult to extract any further determination of any beginning of our world from the fact that it is the beginning of our world as such. At first sight, there isn't anything else present, any content which could be used to make the beginning of our world more determinate. It can be noted, however, that the beginning of our world in emptiness, in nothing else but the empty negative, is equally in its first manifestation in fanatical hostility towards an end, it is fearful of being lost in an end, it is fearful of being captured for ever by an end. The beginning of our world is equally within itself the end of an end, the end of an end in which the end is also the begin and the begin is also the end, the beginning of our world is thus the beginning of everything. In so far, that from which a movement began has united with itself, in the beginning an end ends and equally in such an end the beginning begins. The beginning of our world on its own accord determines thus itself as the other of itself, the beginning is thus the local hidden variable of an end, it is a simplicity into which an end has withdrawn. The beginning of our world contains as such within itself thus the beginning of any further self-governed advance and development. In its last manifestation, the beginning of our world seems to be equally the foundation on which everything other is built, it is the simplest, the simple itself, quite general, without any content and still undeveloped. The beginning of our world is the foundation which is present and preserved throughout the entire subsequent development, remaining completely immanent in its further determinations. That which forms the starting point of the development of our world remains at the base of all that follows and does not vanish from it. Enclosed in the beginning of our world is thus the entire development that follows. The further necessary development of our world started right from the beginning itself. The beginning of our world in its own necessary development brings with its own self the demand of further development. The beginning of our world starts from itself and advances to the other of itself, it is a movement through which the latter at the end returns to the first. The progress that follows is more or less only a further determination of the beginning of our world, every further progress is equally a fresh beginning too, it is the sublation of the very first beginning of our world. In so far, while getting further away from the beginning of our world, the development of our world is equally getting back nearer to it. Consequently, after the contradictions contained in the beginning of our world have been developed, the final result is the relationship which formed the beginning as such, is the infinite progress, the same contradiction with which our world began. However it may be, once the beginning of our world has inwardly reconstituted itself, all attempts to preserve the end are utterly in vain. In so far, the beginning as such remains to some extent a matter of indifference. Contrary to all, both sides of the beginning of our world constitutes the beginning of our world. The beginning of our world has thus its own result, its own negation in itself and passes thus into a higher space-time dimension, into a new unity and struggle between energy and time.

5. Conclusion

The big bang as an explanation of the beginning of our world is not the only conceivable logical possibility to explain the beginning of our world. The beginning of our world out of the empty negative, out of

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{279}$$

is theoretically possible too.

Acknowledgments

No funding or any financial support by a third party was received.

6. Patient consent for publication

Not required.

Conflict of interest statement

No conflict of interest to declare.

Abbreviations

BMI, body mass index; ESRD, end-stage renal disease; HD, hemodialysis; CABG, coronary artery bypass graft; CCI, Charlson comorbidity index; AMI, acute myocardial infarction; MACE, major adverse cardiovascular events; LDL-C, low-density lipoprotein cholesterol; GFR, glomerular filtration rate; STEMI, ST elevation myocardial infarction; NSTEMI, non-ST elevation myocardial infarction; MI, myocardial infarction; ARB, angiotensin receptor blocker; ACEI, angiotensin-converting enzyme inhibitors; CCBs, calcium channel blockers; RR (nc), relative risk (necessary condition); RR (sc), relative risk (sufficient condition); OR, odds ratio; IOR, index of relationship; p(IOU), index of unfairness; p(IOI), index of independence.

Erratum

Unfortunately, some misprints appeared in the previous publications, especially in the section of definitions. Some of the misprints have been brought up to date in this publication as far as possible.

Private note

The definition section of a paper need not and does not necessarily contain new scientific aspects. Above all, it also serves to better understand a scientific publication, to follow every step of the arguments of an author and to explain in greater details the fundamentals on which a publication is based. Therefore, there is no objective need to force authors to reinvent a scientific wheel once and again unless such a need appears obviously factually necessary. The effort to write about a certain subject in an original way in multiple publications does not exclude the necessity simply to cut and paste from an earlier work, and has nothing to do with self-plagiarism. However, such an attitude cannot simply be transferred to the sections' introduction, results, discussion and conclusions et cetera.

References

Thomas Aquinas. *On the eternity of the world; (De aeternitate mundi)*. Milwaukee, Marquette University Press, 1964. ISBN 978-0-87462-216-4. URL http://archive.org/details/oneternityofworl0000thom.

Aristotle, of Stageira (384-322 B.C.E). *Metaphysica. Volume VIII. Translated by William David Ross and John Alexander Smith*. The works of Aristotle. At The Clarendon Press, Oxford, 1908. URL http://archive.org/details/worksofaristotle12arisuoft. Archive.org Zenodo.

A. J. Ayer. Negation. *The Journal of Philosophy*, 49(26):797–815, January 1952. doi: 10.2307/2020959. JSTOR.

Ilija Barukčić. The beginning of our world. *Causation*, 2(5):5–50, 2007.

Ilija Barukčić. The Equivalence of Time and Gravitational Field. *Physics Procedia*, 22:56–62, January 2011. ISSN 18753892. doi: 10.1016/j.phpro.2011.11.008. ICPST 2011. Web of Science. Free full text: Elsevier.

Ilija Barukčić. The relativistic wave equation. *International Journal of Applied Physics and Mathematics*, 3(6):387–391, 2013. ISSN 2010362X. doi: 10.7763/IJAPM.2013.V3.242.

Ilija Barukčić. Anti einstein – refutation of einstein's general theory of relativity. *International Journal of Applied Physics and Mathematics*, 5(1):18–28, 2015a. ISSN 2010362X. doi: 10.17706/ijapm.2015.5.1.18-28.

Ilija Barukčić. Anti Newton — Refutation of the Constancy of Newton's Gravitational Constant Big G. *International Journal of Applied Physics and Mathematics*, 5(2):126–136, 2015b. doi: 10.17706/ijapm.2015.5.2.126-136.

Ilija Barukčić. Newton's gravitational constant big g is not a constant. *Journal of Modern Physics*, 7(66):510–522, 3 2016a. doi: 10.4236/jmp.2016.76053.

Ilija Barukčić. The geometrization of the electromagnetic field. *Journal of Applied Mathematics and Physics*, 4(1212):2135–2171, 12 2016b. doi: 10.4236/jamp.2016.412211.

Ilija Barukčić. Unified field theory. *Journal of Applied Mathematics and Physics*, 4(88):1379–1438, 8 2016c. doi: 10.4236/jamp.2016.48147.

Ilija Barukčić. The Physical Meaning of the Wave Function. *Journal of Applied Mathematics and Physics*, 04(06):988–1023, 2016d. ISSN 2327-4352. doi: 10.4236/jamp.2016.46106.

Ilija Barukčić. Aristotle's law of contradiction and einstein's special theory of relativity. *Journal of Drug Delivery and Therapeutics*, 9 (22):125–143, 3 2019. ISSN 2250-1177. doi: 10.22270/jddt.v9i2.2389.

Ilija Barukčić. Einstein's field equations and non-locality. *International Journal of Mathematics Trends and Technology*, 66(6):146–167, June 2020a. doi: 10.5281/zenodo.3907238. URL https://doi.org/10.5281/zenodo.3907238. Zenodo.

Ilija Barukčić. Einstein's field equations and non-locality. *International Journal of Mathematics Trends and Technology IJMTT*, 66(6): 146–167, 2020b. doi: 10.14445/22315373/IJMTT-V66I6P515. IJMTT.

Ilija Barukčić. The field equations for gravitation and electromagnetism. *Causation*, 15(7):5–25, July 2020c. doi: 10.5281/zenodo.3935948. URL https://doi.org/10.5281/zenodo.3935948. Zenodo.

Ilija Barukčić. *N-th index D-dimensional Einstein gravitational field equations. Geometry unchained.*, volume 1. Books on Demand GmbH, Hamburg-Norderstedt, 1 edition, 2020d. ISBN 978-3-7526-4490-6. ISBN-13: 9783752644906 . Free full text (preprint): ZENODO.

Ilija Barukčić. Quantum gravity. *Causation*, 17(9):1–157, 9 2021. doi: 10.5281/zenodo.5717335. URL https://doi.org/10.5281/zenodo.5717335. Zenodo.

George Boole. *An investigation of the laws of thought, on which are founded mathematical theories of logic and probabilities*. New York, Dover, 1854. Free full text: archive.org, San Francisco, CA 94118, USA.

Denis Brian. *Einstein: a life*. J. Wiley, New York, N.Y, 1996. ISBN 978-0-471-11459-8.

A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos. Aspects of the grand unification of strong, weak and electromagnetic interactions. *Nuclear Physics B*, 135(1):66–92, 3 1978. ISSN 0550-3213. doi: 10.1016/0550-3213(78)90214-6.

Walter A. Carnielli and João Marcos. Ex contradictione non sequitur quodlibet. *Bulletin of Advanced Reasoning and Knowledge*, 7 (1):89–109, 2001. URL https://www.researchgate.net/publication/236647971_Ex_contradictione_non_sequitur_quodlibet.

Rudolf Julius Emanuel Clausius. *The mechanical theory of heat: with its applications to the steam-engine and to the physical properties of bodies*. John van Voorst, London, first edition edition, 1867. URL https://archive.org/details/mechanicaltheor04claugoog. archive.org, San Francisco, CA 94118, USA.

Newton C. A. da Costa. On the theory of inconsistent formal systems. *Notre Dame Journal of Formal Logic*, 15(4):497–510, October 1974. ISSN 0029-4527. doi: 10.1305/ndjfl/1093891487.

Newton Carneiro Alfonso da Costa. Nota sobre o conceito de contradição. *Anuário da Sociedade Paranaense de Matemática*, 1(2):6–8, 1958. URL Portuguese.

Chandler Davis. The norm of the schur product operation. *Numerische Mathematik*, 4(1):343–344, 12 1962. ISSN 0945-3245. doi: 10.1007/BF01386329.

Bernhard Jacob Degen. *Principium identitatis indiscernibilium*. Meyer, 1741.

Paul Adrien Maurice Dirac. Quantum mechanics and a preliminary investigation of the hydrogen atom. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 110(755):561–579, 3 1926. doi: 10.1098/rspa.1926.0034. The Royal Society.

John Donnelly. Creation ex nihilo. In *Proceedings of the American Catholic Philosophical Association*, volume 44, pages 172–184, 1970. DOI: 10.5840/acpaproc19704425 .

Gabrielle-Émilie Le Tonnelier de Breteuil du Châtelet. *Institutions de physique*. Chez Prault fils, Paris, premier tome edition, 1740. URL https://archive.org/details/institutionsdeph00duch. archive.org, San Francisco, CA 94118, USA.

Kenny Easwaran. The role of axioms in mathematics. *Erkenntnis*, 68(3):381–391, 2008. DOI: 10.1007/s10670-008-9106-1.

Meister Eckhart. *Meister Eckhart: Die deutschen Werke, Band 1: Predigten. Editor Josef Quint*, volume 2. W.Kohlhammer Verlag, 1986. ISBN: 978-3-17-061210-5.

Arnold Ehrhardt. Creatio ex nihilo. *Studia Theologica - Nordic Journal of Theology*, 4(1):13–43, 1950. doi: 10.1080/00393385008599697. URL https://doi.org/10.1080/00393385008599697. DOI: 10.1080/00393385008599697 .

Albert Einstein. Zur Elektrodynamik bewegter Körper. *Annalen der Physik*, 322(10):891–921, 1905a. ISSN 1521-3889. doi: https://doi.org/10.1002/andp.19053221004. Wiley Online Library.

Albert Einstein. Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig? *Annalen der Physik*, 323(13):639–641, 1905b. ISSN 1521-3889. doi: 10.1002/andp.19053231314. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/andp.19053231314.

Albert Einstein. Relativität und Gravitation. Erwiderung auf eine Bemerkung von M. Abraham. *Annalen der Physik*, 343(10):1059–1064, January 1912. ISSN 1521-3889. doi: 10.1002/andp.19123431014. URL https://doi.org/10.1002/andp.19123431014.

Albert Einstein. Die Feldgleichungen der Gravitation. *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 844-847.*, 1915. URL http://adsabs.harvard.edu/abs/1915SPAW.......844E.

Albert Einstein. Die grundlage der allgemeinen relativitätstheorie. *Annalen der Physik*, 354(7):769–822, 1916. ISSN 1521-3889. doi: https://doi.org/10.1002/andp.19163540702.

Albert Einstein. Kosmologische betrachtungen zur allgemeinen relativitätstheorie. *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin)*, page 142–152, 1917.

Albert Einstein. Prinzipielles zur allgemeinen Relativitätstheorie. *Annalen der Physik*, 360(4):241–244, 1918. ISSN 1521-3889. doi: 10.1002/andp.19183600402. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/andp.19183600402. AdP. Free full text: Wiley Online Library.

Albert Einstein. Induktion and Deduktion in der Physik. *Berliner Tageblatt and Handelszeitung*, page Suppl. 4, December 1919. URL https://einsteinpapers.press.princeton.edu/vol7-trans/124. Berliner Tageblatt and Handelszeitung.

Albert Einstein. Fundamental ideas and problems of the theory of relativity. nobel lecture. lecture delivered to the nordic assembly of naturalists at gothenburg july 11, 1923. *Nordic Assembly of Naturalists*, page 482–490, 7 1923a.

Albert Einstein. *The meaning of relativity. Four lectures delivered at Princeton University, May, 1921*. Princeton University Press, Princeton, 1923b.

Albert Einstein. Elementary Derivation of the Equivalence of Mass and Energy. *Bulletin of the American Mathematical Society*, 41(4): 223–230, 1935. URL https://projecteuclid.org/download/pdf_1/euclid.bams/1183498131.

Albert Einstein. Physics and reality. *Journal of the Franklin Institute*, 221(3):349–382, 3 1936. ISSN 0016-0032. doi: 10.1016/S0016-0032(36)91047-5.

Albert Einstein and Marcel Grossmann. *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation: Physikalischer Teil von Albert Einstein. Mathematischer Teil von Marcel Grossmann*. B. G. Teubner, Leipzig, 1913.

Albert Einstein and Willem de Sitter. On the Relation between the Expansion and the Mean Density of the Universe. *Proceedings of the National Academy of Sciences of the United States of America*, 18(3):213–214, March 1932. ISSN 0027-8424. URL https://www.ncbi.nlm.nih.gov/pmc/articles/PMC1076193/.

C. A. Escobar and L. F. Urrutia. Invariants of the electromagnetic field. *Journal of Mathematical Physics*, 55(3):032902, March 2014. ISSN 0022-2488. doi: 10.1063/1.4868478. URL https://aip.scitation.org/doi/10.1063/1.4868478. Publisher: American Institute of Physics.

Johann Gottlieb Fichte. *Science of knowledge*. The english and foreign philosophical library. Trübner & Co., London, 1889.

Paul Finsler. *Über Kurven und Flächen in allgemeinen Räumen*. PhD thesis, Georg-August Universität, Göttingen, 1918.

George Francis FitzGerald. The ether and the earth's atmosphere. *Science (New York, N.Y.)*, 13(328):390, 5 1889. ISSN 0036-8075. doi: 10.1126/science.ns-13.328.390.

Lewis S Ford. An alternative to creatio ex nihilo. *Religious Studies*, 19(2):205–213, 1983. DOI: 10.1017/S0034412500015031 .

Eckart Förster and Yitzhak Y Melamed. *"Omnis determinatio est negatio" – Determination, Negation and Self-Negation in Spinoza, Kant, and Hegel. In: Spinoza and German idealism. Eckart Forster & Yitzhak Y. Melamed (eds.)*. Cambridge University Press, Cambridge [England]; New York, 2012. ISBN 978-1-283-71468-6. URL https://doi.org/10.1017/CBO9781139135139.

Howard Georgi and S. L. Glashow. Unity of all elementary-particle forces. *Physical Review Letters*, 32(8):438–441, 2 1974. doi: 10.1103/PhysRevLett.32.438.

Jacques Hadamard. Rèsolution d'une question relative aux dèterminants. *Bulletin des Sciences Mathématiques*, 2(17):240–246, 1893.

Klaus Hedwig. Negatio negationis: Problemgeschichtliche Aspekte einer Denkstruktur. *Archiv für Begriffsgeschichte*, 24(1):7–33, 1980. ISSN 0003-8946. URL www.jstor.org/stable/24359358.

Hegel, Georg Wilhelm Friedrich. *Wissenschaft der Logik. Erster Band. Erstes Buch*. Johann Leonhard Schrag, Nürnberg, December 1812a. doi: 10.5281/zenodo.5917182. URL https://doi.org/10.5281/zenodo.5917182. Online at: Archive.org Zenodo.

Hegel, Georg Wilhelm Friedrich. *Wissenschaft der Logik. Erster Band. Erstes Buch*. Johann Leonhard Schrag, Nürnberg, December 1812b. doi: 10.5281/zenodo.5917182. URL https://doi.org/10.5281/zenodo.5917182. Online at: Archive.org Zenodo.

Hegel, Georg Wilhelm Friedrich. *Hegelś Science of Logic*. Prometheus Books, New York, USA, 1991. ISBN 13: 9781573922807.

Hegel, Georg Wilhelm Friedrich. *The Science of Logic. Translated and edited by George Di Giovanni*. Cambridge University Press, Cambridge, USA, 2010. ISBN-13: 978-0-511-78978-6.

Heinemann, Fritz H. The Meaning of Negation. *Proceedings of the Aristotelian Society*, 44:127–152, 1943. ISSN 0066-7374. Oxford University Press.

Laurence R. Horn. *A natural history of negation*. University of Chicago Press, Chicago, 1989. ISBN 978-0-226-35337-1. ISBN: 978-0-226-35337-1.

L. P. Hughston and K. P. Tod. *An introduction to general relativity*. Number 5 in London Mathematical Society student texts. Cambridge University Press, Cambridge ; New York, 1990. ISBN 978-0-521-32705-3.

Dragan Huterer and Michael S Turner. Prospects for probing the dark energy via supernova distance measurements. *Physical Review D*, 60(8):081301, 1999.

M. W. Kalinowski. The program of geometrization of physics: Some philosophical remarks. *Synthese*, 77(2):129–138, 1988. ISSN 00397857, 15730964. URL https://www.jstor.org/stable/20116587.

David C. Kay. *Schaum's outline of theory and problems of tensor calculus*. Schaum's outline series. Schaum's outline series in mathematics. McGraw-Hill, New York, 1988. ISBN 978-0-07-033484-7.

Anton Friedrich Koch. Die Selbstbeziehung der Negation in Hegels Logik. *Zeitschrift für philosophische Forschung*, 53(1):1–29, 1999. ISSN 0044-3301. URL www.jstor.org/stable/20484868.

Helge Kragh. Big bang: the etymology of a name. *Astronomy & Geophysics*, 54(2):2.28–2.30, 4 2013. ISSN 1366-8781. doi: 10.1093/astrogeo/att035.

Leopold Kronecker. Ueber bilineare Formen. *Journal für die reine und angewandte Mathematik (Crelle's Journal)*, 68:273 – 285, 1868.

Kenneth Kunen. Negation in logic programming. *The Journal of Logic Programming*, 4(4):289–308, December 1987. ISSN 0743-1066. doi: 10.1016/0743-1066(87)90007-0. URL http://www.sciencedirect.com/science/article/pii/0743106687900070.

Joseph Larmor. IX. A dynamical theory of the electric and luminiferous medium.— Part III. Relations with material media. *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, 190:205–300, 1 1897. doi: 10.1098/rsta.1897.0020.

M. Laue. Zur Dynamik der Relativitätstheorie. *Annalen der Physik*, 340(8):524–542, 1911. ISSN 1521-3889. doi: 10.1002/andp.19113400808. URL https://onlinelibrary.wiley.com/doi/pdf/10.1002/andp.19113400808.

D. Lehmkuhl. Mass-Energy-Momentum: Only there Because of Spacetime? *The British Journal for the Philosophy of Science*, 62(3): 453–488, September 2011. ISSN 0007-0882, 1464-3537. doi: 10.1093/bjps/axr003. URL https://academic.oup.com/bjps/article-lookup/doi/10.1093/bjps/axr003.

Gottfried Wilhelm Leibniz. *Oeuvres philosophiques latines & françoises de feu Mr. de Leibnitz*. Chez Jean Schreuder, Amsterdam (NL), 1765. URL https://archive.org/details/oeuvresphilosoph00leibuoft/page/n9.

Leibniz, Gottfried Wilhelm. *La monadologie (Nouvelle édition) / Leibniz ; nouvelle édition, avec une introduction, des sommaires, un commentaire perpétuel extrait des autres ouvrages de Leibniz, des exercices et un lexique de la terminologie leibnizienne. Bertrand, Alexis (1850-1923). Éditeur scientifique: Alexis Bertrand*. Vve E. Belin et fils, Paris, 1886. URL https://gallica.bnf.fr/http://catalogue.bnf.fr/ark:/12148/cb30781197z. Bibliothèque nationale de France.

Abbé G Lemaître. Contributions to a british association discussion on the evolution of the universe. *Nature*, 128(3234):704–706, 1931a.

Georges Lemaître. The beginning of the world from the point of view of quantum theory. *Nature*, 127(3210):706–706, 1931b.

Hendrik Anton Lorentz. De relatieve beweging van de aarde en den aether. *Verslagen der Afdeeling Natuurkunde van de Koninklijke Akademie van Wetenschappen*, 1:74–79, 1892.

Hendrik Antoon Lorentz. Simplified theory of electrical and optical phenomena in moving systems. *Verhandelingen der Koninklijke Akademie van Wetenschappen*, 1:427–442, 1899.

Frédéric Moulin. Generalization of Einstein's gravitational field equations. *The European Physical Journal C*, 77(12):878, December 2017. ISSN 1434-6052. doi: 10.1140/epjc/s10052-017-5452-y. URL https://doi.org/10.1140/epjc/s10052-017-5452-y.

Tomáš Málek. *General Relativity in Higher Dimensions*. PhD thesis, Institute of Theoretical Physics. Faculty of Mathematics and Physics. Charles University, Prague, 2012. URL https://arxiv.org/pdf/1204.0291.pdf.

Russell Newstadt. *Omnis Determinatio est Negatio: A Genealogy and Defense of the Hegelian Conception of Negation*. Loyola University Chicago, Chicago (IL), dissertation edition, 2015. Free full text: Loyola University Chicago, USA.

A. A. Penzias and R. W. Wilson. A measurement of excess antenna temperature at 4080 mc/s. *The Astrophysical Journal*, 142:419–421, 7 1965. ISSN 0004-637X. doi: 10.1086/148307.

Max Karl Ernst Ludwig Planck. Ueber das Gesetz der Energieverteilung im Normalspectrum. *Annalen der Physik*, 309(3):553–563, 1901. ISSN 1521-3889. doi: 10.1002/andp.19013090310. Wiley Online Library.

Greek philosopher Plato. *Parmenides. Ins Deutsche übertragen von Otto Kiefer*. Verlegt bei Eugen Diederichs, 1910. Archive.org.

Jules Henri Poincaré. Sur la dynamique de l'électron. *Comptes rendus hebdomadaires des séances de l'Académie des sciences*, 140: 1504–1508, 1905.

Graham Priest. What is so Bad about Contradictions? *The Journal of Philosophy*, 95(8):410–426, 1998. ISSN 0022-362X. doi: 10.2307/2564636. URL https://www.jstor.org/stable/2564636.

Graham Priest, Richard Sylvan, Jean Norman, and A. I. Arruda, editors. *Paraconsistent logic: essays on the inconsistent*. Analytica. Philosophia, München ; Hamden [Conn.], 1989. ISBN 978-3-88405-058-3.

Francisco Miró Quesada, editor. *Heterodox logics and the problem of the unity of logic. In: Non-Classical Logics, Model Theory, and Computability: Proceedings of the Third Latin-American symposium on Mathematical Logic, Campinas, Brazil, July 11-17, 1976. Arruda, A. I., Costa, N. C. A. da, Chuaqui, R. (Eds.).*, volume 89 of *Studies In Logics And The Foundations Of Mathematics*. North-Holland, Amsterdam ; New York : New York, February 1977. ISBN 978-0-7204-0752-5.

Gregorio Ricci-Curbastro and Tullio Levi-Civita. Méthodes de calcul différentiel absolu et leurs applications. *Mathematische Annalen*, 54(1):125–201, 3 1900. ISSN 1432-1807. doi: 10.1007/BF01454201.

Connie Robertson. *The Wordsworth Dictionary of Quotations. Edited by Connie Robertson*. Wordsworth, Ware, Hertfordshire, 1998. ISBN 978-1-85326-751-2. ISBN: 1-85326-489-X.

Josiah Royce. *Negation*, volume 9 of *Encyclopaedia of Religion and Ethics. J. Hastings (ed.)*. Charles Scribner's Sons, New York (USA), 1917. Free full text: archive.org, San Francisco, CA 94118, USA.

Bertrand Russell. *The problems of philosophy*. H. Holt, 1912. archive.org.

J. Schur. Bemerkungen zur theorie der beschränkten bilinearformen mit unendlich vielen veränderlichen. *Journal für die reine und angewandte Mathematik (Crelles Journal)*, 1911(140):1–28, 7 1911. ISSN 0075-4102, 1435-5345. doi: 10.1515/crll.1911.140.1.

J. L. Speranza and Laurence R. Horn. A brief history of negation. *Journal of Applied Logic*, 8(3):277–301, September 2010. ISSN 1570-8683. DOI: 10.1016/j.jal.2010.04.001 ScienceDirect.

Benedictus de Spinoza. *Opera quae supersunt omnia / iterum edenda curavit, praefationes, vitam auctoris, nec non notitias, quae ad historiam scriptorum pertinent*. in bibliopolio academico, June 1674. doi: 10.5281/zenodo.5651174. URL https://doi.org/10.5281/zenodo.5651174. Zenodo.

Hans Stephani, editor. *Exact solutions of Einstein's field equations*. Cambridge monographs on mathematical physics. Cambridge University Press, Cambridge, UK ; New York, 2nd ed edition, 2003. ISBN 978-0-521-46136-8.

Richard C. Tolman. XXXIII. Non-Newtonian Mechanics, The Mass of a Moving Body. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 23(135):375–380, 1912. doi: 10.1080/14786440308637231. URL https://www.tandfonline.com/doi/full/10.1080/14786440308637231.

Marie-Antoinette Tonnelat, Février P., and André Lichnerowicz. *La théorie du champ unifié d'Einstein et quelques-uns de ses développements*. Gauthier-Villars, 1955.

Tamar Tsopurashvili. Negatio negationis als Paradigma in der Eckhartschen Dialektik. In *Universalità della Ragione. A. Musco (ed.)*, volume II.1, pages 595–602, Palermo, 17-22 settembre 2007, 2012. Luglio.

Woldemar Voigt. *Über das Doppler'sche Princip*, volume 8 of *Nachrichten von der Köönigl. Gesellschaft der Wissenschaften und der Georg-Augusts-Universität zu Göttingen*. Dieterichsche Verlags-Buchhandlung, 1887. URL http://archive.org/details/nachrichtenvond04gtgoog. archive.org.

Woldemar Voigt. *Die fundamentalen physikalischen Eigenschaften der Krystalle in elementarer Darstellung*. Verlag von Veit und Companie, 1898. URL http://archive.org/details/bub_gb__Ps4AAAAMAAJ.

Michael V Wedin. Negation and quantification in aristotle. *History and Philosophy of Logic*, 11(2):131–150, 1990a. Taylor & Francis.

Michael V. Wedin. Negation and quantification in aristotle. *History and Philosophy of Logic*, 11(2):131–150, January 1990b. ISSN 0144-5340. doi: 10.1080/01445349008837163. DOI: 10.1080/01445349008837163.

G Zehfuss. Über eine gewisse Determinante. *Zeitschrift für Mathematik und Physik*, 3:298–301, 1858.

Fritz Zwicky. Die Rotverschiebung von extragalaktischen Nebeln. *Helvetica physica acta*, 6:110–127, 1933.

I was born October, 1st 1961 in Novo Selo, Bosnia and Herzegovina, former Yogoslavia. I am of Croatian origin. From 1982-1989 C.E., I studied human medicine at the University of Hamburg, Germany. Meanwhile, I am working as a specialist of internal medicine. My basic field of research since my high school days at the Wirtschaftsgymnasium Bruchsal, Baden Württemberg, Germany is the mathematization of the relationship between a cause and an effect valid without any restriction under any circumstances including the conditions of classical logic, probability theory, quantum mechanics, special and general theory of relativity, human medicine et cetera. I endeavour to investigate positions of quantum mechanics, relativity theory, mathematics et cetera, only insofar as these positions put into question or endanger **the general validity of the principle of causality**.

[a]https://orcid.org/0000-0002-6988-2780

[b]https://cel.webofknowledge.com/InboundService.do?app=wos&product=CEL&Func=Frame&SrcApp=Publons&SrcAuth=Publons_CEL&locale=en-US&SID=F4r5Tsr3OcrmFbYrqiF&customersID=Publons_CEL&smartRedirect=yes&mode=FullRecord&IsProductCode=Yes&Init=Yes&action=retrieve&UT=WOS%3A000298855300006

[c]https://publons.com/researcher/3501739/ilija-barukcic/

[d]https://www.scopus.com/authid/detail.uri?authorId=37099674500

[e]https://www.scopus.com/authid/detail.uri?authorId=54974181600

[f]https://www.mendeley.com/search/?authorFullName=Ilija%20Baruk%C4%8Di%C4%87&page=1&query=Barukcic&sortBy=relevance

[g]https://www.researchgate.net/profile/Ilija-Barukcic-2

[h]https://zenodo.org/search?page=1&size=20&q=keywords:%22Baruk%C4%8Di%C4%87%22&sort=mostviewed

[i]https://zenodo.org/search?page=1&size=20&q=keywords:%22Baruk%C4%8Di%C4%87,%20Conference%22

[j]https://twitter.com/ilijabarukcic?lang=de

[k]https://twitter.com/Causation_Journ

[l]https://vixra.org/author/ilija_barukcic

[m]https://www.youtube.com/channel/UCwf3w1IngcukI0Ojpw8HTwg

[n]https://portal.dnb.de/opac/showNextResultSite?currentResultId=%22Barukcic%22%26any&currentPosition=30