The Distribution of Prime Numbers

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Abstract. In this paper, we discovered a new sequence of all prime numbers which ends with a 1 or 9, the sequence defined by $a(n) = (n^2 - n - 1)/\text{gcd}\,(b(n),\ n^2 - n - 1)$, with $b(n)$ satisfay the recurcive formula $b(n) = (n-1).b(n-1) - n.b(n-2)$, and $b(1) = b(2) = -1$, the sequence $a(n)$ takes only 1's and primes.

Keywords. Prime numbers, sequence, Rowland sequence.

1. Introduction

A number is said to be a prime number if the number is divisible by 1 and itself; otherwise it's composite.

Some prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,..

The distribution of prime numbers which was treated in many researche for a formula can be helps in generating the prime numbers or testing if the given numbers is prime. In this paper, we present some known formulas.

Mill's showed that there existe a real number $A > 1$ such that $f(n) = \left[\, A^{3^n} \,\right]$ is a prime number for any integers n, approximately A=1.306377883863,.. (see A051021). The first few values

$f(n) =$ {2, 11, 1361, 2521008887, 16022236204009818131831320183,..}, (see A051254)

Euler's quadratic polynomial $n^2 + n + 41$ is prime for all n between 0 and 39, however, it is not prime for all integers.

The Rowland sequence of prime numbers composed entirely of 1's and primes, the sequence defined by the recurrence relation

$$r(n) = r(n-1) + \text{gcd}\,(n\,, r(n-1));\ \ r(1) = 7$$

The sequence of differences $r(n+1) - r(n)$

1, 1, 1, 5, 3, 1, 1, 1, 1, 11, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 23, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,1, 1, 1, 47, 3, 1, 5, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 101, 3, 1, 1, 7, 1, 1, 1, 1, 11, 3, 1, 1, 1, 1, 1, 13, 1, 1, 1, 1, 1, 1, 1, 1, 1,..

(sequence A132199 in the OEIS).

For more details and formulas see[2]. In this paper, we present an interesting sequence which play the same role of Rowland sequence composed by a prime number or 1. Moreover, our sequence give all prime numbers with the last digits 1 or 9.

2. The sequence of $b(n)$ and $a(n)$

The sequence $b(n)$ satisfy the following recursive formula

$$b(n) = (n-1)b(n-1) - nb(n-2)$$

With the starting conditions $b(1) = b(2) = -1$.

The first few values of b(n).

b(n)={-1, -1, 1, 7, 23, 73, 277, 1355, 8347, 61573, 523913, 5024167, 53479135, 624890417, 7946278813,...}, (see A356684)

Other formula of $b(n)$ as continued fraction

$$\frac{b(n)}{n^2-n-1} = \cfrac{1}{2-\cfrac{3}{3-\cfrac{4}{4-\cfrac{5}{\cfrac{\cdots}{(n-1)-\cfrac{n}{n-(n+1)}}}}}} \ ;\ for\ n \geq 2$$

In this section, we present our sequence of prime numbers defined in the conjecture as follows.

Conjecture 1. The sequence $a(n)$ satisfy the following formula

$$a(n) = \frac{n^2-n-1}{gcd(b(n),\ n^2-n-1)} \ ;\ for\ n \geq 2$$

Where gcd(x, y) denotes the greatest common divisor of x and y.

The values of $a(n)$.

1, 5, 11, 19, 29, 41, 11, 71, 89, 109, 131, 31, 181, 19, 239, 271, 61, 31, 379, 419, 461, 101, 29, 599, 59, 701, 151, 811, 79, 929, 991, 211, 59, 41, 1259, 1, 281, 1481, 1559, 149, 1721, 1, 61, 1979, 2069, 2161, 1, 2351, 79, 2549, 241, 1, 2861, 2969, 3079, 3191,…(see A356247)

Every term of this sequence is either a prime number or equal to 1.

Conjecture 2. The sequence a(n) contains all prime numbers which ends with a 1 or 9.

Example 1: 11, 19, 29, 31, 41, 59, 61, 71, 79, 89,…

Conjecture 3. Except for 5, the prime numbers of this sequence are all repeated only once.

Consequently, let us consider the values of n and m such that we get:

$$i) \qquad a(n) = a(m) = n + m - 1, with\ n \neq m$$

$$ii) \qquad a(n) = a(m) = \gcd(n^2 - n - 1, m^2 - m - 1)$$

From (i) we have $m = a(n) - n + 1$, then we get the sequence a(n) in term n as follow

$$iii) \qquad a(n) = a(a(n) - n + 1)$$

The above formula helps us to find the terms which have the same value.

Example 2:

i) $a(18) = a(44) = 18 + 44 - 1 = 61$

ii) $a(18) = a(44) = gcd(18^2 - 18 - 1, 44^2 - 44 - 1) = gcd(305, 1891) = 61$

iii) $a(18) = a(a(18) - 18 + 1) = a(61 - 18 + 1) = a(44)$

References

[**1**] Richard Guy, Unsolved Problems in Number Theory, Springer science (2004).

[**2**] Eric S. Rowland, A Natural Prime-Generating Recurrence, Journal of Integer Sequences, Vol. 11 (2008).

[**3**] N. J. A. Sloane, The On-line Encyclopedia of integers sequences, https://oeis.org

(Concerned with the sequence A051021, A051254, A132199, A356247, A356684)