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Numerical solution of generalized Burgers-Huxley equations using wavelet based lifting schemes

L. M. Angadi*
Department of Mathematics, Government First Grade College, Chikodi – 591201, India

****Corresponding author:***
E-mail add: *angadi.lm@gmail.com*

ABSTRACT
Nonlinear partial discriminational equations are extensively studied in Applied Mathematics and Physics. The generalized Burgers- Huxley equations play important places in different nonlinear drugs mechanisms. In this paper, we presented numerical solution of generalized Burgers-Huxley equations by Lifting schemes using different wavelet filter coefficients. The numerical solution obtained by this scheme is compared with the exact solution to demonstrate the accuracy and also faster convergence in lesser computational time as compared with existing scheme. Some of the problems are taken to demonstrate the applicability and of validity the scheme.

Keywords: *Lifting scheme; Wavelet filter coefficients; Burgers–Huxley equation.*

Introduction
Non-linear partial differential equations usually occur in modeling of various phenomena in most of the engineering and physical science branches. In the spatially homogeneous media, behavior of bifurcations and periodic trawling waves in excitable media are different. This difference cause by the strongly non-linearity and singular characteristics of the local reaction kinetics play considerable role. Indeed, singular perturbation theory utilizes the mentioned characteristics of excitable media. We recall that KKP-Fisher [1] equation can be utilized successfully in modeling the diffusion phenomena which admits a traveling front solution involving the two steady-states. Among possible conceptions of the Fisher equation, the Burgers-Huxley (BH) equation is most important one. It is well-known that a large class of physical phenomena such as the interaction between convection effects, reaction mechanism, and diffusion transports can be described by the BH equation [2]. However, with the advent of modern computers and sophisticated software, we are now able to solve such kind of problems using approximate analytical or numerical methods [3]. Some of the iterative methods are used for the numerical and analytical solutions of generalized Burgers-Huxley equations. For example, Spectral Collocation method [4], New exact solutions [5], Haar wavelet method [6] etc.

Beginning from 1980s, wavelets have been used for solution of partial differential equations (PDEs). The good features of this approach are possibility to detect singularities, irregular structure and transient phenomena exhibited by the analyzed equations. "Wavelets" have been very popular topic of conversations in several scientific and engineering gatherings these days. Some of the researchers have decided that, wavelets are new basis for representing continuous functions, as a technique for time-frequency analysis, and as a new mathematical subject. Of course, "wavelets" is a versatile tool with very rich mathematical content and great potential for applications. However, wavelet analysis is a numerical concept which allows one to represent a function in terms of a set of bases functions, called wavelets, which are localized both in location and scale [7].

In recent times, some of the works on wavelet based methods are the discrete wavelet transforms (DWT) and the full approximation scheme (FAS) were introduced recently in [8 - 9]. The wavelet based full approximation scheme (WFAS) has exposed to be a very efficient and favorable method for numerous problems related to computational science and engineering fields [10]. These styles can be either used as an iterative solver or as a preconditioning fashion, offering in numerous cases a better performance than some of the most innovative and living FAS algorithms.

Due to the efficiency and potentiality of WFAS, researches further have been carried out for its enrichment. In order to realize this task, work build that is orthogonal/biorthogonal discrete wavelet transform using lifting scheme [11]. Wavelet based lifting technique is introduced by Sweldens [12], which permits some improvements on the properties of existing wavelet transforms. Wavelet grounded numerical result of elasto- hydrodynamic lubrication problems via lifting scheme was introduced by Shiralashetti et al. [13] .The technique has some numerical benefits as a reduced number of operations which are fundamental in the context of the iterative solvers. Evidently all attempts to simplify the wavelet solutions for PDE are welcome. In PDE, matrices arising from system are dense with non-smooth diagonal and smooth away from the diagonal. This smoothness of the matrix transforms into smallness using sea transfigure and it leads to design the effective ripples grounded lifting scheme.

Lifting scheme is a new approach to construct the so-called second generation wavelets that are not necessarily translations and dilations of one function. The latter we refer to as a first generation wavelets or classical methods. The lifting scheme has some fresh advantages in comparison with the classical ripples. This transfigure workshop for signals of an arbitrary size with correct treatment of boundaries. Another point of the lifting scheme is that all constructions are deduced in the spatial sphere. This is in discrepancy to the traditional approach, which relies heavily on the frequency sphere. The two major advantageous are:

- It leads to a more intuitively appealing treatment better suited to those in interested in applications than mathematical foundations.
- It makes a computational time optimal and sometimes increasing the speed of calculations.

The lifting scheme starts with a set of well-known filters, thereafter lifting steps are used an attempt to improve (lift) the properties of a corresponding wavelet decomposition. This procedure has some fine benefits as a reduced number of operations which are essential in the environment of the iterative solvers. In addition to this, the present paper illustrates that the application of the lifting scheme for the numerical solution of generalized Burgers-Huxley equations.

The present paper is organized as follows: In section 1, Preliminaries of wavelet filter coefficients and lifting scheme. The method of solution describes in section 2. In section 3 provides numerical results of the illustrative problem and finally, in section 4 conclusion of the proposed work is given.

1. Preliminaries of wavelet filter coefficients and Lifting scheme

The lifting scheme starts with a set of well-known filters; thereafter lifting steps are used in attempt to improve the properties of corresponding wavelet decomposition.

Now, we have discussed about different wavelet filters as follows:

a) Haar wavelet filter coefficients

We know that low pass filter coefficients $\left[a_0,\ a_1\right]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ and high pass filter coefficients $\left[b_0,\ b_1\right]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ play an important role in decomposition.

b) Daubechies wavelet filter coefficients

Daubechies introduced scaling functions having the shortest possible support. The scaling function ϕ_N has support $\left[0,\ N-1\right]$, while the corresponding wavelet ψ_N has support in the interval $\left[1-N/2,\ N/2\right]$.

We have low pass filter coefficients $\left[a_0,\ a_1,\ a_2,\ a_3\right]^T = \left[\frac{1+\sqrt{3}}{4\sqrt{2}},\ \frac{3+\sqrt{3}}{4\sqrt{2}},\ \frac{3-\sqrt{3}}{4\sqrt{2}},\ \frac{1-\sqrt{3}}{4\sqrt{2}}\right]^T$

and high pass filter coefficients $\left[b_0,\ b_1,\ b_2,\ b_3\right]^T = \left[\frac{1-\sqrt{3}}{4\sqrt{2}},\ -\frac{3-\sqrt{3}}{4\sqrt{2}},\ \frac{3+\sqrt{3}}{4\sqrt{2}},\ -\frac{1+\sqrt{3}}{4\sqrt{2}}\right]^T$

c) Biorthogonal (CDF (2,2)) wavelets

Let's consider the (5, 3) biorthogonal spline wavelet filter pair, the low pass filter pair are $(\tilde{a}_{-1}, \tilde{a}_0, \tilde{a}_1) = \left(\frac{1}{2\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$ and $(a_{-2}, a_{-1}, a_0, a_1, a_2) = \left(\frac{-1}{4\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{-1}{4\sqrt{2}}\right)$.

But, we have $b_k = (-1)^k \tilde{a}_{1-k}$ and $\tilde{b}_k = (-1)^k a_{1-k}$, the high pass filter pair are

$b_0 = \frac{1}{2\sqrt{2}},\ b_1 = \frac{-1}{\sqrt{2}},\ b_2 = \frac{1}{2\sqrt{2}}$ & $\tilde{b}_{-1} = \frac{1}{4\sqrt{2}},\ \tilde{b}_0 = \frac{1}{2\sqrt{2}},\ \tilde{b}_1 = \frac{-3}{2\sqrt{2}},\ \tilde{b}_2 = \frac{1}{2\sqrt{2}},\ \tilde{b}_3 = \frac{1}{4\sqrt{2}}$

Foundations of lifting scheme:

Consider to numbers *a*, *b* as two neighboring samples of a sequence and then these have some correlation which we would like to take advantage. The simple linear transform which replaces *a* and *b* by average ***s*** and difference *d* i.e.

$$s = \frac{a+b}{2} \quad \& \quad d = \frac{a-b}{2}.$$

The idea is that if *a* and *b* are highly correlated, the expected absolute value of their difference d will be small and can be represented with fever bits. In case that a = b, the difference is simply zero. We haven't lost any information because we can always recover a and b from the gives s and d as

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$$a = s - \frac{d}{2} \quad \& \quad b = s + \frac{d}{2}$$

Finally, a wavelet transform built through lifting consists of three steps: split. Predict and update as given in the Figure 1 [14].

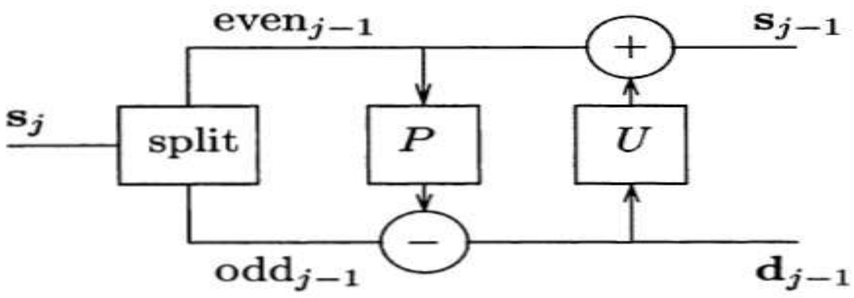


Fig. 1. *Steps in lifting scheme*

Split: Splitting the signal into two disjoint sets of samples.

Predict: If the signal contains some structure, then we can expect correlation between a sample and its nearest neighbors. i. e. $d_{j-1} = \text{odd}_{j-1} - \text{P}(\text{even}_{j-1})$

Update: Given an even entry, we have predicted that the next odd entry has the same value, and stored the difference. We then update our even entry to reflect our knowledge of the signal. i.e. $s_{j-1} = \text{even}_{j-1} + \text{U}(d_{j-1})$

The detailed algorithm using different wavelets is given in the next section. The general lifting stages for decomposition and reconstruction of a signal are given in Figure 2.

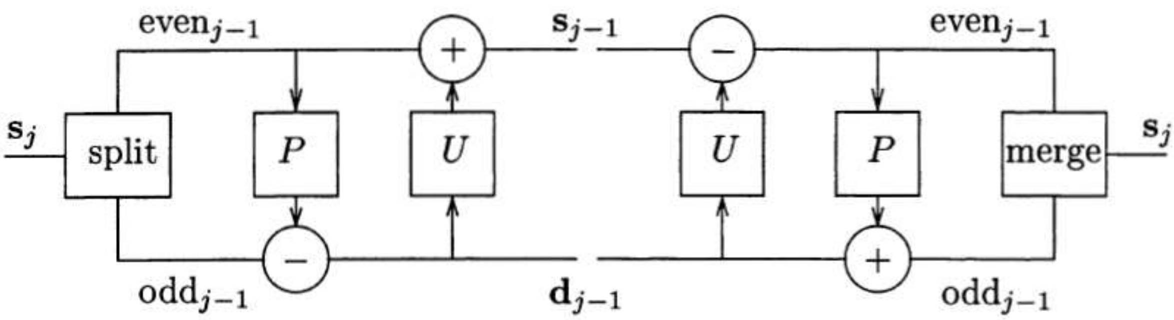


Fig. 2. *Lifting wavelet algorithm*

The detailed algorithm using different wavelets is given in the next section.

2. Method of solution

The generalized Burgers-Huxley equation is investigated by Satsuma [15] as:

$$u_t = u_{xx} - \alpha u^n u_x + \beta u\left(1 - u^n\right)\left(u^n - \gamma\right), \quad 0 \le x \le 1 \;\&\; t > 0 \quad (3.1)$$

Where α , $\beta \ge 0$ are real constants, n is positive integer and $\gamma \in [0 , 1]$.

After discretizing the equation (3.1) through the finite difference method (FDM), we get system of algebraic equations. Through this system we can write the system as

$$A\,u \;=\; b \tag{3.2}$$

where A is $N \times N$ coefficient matrix, b is $N \times N$ matrix and u is $N \times N$ matrix to be determined.

where $N = 2^J$, N is the number of grid points and J is the level of resolution.

Solve Eq. (3.2) through the iterative method, we get approximate solution. Approximate solution containing some error, therefore required solution equals to sum of approximate solution and error. There are numerous styles to minimize similar error to get the accurate result. Some of them are HWLS, DWLS BWLS etc.

Now we are using the advanced technique based on different wavelets called as lifting scheme. Lately, lifting schemes are useful in the signal analysis and image processing in the area of wisdom and engineering. But currently it extends to approximations in the numerical analysis [10]. Here, we are discussing the algorithm of the lifting schemes as follows:

2.1. Haar wavelet Lifting scheme (HWLS)

Daubechies and Sweldens have shown that every wavelet filter can be decomposed into lifting steps [11]. More details of the advantages as well as other important structural advantages of the lifting technique can be available in [12]. The representation of Haar wavelet via lifting form presented as;

Decomposition:

Consider approximate solution $S = \tilde{P}_j$ like as signal and then apply the HWLS decomposition (finer to coarser) procedure as,

$$\left.\begin{aligned} & d^{(1)} = S_{2j} - S_{2j-1}, \quad s^{(1)} = S_{2j-1} + \frac{1}{2} d^{(1)}, \quad S_1 = \sqrt{2}\, s^{(1)} \\ & \text{and} \quad D = \frac{1}{\sqrt{2}} d^{(1)} \end{aligned}\right\} \tag{3.3}$$

In this stage finally, we get new approximation as,

$$S = [S_1 \;\; D] \tag{3.4}$$

Reconstruction:

Consider Eq. (3.2) and then apply the HWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{aligned} & d^{(1)} = \sqrt{2}\; D, \quad s^{(1)} = \frac{1}{\sqrt{2}} S_1, \quad S_{2j-1} \;=\; s^{(1)} - \frac{1}{2} d^{(1)} \\ & \text{and} \quad S_{2j} = d^{(1)} \;+\; S_{2j-1} \end{aligned}\right\} \tag{3.5}$$

which is the required solution of the given equation.

2.2. Daubechies wavelet Lifting scheme (DWLS)

As bandied in the former section3.1, we follow the same procedure but we used different sea i.e., Daubechies 4th order sea measure. The DWLS procedure is as follows;

Decomposition:

$$\left.\begin{aligned}
&s^{(1)} = S_{2j-1} + \sqrt{3}\,S_{2j}, \quad d^{(1)} = S_{2j} - \frac{\sqrt{3}}{4}s^{(1)} - \left(\frac{\sqrt{3}-2}{4}\right)s_1^{(j-1)}, \\
&s^{(2)} = s^{(1)} - d_1^{(j+1)}, \quad S_1 = \frac{\sqrt{3}-1}{\sqrt{2}}s^{(2)} \quad \text{and} \\
&D = \frac{\sqrt{3}+1}{\sqrt{2}}d^{(1)}
\end{aligned}\right\} \tag{3.6}$$

Here, we get new approximation as,

$$S = [S_1 \;\; D] \tag{3.7}$$

Reconstruction:

Consider Eq. (3.5), then apply the DWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{aligned}
&d^{(1)} = \frac{\sqrt{2}}{\sqrt{3}+1}D, \\
&s^{(2)} = \frac{\sqrt{2}}{\sqrt{3}-1}S_1, \\
&s_1^{(j)} = s^{(2)} + d_1^{(j+1)}, \\
&S_{2j} = d^{(1)} + \frac{\sqrt{3}}{4}s_1^{(j)} + \frac{\sqrt{3}-2}{4}s_1^{(j-1)} \quad \text{and} \\
&S_{2j-1} = s^{(1)} - \sqrt{3}\,S_{2j}
\end{aligned}\right\} \tag{3.8}$$

which is the required solution of the given equation.

2.3. Biorthogonal wavelet Lifting scheme (BWLS)

As discussed in the previous sections 3.1 and 3.2, we follow the same procedure here we used another wavelet i.e., biorthogonal wavelet (CDF(2,2)). The BWLS procedure is as follows;

Decomposition:

$$\left.\begin{aligned}
&d^{(1)} = S_{2j} - \frac{1}{2}\left[S_{2j-1} + S_{2j+2}\right], \\
&s^{(1)} = S_{2j-1} + \frac{1}{4}\left[d_{j-1}^{(1)} + d^{(1)}\right], \\
&D = \frac{1}{\sqrt{2}}d^{(1)}, \\
&S_1 = \sqrt{2}\,s^{(1)}
\end{aligned}\right\} \tag{3.9}$$

In this stage finally, we get new signal as,

$$S = [S_1 \;\; D] \tag{3.10}$$

Reconstruction:

Consider Eqn. (3.10), then apply the DWLS reconstruction (coarser to finer) procedure as

$$\left.\begin{aligned} s^{(1)} &= \frac{1}{\sqrt{2}} S_1, \\ d^{(1)} &= \sqrt{2}\, D, \\ S_{2j-1} &= s^{(1)} - \frac{1}{4}\left[d^{(1)}_{j-1} + d^{(1)} \right] \\ S_{2j} &= d^{(1)} + \frac{1}{2}\left[S_{2j-1} + S_{2j+2}) \right], \end{aligned}\right\} \quad (3.11)$$

which is the required solution of the given equation.

The coefficients $s_1^{(j)}$ and $d_1^{(j)}$ are the average and detailed coefficients respectively of the approximate solution u_a .

The new approaches are tested through some of the numerical problems and the results are shown in next section.

3. Numerical illustration

In this section, we applied Lifting scheme for the numerical solution of Cahn-Allen equations and also show the capability and applicability of HWLS, DWLS and BWLS. The error is computed by

$$E_{\max} = \max \left| u_e(x,\ t) - u_a(x,\ t) \right|,$$

where $u_e(x,\ t)$ and $u_a(x,\ t)$ are exact and approximate solution respectively.

Problem 4.1: Consider the generalized Burgers-Huxley equation $\left(\text{In Eq. } (3.1)\ \alpha = -2,\ \beta = 1,\ n = 1\ \&\ \gamma = 3\right)$ i.e.

$$u_t = u_{xx} + 2uu_x + u(1-u)(u-3),\ 0 \le x \le 1\ \&\ t > 0 \quad (4.1)$$

subject to the I.C.: $$u(x,0) = \frac{3}{2} - \frac{3}{2}\tanh\left[\left(\frac{3\sqrt{3}-3}{4}\right)x\right] \quad (4.2)$$

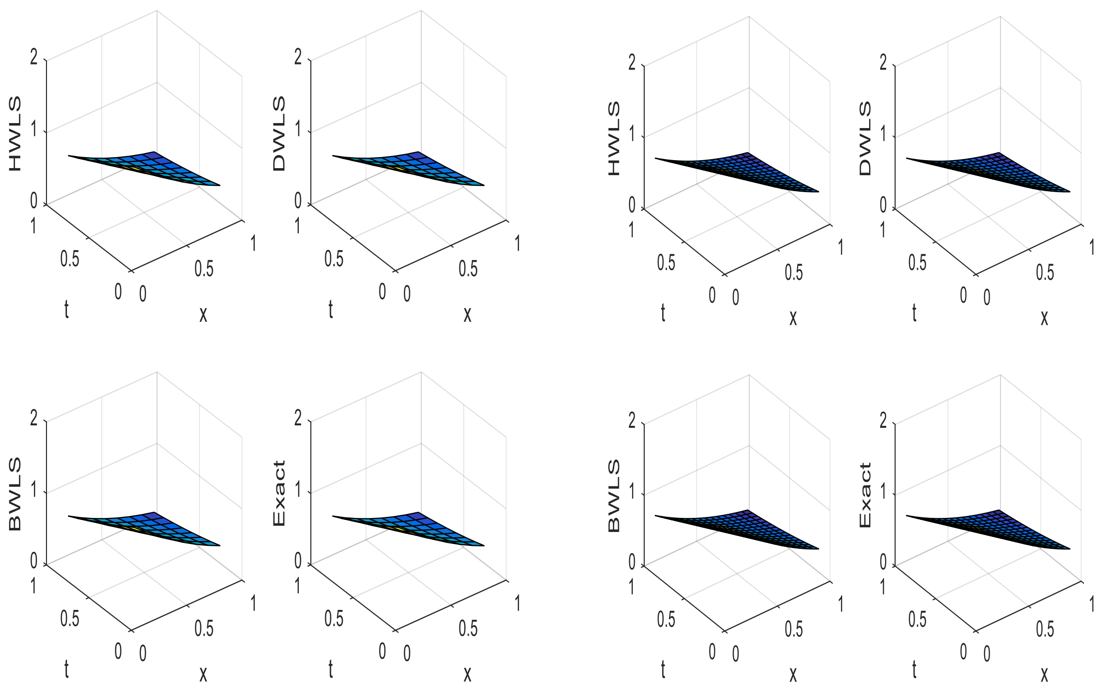
and B.C.s: $$\left.\begin{aligned} u(0,\ t) &= \frac{3}{2} - \frac{3}{2}\tanh\left[\left(\frac{3\sqrt{3}-3}{4}\right)\left(\frac{5-\sqrt{3}}{2}t\right)\right] \\ u(1,\ t) &= \frac{3}{2} - \frac{3}{2}\tanh\left[\left(\frac{3\sqrt{3}-3}{4}\right)\left(1 + \frac{5-\sqrt{3}}{2}t\right)\right] \end{aligned}\right\} \quad (4.3)$$

Which has the exact solution $u(x,\ t) = \frac{3}{2} - \frac{3}{2}\tanh\left[\left(\frac{3\sqrt{3}-3}{4}\right)\left(x + \frac{5-\sqrt{3}}{2}t\right)\right]$ [16].

By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solutions are presented in table 1 and figure 3. The maximum absolute errors with CPU time of the methods are presented in table 2.

Table 1. *Comparison of numerical solutions with exact solution of problem 4.1.*

x	t	FDM	HWLS	DWLS	BWLS	EXACT
0.2	0.2	1.077612	1.077612	1.077612	1.077612	1.077884
0.4		0.931885	0.931885	0.931885	0.931885	0.931344
0.6		0.797815	0.797815	0.797815	0.797815	0.796470
0.8		0.676108	0.676108	0.676108	0.676108	0.674757
0.2	0.4	0.84454	0.84454	0.84454	0.84454	0.844367
0.4		0.719072	0.719072	0.719072	0.719072	0.717719
0.6		0.607077	0.607077	0.607077	0.607077	0.604738
0.8		0.507707	0.507707	0.507707	0.507707	0.505602
0.2	0.6	0.645863	0.645863	0.645863	0.645863	0.644477
0.4		0.543007	0.543007	0.543007	0.543007	0.540296
0.6		0.453129	0.453129	0.453129	0.453129	0.449741
0.8		0.374734	0.374734	0.374734	0.374734	0.372067
0.2	0.8	0.483895	0.483895	0.483895	0.483895	0.481341
0.4		0.403008	0.403008	0.403008	0.403008	0.399064
0.6		0.333209	0.333209	0.333209	0.333209	0.329013
0.8		0.272973	0.272973	0.272973	0.272973	0.269983



a) $N \times N = 8 \times 8$ b) $N \times N = 16 \times 16$

Fig. 3. *Comparison of numerical solutions with exact solution of problem 4.1 for a)* $N \times N = 8 \times 8$ *and b)* $N \times N = 16 \times 16$.

Table 2. *Maximum error and CPU time (in seconds) of the methods of problem 4.1.*

$N \times N$	Method	E_{max}	Setup time	Running time	Total time
4×4	FDM	4.1961e-03	4.1486	0.0019	4.1505
	HWLS	4.1961e-03	0.0009	0.0029	0.0038
	DWLS	4.1961e-03	0.0003	0.0098	0.0101
	BWLS	4.1961e-03	0.0004	0.0041	0.0045
16×16	FDM	1.8461e-03	5.3062	0.0022	5.3084
	HWLS	1.8461e-03	0.0009	0.0029	0.0038
	DWLS	1.8461e-03	0.0003	0.0097	0.0100
	BWLS	1.8461e-03	0.0003	0.0041	0.0044
64×64	FDM	4.8834e-04	9.6336	0.0042	9.6378
	HWLS	4.8834e-04	0.0009	0.0030	0.0039
	DWLS	4.8834e-04	0.0003	0.0098	0.0101
	BWLS	4.8834e-04	0.0004	0.0041	0.0045

Problem 4.2: Consider the generalized Burgers-Huxley equation $\left(\text{In Eq. } (3.1)\ \alpha = -1,\ \beta = 1,\ n = 1\ \&\ \gamma = 1\right)$ i.e.

$$u_t = u_{xx} + u u_x + u(1-u)(u-1),\ 0 \le x \le 1\ \&\ t > 0 \tag{4.4}$$

subject to the I.C.: $$u(x,0) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right) \tag{4.5}$$

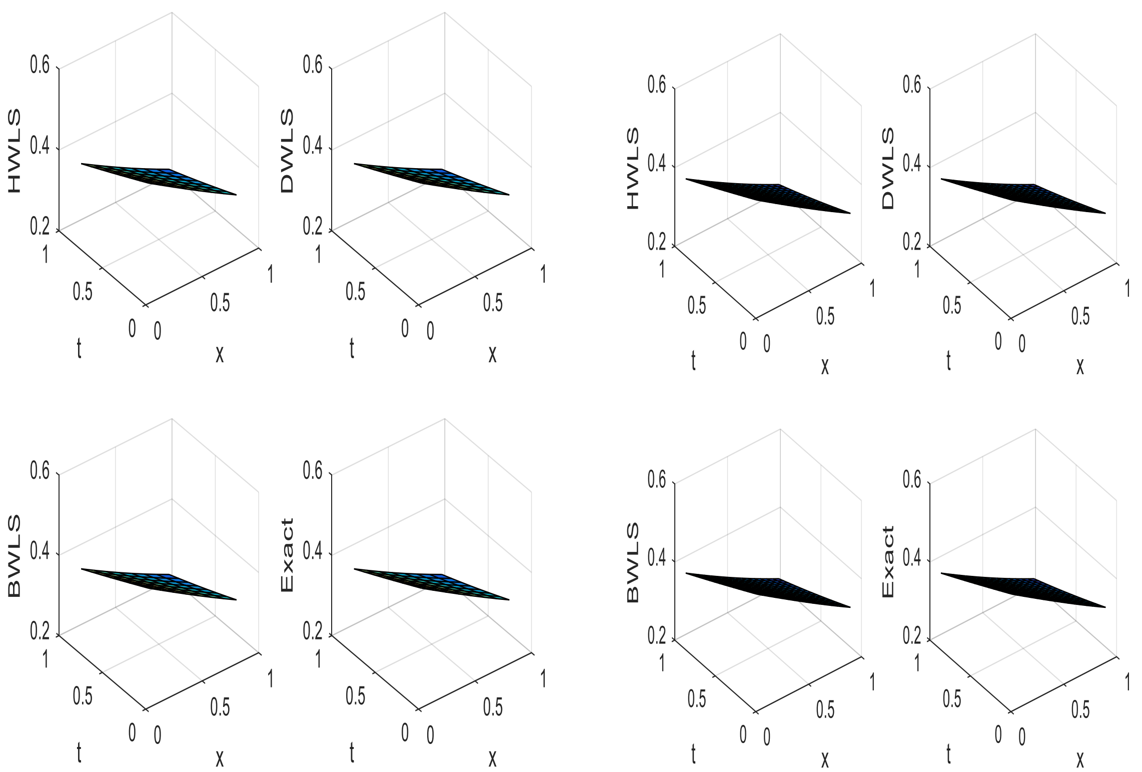
and B.C.s: $$\left.\begin{aligned} u(0,t) &= \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{3}{8}t\right) \\ u(1,t) &= \frac{1}{2} - \frac{1}{2}\tanh\left[\frac{1}{4}\left(1 + \frac{3}{2}t\right)\right] \end{aligned}\right\} \tag{4.6}$$

Which has the exact solution $u(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left[\frac{1}{4}\left(x + \frac{3}{2}t\right)\right]$ [17].

By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solutions are presented in table 3 and figure 4. The maximum absolute errors with CPU time of the methods are presented in table 4.

Table 3. *Comparison of numerical solutions with exact solution of problem 4.2.*

x	t	FDM	HWLS	DWLS	BWLS	EXACT
0.2	0.2	0.437838	0.437838	0.437838	0.437838	0.437823
0.4		0.413440	0.413440	0.413440	0.413440	0.413382
0.6		0.389458	0.389458	0.389458	0.389458	0.389361
0.8		0.365956	0.365956	0.365956	0.365956	0.365864
0.2	0.4	0.401242	0.401242	0.401242	0.401242	0.401312
0.4		0.377515	0.377515	0.377515	0.377515	0.377541
0.6		0.354405	0.354405	0.354405	0.354405	0.354344
0.8		0.331908	0.331908	0.331908	0.331908	0.331812
0.2	0.6	0.365683	0.365683	0.365683	0.365683	0.365864
0.4		0.342835	0.342835	0.342835	0.342835	0.342990
0.6		0.320798	0.320798	0.320798	0.320798	0.320821
0.8		0.299501	0.299501	0.299501	0.299501	0.299432
0.2	0.8	0.331570	0.331570	0.331570	0.331570	0.331812
0.4		0.309794	0.309794	0.309794	0.309794	0.310025
0.6		0.288973	0.288973	0.288973	0.288973	0.289050
0.8		0.268990	0.268990	0.268990	0.268990	0.268941



b) $N \times N = 8 \times 8$ b) $N \times N = 16 \times 16$

Fig. 4. *Comparison of numerical solutions with exact solution of problem 4.2 for b)* $N \times N = 8 \times 8$ *and b)* $N \times N = 16 \times 16$.

c)

Table 4. *Maximum error and CPU time (in seconds) of the methods of problem 4.2.*

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
4×4	FDM	2.4213e-04	3.7477	0.0025	3.7502
	HWLS	2.4213e-04	0.0011	0.0049	0.0060
	DWLS	2.4213e-04	0.0005	0.0135	0.0140
	BWLS	2.4213e-04	0.0005	0.0053	0.0058
16×16	FDM	1.1017e-04	5.7200	0.0024	5.7224
	HWLS	1.1017e-04	0.0009	0.0029	0.0038
	DWLS	1.1017e-04	0.0003	0.0098	0.0101
	BWLS	1.1017e-04	0.0005	0.0040	0.0045
64×64	FDM	3.3808e-05	7.6697	0.0040	7.6737
	HWLS	3.3808e-05	0.0010	0.0030	0.0040
	DWLS	3.3808e-05	0.0003	0.0097	0.0100
	BWLS	3.3808e-05	0.0003	0.0042	0.0045

Problem 4.3: Consider the generalized Burgers-Huxley equation $\left(\text{In Eq. } (3.1)\ \alpha = 1,\ \beta = \frac{2}{3},\ n = 2\ \&\ \gamma = 0\right)$ i.e.

$$u_t = u_{xx} - u^2 u_x + \frac{2}{3} u^3 \left(1 - u^2\right),\ 0 \le x \le 1\ \&\ t > 0 \qquad (4.7)$$

subject to the I.C.: $u(x,0) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x}{3}\right)\right]^{1/2}$ (4.8)

and B.C.s:

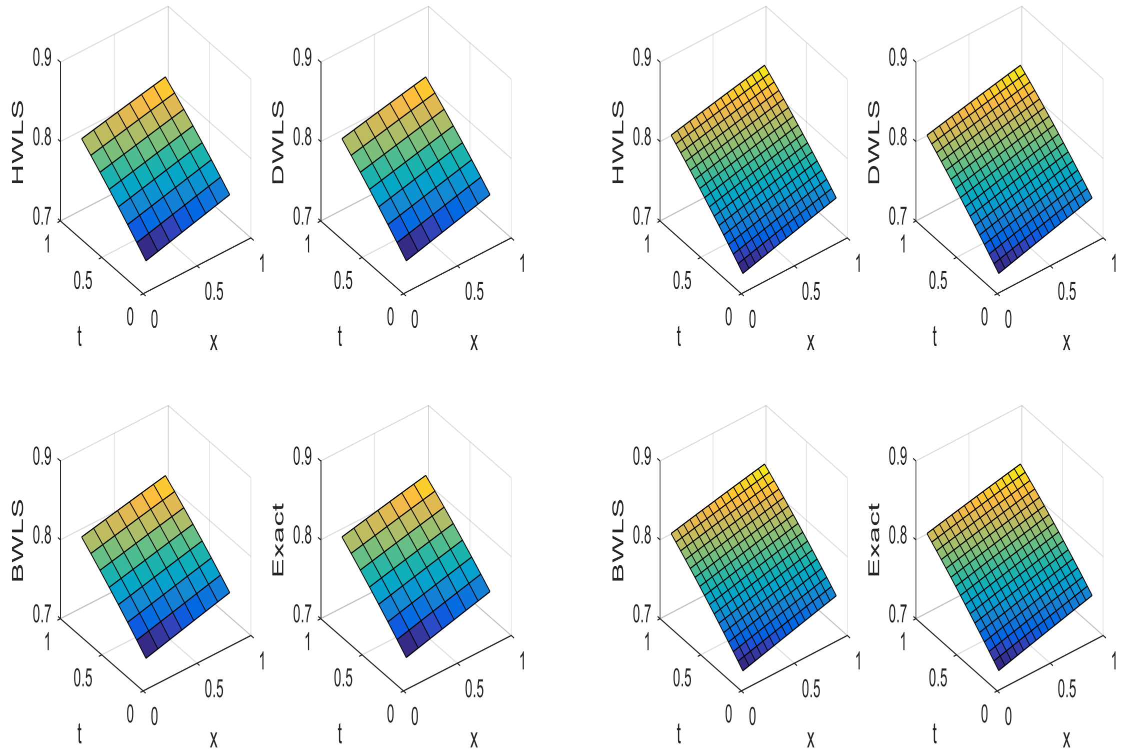
$$\left.\begin{aligned} u(0,t) &= \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{t}{9}\right)\right]^{1/2} \\ u(1,t) &= \left[\frac{1}{2} + \frac{1}{2}\tanh\left\{\frac{1}{9}(3+t)\right\}\right]^{1/2} \end{aligned}\right\} \qquad (4.9)$$

Which has the exact solution $u(x,t) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left\{\frac{1}{9}(3x+t)\right\}\right]^{1/2}$ [18].

By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solutions are presented in table 5 and figure 5. The maximum absolute errors with CPU time of the methods are presented in table 6.

Table 5. *Comparison of numerical solutions with exact solution of problem 4.3.*

x	t	FDM	HWLS	DWLS	BWLS	EXACT
0.2	0.2	0.735341	0.735341	0.735341	0.735341	0.737785
0.4		0.758032	0.758032	0.758032	0.758032	0.759708
0.6		0.779505	0.779505	0.779505	0.779505	0.780587
0.8		0.799788	0.799788	0.799788	0.799788	0.800347
0.2	0.4	0.742238	0.742238	0.742238	0.742238	0.745202
0.4		0.764466	0.764466	0.764466	0.764466	0.766788
0.6		0.785675	0.785675	0.785675	0.785675	0.787301
0.8		0.805805	0.805805	0.805805	0.805805	0.806674
0.2	0.6	0.749341	0.749341	0.749341	0.749341	0.752512
0.4		0.771136	0.771136	0.771136	0.771136	0.773748
0.6		0.791999	0.791999	0.791999	0.791999	0.793889
0.8		0.811847	0.811847	0.811847	0.811847	0.812868
0.2	0.8	0.756414	0.756414	0.756414	0.756414	0.759708
0.4		0.777804	0.777804	0.777804	0.777804	0.780587
0.6		0.798306	0.798306	0.798306	0.798306	0.800346
0.8		0.817822	0.817822	0.817822	0.817822	0.818929



c) $N \times N = 8 \times 8$ b) $N \times N = 16 \times 16$

Fig. 5. *Comparison of numerical solutions with exact solution of problem 4.3 for d)* $N \times N = 8 \times 8$ *and b)* $N \times N = 16 \times 16$.

Table 6. *Maximum error and CPU time (in seconds) of the methods of problem 4.3.*

$N \times N$	Method	E_{max}	Setup time	Running time	Total time
4×4	FDM	3.2939e-03	2.9781	0.0019	2.9800
	HWLS	3.2939e-03	0.0009	0.0028	0.0037
	DWLS	3.2939e-03	0.0003	0.0096	0.0099
	BWLS	3.2939e-03	0.0003	0.0042	0.0045
16×16	FDM	3.9987e-04	3.1827	0.0024	3.1851
	HWLS	3.9987e-04	0.0009	0.0028	0.0037
	DWLS	3.9987e-04	0.0003	0.0097	0.0100
	BWLS	3.9987e-04	0.0003	0.0040	0.0043
64×64	FDM	6.4417e-05	7.3539	0.0041	7.3580
	HWLS	6.4417e-05	0.0010	0.0028	0.0038
	DWLS	6.4417e-05	0.0003	0.0097	0.0100
	BWLS	6.4417e-05	0.0003	0.0042	0.0045

4. Conclusions

In this paper, we applied wavelets based Lifting schemes for the numerical solution of generalized Burgers-Huxley equations using different wavelet filters. From the above figures (3-5) and tables (1, 3, 5), we observe that the numerical solutions obtained by different Lifting schemes are agrees with the exact solution.

Also from the tables (2, 4, 6), the convergence of the presented schemes i.e. the error decreases when the level of resolution N increases. In addition, the calculations involved in Lifting schemes are simple, straight forward and low computation cost compared to classical method i.e. FDM.

Hence the presented Lifting schemes in particular HWLS & BWLS are very effective for solving non-linear partial differential equations.

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