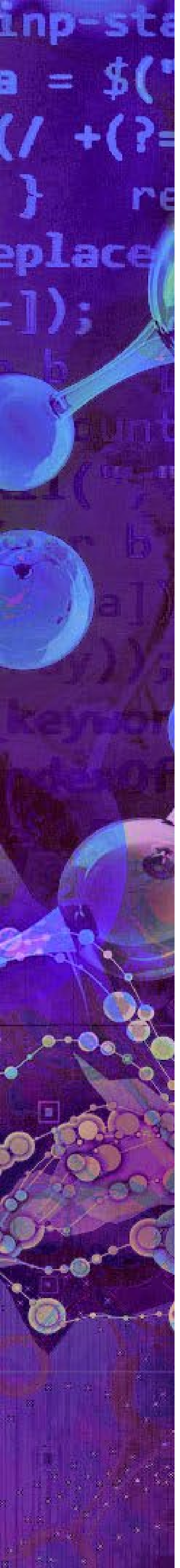

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A comparative study of dark matter flow & hydrodynamic turbulence and its applications

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Zhijie (Jay) Xu
Multiscale Modeling Team
Computational Mathematics Group
Physical & Computational Science Directorate
Zhijie.xu@pnnl.gov; zhijiexu@hotmail.com

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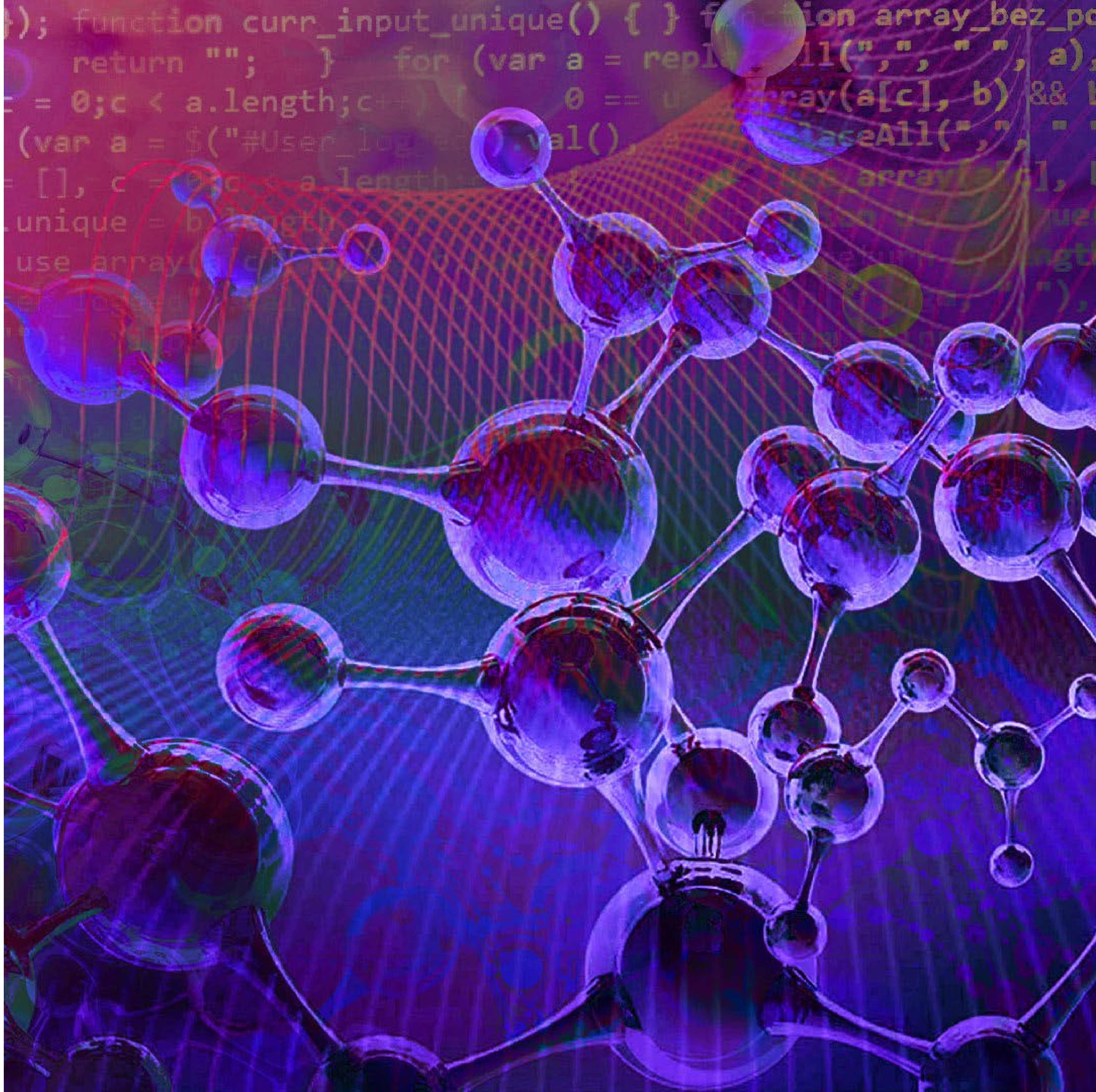


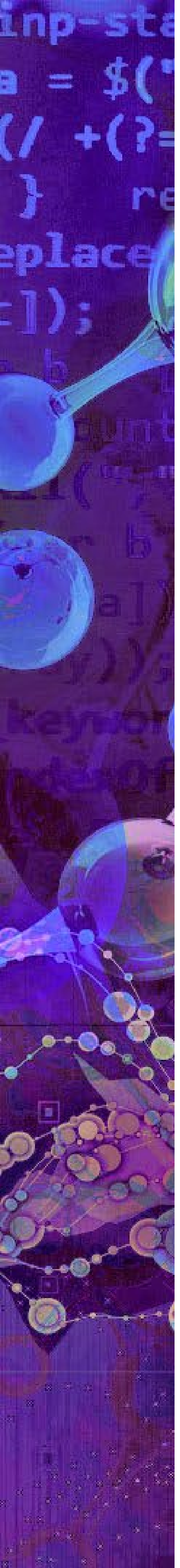
Preface

Dark matter, if exists, accounts for five times as much as ordinary baryonic matter. Therefore, dark matter flow might possess the widest presence in our universe. The other form of flow, hydrodynamic turbulence in air and water, is without doubt the most familiar flow in our daily life. During the pandemic, we have found time to think about and put together a systematic comparison for the connections and differences between two types of flow, both of which are typical non-equilibrium systems.

The goal of this presentation is to leverage this comparison for a better understanding of the nature of dark matter and its flow behavior on all scales. Science should be open. All comments are welcome.

Thank you!

(all slides available at zenodo.org by searching “dark matter flow”)



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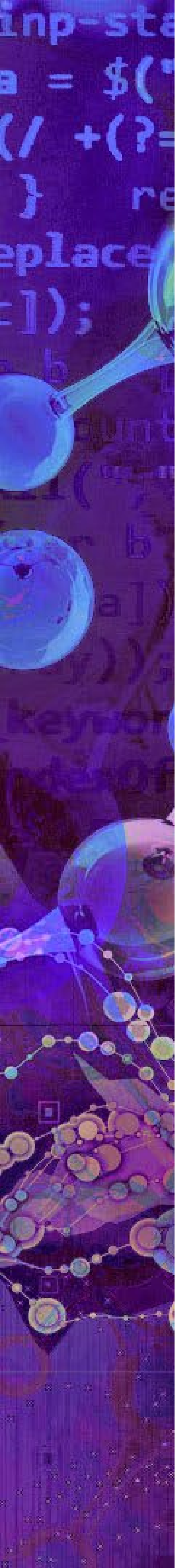
Data repository and relevant publications

Structural (halo-based) approach:

0.	Data https://dx.doi.org/10.5281/zenodo.6541230
1.	Inverse mass cascade in dark matter flow and effects on halo mass functions https://doi.org/10.48550/arXiv.2109.09985
2.	Inverse mass cascade in dark matter flow and effects on halo deformation, energy, size, and density profiles https://doi.org/10.48550/arXiv.2109.12244
3.	Inverse energy cascade in self-gravitating collisionless dark matter flow and effects of halo shape https://doi.org/10.48550/arXiv.2110.13885
4.	The mean flow, velocity dispersion, energy transfer and evolution of rotating and growing dark matter halos https://doi.org/10.48550/arXiv.2201.12665
5.	Two-body collapse model for gravitational collapse of dark matter and generalized stable clustering hypothesis for pairwise velocity https://doi.org/10.48550/arXiv.2110.05784
6.	Evolution of energy, momentum, and spin parameter in dark matter flow and integral constants of motion https://doi.org/10.48550/arXiv.2202.04054
7.	The maximum entropy distributions of velocity, speed, and energy from statistical mechanics of dark matter flow https://doi.org/10.48550/arXiv.2110.03126
8.	Halo mass functions from maximum entropy distributions in collisionless dark matter flow https://doi.org/10.48550/arXiv.2110.09676

Statistics (correlation-based) approach:

0.	Data https://dx.doi.org/10.5281/zenodo.6569898
1.	The statistical theory of dark matter flow for velocity, density, and potential fields https://doi.org/10.48550/arXiv.2202.00910
2.	The statistical theory of dark matter flow and high order kinematic and dynamic relations for velocity and density correlations https://doi.org/10.48550/arXiv.2202.02991
3.	The scale and redshift variation of density and velocity distributions in dark matter flow and two-thirds law for pairwise velocity https://doi.org/10.48550/arXiv.2202.06515
4.	Dark matter particle mass and properties from two-thirds law and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2202.07240
5.	The origin of MOND acceleration and deep-MOND from acceleration fluctuation and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2203.05606
6.	The baryonic-to-halo mass relation from mass and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2203.06899
7.	Universal scaling laws and density slope for dark matter halos from rotation curves and energy cascade https://doi.org/10.48550/arXiv.2209.033



Overview

- Some fundamentals of dark matter research
- Basic concepts in hydrodynamic turbulence
- Dark matter flow (SG-CFD) vs. hydrodynamic turbulence

- Theory of dark matter flow
  - Structural (halo-based) approach
  - Statistical (correlation-based) approach

- Applications of dark matter flow
  - Predicting dark matter particle properties
  - Understanding the origin of MOND
  - The baryonic-halo mass ratio and total baron faction
  - Universal scaling laws and halo density slope

Applications of dark matter flow

Universal scaling laws and scales for dark matter halos from rotation curves and energy cascade

Xu, Zhijie arXiv:2209.03313 [astro-ph.GA]
https://doi.org/10.48550/arXiv.2209.03313

Introduction

- Standard CDM Models' successes for large scale structure formation and evolution.

- Small scale challenges suggest missing pieces:
  - Galaxy scale (<1Mpc)
  - Core-cusp problem
  - Missing satellite
  - Too-big-to-fail
  - Baryonic Tully-Fisher and MOND

- Core-cusp problem
  - Dark matter halo density $\rho(r) \propto r^{\gamma}$
  - Cored density (γ =0) from observational data
  - Cuspy density (γ ~-1) from N-body simulations
    - No consensus
    - γ =-1.0 in NFW profile
    - γ =-1.2 (Diemand & Moore 2011)
    - $\gamma = -1.3$ (Governato et al. 2010)
    - $\gamma = -1.3$ (McKeown et al. 2022)

- Is there an asymptotic density slope for halos?
- Why exists a nearly universal density profile?
- Why different inner slopes γ exist in simulations?

- Core-cusp solutions
  - Within CDM framework
    - Baryonic feedback processes
  - Beyond CDM
    - Self-interacting dark matter

No matter collisionless or self-interacting

- What are the critical length or density scales for dark matter if exist?
- What is the effect of self-interaction on these scales?
- What are the fundamental properties (mass, cross-section etc.) of dark matter?

Energy cascade in hydrodynamic turbulence

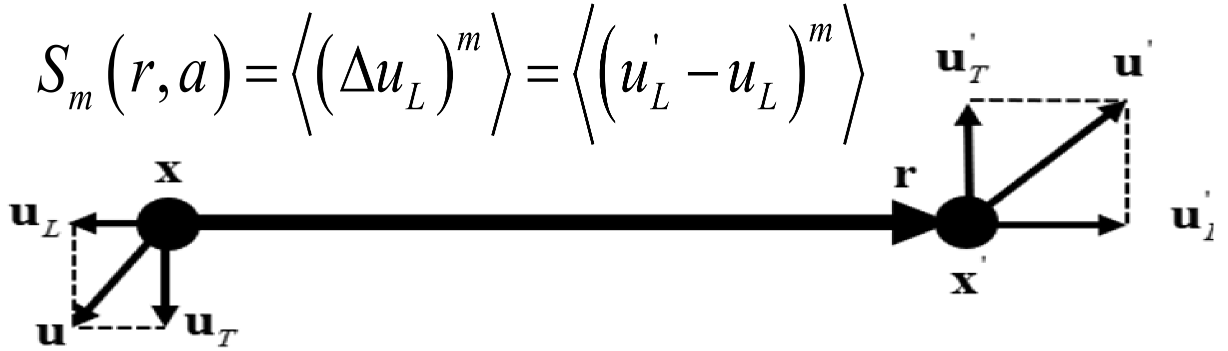
- There exist an inertial range with a scale-independent rate of energy cascade (ε does not depend on eddy size *l*) for eddy size η< *l* <L. η is a dissipative scale determined by viscosity ν and ε.

- In inertial range, inertial force is dominant over viscous force. A general scaling for velocity structure functions $S_m(r)$ for pairwise velocity Δu_L can be identified:

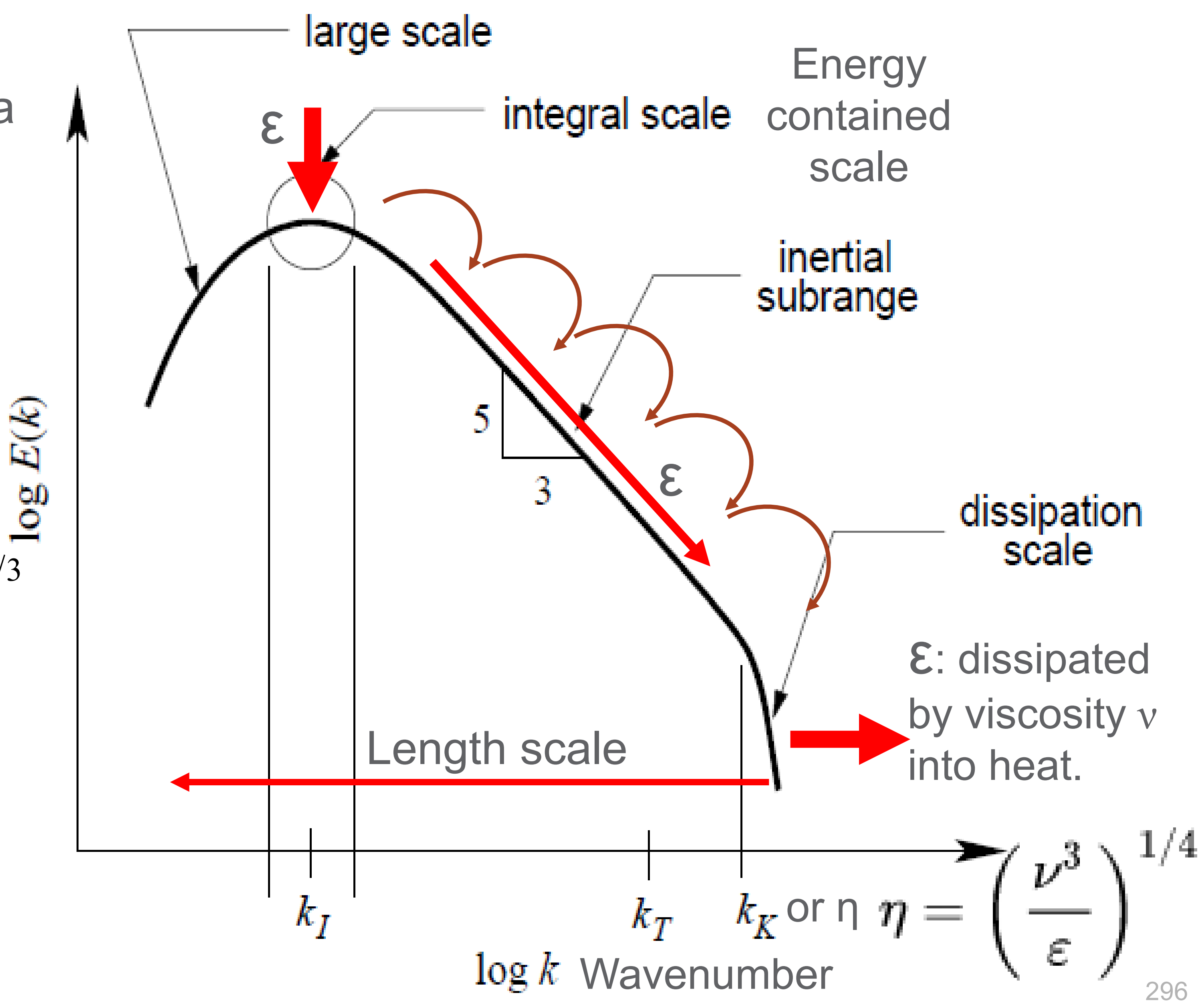
$$S_m(r) \propto (\varepsilon_u)^{m/3} r^{m/3} \xrightarrow{m=2} S_2 \propto (-\varepsilon_u)^{2/3} r^{2/3}$$

Two-thirds law

$$S_m(r,a) = \left\langle (\Delta u_L)^m \right\rangle = \left\langle (u'_L - u_L)^m \right\rangle$$



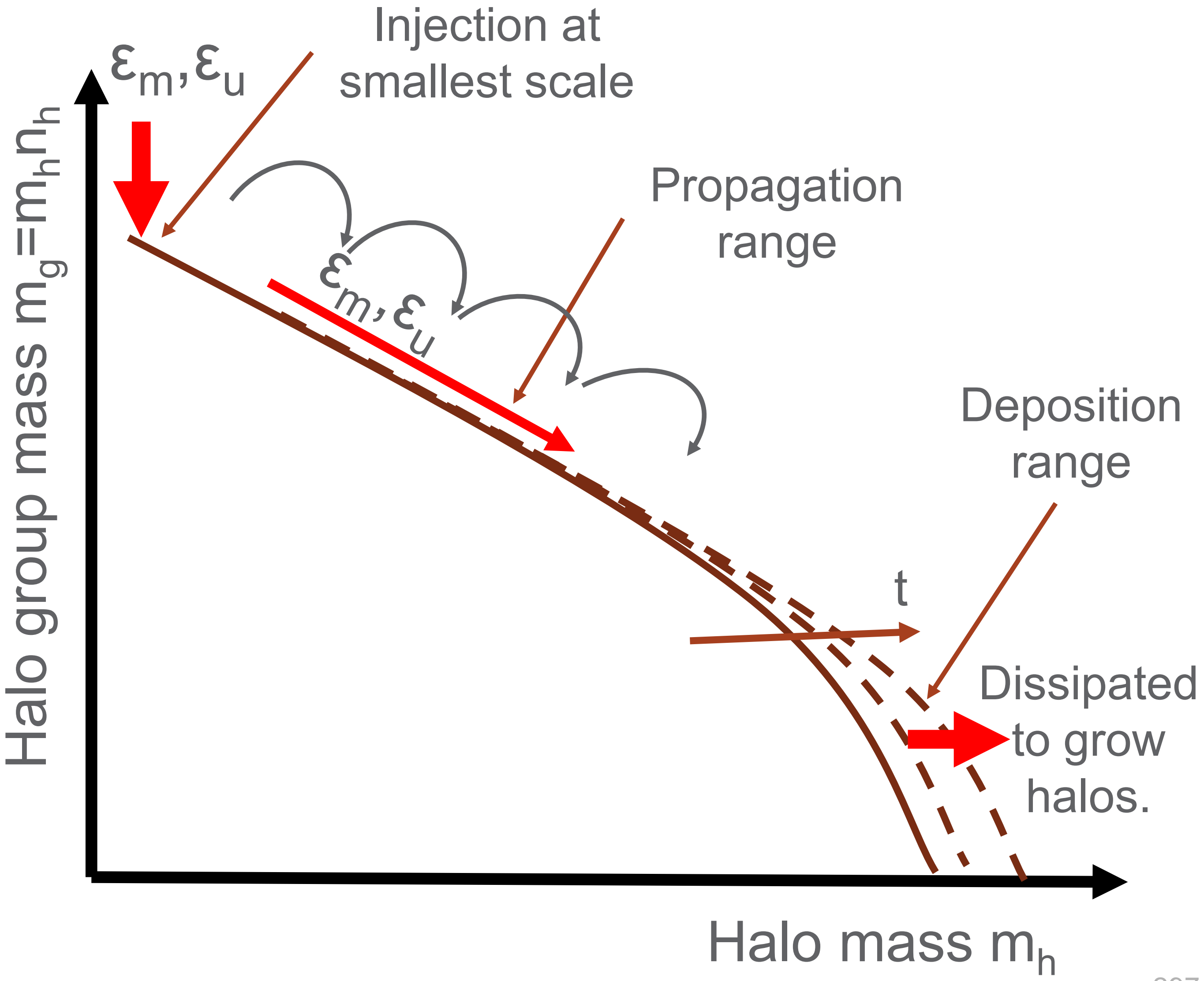
Big whirls have little whirls, That feed on their velocity;
And little whirls have lesser whirls, And so on to viscosity.



Mass/Energy cascade in dark matter flow (SG-CFD)

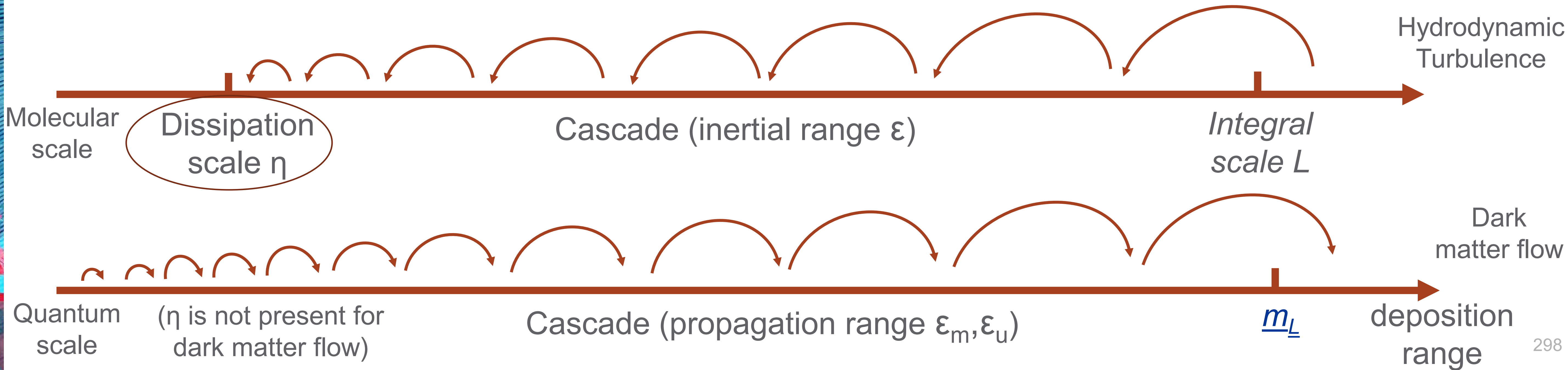
- Collisionless nature and long-range interaction.
- Long-range gravity requires a broad spectrum of halos to be formed to maximize system entropy.
- A continuous cascade of mass/energy from smaller to larger mass scales with a scale-independent rate of mass transfer ε_m and ε_u in a certain range of mass scales (propagation range).
- The mass/energy cascade is an intermediate statistically steady state for non-equilibrium systems to continuously maximize system entropy.
- The maximum entropy distribution of dark matter flow.

Little halos have big halos, That feed on their mass;
And big halos have greater halos, And so on to growth



Mass/Energy cascade in dark matter flow (SG-CFD)

- Collisionless, no dissipation range in SG-CFD.
- The smallest length scale of inertial range is not limited by viscosity.
- This enable us to extend the scale-independent ε_u down to the smallest scale, where quantum effects become important
- Dark matter flow exhibits scale-dependent flow behaviors for peculiar velocity, i.e. a constant divergence flow on small scales and an irrotational flow on large scales.
- The constant divergence flow shares the same even order kinematic relations with those of incompressible (divergence free) flow. This hints to similar scaling laws holds for dark matter.



Hydrodynamic Turbulence

Molecular scale

Dissipation scale η

Cascade (inertial range ε)

Integral scale L

Dark matter flow

Quantum scale

(η is not present for dark matter flow)

Cascade (propagation range ε_m,ε_u)

m_L

deposition range

Constant (time and scale independent) rate of energy cascade

Power-law time evolution for energy in terms of rate of energy cascade ε_u:

$$K_p = -\varepsilon_{\mathbf{u}} t$$

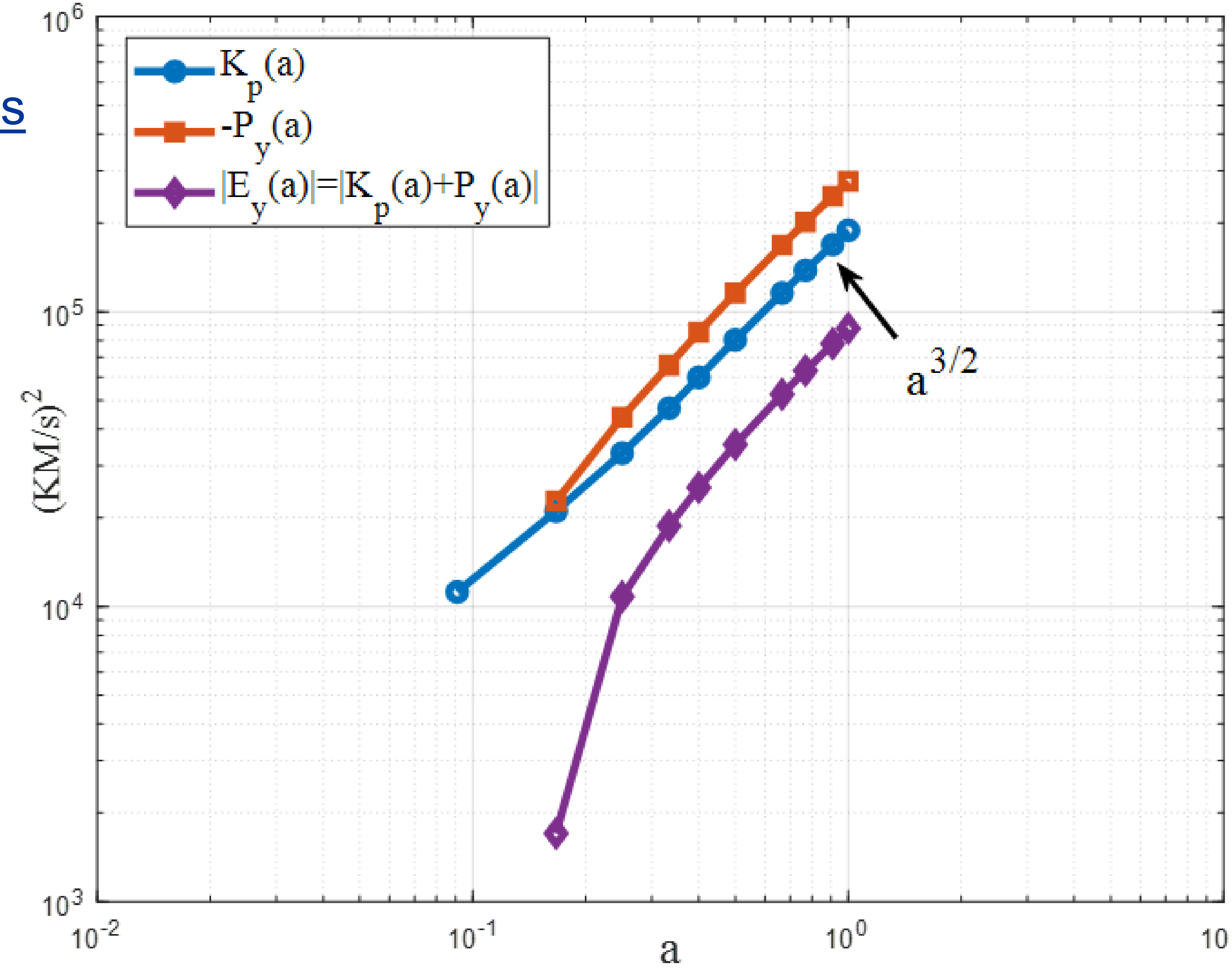
Power-law for Peculiar kinetic energy

$$P_y = \frac{7}{5}\varepsilon_{\mathbf{u}} t$$

Power-law for potential energy

$$\varepsilon_u = -\frac{K_p}{t} = -\frac{3}{2}\frac{u_0^2}{t_0} \approx -4.6\times10^{-7}\frac{m^2}{s^3}$$

Also see detail analysis for inverse kinetic energy cascade.



The time variation of specific kinetic and potential energies from *N*-body simulation.

The two-thirds law on small scales from N-body simulations

Odd order moment (generalized stable clustering hypothesis):

$$S_{2n+1}^{lp}\left(r\right)=\left(2n+1\right)S_{1}^{lp}\left(r\right)S_{2n}^{lp}\left(r\right)\propto r^{1}$$

Even order (two-thirds law):

$$S_{2n}^{lp}\left(r\right)-2^{n}u^{2n}K_{2n}\left(\Delta u_{L},0\right)=\beta_{2n}^{*}\left(r/r_{s}\right)^{2/3}\propto r^{2/3}$$

Second order (two-thirds law):

$$S_{2}^{lp}\left(r\right)-2u^{2}=S_{2r}^{lp}=\beta_{2}^{*}\left(r/r_{s}\right)^{2/3}\propto r^{2/3}$$

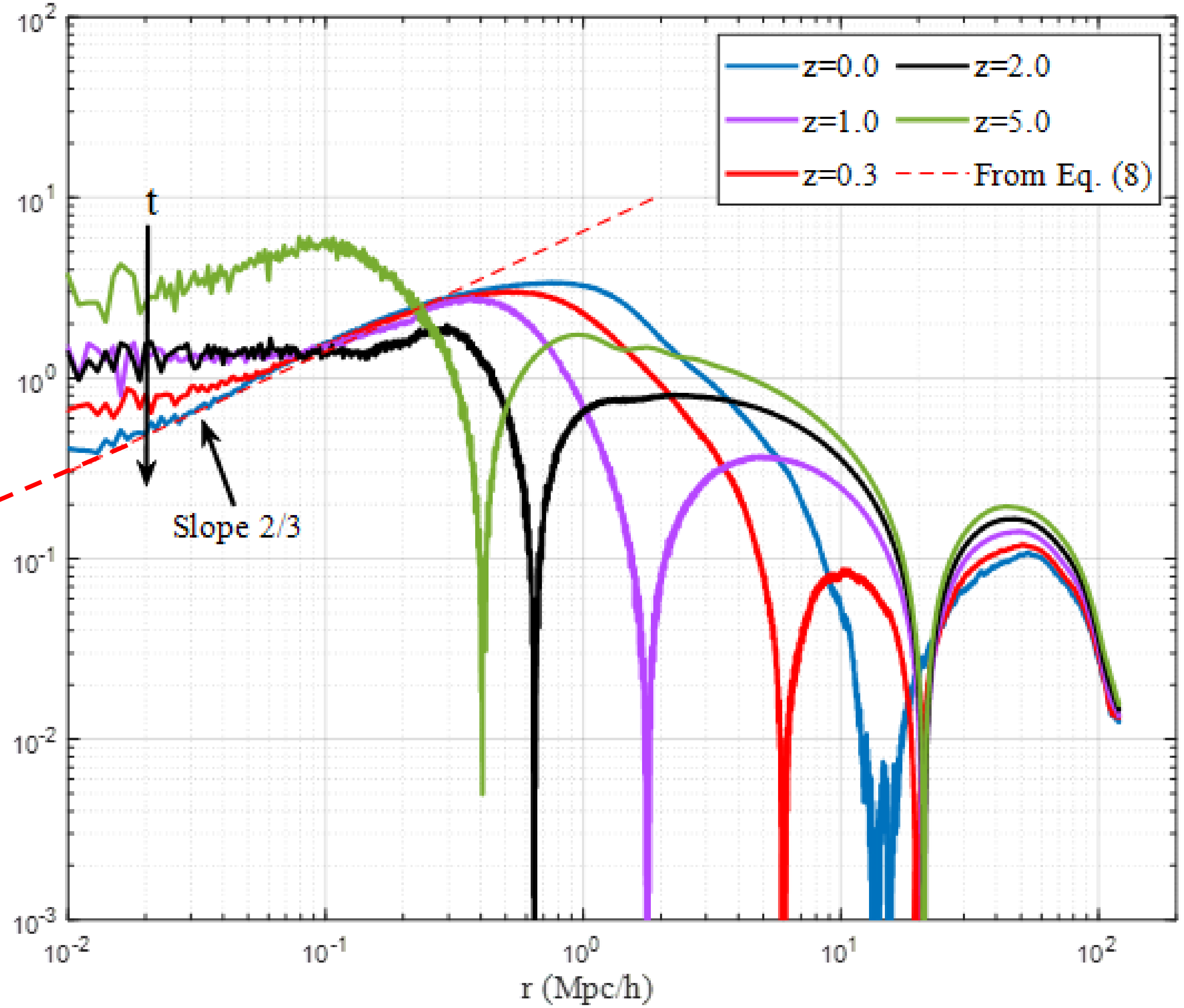
Introduce a velocity scale:

$$v_{r}^{2}=S_{2r}^{lp}\left(r\right)\Big/\left(2^{2/3}\beta_{2}^{*}a^{3/2}\right)$$

$$\left(-\varepsilon_{u}\right)=\frac{2v_{r}^{2}}{r}v_{r}=\frac{2v_{r}^{2}}{r/v_{r}}=\frac{2v_{r}^{3}}{r}$$

Acceleration

Turnaround time



Extend all the way to the smallest scale

Variation of normalized reduced longitudinal structure function and two-thirds law

Testing -4/3 law from rotation curves

$(-\varepsilon_u) = 2v_r^3/r$ Constant energy cascade

$v_r^2 \propto Gm_r/r$ Virial theorem

All relevant quantities determined by G, scale r and ε_u :

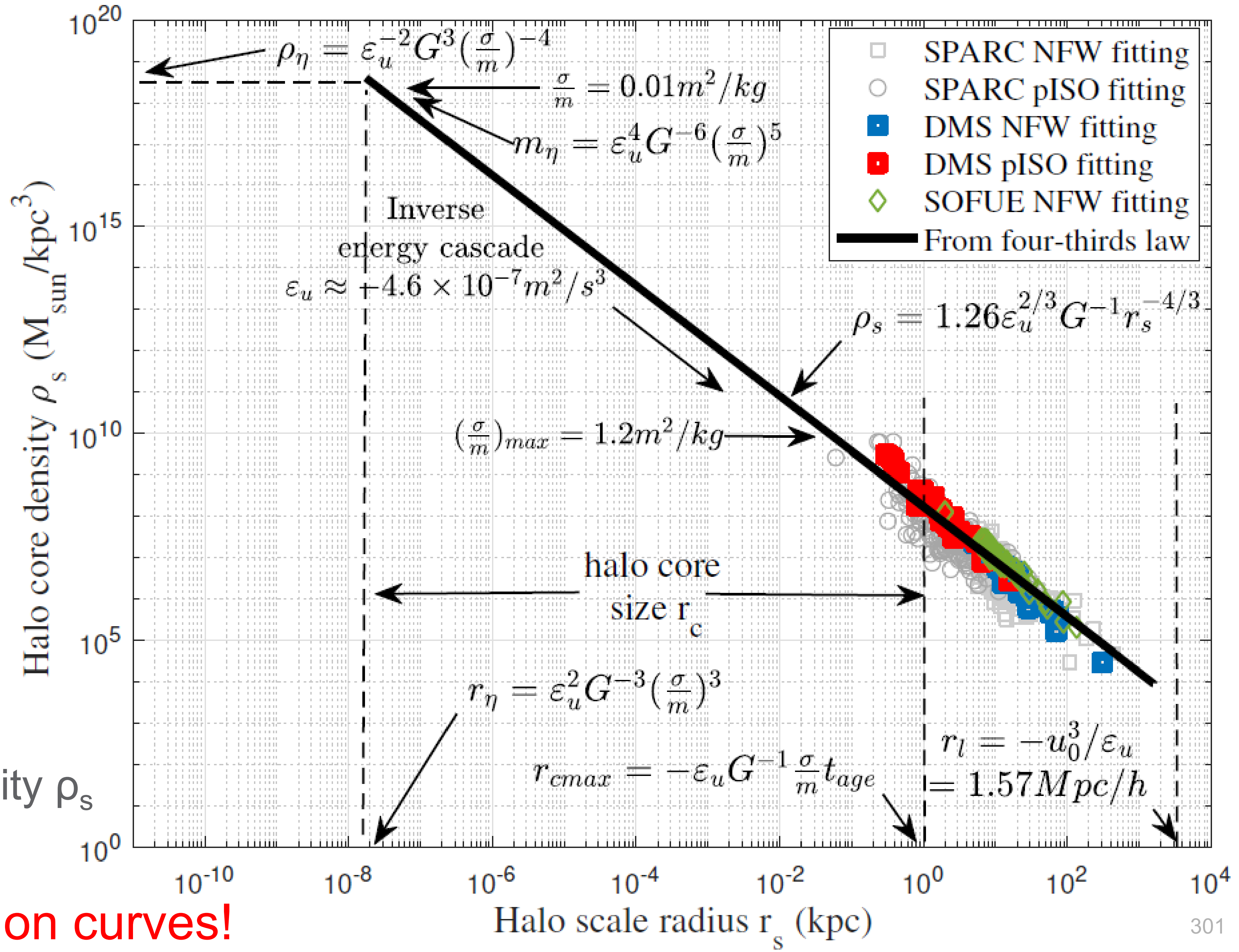
Mass: $m_r = \alpha_r \varepsilon_u^{2/3} G^{-1} r^{5/3}$

Density: $\boxed{\rho_r = \beta_r \varepsilon_u^{2/3} G^{-1} r^{-4/3}}$

Velocity: $v_r = (\gamma_s \varepsilon_u r)^{1/3}$

Time: $t_r \propto \varepsilon_u^{-1/3} r^{2/3}$

Halo scale r_s radius and core density ρ_s fitted from rotation curves



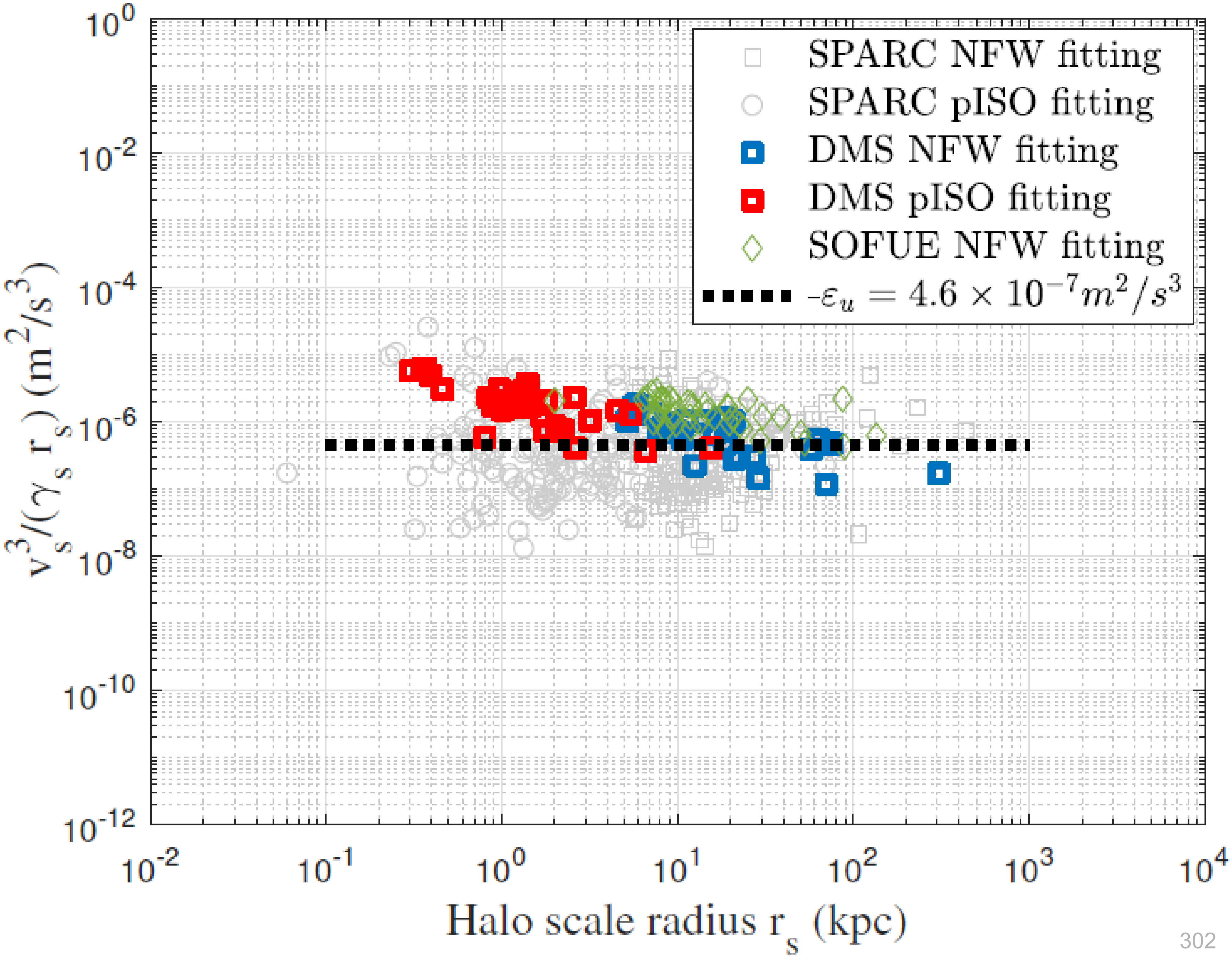
-4/3 law is confirmed by rotation curves!

Constant rate of cascade from galaxy rotation curves

$$v_s^2 = Gm_r\left(r_s\right)/r_s \qquad \left(-\varepsilon_u\right) = \frac{v_s^3}{\gamma_s r_s}$$

$$\gamma_s \approx 6.83$$

- Confirm the existence of a constant rate of cascade;
- Dispersion from spatial intermittence of energy cascade;
- Halos in different local environment may have different ε_u
- Dwarf galaxies tends to have smaller ε_u due to tidal stripping.



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Relevant scales for self-interacting dark matter

The largest length scale: $r_l = -u_0^3/\varepsilon_u$

The smallest length scale:

$\rho_r(\sigma/m)v_r t_r = 1$ Elastic scatter

$v_s^2 = G m_r(r_s)/r_s$ Virial theorem

$(-\varepsilon_u) = v_s^3/\gamma_s r_s$ Constant energy cascade

All relevant quantities determined by G, cross-section σ/m and ε_u :

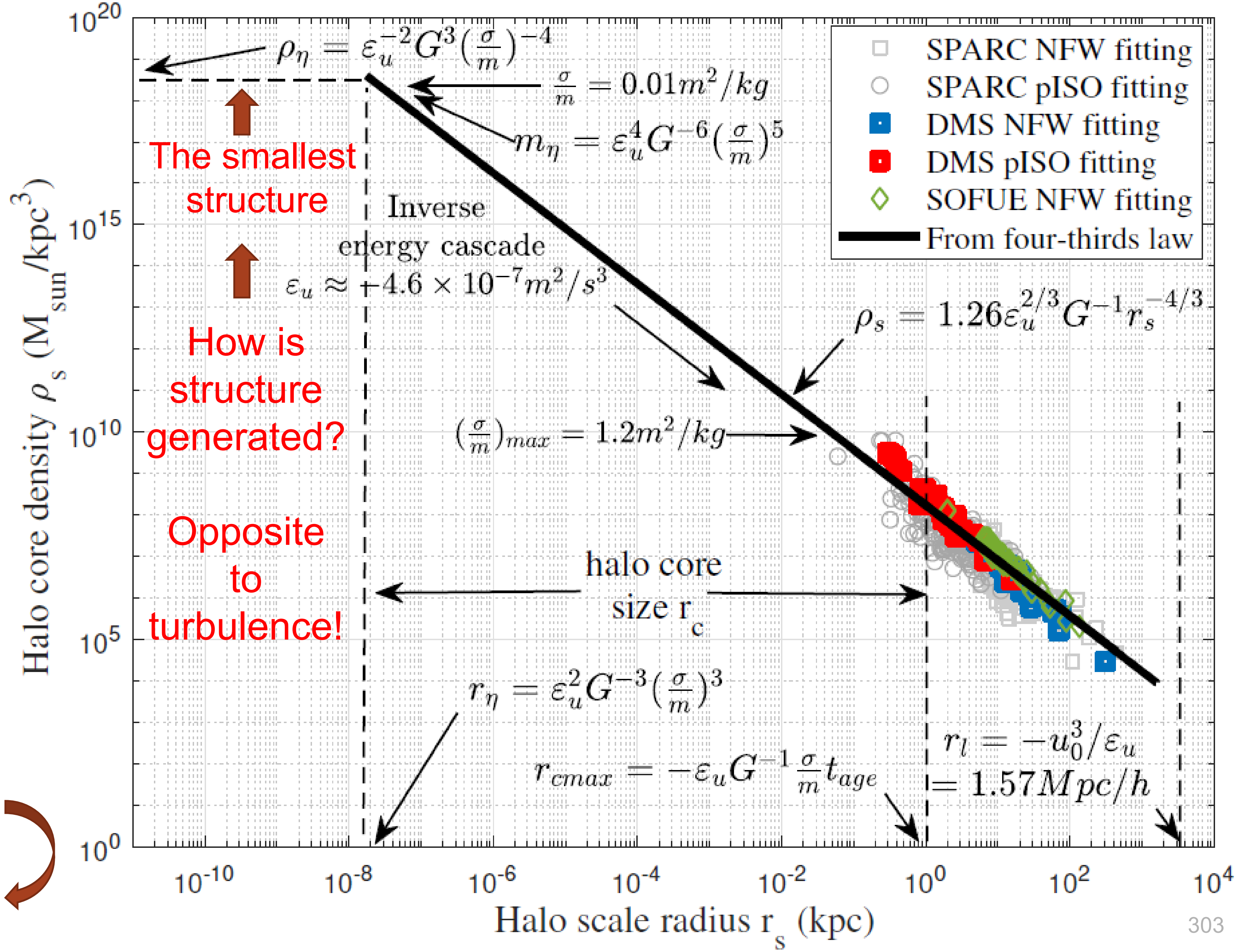
Length: $r_\eta = \varepsilon_u^2 G^{-3}(\sigma/m)^3$

Mass: $m_\eta = \varepsilon_u^4 G^{-6}(\sigma/m)^5$

Density: $\rho_\eta = \varepsilon_u^{-2} G^3(\sigma/m)^{-4}$

Maximum core size: $\rho_r \frac{\sigma}{m} v_r t_{age} = 1$

$$\frac{r_{c\max}}{(\sigma/m)} = -\varepsilon_u G^{-1} t_{age} \approx 100kpc\frac{kg}{m^2}$$



The smallest structure

How is structure generated?

Opposite to turbulence!

Relevant scales for collisionless dark matter

The largest length scale: $r_l = -u_0^3/\varepsilon_u$

The smallest length scale:

$m_X v_X \cdot l_X/2 = \hbar$ Uncertainty principle

$v_s^2 = G m_r(r_s)/r_s$ Virial theorem

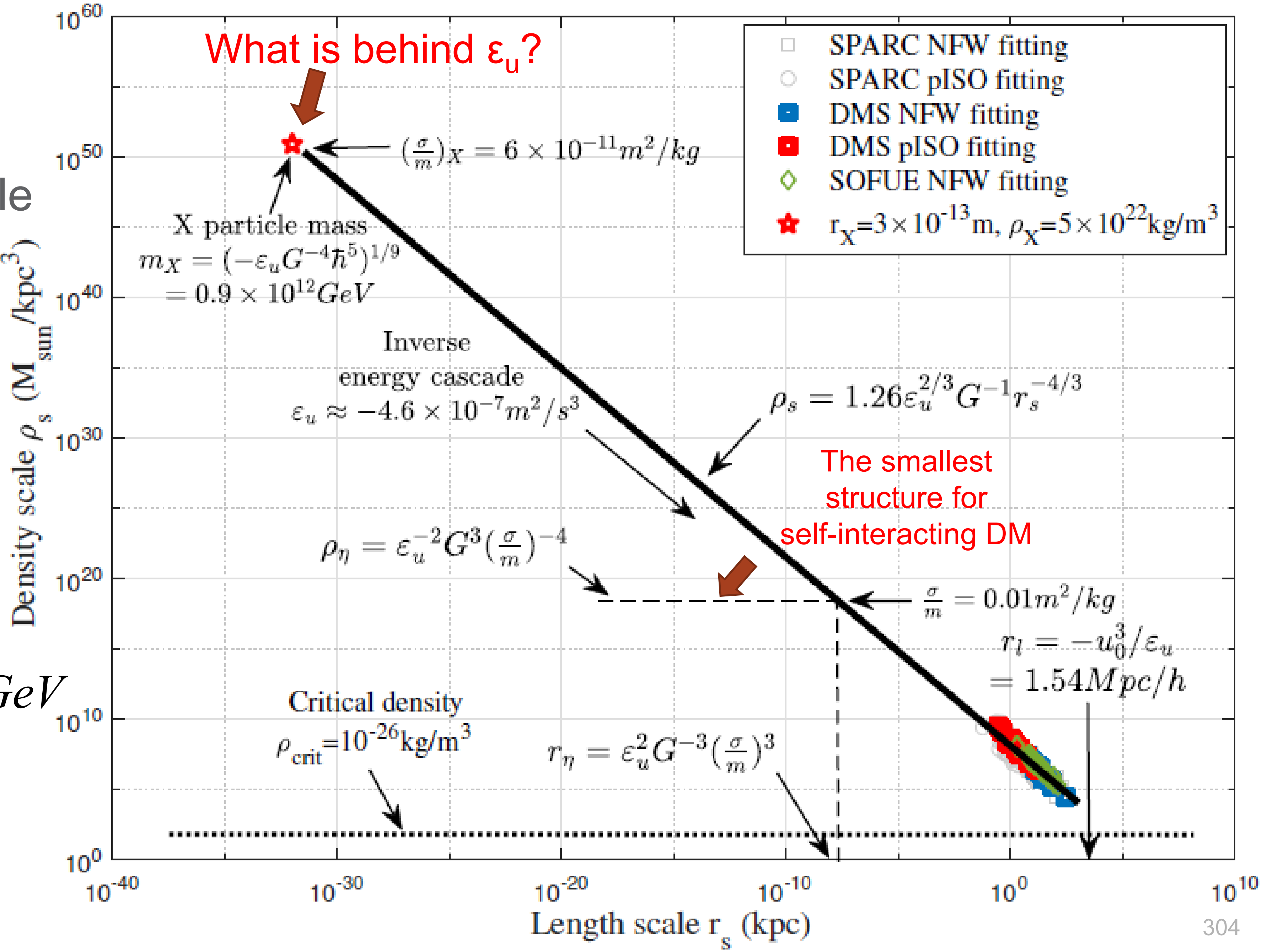
$(-\varepsilon_u) = v_s^3/\gamma_s r_s$ Constant energy cascade

All relevant quantities determined by G, Planck constant *h* and ε_u :

Mass scale: $m_X \propto \left(-\varepsilon_u \hbar^5/G^4\right)^{\frac{1}{9}} \approx 10^{12}\,GeV$

Length scale: $l_X \propto \left(-G\hbar/\varepsilon_u\right)^{\frac{1}{3}} \approx 10^{-13}\,m$

Time scale: $t_X \propto \left(G^2\hbar^2/\varepsilon_u^5\right)^{\frac{1}{9}} \approx 10^{-7}\,s$


What is behind ε_u?

The smallest structure for self-interacting DM

Uncertainty principles & energy cascade

Position (**x**), Velocity (**v** = d**x**/dt), Acceleration (**a** = d**v**/dt)

For fully collisionless dark matter:

1) A unique "symmetry" between **x** and **v** in phase space:
- At given **x** , particles can have multiple **v** (multi-stream)
- With given **v,** particles can be at different **x**
- NOT possible for non-relativistic baryons

2) Due to the long-rang gravitational interaction,
- Fluctuations (uncertainty) in **x**
- Fluctuations (uncertainty) in **v**
- Fluctuations (uncertainty) in **a**

3) Two pairs of conjugate variables:
- Position **x** and momentum **p**
- Momentum **p** and acceleration **a**

Wave function for position: $\psi(x)$

Wave function for momentum: $\varphi(p)$

Wave function for acceleration: $\mu(a)$

$$\mu_X = -m_X\varepsilon_u = 7.44\times10^{-22}\,kg\cdot m^2/s^3$$

$$\boxed{\begin{aligned}\psi(x) &= \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\varphi(p)e^{ipx/\hbar}dp\\ \varphi(p) &= \frac{1}{\sqrt{2\pi\mu_X}}\int_{-\infty}^{\infty}\mu(a)e^{ipa/\mu_X}da\end{aligned}}$$

$$\Downarrow$$

Uncertainty principles: $\sigma_x\sigma_p \ge \hbar/2$ $\quad\sigma_p\sigma_a \ge \mu_X/2$

$$\Downarrow$$

Postulated uncertainty principle for ***a*** and ***p***
leads to the constant rate of energy cascade: $\varepsilon_u = \mu_X/m_X = a_X v_X$

Universal halo density from energy cascade

Reduced spatial/ temporal coordinate: $x(r,t)=\dfrac{r}{r_s(t)}=\dfrac{c(t)r}{r_h(t)}$

Function *F(x)* for enclosed mass at given *r*: $m_r(r,t)=m_h(t)\dfrac{F(x)}{F(c)}$

Halo density: $\rho_h(r,t)=\dfrac{1}{4\pi r^2}\dfrac{\partial m_r(r,t)}{\partial r}=\dfrac{m_h(t)}{4\pi r_h^3}\dfrac{c^3F'(x)}{x^2F(c)}$

Radial continuity equation: $\dfrac{\partial\rho_h(r,t)}{\partial t}+\dfrac{1}{r^2}\dfrac{\partial\left[r^2\rho_h(r,t)u_r(r,t)\right]}{\partial r}=0$

Radial flow equation: $u_h(x)=u_r\dfrac{t}{r_s}=\left[x\dfrac{\partial\ln r_s}{\partial\ln t}+\left(\dfrac{\partial\ln F(c)}{\partial\ln t}-\dfrac{\partial\ln m_h}{\partial\ln t}\right)\dfrac{F(x)}{F'(x)}\right]$

Density slope: $\gamma=\dfrac{\partial\ln\rho_h}{\partial\ln x}=\dfrac{\dfrac{\partial u_h}{\partial x}+\dfrac{\partial\ln m_r(r_s,t)}{\partial\ln t}-\dfrac{\partial\ln r_s}{\partial\ln t}}{\dfrac{\partial\ln r_s}{\partial\ln t}-\dfrac{u_h}{x}}-2$

Radial flow $u_h(x)$ ⟷ $F(x)$ ⟷ Density ρ_h

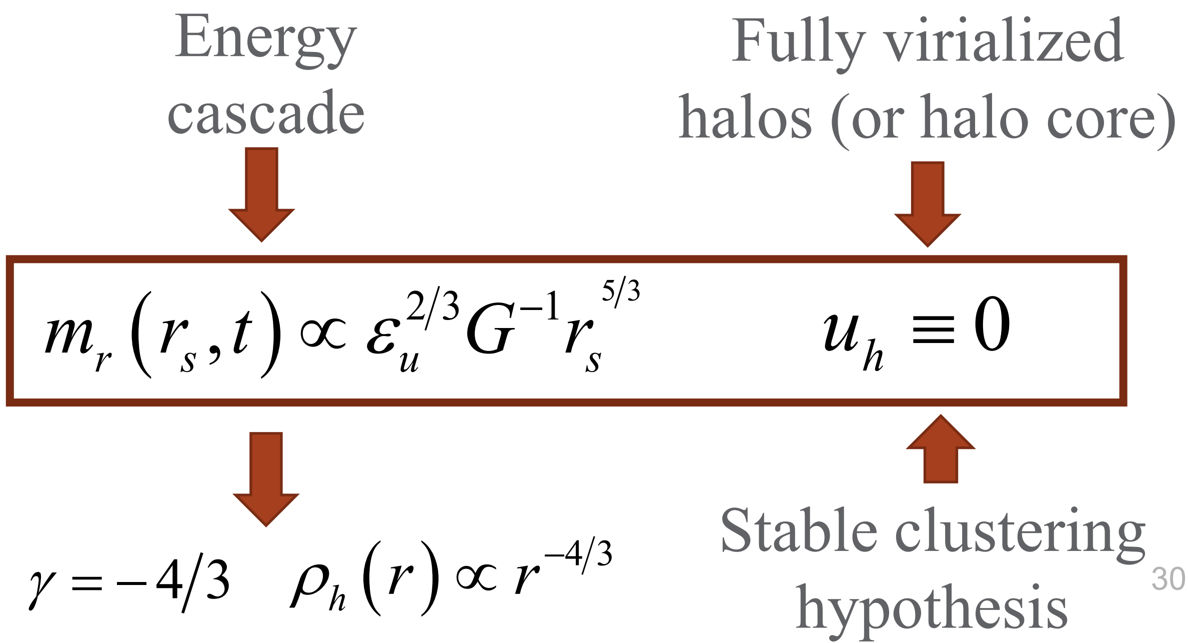
Isothermal: $F(x)=x/c$

NFW: $F(x)=\ln(1+x)-x/(1+x)\propto x^2$

Einasto: $F(x)=\Gamma(3/\alpha)-\Gamma\left(3/\alpha,2x^\alpha/\alpha\right)\propto x^3$

Density slope is dependent on

- Gradient of radial flow (spatial variation)
- Mass accretion (time variation)



Energy cascade

Fully virialized halos (or halo core)

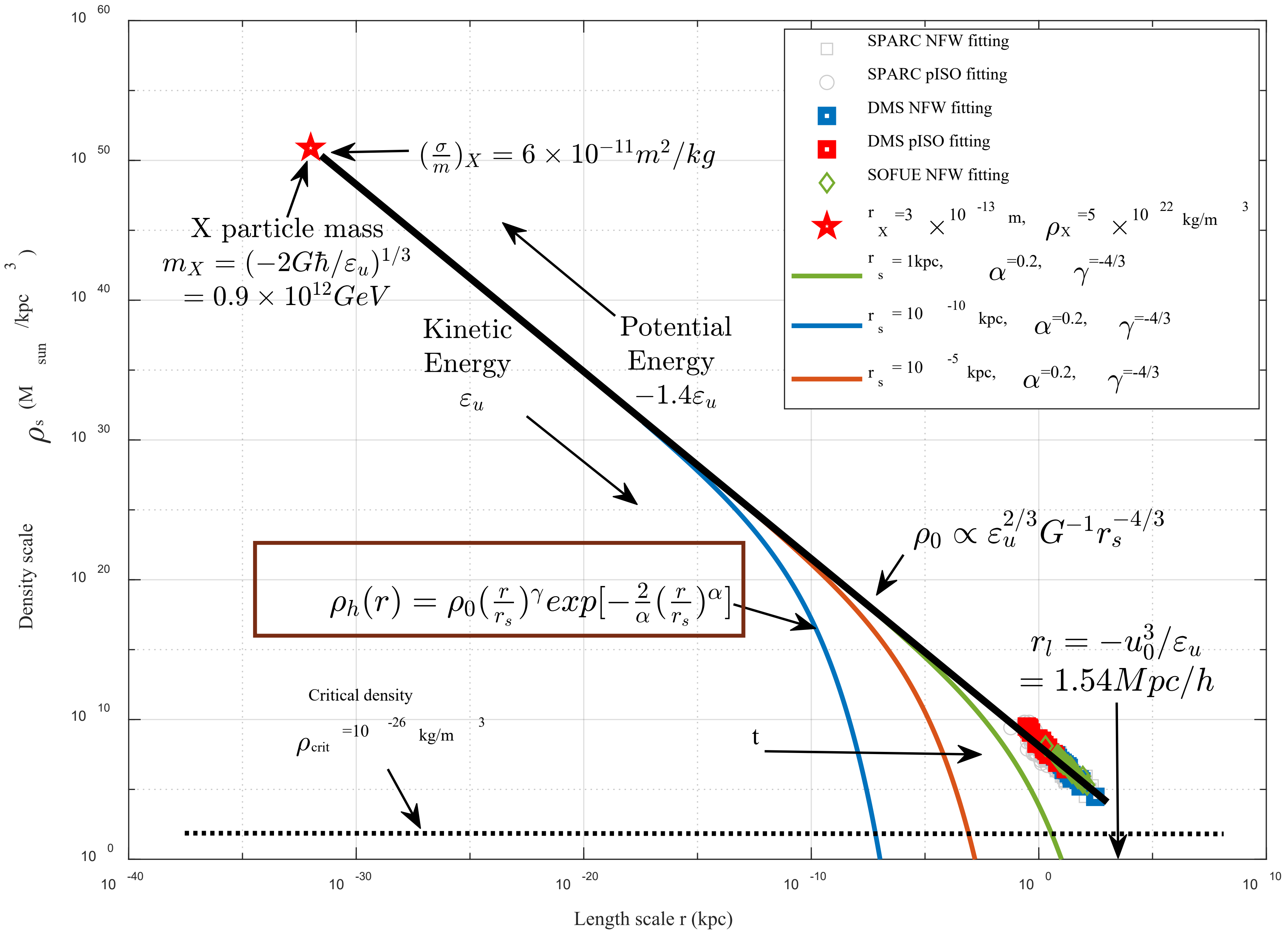
$m_r(r_s,t)\propto\varepsilon_u^{2/3}G^{-1}r_s^{5/3}$ $\quad u_h\equiv 0$

$\gamma=-4/3\quad\rho_h(r)\propto r^{-4/3}$

Stable clustering hypothesis

Evolution of halo density profile

- Nearly universal density profile for fully virialized halos with $u_h=0$;
- Asymptotic density slope γ=-4/3;
- Simulated halos have different slope due to u_h and mass accretion
- Halos in different local environment may have different γ
- A modified Einasto profile is proposed to reflect different slopes γ



Summary and keywords

Small scale challenge	Two-thirds law	Four-thirds law
Core-cusp	Density slope	Uncertainty principle

- Smalls scale challenges suggest missing pieces in our current understandings of dark matter
- Review inverse mass and energy cascade in dark matter flow with a constant rate ε_u
- Energy cascade leads to a 2/3 law for kinetic energy $\propto r^{2/3}$ on scale r, as confirmed by N-body simulation
- Energy cascade leads to a -4/3 law for halo density $\propto r^{-4/3}$ on scale r, as confirmed by rotation curves.
- The largest halo scale is determined by u_0^3/ε_u
- The smallest halo scale is dependent on the nature of dark matter:
  - Collisionless dark matter: $r_\eta \propto (Gh\,\varepsilon_u)^{1/3}$, where h is Planck constant
  - Self-interacting dark matter: $r_\eta \propto G^{-3}\,\varepsilon_u^2\,(\sigma/m)^3$, where (σ/m) is cross-section
- Asymptotic density slope -4/3 for virialized halos with vanishing radial flow
- Simulated halos have different slope due to radial flow and mass accretion

Backup Slides

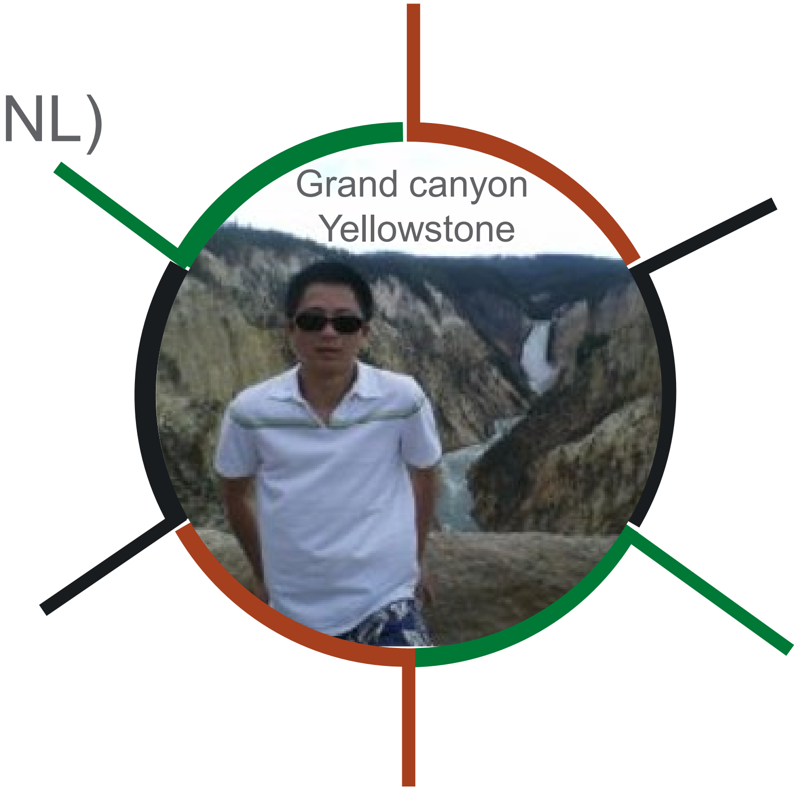
About Me

PROFILE: Zhijie (Jay) Xu

- Computational Scientist
- Team lead

EXPERIENCE:

- Idaho National Laboratory (INL)
- Pacific Northwest National Laboratory (PNNL)


Grand canyon
Yellowstone

EDUCATION:

- Zhejiang University
  Civil Engineering
- National University of Singapore
  Structural Engineering
- Rensselaer Polytechnic Institute
  Mechanical Engineering

INTERESTS:

- Fluid dynamics
- Cosmological flow
- Multiscale Modeling

HOBBIES:

- Travel
- Hiking
- Biking

CONTACT:

- zhijiexu@hotmail.com
- Zhijie.xu@pnnl.gov