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A FIREFLY METAHEURISTIC ALGORITHM FOR STRUCTURAL SIZE AND SHAPE OPTIMIZATION WITH DYNAMIC CONSTRAINTS

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Abstract. In this paper, a structural mass optimization on shape and size is performed taking into account dynamic constraints. Mass reduction especially conflicts with frequency constraints when they are lower bounded since vibration modes may easily switch due to shape modifications. Here it is investigated the use of a Firefly Methaheuristic Algorithm (FMA) as an optimization engine. One important feature of the algorithm is based on the fact that it is not gradient based, but just based on simple objective functions evaluations. This is of major importance when dealing of highly non-linear dynamic optimization problems with several constraints avoiding bad numerical behavior due to gradient evaluations. The algorithm is briefly revised, highlighting its most important features. Some new implementations are performed on the algorithm based on literature reports in order to improve the algorithm performance. It is presented several examples regarding the optimization on shape and sizing with frequency constraints of complex trusses that are widely reported in the literature as benchmark examples solved with several non-heuristic algorithms. The results show that the algorithm performed similar to other methaheuristic methods and better in other cases when compared with traditional gradient-based methods.

1 INTRODUCTION

The optimization was always present in the day-life of the man since ancient times. In order to deal with restrictions on food and shelter, mankind had to develop solutions that cope with source limitations and knowledge lack of the problems. Many of these solutions initially were based on nature observations. Gradually, the knowledge was advancing based on these nature observations and past experience on mistakes and successes. All the solutions found to solve these problems are in fact being put the test by day life and eventually can be supplanted by better ones.

In the last two centuries, the development of algorithms that aims at optimizing problems developed most based on the differential calculus. Methods like gradient descent, SIMPLEX, BFGS, Linear and Quadratic Sequential Programming are broadly used to solve a variety of engineering problems. The basic idea shared by these methods is that the gradient of the function to be optimized has important information to quickly find and optimum solution for a specific problem. However, when dealing with highly non-linear, non-convex, non-differentiable, non-smooth problems (that are the opposite of the necessary conditions to the applicability of the methods), these methods had presented some difficulties on convergence sometimes getting stuck on local optima. Some of the challenging actual engineering problems in fact may present such behavior.

Nowadays, a set of algorithms based on natural behavior of swarms, ants, bees, beetles and birds had emerged as alternative to overcome difficulties presented by traditional methods in some of those optimization problems. They all share the idea of using some social behavior presented by species to solve problems regarding survival in the nature and they are put in some way as mathematical codes to solve engineering problems using some heuristic rules. The advantage of such methods lies on the fact that they do not need information regarding gradients of the function to be optimized, and some of them may easily take advantage of parallel processing to solve hard problems quickly.

So, in this paper, a structural mass optimization on shape and size is performed taking into account dynamic constraints. In this kind of problem, mass reduction especially conflicts with frequencies constraints when they are lower bounded since vibration modes may easily switch due to shape modifications. It is investigated the use of a Firefly Methaheuristic Algorithm (FMA) as an optimization engine. One important feature of the algorithm is based on the fact that it is non-gradient based, but just based on simple objective functions evaluations. This is of major importance when dealing of highly non-linear dynamic optimization problems with several constraints avoiding bad numerical behavior due to gradient evaluations. The algorithm is briefly revised, highlighting its most important features. Some new implementations are performed on the algorithm based on literature reports in order to improve the algorithm performance. It is presented several examples regarding the optimization on shape and sizing with frequency constraints of complex trusses that are widely reported in the literature as benchmark examples. The results show that the algorithm performed similar to other methaheuristic methods and better in other cases when compared with traditional gradient-based methods.

2 THE FIREFLY METAHEURISTIC ALGORITHM

The algorithm was firstly proposed by Yang (2007) at Cambridge University on his PhD Thesis (Yang, 2010b). It is based in the observation of the flashing light of fireflies. According to Day (2010) there are about to thousand firefly species and most of them produces short and rhythmic flashes. It should be emphasized that the fireflies are beetles of the family of *lampyridae* and beetle c*oleoptera.* They are capable of producing a cold light thanks to special photogenic organs situated very close to the body surface behind a window of translucent cuticle. This phenomenon is called bioluminescence. The larval phase of the fireflies (glowworms) presents the bioluminescence phenomenon too. The majority of fireflies and glow-worms have lanterns in at least or both sexes. As cited by Day (2010) a generalized definition of bioluminescence often cited is the production and emission of light by a living organism. But this is an over simplification and there are many terms, such as fluorescence, phosphorescence, luminescence, chemiluminescence and most recently biofluorescence which have added to the confusion. To avoid confusion he has expanded upon the general description to define bioluminescence as: "*the direct production of light from a chemical reaction occuring within a living organism"*. This excludes the group of living organisms which produce and emit light using a fluorescent protein such as the jellyfish *Aequorea victoria*. According to Wikepedia (2010), these proteins exhibit bright green fluorescence when exposed to blue light and are therefore not a component of an enzyme catalysed luminescent reaction. Day (2010) stated that in the case of bioluminescense, the enzyme luciferase acts on the luciferin, in the presence of magnesium ions, ATP, and oxygen to produce light.

Lukasik and Zak (2008) stated that bioluminescent signals are known to serve as elements of courtship rituals, methods of prey attraction, social orientation or as a warning signal to predators (in case of immature firefly forms commonly referred to as glowworms). The phenomenon of firefly glowing is an area of continuous research considering both its biochemical and social aspects. Firefly Methaheuristic Algorithm developed recently by Yang (2007) at Cambridge University follows this approach. The rhythmic flash, the rate of flashing and the amount of time it remains on, form part of the system that brings both sex together. Females respond to male's unique pattern of flashing in the same species. Some tropic fireflies can even synchronize their flashes, thus forming emerging biological self-organized behavior.

Since the light intensity decays with the square of the distance, the fireflies are limited visible to other fireflies. This plays an important role in the communication of the fireflies and the attractiveness, which may be impaired by the distance. The flashing light can be formulated in such a way that it is associated with the objective function to be optimized, which makes it possible to formulate new optimization algorithms.

In the development of the algorithm, some simplifications are assumed in order to develop the firefly-inspired algorithms. Such simplifications can be summarized as: a) it is assumed that all fireflies are unisex so they will be attracted to each other regardless of their sex; b) the attractiveness is proportional do their brightness and they both decrease as the distance increases and c) in the case of no existence of no brighter firefly on then, the fireflies will move randomly. The brightness of a firefly is affected by their fitness (landscape of the objective function). In a maximization problem, the brightness is proportional to the objective function. Figure 1 shows the pseudo-code of the firefly algorithm in its simplest form.

2.1 Attractiveness and Light Intensity

Accordingly to Yang (2010b), in the firefly algorithm, there are two important issues: a) the variation of light intensity and b) formulation of the attractiveness. For simplicity, one can always assume that the attractiveness of a firefly is determined by its brightness which in turn is associated with the encoded objective function $f({}^{i}\mathbf{x})$. In the simplest case for maximum optimization problems, the brightness *I* of a firefly *i* at a particular position ${}^{i}\mathbf{x} = (x_1, x_2, ..., x_d)^T$ can be chosen as $I({}^{i}\mathbf{x}) \propto f({}^{i}\mathbf{x})$. However, the attractiveness β is relative; it should be seen in the eyes of the beholder or judged by the other fireflies. Thus, it will vary with the distance r_{ij} between firefly *i* and firefly *j*. In addition, light intensity decreases with the distance from its source, and light is also absorbed by the media, so we should allow the attractiveness to vary with the degree of absorption (γ). In the simplest form, the light intensity $I(r_{ij})$ varies according to the inverse square law (Eq.(1)):

$$I(r_{ij}) = I_s \,/\, r_{ij}^2 \tag{1}$$

where I_s is the intensity at the source. For a given medium with a fixed light absorption coefficient γ, the light intensity *I* varies with the distance r_{ij} in the following form (Eq.(2)):

$$I(r_{ij}) = I_0 \exp(-\gamma r_{ij}^2) \tag{2}$$

where I_0 is the original light intensity.

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Objective Function f(x),  with  x = (x_1, x_1, ..., x_d)^T    d = no. of design variables
Generate initial population of fireflies randomly  ^i x,  i = 1,2,...,n     n = no. of fireflies
Light intensity I at ^i x  is determined by f(^i x)
Define light absorption coefficient γ
While t < maximum number of generations or convergence criteria are met
 For i=1 to n
   For j=1 to n
      If (I_i < I_j), move i towards j
     Calculate the distance r_ij=|| ^i x − ^j x ||
      Calculate β = β_0 exp(−γ r_ij)
     Generate random number vector ^i u = random(α min(^i x, ^j x), α max(^i x, ^j x))
      Update design variables ^i x = ^i x + β_0 exp(−γ r_ij^2)(^i x − ^j x) + ^j u
   End For j
  End For i
  Rank fireflies and find the current global best
End While
Postprocess results
```

Figure 1: Pseudo-Code for Firefly Methaheuristic Algoritm. (Adapted from Lukasik and Zak (2009), and Yang (2010b))

As the firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, one can define the attractiveness β of a firefly by (Eq.(3)):

$$\beta(r_{ij}) = \beta_0 \exp(-\gamma r_{ij}^2) \tag{3}$$

where β_0 is the attractiveness at r_{ij}=0. Equation (3) defines a characteristic distance Γ=l/$\sqrt{\gamma}$ over which the attractiveness changes significantly from β_0 to $\beta_0$exp(-1). In the actual implementation, the attractiveness function $\beta(r_{ij})$ can be any monotonically decreasing functions such as $\beta(r_{ij}) = \beta_0 \exp(-\gamma r_{ij}^m)$. For a fixed γ, the characteristic length becomes $\Gamma = \gamma^{-1} \to 1 \ \ when \ \ m \to \infty$. Conversely, for a given characteristic length scale Γ in an optimization problem, the parameter γ *is* used as a typical initial value $\gamma = 1/\Gamma^m$.

The movement of a firefly *i* is attracted to another more attractive (brighter) firefly *j* is determined by (Eq.(4)):

$$ {}^i\mathbf{x} = {}^i\mathbf{x} + \beta_0 \exp(-\gamma r_{ij}^2)({}^i\mathbf{x} - {}^j\mathbf{x}) + {}^j\mathbf{u} \tag{4} $$

where the second term is due to the attraction. The third term ${}^j\mathbf{u}$ is randomization vector of random numbers drawn from a uniform distribution and obtained from an augmented interval ($\alpha > 1$) from maximum and minimum design variables values for fireflies *i* and *j*. For most of implementations, one can take β_0=1 and ${}^i\mathbf{u} = random(\alpha \min({}^i\mathbf{x}, {}^j\mathbf{x}), \alpha \max({}^i\mathbf{x}, {}^j\mathbf{x}))$. It is worth pointing out that equation (4) is a random walk biased towards the brighter fireflies. If β_0=0, it becomes a simple random walk. The parameter γ now characterizes the variation of the attractiveness, and its value is crucially important in determining the speed of the convergence and how the FMA algorithm behaves. $\gamma \in [0, \infty)$ but for most application, due to Γ values of the system to be optimized, it typically varies from 0.1 to 10.

3 BRIEF BIBLIOGRAPHICAL REVIEW

The use of Fireflies as an optimization tool has initially proposed by Yang (2007) when he conceived the algorithm. New researchers had used the basic algorithm and since some improvements had been proposed in order to compare the method with other methaheuristic algorithms. There are few books that deal with this theme, such as Yang (2008) and Yang (2010b).

Most of the papers that use the algorithm perform validation and comparisons against other methaheuristic such as particle swarm optimization (PSO). Most of problems are benchmark De Jung´s test suite functions used to validate optimization algorithms.

Krishnanand and Ghose (2009b) applied the Glowworm Swarm Optimization algorithm (a variant of the firefly optimization algorithm) for optimizing multi-modal functions. In this paper, wellknown "J*i*" functions are used to compare the performance of the Glowworm algorithm with a nich-particle swarm algorithm for optimization. The implementation of the algorithms envisions the use on robotic implementation. The paper concludes that the GSO could find multiple optima of multimodal functions and performers similar to Particle Swarm. In the same way the paper of Yang, (2009 and 2010a) concludes that the FMA could carry out a nonlinear design optimization using stochastic test functions with singularities and stochastic components and that it can be potentially more powerful than other existing algorithms such as PSO. He warns that the convergence analysis still requires theoretical framework.

More recently Chai-ead *et al* (2011) applied bee colony algorithm and Firefly algorithm to noisy non-linear optimization problems. They conducted numerical experimental tests that were analyzed in terms of best solutions found so far, mean and standard deviation on both the actual yields and execution time to converge to the optimum. The Firefly algorithm seems to perform better when the noise levels increase. The Bees algorithm provided the better levels of computation time and the speed of convergence. They concluded that the Firefly algorithm was more suitable to exploit a search space by improving individuals' experience and simultaneously obtaining a population of local optimal solutions.

Sayadia *et al* (2010) successfully applied the FMA for minimization in permutation flow shop scheduling problems. Since the permutation flow shop is formulated as a mixed integer programming and it is classified as NP-Hard problem. Therefore, a direct solution is not available and Methaheuristic approaches need to be used to find the near-optimal solutions. The results of implementation of the proposed method were compared with other existing ant colony optimization technique. The results indicated that the new proposed method performed better than the ant colony for some well known benchmark problems.

Apostolopoulos and Vlachos (2011) applied the firefly algorithm for solving the Economic emissions load dispatch problem. In their paper, the algorithm is used to minimize both fuel cost and emission of generating units. A general formulation of this algorithm is presented together with an analytical mathematical modeling to solve this problem by a single equivalent objective function. The results are compared with those obtained by alternative techniques and it is shown that it is capable of yielding good optimal solutions with proper selection of control parameters.

In fact, Krishnanand and Ghose (2008a, 2008b 2009a, 2009b, 2009c, 2009d) presented a series of use of the firefly algorithm to a sort of problems like high dimensional spaces optimization, multimodal functions, hazard sensing in ubiquitous environments, etc, showing that the algorithm and variants can be successfully applied to several types of problems.

Finally a claimed superior variant of the algorithm was proposed by Yang (2010c), called Lévy-flight Firefly Algorithm. In the paper it is claimed that the Lévy-flight firefly algorithm converges more quickly and deals with global optimization more naturally. In this paper it is demonstrated that the Particle Swarm Algorithm is a special class of FMA. However in another paper, Lukasik and Zak (2009) had performed numerical studies using benchmark constrained optimization problems and concluded that PSO had performed better for 11 benchmark instances out of 14 being used against FMA (standard deviation of the best solution was used since same population size and iterations were fixed for both methods), so there are still some controversies regarding the performance of the algorithm.

4 STRUCTURAL SIZE AND SHAPE OPTIMIZATION WITH DYNAMIC CONSTRAINTS

So, in terms of truss optimization, the problem can be mathematically stated as:

$$\begin{aligned}
&\textit{minimize}\ \ Mass = \sum_{i=1}^{n} L_i \rho_i A_i \qquad i = 1,\ldots,n \ \ \textit{for all bars} \\
&\textit{Subjected to}\ \ \omega_j \geq \omega_j^* \ \ \textit{for some eigenvalues}\ \ j \\
&\qquad\qquad\qquad \omega_k \leq \omega_k^* \ \ \textit{for some eigenvalues}\ \ k \\
&\textit{and} \\
&A_{l\,\min} \leq A_l \qquad\qquad \textit{for some bar cross sectional areas}\ l \\
&\mathbf{x}_{q\min} \leq \mathbf{x}_q \leq \mathbf{x}_{q\max} \quad \textit{for some node coordinates}\ \ q
\end{aligned} \tag{5}$$

In this paper the constraints violations will be treated with the penalty function technique so the objective function to be minimized is modified to:

$$Mass = \Big(\sum_{i=1}^{n} L_i \rho_i A_i\Big)(1 + PF) \qquad \textit{for all bars} \tag{6}$$

where the Penalization Factor (PF) is defined as the sum of all active constraints violations as indicated (not only to frequency constraints but to stress and displacements constraints when appropriate).

$$PF = \sum_{i=1}^{nc} \left| \frac{\omega_i}{\omega_i^*} - 1 \right| \qquad \textit{for all active constraints} \tag{7}$$

This formulation allows, for solutions with violated constraints, objective function always greater than the non-violated one.

4.1 Ten bar Truss

This example was first solved by Grandhi and Venkayya (1988) using the optimality algorithm. Sedeghati *at al.* (2002) used a Sequential Quadratic Programming (SQP) with conjunction with finite element force method to solve the problem. Wang et al. (2004) used an evolutionary node shift method and Lingyum et al. (2005) used Niche Hybrid Genetic Algorithm. Gomes (2009, 2011) used a PSO algorithm to solve the same problem. This paper addresses this problem using the Firefly Methaheuristic Algorithm previously described. It is a simple 10-bar truss with fixed shape and variable continuous bar sizes. At each free node it is attached a non-structural mass of 454.0 kg as depicted by Figure 2. The material properties as design variable ranges are listed in Table 1. So this is a truss optimization on size with three frequency constraints and ten design variables.

Table 2 shows the design variables results and the final mass for the optimized truss. It should be highlighted the good results obtained with the FMA algorithm and with PSO. The truss mass obtained by the PSO was a little worse than Sedaghati et al. (2002) results and the FMA algorithm performed better just than Grandhi (1993).

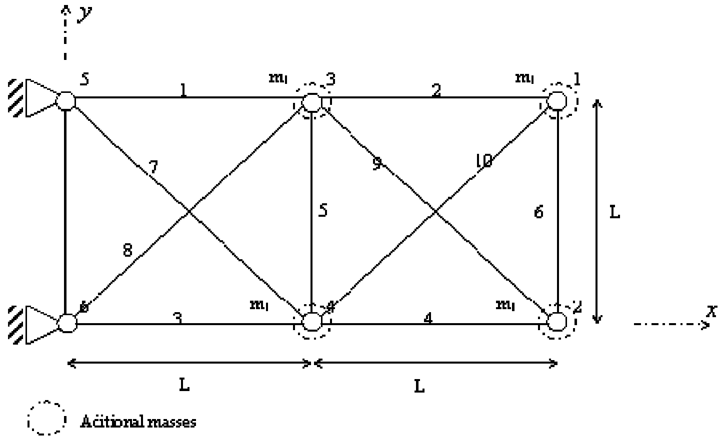

Figure 2: 10-bar truss structure with added masses.

Property	Value	Unit
E (Young Modulus)	6.98×10^{10}	N/m^2
ρ (Material density)	2770.0	kg/m^3
Added Mass	454.0	kg
Design Variable Lower Bound	0.645×10^{-4}	m^2
Main bar's Dimension	9.144	m
Constraints on first 3 frequencies	$\omega_1 \geq 7$, $\omega_2 \geq 15$, $\omega_3 \geq 20$	Hz

Table 1: Material properties and frequency constraints for 10-bar truss structure.

Element No.	Wang (2004)	Grandhi (1993)	Sedaghati (2002)	Lingyum (2005)	Gomes (2009)	Present Work
1	32.456	36.584	38.245	42.234	37.712	34.00
2	16.577	24.658	9.916	18.555	9.959	16.00
3	32.456	36.584	38.619	38.851	40.265	50.00
4	16.577	24.658	18.232	11.222	16.788	22.00
5	2.115	4.167	4.419	4.783	11.576	6.00
6	4.467	2.070	4.419	4.451	3.955	6.00
7	22.810	27.032	20.097	21.049	25.308	30.00
8	22.810	27.032	24.097	20.949	21.613	14.00
9	17.490	10.346	13.890	10.257	11.576	7.00
10	17.490	10.346	11.452	14.342	11.186	16.00
Weight(kg)	553.8	594.0	537.01	542.75	537.98	579.40

Table 2: Optimal design cross sections (cm^2) for several methods (Weight does not consider added masses).

Table 3 shows the dynamic constraints on frequency obtained by each of the methods showing that any o f them are violated.

Frequency No.	Wang (2004)	Grandhi (1993)	Sedaghati (2002)	Lingyum (2005)	Gomes (2009)	Present Work
1	7.011	7.059	6.992	7.008	7.000	7.030
2	17.302	15.895	17.599	18.148	17.786	18.717
3	20.001	20.425	19.973	20.000	20.000	20.959
4	20.100	21.528	19.977	20.508	20.063	23.026
5	30.869	28.978	28. 173	27.797	27.776	28.416
6	32.666	30.189	31.029	31.281	30.939	32.894
7	48.282	54.286	47.628	48.304	47.297	48.710

Table 3: Optimized frequencies (Hz) with several methods for the 10-bar truss structure.

Table 4 shows the results for 5 independent runs using the FMA method and the parameters used in the simulations. It can be noticed that there are some deviation of the optimum value obtained for different runs.

Mean Mass of Fireflies (kg) μ	Standard Deviation σ	No. of Fireflies n	Alpha Coefficient α	Absorption Coefficient γ	Minimum Attractiveness β_0	Mean No. of Iterations	Tolerance for Convergence
580	2.37	50	0.3	0.1	0.5	63	10^{-3}

Table 4: Statistical results for the 5 independent runs of FMA in 10-bar truss structure problem.

4.2 Simple Supported 37-bar truss

This example has been investigated by Wang et al (2004) using the evolutionary node shift method, by Lingyum *et al* (2005) using the NHGA algorithm and by Gomes (2009, 2011) using a PSO algorithm. It is a simple supported Pratt Type 37-bar truss as indicated by Figure 3. There are non-structural masses of m=10 kg attached to each of the bottom nodes of the lower chord, which are modeled as bar elements with rectangular cross sectional areas of $4\text{x}10^{-3}$ m^2. The other bars are modeled as simple bar elements with initial sectional areas of $1\text{x}10^{-4}$ m^2. The material property for the bar elements are set as E=$2.1\text{x}10^{11}$ N/m^2 and ρ=7800 kg/m^3. Used parameters for the FMA algorithm are listed on Table 7. This is considered a truss optimization on size and shape since all nodes of the upper chord are allowed to vary in the y-axis in a symmetrical way and all the diagonal and upper chord bars are allowed to vary its cross sectional are starting from A=$1\text{x}10^{-4}$ m^2. There are three constraints in the first three natural frequencies so that $\omega_1 \geq 20$ Hz, $\omega_2 \geq 40$ Hz, $\omega_3 \geq 60$ Hz. So, it is considered a truss optimization problem with three frequency constraints and nineteen design variables (five shape variables plus fourteen sizing variables). Figure 3 shows a sketch of the 37-bar truss with dimensions.

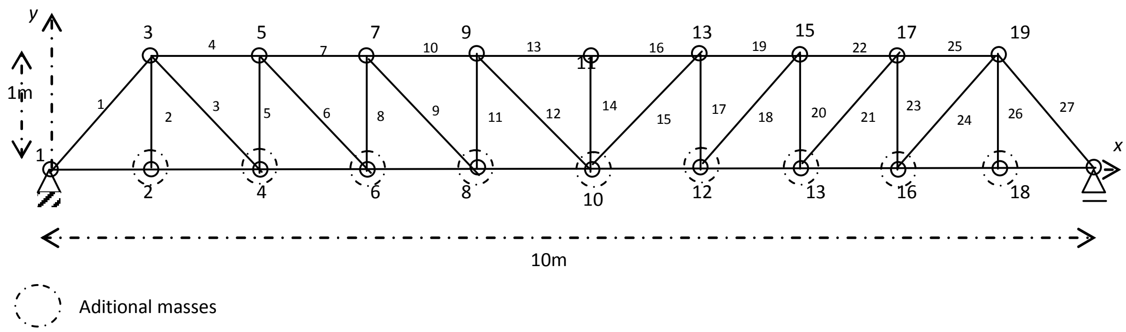

Figure 3: 37-bar truss structure with added masses.

Table 5 shows a comparison among the optimal design cross sections of several methods including the present work (FMA). It can be noticed that in this case, the FMA method did not perform as well as the PSO method indicated by Gomes (2009). There is a difference in mass about 1kg related to Gomes (2009) results.

Table 6 shows the optimized frequencies for several methods and the present work. It can be noticed that all the constraints are not violated.

Variable No.	Initial Design	Wang (2004)	Lingyum (2005)	Gomes (2009)	Present Work
Y_3 , Y_{19}(m)	1.0	1.2086	1.1998	0.9541	0.9311
Y_5 , Y_{17}(m)	1.0	1.5788	1.6553	1.3362	1.2978
Y_7 , Y_{15}(m)	1.0	1.6719	1.9652	1.4806	1.4694
Y_9 , Y_{13}(m)	1.0	1.7703	2.0737	1.6195	1.5948
Y_{11}(m)	1.0	1.8502	2.3050	1.6785	1.7069
A_1 , A_{27}(cm^2)	1.0	3.2508	2.8932	2.9680	2.6783
A_2 , A_{26}(cm^2)	1.0	1.2364	1.1201	1.0775	0.8199
A_3 , A_{24}(cm^2)	1.0	1.0000	1.0000	0.5000	1.1091
A_4 , A_{25}(cm^2)	1.0	2.5386	1.8655	2.7440	2.5139
A_5 , A_{23}(cm^2)	1.0	1.3714	1.5962	1.2833	1.5218
A_6 , A_{21}(cm^2)	1.0	1.3681	1.2642	1.6393	1.4284
A_7 , A_{22}(cm^2)	1.0	2.4290	1.8254	2.5819	3.0302
A_8 , A_{20}(cm^2)	1.0	1.6522	2.0009	1.2201	1.6517
A_9 , A_{18}(cm^2)	1.0	1.8257	1.9526	1.3047	1.3091
A_{10} , A_{19}(cm^2)	1.0	2.3022	1.9705	2.8291	3.4008
A_{11} , A_{17}(cm^2)	1.0	1.3103	1.8294	1.6442	1.2592
A_{12} , A_{15}(cm^2)	1.0	1.4067	1.2358	1.4356	1.3585
A_{13} , A_{16}(cm^2)	1.0	2.1896	1.4049	3.5753	2.2984
A_{14}(cm^2)	1.0	1.0000	1.0000	0.5193	2.0156
Weight(kg)	336.3	366.50	368.84	362.27	363.14

Table 5: Optimal cross section designs for several methods for the 37-bar truss structure (Weight does not consider added masses).

Frequency No.	Initial Design	Wang (2004)	Lingyum (2005)	Gomes (2009)	Present Work
1	8.89	20.0850	20.0013	20.0335	20.0005
2	28.82	42.0743	40.0305	41.0167	40.0168
3	46.92	62.9383	60.0000	60.0224	60.0561
4	63.62	74.4539	73.0444	73.9039	76.6520
5	76.87	90.0576	89.8244	85.0975	92.4318

Table 6: Optimized frequencies (Hz) with several methods for the 37-bar truss.

Table 7 shows the statistical results regarding 5 independent runs of the FMA algorithm in the 37-bar truss problem and the used FMA heuristic parameters.

Mean Mass of Fireflies (kg) μ	Standard Deviation σ	No. of Fireflies n	Alpha Coefficient α	Absorption Coefficient γ	Minimum Attractiveness β_0	Mean No. of Iterations	Tolerance for Convergence
367.1	4.26	40	0.3	0.1	0.5	40	10^{-3}

Table 7: Statistical results for 5 independent runs of FMA in the 37-bar truss problem.

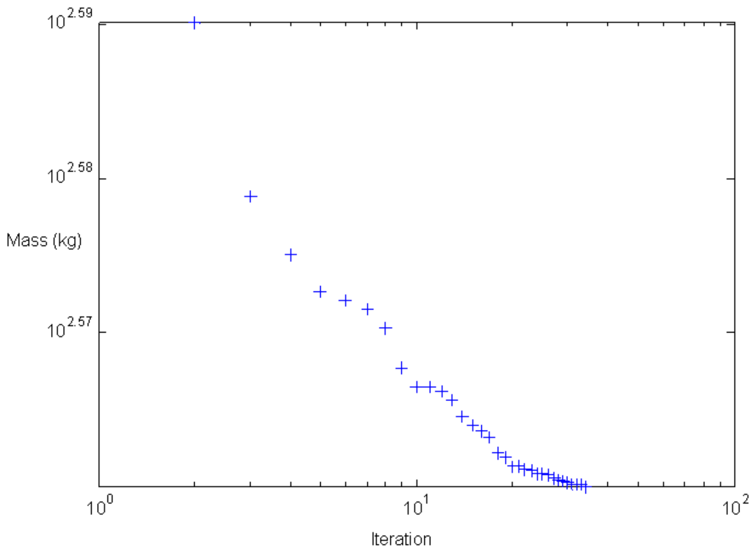

Figure 4: Iterations for mass optimization in 37-bar truss.

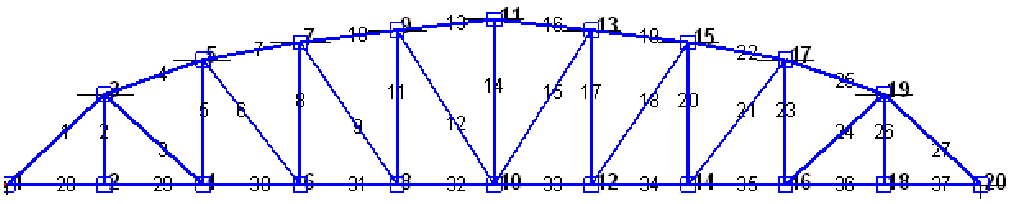

Figure 5: Final configuration for the 37-bar truss using FMA.

4.3 120-bar truss

In this example a hemispherical space truss (like a dome) is optimized on shape and size with constraints in the first two natural frequencies. The space truss has 52 bars and non-structural masses of m=50kg are added to the free nodes. The cross-sectional areas are permitted to vary between 0.0001 m^2 and 0.001 m^2. The shape optimization is performed taking into account that the symmetry should be kept in the design process. Each movable node is allowed to vary ± 2 m. For the frequency constraint it is set that $\omega_1 \le 15.916\,\text{Hz}$ and $\omega_2 \ge 28.649\,\text{Hz}$. A sketch of the initial design is shown in Figure 6 and 7. This example is considered to be a truss optimization problem with two natural frequency constraints and thirteen design variables (five shape variables plus eight size variables).

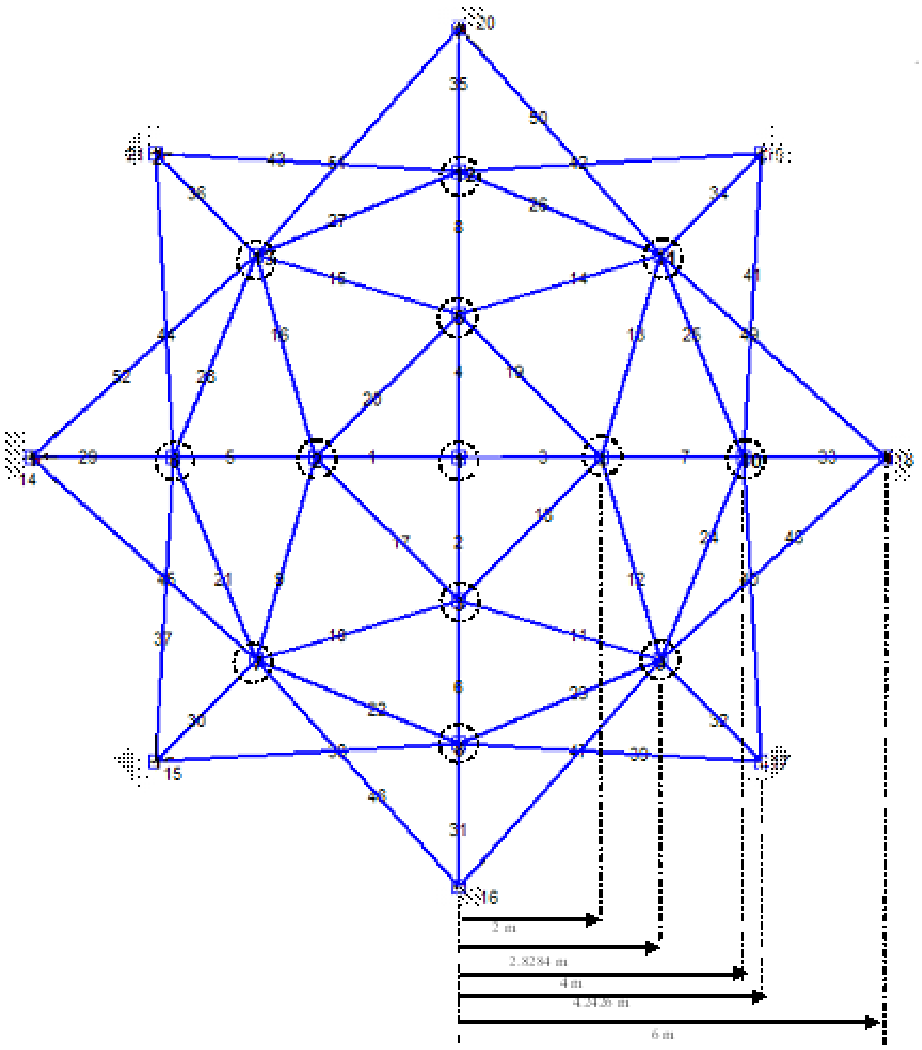

Figure 6: Initial design of the 52-bar dome truss structure.

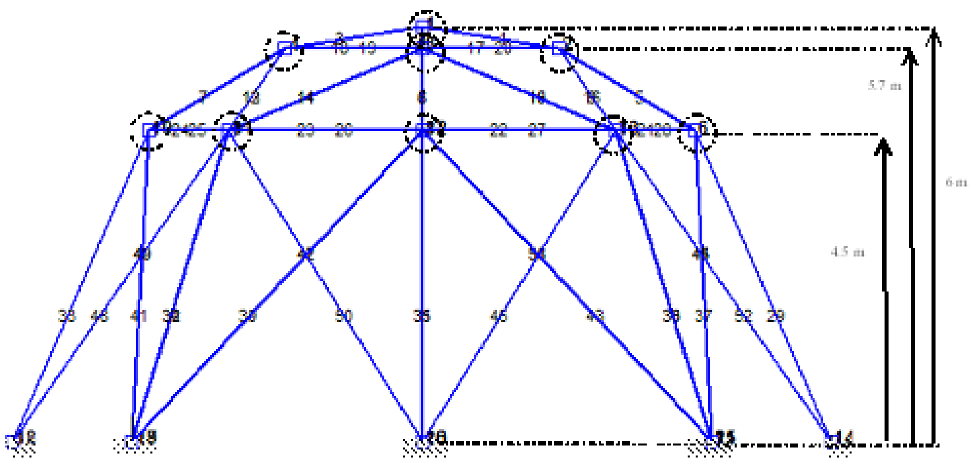

Figure 7: Initial design of the 52-bar dome truss structure (lateral view).

Table 8 shows the initial and final optimized coordinates and cross sectional areas and final mass, as well. FMA optimum mass is about 8 kg heavier than Gomes (2009) optimum using PSO. It can be noticed that the PSO perform better than the other methods. FMA performed better than Lin(1982) and Lingyum(2005).

Variable No.	Initial Design	Lin (1982)	Lingyum (2005)	Gomes (2009)	Present Work
Z_A (m)	6.000	4.3201	5.8851	4.000	4.1700
X_B(m)	2.000	1.3153	1.7623	2.8538	2.7575
Z_B (m)	5.700	4.1740	4.4091	3.7000	3.8028
X_F (m)	2.828	2.9169	3.4406	4.3517	4.2988
Z_F(m)	4.500	3.2676	3.1874	2.5000	2.6011
A_1(cm^2)	2.0	1.00	1.0000	1.0000	1.0000
A_2(cm^2)	2.0	1.33	2.1417	1.6690	1.6905
A_3(cm^2)	2.0	1.58	1.4858	1.2434	1.4776
A_4(cm^2)	2.0	1.00	1.4018	1.3478	1.2130
A_5(cm^2)	2.0	1.71	1.911	1.0186	1.3697
A_6(cm^2)	2.0	1.54	1.0109	1.0000	1.007
A_7(cm^2)	2.0	2.65	1.4693	1.0000	1.3383
A_8(cm^2)	2.0	2.87	2.1411	2.0703	1.6682
Weight(kg)	338.69	298.0	236.046	193.942	202.842

Table 8: Optimal design cross section for several methods in the 52-bar space truss problem (weights does not consider added masses).

Table 9 shows the final optimized frequencies (Hz) for the methods. It is noticed that any of the frequency constraints were violated.

Frequency No.	Initial Design	Lin (1982)	Lingyum (2005)	Gomes (2009)	Present work
1	22.69	15.22	12.81	10.255	13.242
2	25.17	29.28	28.65	28.649	28.671
3	25.17	29.28	28.65	28.649	28.671
4	31.52	31.68	29.54	28.809	29.245
5	33.80	33.15	30.24	28.749	29.342

Table 9: Optimized frequencies (Hz) for several methods for the 52-bar space truss.

Table 10 shows the statistics of 5 independent runs for the 52-bars truss example and the parameters used for the FMA algorithm.

Mean Mass of Fireflies (kg) μ	Standard Deviation σ	No. of Fireflies n	Alpha Coefficient α	Absorptio n Coefficient γ	Minimum Attractiveness β_0	Mean No. of Iterations	Tolerance for Convergence
213.2	4.29	50	0.3	0.1	0.5	200	10^{-3}

Table 10: Statistical results for 5 independent runs of FMA in the 52-bar space truss problem.

In the following Figures 8 and 9 the shape of initial design and the optimized solutions proposed by the literature are compared with that obtained in the present work. Again, it is noticed the similar shapes of the final truss of the present work and that presented by Gomes (2009, 2011).

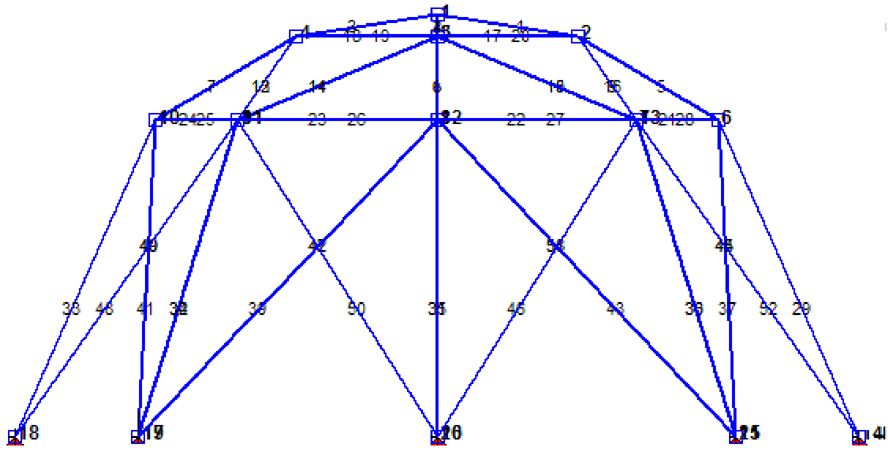

Figure 8: Initial Design of a 52-bar dome structure.

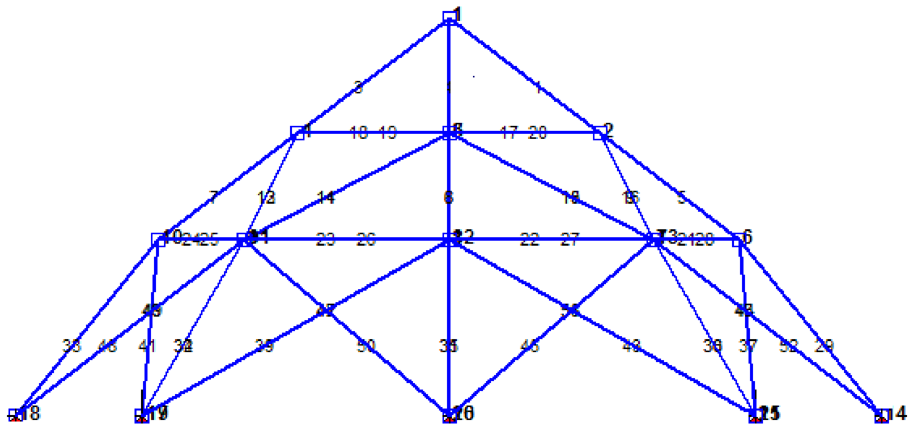

Figure 9: Optimized design of a 52-bar dome structure by Gomes (2009) (PSO).

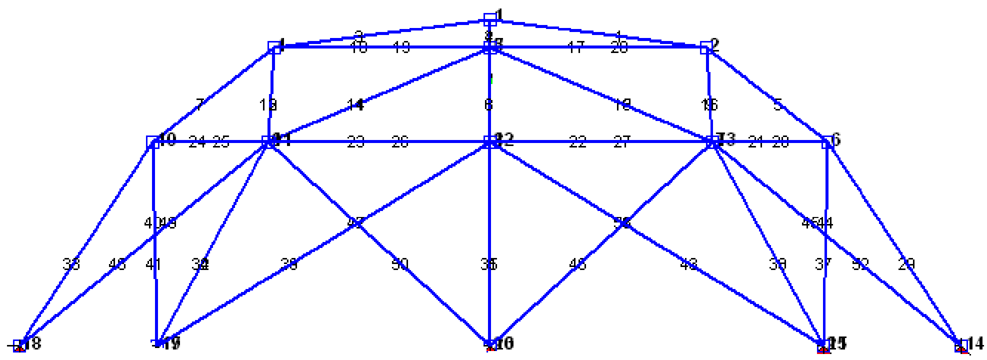

Figure 10: Optimized design of a 52-bar dome structure by present work (FMA).

5 CONCLUSIONS

In this paper the problem of truss design optimization with frequency constraints was addressed. The constraints were treated as usual with penalty functions. It is well-known that this kind of optimization problem has high-nonlinear behavior regarding the frequency constraints especially for shape optimization, since eigenvalues are very sensitive to shape modifications. In the literature it was reported several methods that try to bypass this problem. In this paper a methodology is proposed based on a Firefly Methaheuristic Algorithm (FMA). The FMA is referred in the literature as a global optimizer with advantages in relation to other heuristic algorithms like PSO, since it is claimed that PSO is a subset of a general FMA. Some capabilities that make this heuristic algorithm attractive are its less parameter necessary to set and its floating point treatment. Another common feature to heuristic algorithms is that it requires just objective functions evaluations (it is not required gradients), which allows the method to treat this kind of problem (symmetrical trusses with equal eigenvalues) without any modifications. Another important feature is the fact that the algorithm works with a population and random parameters which allows exploration/exploitation capabilities and escape from local minima in the search process.

It was present three examples of increasing difficulty which were compared with results in the literature. In an engineering point of view, the method performed well in the three cases, showing to be promising. Besides, the method presented results worst than that reported in the literature for PSO.

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