

Binomial Theorem on the Coefficients for Combinatorial Geometric Series

Chinnaraji Annamalai

School of Management, Indian Institute of Technology, Kharagpur, India

Email: anna@iitkgp.ac.in

<https://orcid.org/0000-0002-0992-2584>

Abstract: This paper presents a binomial and factorial theorem on the binomial coefficients for combinatorial geometric series. The coefficient for each term in combinatorial geometric series refers to a binomial coefficient. These ideas can enable the scientific researchers to solve the real life problems.

MSC Classification codes: 05A10, 40A05 (65B10)

Keywords: computation, combinatorics, binomial coefficient

1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series, a new idea was stimulated his mind to create a combinatorial geometric series [1-10]. The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r . In this article, binomial identities and multinomial theorem is provided using the binomial coefficients for combinatorial geometric series.

2. Combinatorial Geometric Series

The combinatorial geometric series [1-10] is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial refers to the binomial coefficient V_n^r .

$$\sum_{i_1=0}^n \sum_{i_2=i_1}^n \sum_{i_3=i_2}^n \cdots \sum_{i_r=i_{r-1}}^n x^{i_r} = \sum_{i=0}^n V_i^r x^i \quad \& \quad V_n^r = \frac{(n+1)(n+2)(n+3) \cdots (n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1$ and $n, r \in N = \{0, 1, 2, 3, \dots\}$.

Here, $\sum_{i=0}^n V_i^r x^i$ refers to the combinatorial geometric series and

V_n^r is the binomial coefficient for combinatorial geometric series.

Theorem 2.1: $\frac{(n+r)!}{n!r!} = V_n^r$, where $n, r \geq 0$ & $n, r \in N$.

$$\text{Proof. } \binom{n+r}{n} = \frac{(n+r)!}{n!(n+r-n)!} = \frac{(n+r)!}{n!r!} = \frac{(n+1)(n+2)(n+3) \cdots (n+r)}{r!} = V_n^r.$$

Hence, theorem is proved.

Theorem 2.2: $\frac{((p+1)n)!}{(pn)!(n)!} = (p+1) \frac{((p+1)n-1)!}{n!(n-1)!}$, $(p \geq 1; n \geq 0 \& p, n \in N)$.

Proof. Let us use the binomial coefficient for combinatorial geometric series

$$\begin{aligned} V_{pn}^n &= \frac{(pn+1)(pn+2)(pn+3)\cdots(pn+n-1)(pn+n)}{(n)!} \\ &= \frac{(pn+1)(pn+2)(pn+3)\cdots(pn+n-1)((p+1)n)}{(n-1)!n} \\ &= \frac{(p+1)(pn+1)(pn+2)(pn+3)\cdots(pn+n-1)}{(n-1)!} = (p+1)V_{pn}^{n-1}. \end{aligned}$$

By applying Theorem 2.1 to the equation $V_{pn}^n = (p+1)V_{pn}^{n-1}$, we get

$$\frac{(pn+n)}{(pn)!n!} = (p+1) \frac{(pn+n-1)}{(pn)!(n-1)!} \Rightarrow \frac{((p+1)n)!}{(pn)!(n)!} = (p+1) \frac{((p+1)n-1)!}{n!(n-1)!}.$$

Hence, theorem is proved.

Conclusion

In this article, a binomial and factorial theorem was built using the binomial theorem on the binomial coefficients for combinatorial geometric series. These ideas can enable the scientific researchers for research and development further.

References

- [1] Annamalai, C. (2022) Binomial Coefficients and Identities in Combinatorial Geometric Series. OSF Preprints. <http://dx.doi.org/10.31219/osf.io/4ha3c>.
- [2] Annamalai, C. (2022) Multinomial Theorem on the Binomial Coefficients for Combinatorial Geometric Series. Zenodo. <http://dx.doi.org/10.5281/zenodo.7029149>.
- [3] Annamalai, C. (2020) Optimized Computing Technique for Combination in Combinatorics. *hal-0286583*. <https://doi.org/10.31219/osf.io/9p4ek>.
- [4] Annamalai, C. (2020) Novel Computing Technique in Combinatorics. *hal-02862222*. <https://doi.org/10.31219/osf.io/m9re5>.
- [5] Annamalai, C. (2022) Computation of Combinatorial Geometric Series and its Combinatorial Identities for Machine Learning and Cybersecurity. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2022-b6mks>.
- [6] Annamalai, C. (2022) Annamalai's Binomial Identity and Theorem, *SSRN Electronic Journal*. <http://dx.doi.org/10.2139/ssrn.4097907>.
- [7] Annamalai, C. (2022) Computation Method for Combinatorial Geometric Series and its Applications. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2022-pnx53-v22>.
- [8] Annamalai, C. (2022) Computing Method for Combinatorial Geometric Series and Binomial Expansion. *SSRN Electronic Journal*. <http://dx.doi.org/10.2139/ssrn.4168016>.

- [9] Annamalai, C. (2022) Factorials and Integers for Applications in Computing and Cryptography. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2022-b6mks>.
- [10] Annamalai, C. (2022) Computation of Combinatorial Geometric Series and its Combinatorial Identities for Cryptographic Algorithm and Machine Learning. *COE, Cambridge University Press*. <https://doi.org/10.33774/coe-2022-b6mks-v8>.