Summation of Series of Binomial Coefficients

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Abstract: This paper presents a summation of series of binomial coefficients in combinatorial geometric series. The coefficient for each term in combinatorial geometric series refers to a binomial coefficient. This idea can enable the scientific researchers to solve the real life problems.

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1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series, a new idea was stimulated his mind to create a combinatorial geometric series [1-10]. The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r. In this article, binomial identities and multinomial theorem is provided using the binomial coefficients for combinatorial geometric series.

2. Combinatorial Geometric Series

The combinatorial geometric series [1-10] is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial refers to the binomial coefficient V_n^r.

$$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r} = \sum_{i=0}^{n} V_i^r x^i \ \ \& \ \ V_n^r = \frac{(n+1)(n+2)(n+3)\cdots(n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1 \ \ and \ \ n, r \in N = \{1, 2, 3, \cdots\}$.

Here, $\sum_{i=0}^{n} V_i^r x^i$ refers to the combinatorial geometric series and

$V_n^r \ is$ the binomial coefficient for combinatorial geometric series and $V_n^r = \frac{(n+r)!}{n!\,r!}$.

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Theorem 2.1: $V_0^r + V_1^r + V_2^r + V_3^r + \cdots + V_{n-1}^r + V_n^r = V_n^{r+1}$.

$Proof.$ $V_0^r + V_1^r + V_2^r + V_3^r + \cdots + V_{n-1}^r + V_n^r = V_n^{r+1} \ \ \Longrightarrow V_{n-1}^{r+1} + V_n^r = V_n^{r+1}$,
where $V_0^r + V_1^r + V_2^r + \cdots + V_{n-1}^r = V_{n-1}^{r+1}$.
Let us prove that $V_{n-1}^{r+1} + V_n^r = V_n^{r+1}$.

$$V_{n-1}^{r+1} + V_n^r = \frac{(n-1+r+1)!}{(n-1)!\,(r+1)!} + \frac{(n+r)!}{r!\,n!} = (n+r)!\left(\frac{n}{n!\,(r+1)!} + \frac{r+1}{n!\,(r+1)!}\right)$$
$$= \frac{(n+r)!\,(n+r+1)}{n!\,(r+1)!} = \frac{(n+r+1)!}{n!\,(r+1)!} = V_n^{r+1}.$$

Hence, theorem is proved.

3. Conclusion

In this article, a binomial theorem was constituted on the binomial coefficients of combinatorial geometric series. This new idea can enable the scientific researchers for research and development further.

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