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Neutrosophic Cycle

Ideas | Approaches | Accessibility | Availability

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Abstract

In this book, some notions are introduced about "Neutrosophic Cycle". Some frameworks are devised as "Different Types" of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs.

New setting is introduced to study different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs. Minimum number and maximum number of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, is a number which is representative based on those vertices or edges. Minimum or maximum neutrosophic number or polynomial of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are called neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number or polynomial. Forming sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable to figure out different types of number of vertices in the sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable sets in the terms of minimum (maximum) number of vertices to get minimum (maximum) number to assign in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs, is key type of approach to have these notions namely different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets. As concluding results, there are some statements, remarks, examples and clarifications about cycle-neutrosophic graphs. The clarifications are also presented in both sections "Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable are addressed in Bibliography. Specially, properties of SuperHyperGraph and neutrosophic SuperHyperGraph by Henry Garrett (2022), is studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book by Henry Garrett (2022).

In this study, there's an idea which could be considered as a motivation.

Question 0.0.1. *Is it possible to use mixed versions of ideas concerning "different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial", "neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial" and "cycle-neutrosophic graphs" to define some notions which are applied to cycle-neutrosophic graphs?*

It's motivation to find notions to use in cycle-neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. In both settings, corresponded numbers or polynomials conclude the discussion. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic

definitions to clarify about preliminaries. In section "Preliminaries", new notions of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial' in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. As concluding results, there are some statements, remarks, examples and clarifications about cycle-neutrosophic graphs. The clarifications are also presented in both sections 'Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for path notions, namely cycle-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

Some frameworks are devised as "Different Types" of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs.

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The words of mind and the minds of words, are too eligible to be in the stage of acknowledgements

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CHAPTER 1

Neutrosophic Notions

1.1 Abstract

New setting is introduced to study different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs. Minimum number and maximum number of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, is a number which is representative based on those vertices or edges. Minimum or maximum neutrosophic number or polynomial of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are called neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number or polynomial. Forming sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable to figure out different types of number of vertices in the sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable sets in the terms of minimum (maximum) number of vertices to get minimum (maximum) number to assign in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs, is key type of approach to have these notions namely different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets. As concluding results, there are some statements, remarks, examples and clarifications about cycle-neutrosophic graphs. The clarifications are also presented in both sections "Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable

AMS Subject Classification: 05C17, 05C22, 05E45

1.2 Background

Different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable are addressed in Bibliography. Specially, properties of SuperHyperGraph and neutrosophic SuperHyperGraph by Henry Garrett (2022), is studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book by Henry Garrett (2022).
In this section, I use two sections to illustrate a perspective about the background of this study.

1.3 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.3.1. *Is it possible to use mixed versions of ideas concerning "different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial", "neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable* *number and polynomial" and "cycle-neutrosophic graphs" to define some notions which are applied to cycle-neutrosophic graphs?*

It's motivation to find notions to use in cycle-neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. In both settings, corresponded numbers or polynomials conclude the discussion. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In section "Preliminaries", new notions of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial' in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. As concluding results, there are some statements, remarks, examples and clarifications about cycle-neutrosophic graphs. The clarifications are also presented in both sections 'Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for complete notions, namely cycle-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.4 Preliminaries

In this section, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.4.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.4.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v\in V}\sum_{i=1}^{3}\sigma_i(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e\in E}\sum_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1}\mu(x_ix_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t}\bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1});$$

(iv) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1}\mu(v_iv_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

Definition 1.4.4. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 1.4.5. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_σ if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH where $x_i x_{i+1} \in E$, $i = 0, 1, \cdots, n-1$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** and denoted by CYC where $x_i x_{i+1} \in E$, $i = 0, 1, \cdots, n-1$, $x_{\mathcal{O}(NTG)} x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1, σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1, σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1, σ_2}.

Remark 1.4.6. Using notations which is mixed with literatures, are reviewed.

1.4.6.1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$, $\mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;

1.4.6.2. $CMT_\sigma, PTH, CYC, STR_{1,\sigma_2}, CMT_{\sigma_1,\sigma_2}, CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$, and WHL_{1,σ_2}.

1.5 Setting of neutrosophic notion number

In this section, I provide some results in the setting of neutrosophic notion number.

Definition 1.5.1. (Zero Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Zero forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(ii) **Zero forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

The set of vertices forms zero forcing number and its zero forcing neutrosophic-number.

Proposition 1.5.2. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}(NTG) = 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex is a neighbor for two vertices. Two vertices which are neighbors, are only members of S is a set of black vertices. Thus the color-change rule implies all vertices are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}(NTG) = 2.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.3. There are two sections for clarifications.

(a) In Figure (1.1), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";
- (ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";
- (iii) if $S = \{n_2\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(v) 2 is zero forcing number and its corresponded sets are
$\{n_1, n_2\}$,$\{n_1, n_3\}$,
$\{n_1, n_4\}$,$\{n_1, n_5\}$,$\{n_1, n_6\}$,$\{n_2, n_3\}$,
$\{n_2, n_4\}$,$\{n_2, n_5\}$,$\{n_2, n_6\}$,$\{n_3, n_4\}$,
$\{n_3, n_5\}$,$\{n_3, n_6\}$,
$\{n_4, n_5\}$,$\{n_4, n_6\}$, and
$\{n_5, n_6\}$;

(vi) 1.3 is zero forcing neutrosophic-number and its corresponded set is $\{n_1, n_5\}$.

(b) In Figure (1.2), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(v) 2 is zero forcing number and its corresponded sets are
$\{n_1, n_2\}$,$\{n_1, n_3\}$,
$\{n_1, n_4\}$,$\{n_1, n_5\}$,
$\{n_2, n_3\}$,$\{n_2, n_4\}$,
$\{n_2, n_5\}$,$\{n_3, n_4\}$,
$\{n_3, n_5\}$, and $\{n_4, n_5\}$;

(vi) 2.7 is zero forcing neutrosophic-number and its corresponded set is $\{n_1, n_5\}$.

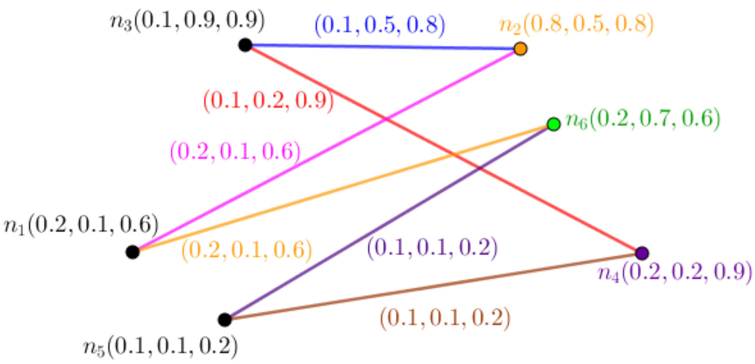

Figure 1.1: A Neutrosophic Graph in the Viewpoint of its Zero Forcing Number. 47NTG5

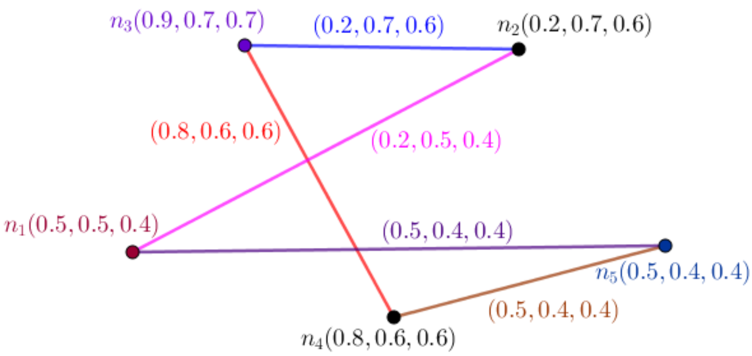

Figure 1.2: A Neutrosophic Graph in the Viewpoint of its Zero Forcing Number. 47NTG6

The main definition is presented in next section. The notions of failed zero-forcing number and failed zero-forcing neutrosophic-number facilitate the ground to introduce new results. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types. New setting is introduced to study failed zero-forcing number and failed zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex but now reverse approach is on demand which is finding biggest set which doesn't force.

Definition 1.5.4. (Failed Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Failed zero-forcing number** $\mathcal{Z}'(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(ii) **Failed zero-forcing neutrosophic-number** $\mathcal{Z}'_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

The set of vertices forms failed zero-forcing number and its failed zero-forcing neutrosophic-number.

Proposition 1.5.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}'(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex is a neighbor for two vertices. Vertices with distance two, are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.6. There are two sections for clarifications.

(a) In Figure (2.3), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(*v*) 3 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(*vi*) 4.9 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

(*b*) In Figure (2.4), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

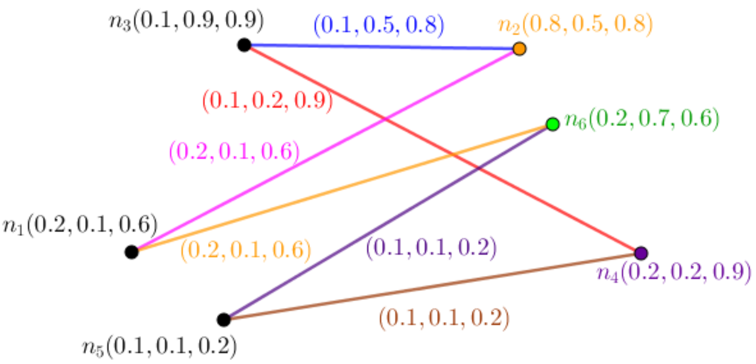

Figure 1.3: A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number. 48NTG5

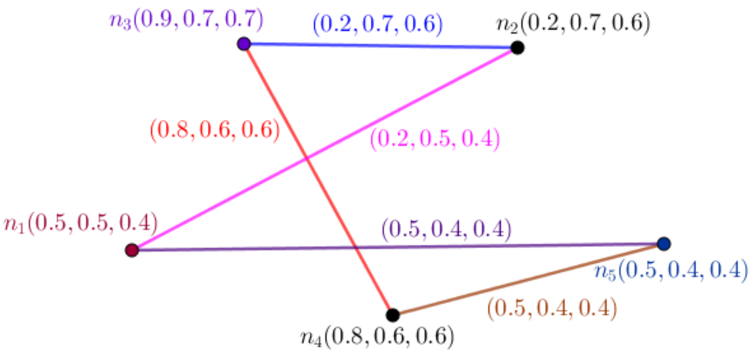

Figure 1.4: A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number. 48NTG6

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4\}$, $\{n_3, n_5\}$, $\{n_2, n_5\}$, $\{n_4, n_1\}$, and $\{n_1, n_3\}$;

(vi) 3.7 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3\}$.

The main definition is presented in next section. The notions of 1-zero-forcing number and 1-zero-forcing neutrosophic-number facilitate the ground to introduce new results. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types. New setting is introduced to study 1-zero-forcing number and 1-zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex and now approach is on demand which is finding smallest set which forces.

Definition 1.5.7. (1-Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-zero-forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

(ii) **1-zero-forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

Proposition 1.5.8. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}(NTG) = 1.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex is a neighbor for two vertices. Two vertices which are neighbors, are only members of S is a set of black vertices through color-change rule. Thus the color-change rule implies all vertices are black vertices but extra condition implies every given vertex is member of S is a set of black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition. So

$$\mathcal{Z}(NTG) = 1.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.9. There are two sections for clarifications.

(a) In Figure (2.5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

   (i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus

n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_2\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices but extra condition implies n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

(*iv*) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices but extra condition implies n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

(*v*) 1 is 1-zero-forcing number and its corresponded sets are $\{n_1\}, \{n_2\}, \{n_3\}, \{n_4\}, \{n_5\}$. and $\{n_6\}$;

(*vi*) 0.4 is 1-zero-forcing neutrosophic-number and its corresponded set is $\{n_5\}$.

(*b*) In Figure (2.6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_2\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices but extra condition implies n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

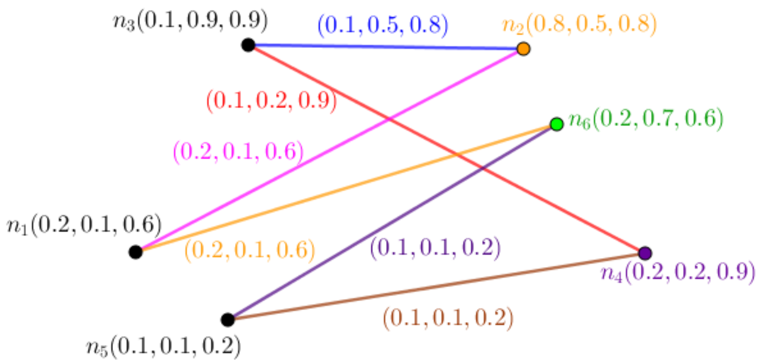

Figure 1.5: A Neutrosophic Graph in the Viewpoint of its 1-Zero-Forcing Number. 49NTG5

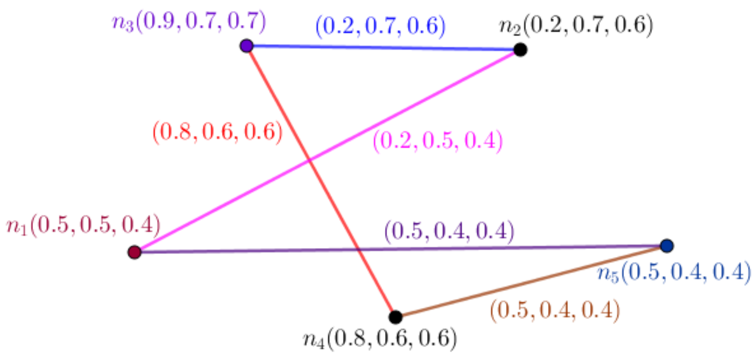

Figure 1.6: A Neutrosophic Graph in the Viewpoint of its 1-Zero-Forcing Number. 49NTG6

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices but extra condition implies n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

(v) 1 is 1-zero-forcing number and its corresponded sets are $\{n_1\}, \{n_2\}, \{n_3\}, \{n_4\}, \{n_5\}$. and $\{n_6\}$;

(vi) 1.3 is 1-zero-forcing neutrosophic-number and its corresponded set is $\{n_5\}$.

Definition 1.5.10. (Independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously;

(ii) **independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S

of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously.

Proposition 1.5.11. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Assume $|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. In other side, for having an edge, there's a need to have two vertices. So by using the members of $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$. It implies that $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$ is corresponded to independent number. Thus

$$\mathcal{I}(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.12. There are two sections for clarifications.

(a) In Figure (2.7), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| \neq \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

  (ii) if $S = \{n_2, n_4, n_6\}$ is a set of vertices, then there's no vertex in S but n_2, n_4 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_2, n_4, n_6\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S and It doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*iv*) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_1, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*v*) 3 is independent number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(*vi*) 3.2 is independent neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

(*b*) In Figure (2.8), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S and it implies that $S = \{n_2, n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*ii*) if $S = \{n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It implies that $S = \{n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 or n_5, n_1 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5 or n_5n_1. There are three edges to have exclusive endpoints from S and It doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*iv*) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to

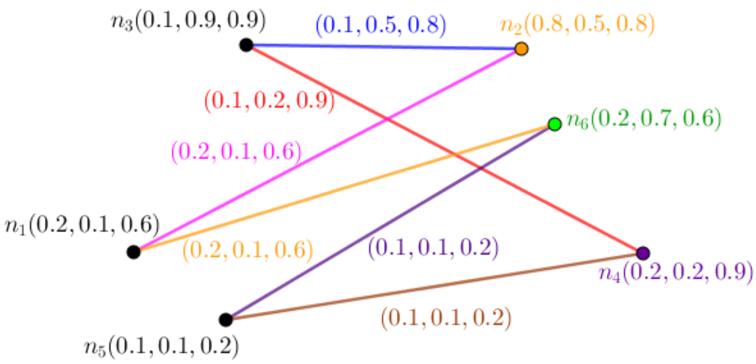

Figure 1.7: A Neutrosophic Graph in the Viewpoint of its Independent Number. 50NTG5

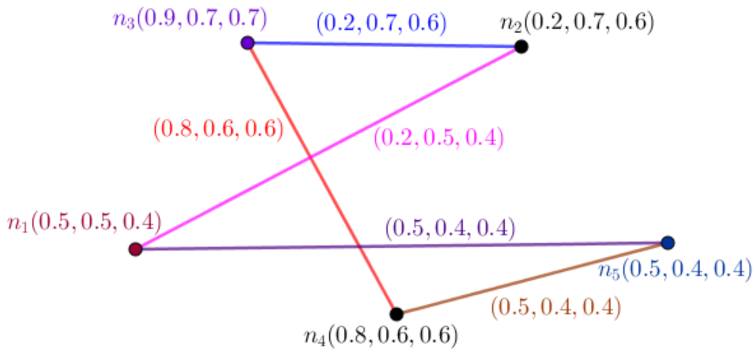

Figure 1.8: A Neutrosophic Graph in the Viewpoint of its Independent Number. 50NTG6

have two vertices. So by using the members of S, it's possible to have endpoints of an edge n_1n_5. There's one edge n_1n_5 to have exclusive endpoints n_1 and n_5 from S hence it implies that $S = \{n_1, n_3, n_5\}$ isn't corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(v) 2 is independent number and its corresponded sets are $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, and $\{n_3, n_5\}$;

(vi) 2.8 is independent neutrosophic-number and its corresponded set is $\{n_2, n_5\}$.

The natural way proposes us to use the restriction "minimum" instead of "maximum."

Definition 1.5.13. (Failed independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(*ii*) **failed independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Thus we replace the term "minimum" by the term "'maximum." Hence,

Definition 1.5.14. (Failed independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **failed independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(*ii*) **failed independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Proposition 1.5.15. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{I}(NTG) = 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. If $|S| > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors. Thus there is no triangle but there's one edge. One edge has two endpoints. These endpoints are corresponded to failed independent number $\mathcal{I}(NTG)$. So

$$\mathcal{I}(NTG) = 2.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.16. There are two sections for clarifications.

(*a*) In Figure (2.9), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either failed independent number $\mathcal{I}(NTG)$ or failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*ii*) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_1, n_3\}$ is corresponded to either failed independent number $\mathcal{I}(NTG)$ or failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither failed independent number $\mathcal{I}(NTG)$ nor failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iv*) if $S = \{n_2, n_3\}$ is a set of vertices, then there's no vertex in S but n_2 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_2, n_3\}$ is corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*v*) 2 is failed independent number and its corresponded set is $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_1, n_6\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_2, n_6\}$, $\{n_3, n_4\}$, $\{n_3, n_5\}$, $\{n_3, n_6\}$, $\{n_4, n_5\}$, $\{n_4, n_6\}$, $\{n_5, n_6\}$, and $\{n_6, n_1\}$;

(*vi*) 4 is failed independent neutrosophic-number and its corresponded set is $\{n_2, n_3\}$.

(*b*) In Figure (2.10), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either failed independent number $\mathcal{I}(NTG)$ or failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*ii*) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_1, n_3\}$ is corresponded to either failed independent number $\mathcal{I}(NTG)$ or failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members

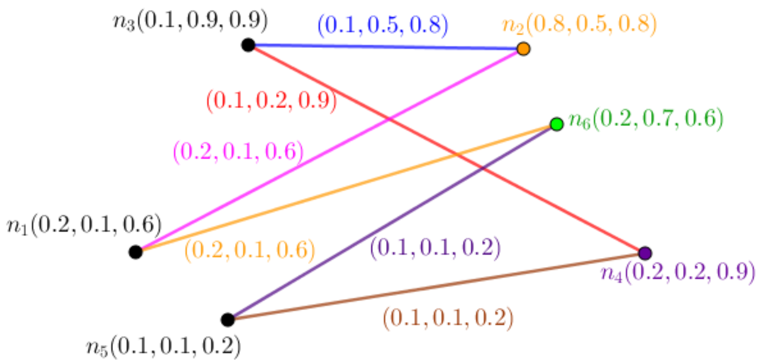

Figure 1.9: A Neutrosophic Graph in the Viewpoint of its Failed Independent Number. 51NTG5

either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither failed independent number $\mathcal{I}(NTG)$ nor failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 2}$ thus it implies that $S = \{n_3, n_4\}$ is corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 2 is failed independent number and its corresponded set is $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_3, n_4\}$, $\{n_3, n_5\}$, $\{n_4, n_5\}$, and $\{n_5, n_1\}$;

(vi) 4.3 is failed independent neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Definition 1.5.17. (1-independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously For one time, one vertex is allowed to be endpoint;

(ii) **1-independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously. For one time, one vertex is allowed to be endpoint.

Definition 1.5.18. (Failed 1-independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

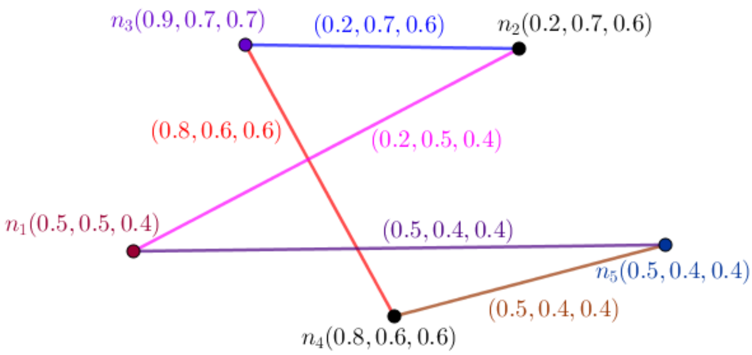

Figure 1.10: A Neutrosophic Graph in the Viewpoint of its Failed Independent Number. 51NTG6

(*i*) **failed 1-independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. For one time, one vertex is allowed not to be endpoint;

(*ii*) **failed 1-independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. For one time, one vertex is allowed not to be endpoint.

Proposition 1.5.19. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Assume $|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$ isn't corresponded to 1-independent number $\mathcal{I}(NTG)$. In other side, for having an edge, there's a need to have two vertices. So by using the members of $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$. But extra condition implies that $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1}$ is corresponded to 1-independent number. Thus

$$\mathcal{I}(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.20. There are two sections for clarifications.

(*a*) In Figure (2.11), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either 1-independent number $\mathcal{I}(NTG)$ or 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| \neq \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1}$;

(*ii*) if $S = \{n_2, n_4, n_6\}$ is a set of vertices, then there's no vertex in S but n_2, n_4 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_2, n_4, n_6\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1}$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. But extra condition implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to 1-independent number $\mathcal{I}(NTG)$ but not 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1}$;

(*iv*) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_1, n_3, n_5\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*v*) 4 is 1-independent number and its corresponded sets are $\{n_2, n_4, n_6, n_1\}$, $\{n_2, n_4, n_6, n_3\}$, $\{n_2, n_4, n_6, n_5\}$, $\{n_1, n_3, n_5, n_2\}$, $\{n_1, n_3, n_5, n_4\}$, and $\{n_1, n_3, n_5, n_6\}$;

(*vi*) 5.1 is 1-independent neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6, n_3\}$.

(b) In Figure (2.12), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_2, n_4\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1}$;

(ii) if $S = \{n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_3, n_5\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1}$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 or n_5, n_1 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5 or n_5n_1. There are three edges to have exclusive endpoints from S. But extra condition implies that $S = \{n_1, n_3, n_4, n_5\}$ isn't corresponded to 1-independent number $\mathcal{I}(NTG)$ and 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1}$;

(iv) if $S = \{n_4, n_2, n_5\}$ is a set of vertices, then there's no vertex in S but n_4, n_2 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge n_4n_5. There's one edge n_4n_5 to have exclusive endpoints n_4 and n_5 from S. But extra condition implies that $S = \{n_4, n_2, n_5\}$ is corresponded to both 1-independent number $\mathcal{I}(NTG)$ and 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|\neq\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(v) 3 is 1-independent number and its corresponded sets are $\{n_1, n_3, n_2\}$, $\{n_1, n_3, n_4\}$, $\{n_1, n_3, n_5\}$, $\{n_1, n_4, n_2\}$, $\{n_1, n_4, n_3\}$, $\{n_1, n_4, n_5\}$, $\{n_2, n_4, n_1\}$, $\{n_2, n_4, n_3\}$, $\{n_2, n_4, n_5\}$, $\{n_2, n_5, n_1\}$, $\{n_2, n_5, n_3\}$, $\{n_2, n_5, n_4\}$, $\{n_3, n_5, n_2\}$, $\{n_3, n_5, n_4\}$, and $\{n_3, n_5, n_1\}$;

(vi) 5.1 is 1-independent neutrosophic-number and its corresponded set is $\{n_2, n_5, n_3\}$.

The natural way proposes us to use the restriction "maximum" instead of "minimum."

Definition 1.5.21. (Clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

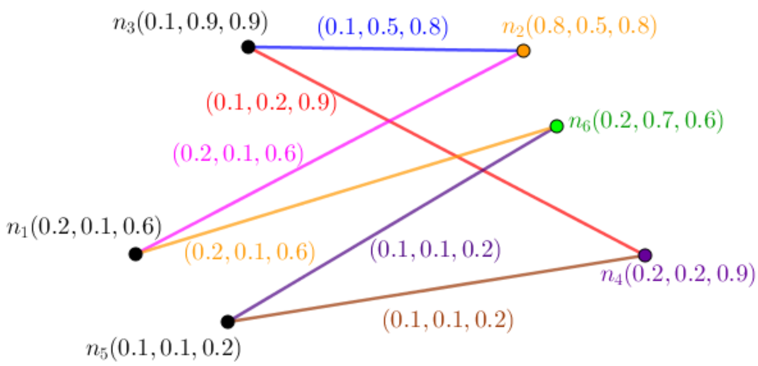

Figure 1.11: A Neutrosophic Graph in the Viewpoint of its 1-Independent Number. 52NTG5

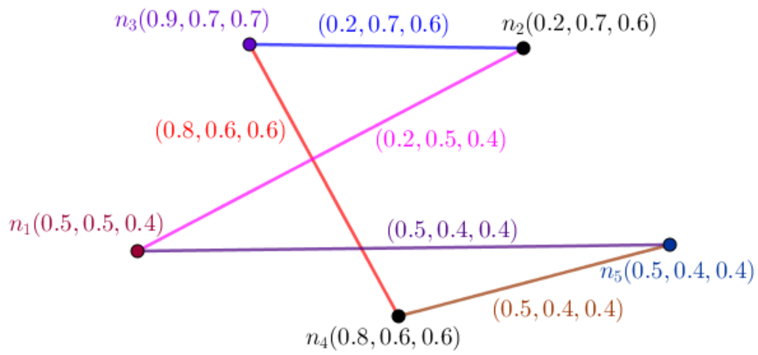

Figure 1.12: A Neutrosophic Graph in the Viewpoint of its 1-Independent Number. 52NTG6

(i) **clique number** $\mathcal{C}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(ii) **clique neutrosophic-number** $\mathcal{C}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Proposition 1.5.22. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{C}(NTG) = 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. If $|S| > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors. Thus there is no triangle but there's one edge. One edge has two endpoints. These endpoints are corresponded to clique number $\mathcal{C}(NTG)$. So

$$\mathcal{C}(NTG) = 2.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.23. There are two sections for clarifications.

(a) In Figure (2.13), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either clique number $\mathcal{C}(NTG)$ or clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_1, n_3\}$ is corresponded to either clique number $\mathcal{C}(NTG)$ or clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither clique number $\mathcal{C}(NTG)$ nor clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iv) if $S = \{n_2, n_3\}$ is a set of vertices, then there's no vertex in S but n_2 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_2, n_3\}$ is corresponded to both clique number $\mathcal{C}(NTG)$ and clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 2 is clique number and its corresponded set is $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_1, n_6\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_2, n_6\}$, $\{n_3, n_4\}$, $\{n_3, n_5\}$, $\{n_3, n_6\}$, $\{n_4, n_5\}$, $\{n_4, n_6\}$, $\{n_5, n_6\}$, and $\{n_6, n_1\}$;

(*vi*) 4 is clique neutrosophic-number and its corresponded set is $\{n_2, n_3\}$.

(*b*) In Figure (2.14), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either clique number $\mathcal{C}(NTG)$ or clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*ii*) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_1, n_3\}$ is corresponded to either clique number $\mathcal{C}(NTG)$ or clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither clique number $\mathcal{C}(NTG)$ nor clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iv*) if $S = \{n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_3, n_4\}$ is corresponded to both clique number $\mathcal{C}(NTG)$ and clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*v*) 2 is clique number and its corresponded set is $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_3, n_4\}$, $\{n_3, n_5\}$, $\{n_4, n_5\}$, and $\{n_5, n_1\}$;

(*vi*) 4.3 is clique neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

The natural way proposes us to use the restriction "minimum" instead of "maximum."

Definition 1.5.24. (Failed Clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **failed clique number** $\mathcal{C}^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously;

(*ii*) **failed clique neutrosophic-number** $\mathcal{C}_n^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S

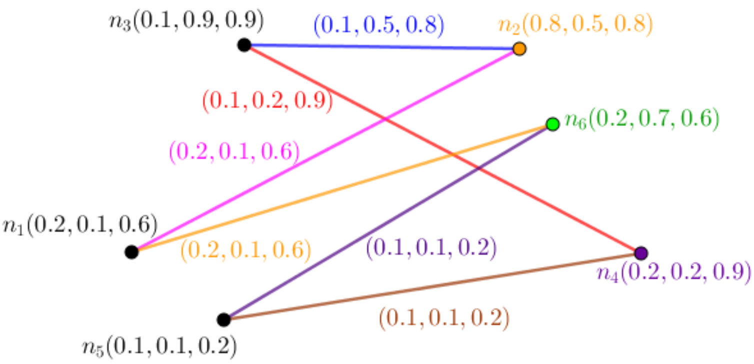

Figure 1.13: A Neutrosophic Graph in the Viewpoint of its clique Number. 53NTG5

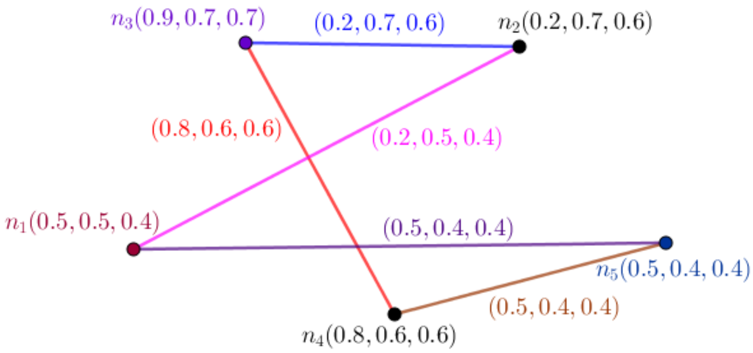

Figure 1.14: A Neutrosophic Graph in the Viewpoint of its clique Number. 53NTG6

of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously.

Proposition 1.5.25. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph. Then*

(i) *if* $\mathcal{O}(NTG) = 0$, *then*
$$\mathcal{C}^{\mathcal{F}}(NTG) = 0;$$

(ii) *if* $\mathcal{O}(NTG) = 1$, *then*
$$\mathcal{C}^{\mathcal{F}}(NTG) = 0;$$

(iii) *if* $\mathcal{O}(NTG) = 2$, *then*
$$\mathcal{C}^{\mathcal{F}}(NTG) = 0;$$

(iv) *if* $\mathcal{O}(NTG) = 3$, *then*
$$\mathcal{C}^{\mathcal{F}}(NTG) = 0;$$

(v) *if* $\mathcal{O}(NTG) \geq 4$, *then*
$$\mathcal{C}^{\mathcal{F}}(NTG) = 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex.

(i) If $\mathcal{O}(NTG) = 0$, then there's no vertex to be considered. So minimum cardinality of a set is zero. It implies

$$\mathcal{C}^{\mathcal{F}}(NTG) = 0;$$

(ii) if $\mathcal{O}(NTG) = 1$, then by using Definition, there aren't two vertices. Thus it implies

$$\mathcal{C}^{\mathcal{F}}(NTG) = 0;$$

(iii) if $\mathcal{O}(NTG) = 2$, then there are two vertices. By it's cycle-neutrosophic graph, it's contradiction. Since if it's cycle-neutrosophic graph, then $\mathcal{O}(NTG) \neq 2$. In other words, it's cycle-neutrosophic graph, then $\mathcal{O}(NTG) \geq 3$. At least two vertices are needed to have new notion but at least three vertices are needed to have cycle-neutrosophic graph. Thus

$$\mathcal{C}^{\mathcal{F}}(NTG) = 0;$$

(iv) if $\mathcal{O}(NTG) = 3$, then, by it's cycle-neutrosophic graph, there aren't two vertices x and y such that x and y aren't endpoints of an edge. It implies

$$\mathcal{C}^{\mathcal{F}}(NTG) = 0;$$

(v) if $\mathcal{O}(NTG) \geq 4$, then, by it's cycle-neutrosophic graph, there are two vertices x and y such that x and y aren't endpoints of an edge. Thus lower bound is achieved for failed clique number. It implies

$$\mathcal{C}^{\mathcal{F}}(NTG) = 2.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.26. There are two sections for clarifications.

(a) In Figure (2.15), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ implies that $S = \{n_2, n_4\}$ is corresponded to failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ but not failed clique neutrosophic-number $\mathcal{C}^{\mathcal{F}}_n(NTG)$;

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ implies that $S = \{n_1, n_3\}$ is corresponded to failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ but not failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. But n_1 and n_3 aren't endpoints for any given edge. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ nor failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iv) if $S = \{n_1, n_5\}$ is a set of vertices, then there's no vertex in S but n_1 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ thus it implies that $S = \{n_1, n_5\}$ is corresponded to both failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(v) 2 is failed clique number and its corresponded set is $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_2, n_6\}$, $\{n_3, n_5\}$, $\{n_3, n_6\}$, and $\{n_4, n_6\}$;

(vi) 1.3 is failed clique neutrosophic-number and its corresponded set is $\{n_1, n_5\}$.

(b) In Figure (2.16), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ implies that $S = \{n_2, n_4\}$ is corresponded to failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ but not failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ implies that $S = \{n_1, n_3\}$ is corresponded to failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ but not failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the

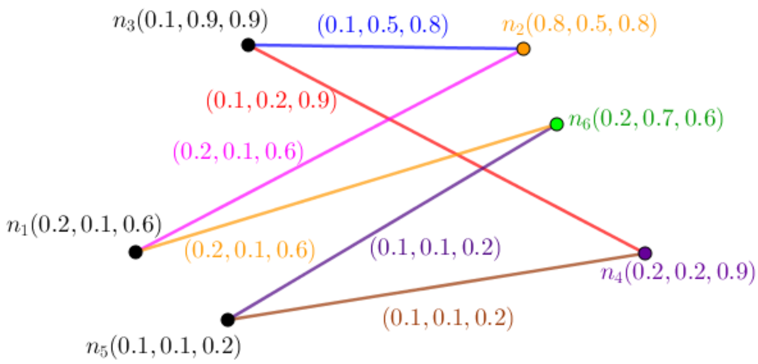

Figure 1.15: A Neutrosophic Graph in the Viewpoint of its Failed Clique Number. 54NTG5

members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. But n_1 and n_3 aren't endpoints for every given edge. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ nor failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iv) if $S = \{n_2, n_5\}$ is a set of vertices, then there's no vertex in S but n_2 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ thus it implies that $S = \{n_2, n_5\}$ is corresponded to both failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(v) 2 is failed clique number and its corresponded set is $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, and $\{n_3, n_5\}$;

(vi) 2.8 is failed clique neutrosophic-number and its corresponded set is $\{n_2, n_5\}$.

Definition 1.5.27. (1-clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-clique number** $\mathcal{C}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time;

(ii) **1-clique neutrosophic-number** $\mathcal{C}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time.

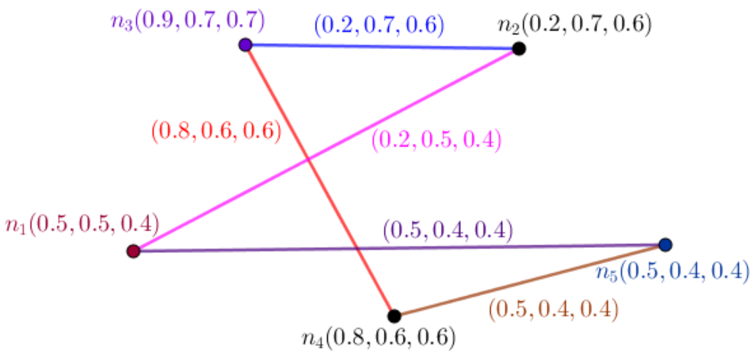

Figure 1.16: A Neutrosophic Graph in the Viewpoint of its Failed Clique Number.

54NTG6

Definition 1.5.28. (Failed 1-clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed 1-clique number** $\mathcal{C}^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time;

(ii) **failed 1-clique neutrosophic-number** $\mathcal{C}_n^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time.

Proposition 1.5.29. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{C}(NTG) = 3.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. If $|S| > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors. Thus there is no triangle but there's one edge. One edge has two endpoints. These endpoints are corresponded to 1-clique number $\mathcal{C}(NTG)$. Two vertices could be satisfied in extra condition. So

$$\mathcal{C}(NTG) = 3.$$

∎

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.30. There are two sections for clarifications.

(a) In Figure (2.17), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ implies that $S = \{n_2, n_4\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ implies that $S = \{n_1, n_3\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iv) if $S = \{n_5, n_6\}$ is a set of vertices, then there's no vertex in S but n_5 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ thus it implies that $S = \{n_5, n_6\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 3 is 1-clique number and its corresponded sets are like $\{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$ which contain two edges;

(vi) 4.9 is 1-clique neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3\}$.

(b) In Figure (2.18), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ implies that $S = \{n_2, n_4\}$ is corresponded to

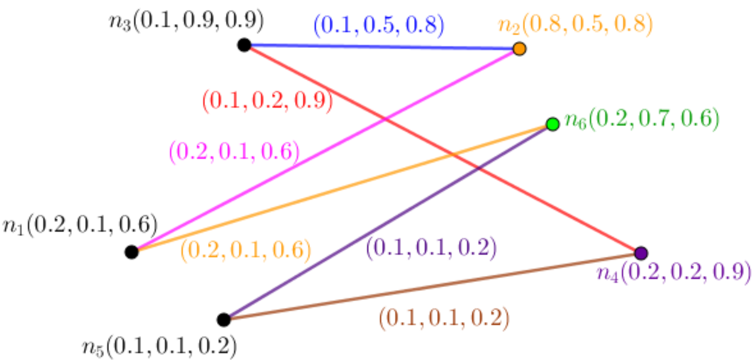

Figure 1.17: A Neutrosophic Graph in the Viewpoint of its 1-Clique Number. 55NTG5

neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ implies that $S = \{n_1, n_3\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iii) if $S = \{n_3, n_4, n_2\}$ is a set of vertices, then there's no vertex in S but n_3, n_4 and n_2. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_2, n_3 of S, it's possible to have endpoints of an edge either n_2n_3 or n_3n_4. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S|=3}$ thus it implies that $S = \{n_3, n_4, n_2\}$ is corresponded to both 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iv) if $S = \{n_5, n_6\}$ is a set of vertices, then there's no vertex in S but n_5 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 4}$ thus it implies that $S = \{n_5, n_6\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 3 is 1-clique number and its corresponded sets are like $\{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$ which contain two edges;

(vi) 6.3 is 1-clique neutrosophic-number and its corresponded set is $\{n_3, n_4, n_2\}$.

Definition 1.5.31. (Matching Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

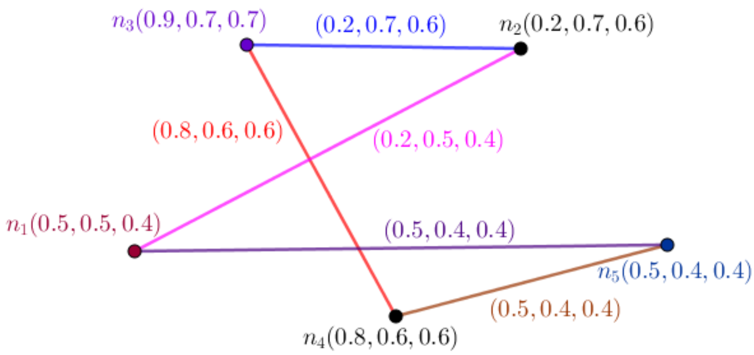

Figure 1.18: A Neutrosophic Graph in the Viewpoint of its 1-Clique Number. 55NTG6

(i) **matching number** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of edges such that every two edges of S don't have any vertex in common;

(ii) **matching neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of edges such that every two edges of S don't have any vertex in common.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 1.5.32. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}}$ be consecutive arrangements of vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O} - 1.$$

Define

$$S = \{x_1x_2, x_3x_4, \cdots, x_ix_{i+1}\}_{i=1}^{\mathcal{O}-1}.$$

In S, there aren't two edges which have common endpoints. S is matching set and it has maximum cardinality amid such these sets which are matching set which is a set in that, there aren't two edges which have common endpoints. So

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.33. There are two sections for clarifications.

(a) In Figure (2.19), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is a set of edges, then there are three edges in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges in S. Cardinality and structure of S implies that $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.5, of S implies $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is a set of edges, then there are three edges in S In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.7, of S implies $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 3 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4, n_5n_6\}$, and $\{n_2n_3, n_4n_5, n_6n_1\}$;

(vi) 2.5 is matching neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4, n_5n_6\}$.

(b) In Figure (2.20), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for

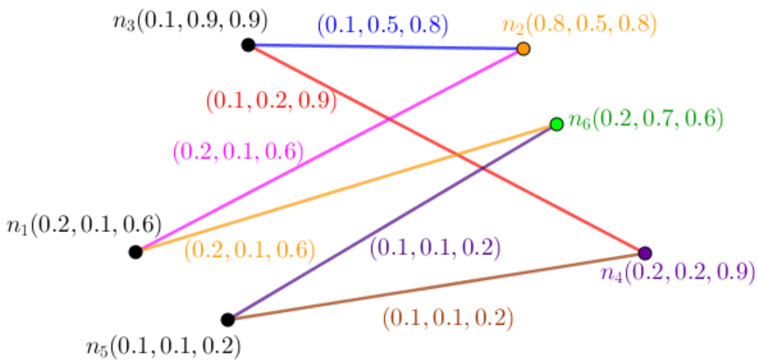

Figure 1.19: A Neutrosophic Graph in the Viewpoint of its Matching Number. 58NTG5

two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_2n_3, n_4n_5\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_4n_5. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.8, of S implies $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 3.1, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, and $\{n_2n_3, n_4n_5\}$;

(vi) 2.8 is matching neutrosophic-number and its corresponded set is $\{n_2n_3, n_4n_5\}$.

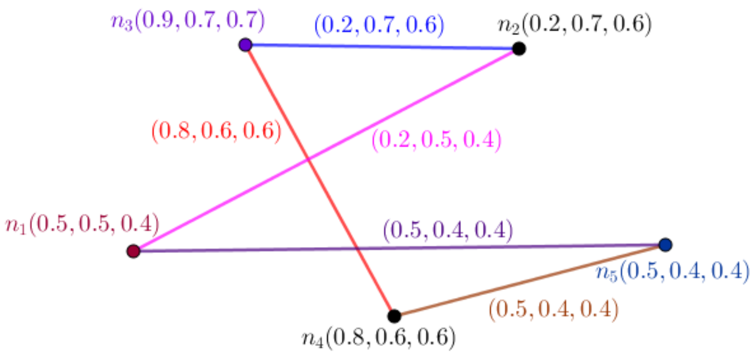

Figure 1.20: A Neutrosophic Graph in the Viewpoint of its Matching Number. 58NTG6

Definition 1.5.34. (Matching Polynomial).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **matching polynomial** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the matching polynomial represent the number of sets of independent edges of various cardinalities in G.

(ii) **matching polynomial neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the matching polynomial represent the number of sets of independent edges of various neutrosophic cardinalities in G.

Proposition 1.5.35. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = 2x^{\lfloor \frac{\mathcal{S}(NTG)}{2} \rfloor} + \cdots + \mathcal{S}(NTG)x + 1.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}}$ be consecutive arrangements of vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \; i = 1, 2, \cdots, \mathcal{O} - 1.$$

Define

$$S = \{x_1x_2, x_3x_4, \cdots, x_ix_{i+1}\}_{i=1}^{\mathcal{O}-1}.$$

In S, there aren't two edges which have common endpoints. S is matching polynomial set and it has maximum cardinality amid such these sets which are matching polynomial set which is a set in that, there aren't two edges which have common endpoints. So

$$\mathcal{M}(NTG) = 2x^{\lfloor \frac{\mathcal{S}(NTG)}{2} \rfloor} + \cdots + \mathcal{S}(NTG)x + 1.$$

∎

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.36. There are two sections for clarifications.

(a) In Figure (2.21), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is corresponded to neither matching polynomial $\mathcal{M}(NTG)$ nor matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;
- (ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching polynomial $\mathcal{M}(NTG)$ nor matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;
- (iii) if $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is a set of edges, then there are three edges in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges in S. Cardinality and structure of S implies that $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.5, of S implies $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;
- (iv) if $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is a set of edges, then there are three edges in S In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.7, of S implies $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;
- (v) $2x^3 + 9x^2 + 6x + 1$ is matching polynomial and its corresponded sets are $\{n_1n_2, n_3n_4, n_5n_6\}$, and $\{n_2n_3, n_4n_5, n_6n_1\}$ for coefficient of biggest term;
- (vi) $x^{2.5} + x^{2.4} + x^{1.4}$ is matching polynomial neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4, n_5n_6\}$.

(b) In Figure (2.22), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to neither matching polynomial $\mathcal{M}(NTG)$ nor matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching polynomial $\mathcal{M}(NTG)$ nor matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_2n_3, n_4n_5\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_4n_5. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.8, of S implies $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ but neutrosophic cardinality, 3.1, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $2x^2 + 5x + 1$ is matching polynomial and its corresponded sets are $\{n_1n_2, n_3n_4\}$, and $\{n_2n_3, n_4n_5\}$ for coefficient of biggest term;

(vi) $x^{2.8} + x^2$ is matching polynomial neutrosophic-number and its corresponded set is $\{n_2n_3, n_4n_5\}$ for coefficient of biggest term.

Definition 1.5.37. (e-Matching Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **e-matching number** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S containing endpoints of edges such that every two edges of S don't have any vertex in common;

(ii) **e-matching neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S containing endpoints of edges such that every two edges of S don't have any vertex in common.

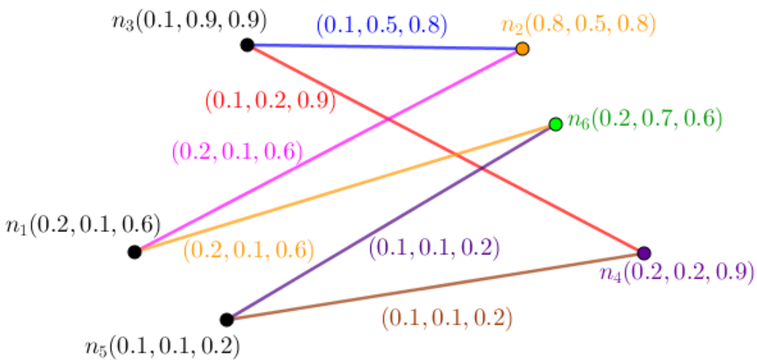

Figure 1.21: A Neutrosophic Graph in the Viewpoint of its Matching Polynomial. 60NTG5

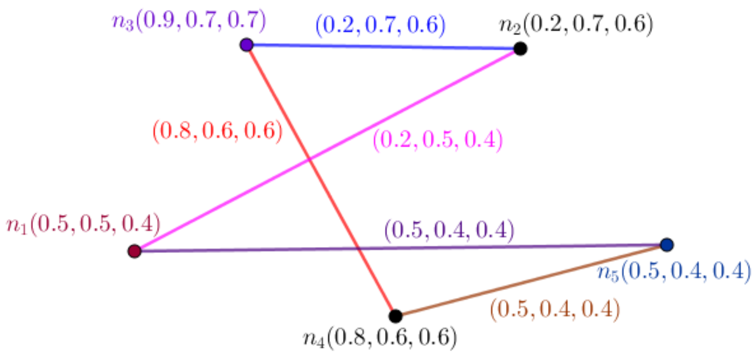

Figure 1.22: A Neutrosophic Graph in the Viewpoint of its Matching Polynomial. 60NTG6

Definition 1.5.38. (e-Matching Polynomial).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **e-matching polynomial** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the e-matching polynomial represent the number of sets of endpoints of independent edges of various cardinalities in G.

(ii) **e-matching polynomial neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the e-matching polynomial represent the number of sets of endpoints of independent edges of various neutrosophic cardinalities in G.

Proposition 1.5.39. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = \mathcal{O}(NTG)$$

where the parity of $\mathcal{O}(NTG)$ is even. And

$$\mathcal{M}(NTG) = \mathcal{O}(NTG) - 1$$

where the parity of $\mathcal{O}(NTG)$ *is odd.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}}$ be consecutive arrangements of vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O} - 1.$$

Define

$$S = \{x_1x_2, x_3x_4, \cdots, x_ix_{i+1}\}_{i=1}^{\mathcal{O}-1}.$$

In S, there aren't two edges which have common endpoints. S is corresponded to e-matching neutrosophic-number and it has maximum cardinality amid such these sets which are corresponded to e-matching neutrosophic-number which is a set in that, there aren't two edges which have common endpoints. So

$$\mathcal{M}(NTG) = \mathcal{O}(NTG)$$

where the parity of $\mathcal{O}(NTG)$ is even. And

$$\mathcal{M}(NTG) = \mathcal{O}(NTG) - 1$$

where the parity of $\mathcal{O}(NTG)$ is odd. ∎

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.40. There are two sections for clarifications.

(a) In Figure (2.23), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If $\{n_1n_3, n_2n_5, n_4n_6\}$ is a set of edges, then there's no edge from S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1, n_3, n_2, n_5, n_4, n_6\}$ is corresponded to neither e-matching number $\mathcal{M}(NTG)$ nor e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

  (ii) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge from S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to neither e-matching number $\mathcal{M}(NTG)$ nor e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $\{n_1, n_2, n_3, n_4, n_5, n_6\}$ is a set of edges, then there are three edges from S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality, $\mathcal{O}(NTG) = 6$, and structure of S implies that

$$S = \{n_1, n_2, n_3, n_4, n_5, n_6\} = V$$

is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, $10.1 = \mathcal{O}_n(NTG)$, of S implies

$$S = \{n_1, n_2, n_3, n_4, n_5, n_6\} = V$$

is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is a set of edges, then there are three edges from S In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S, $\mathcal{O}(NTG) = 6$, implies that

$$S = \{n_2, n_3, n_4, n_5, n_6, n_1\} = V$$

is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, $10.1 = \mathcal{O}_n(NTG)$, of S implies

$$S = \{n_2, n_3, n_4, n_5, n_6, n_1\} = V$$

is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $6 = \mathcal{O}(NTG)$ is e-matching number and its corresponded set is $S = \{n_1, n_2, n_3, n_4, n_5, n_6\} = V$;

(vi) $10.1 = \mathcal{O}_n(NTG)$ is e-matching neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4, n_5, n_6\}$.

(b) In Figure (2.24), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $\{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to neither e-matching number $\mathcal{M}(NTG)$ nor e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to neither e-matching number $\mathcal{M}(NTG)$ nor e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

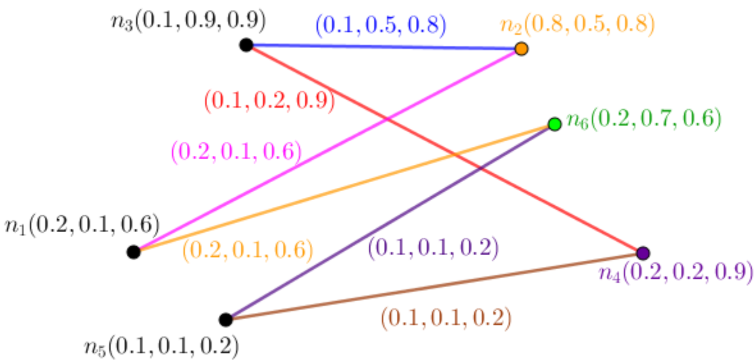

Figure 1.23: A Neutrosophic Graph in the Viewpoint of its e-Matching Number. 61NTG6

(iii) if $\{n_2n_3, n_4n_5\}$ is a set of edges, then there's no edge from S but n_2n_3 and n_4n_5. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_2, n_3, n_4, n_5\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 7.1, of S implies $S = \{n_2, n_3, n_4, n_5\}$ isn't corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $\{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that

$$S = \{n_1, n_2, n_3, n_4\} = V - \{n_5\} \neq V$$

is corresponded to e-matching $\mathcal{M}(NTG)$ and neutrosophic cardinality, 7.2, of S implies

$$S = \{n_1, n_2, n_3, n_4\} = V - \{n_5\} \neq V$$

is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $4 = \mathcal{O}(NTG) - 1 \neq \mathcal{O}(NTG)$ is e-matching number and its corresponded set is $\{n_1, n_2, n_3, n_4\} = V - \{n_5\} \neq V$;

(vi) $7.2 = \mathcal{O}_n(NTG) - \sum_{i=1}^{3} \sigma_i(n_5)$ is e-matching neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4\} = V - \{n_5\} \neq V$;

Definition 1.5.41. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞;

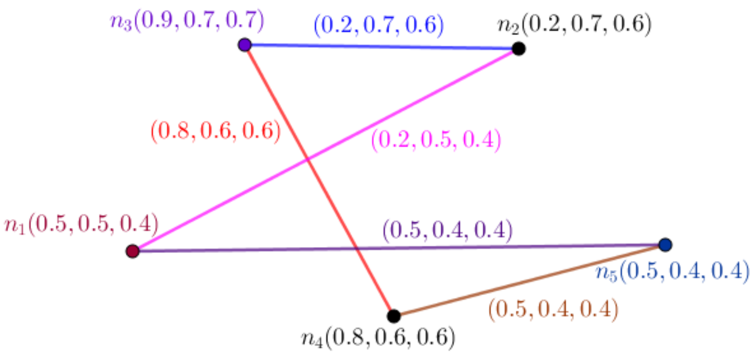

Figure 1.24: A Neutrosophic Graph in the Viewpoint of its e-matching Number. 61NTG7

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞.

Proposition 1.5.42. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(NTG) \geq 3$. *Then*

$$\mathcal{G}(NTG) = \mathcal{O}(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. It implies

$$\mathcal{G}(NTG) = \mathcal{O}(NTG).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.43. There are two sections for clarifications.

(a) In Figure (2.75), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (2.76), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

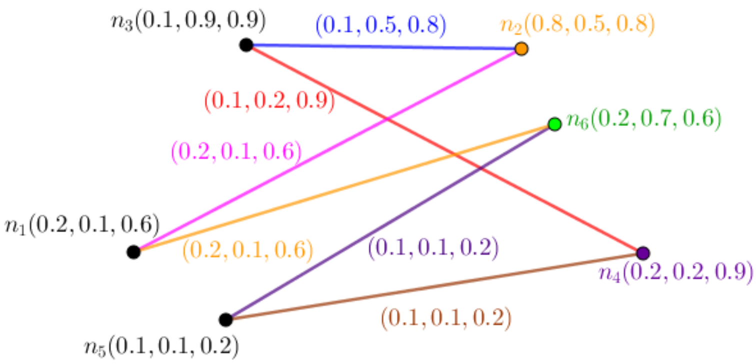

Figure 1.25: A Neutrosophic Graph in the Viewpoint of its Girth.

62NTG5

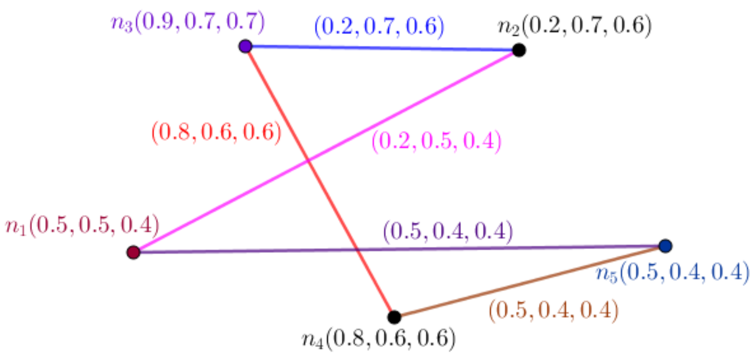

Figure 1.26: A Neutrosophic Graph in the Viewpoint of its Girth.

62NTG6

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Definition 1.5.44. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞;

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞.

63thm

Theorem 1.5.45. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 1.5.46. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(NTG) \geq 3$. Then*

$$\mathcal{G}(NTG) = \mathcal{O}(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. By Theorem (2.5.49),

$$\mathcal{G}(NTG) = \mathcal{O}(NTG).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.47. There are two sections for clarifications.

(a) In Figure (2.27), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (2.28), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only

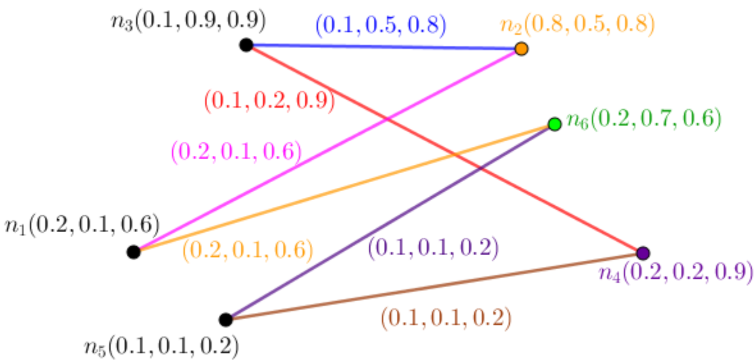

Figure 1.27: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG5

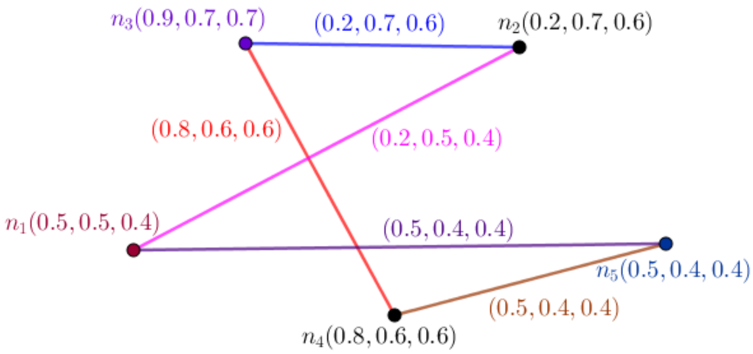

Figure 1.28: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG6

one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Definition 1.5.48. (Girth Polynomial and Neutrosophic Girth Polynomial). Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **girth polynomial** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1x^{m_1} + n_2x^{m_2} + \cdots + n_sx^3$ where n_i is the number of cycle with m_i as its crisp cardinality of the set of vertices of cycle;

(*ii*) **neutrosophic girth polynomial** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1 x^{m_1} + n_2 x^{m_2} + \cdots + n_s x^{m_s}$ where n_i is the number of cycle with m_i as its neutrosophic cardinality of the set of vertices of cycle.

63thm

Theorem 1.5.49. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 1.5.50. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(NTG) \geq 3$. Then*

$$\mathcal{G}(NTG) = x^{\mathcal{O}(NTG)}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. By Theorem (2.5.49),

$$\mathcal{G}(NTG) = x^{\mathcal{O}(NTG)}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.51. There are two sections for clarifications.

(*a*) In Figure (2.29), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*iv*) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth polynomial $\mathcal{G}(NTG)$ and neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*v*) $x^{6=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $x^{8.1=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (2.30), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding

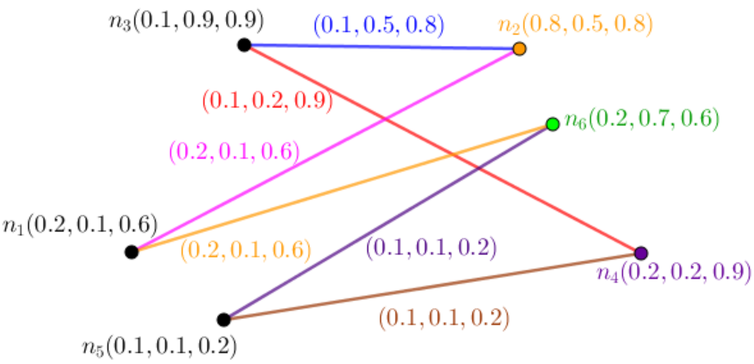

Figure 1.29: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG5

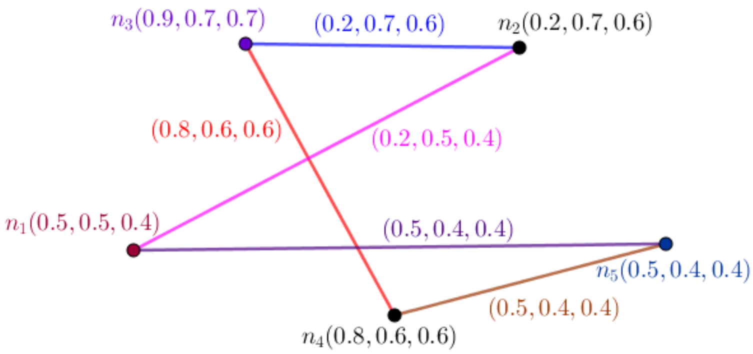

Figure 1.30: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG6

shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth polynomial $\mathcal{G}(NTG)$ and neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^{5=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $x^{8.5=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Definition 1.5.52. (Hamiltonian Neutrosophic Cycle).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **hamiltonian neutrosophic cycle** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive vertices $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ which is neutrosophic cycle;

(*ii*) **n-hamiltonian neutrosophic cycle** $\mathcal{N}(HNC)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive vertices $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ which are neutrosophic cycles.

If we use the notion of neutrosophic cardinality in strong type of neutrosophic graphs, then the next result holds. If not, the situation is complicated since it's possible to have all edges in the way that, there's no value of a vertex for an edge.

66thm

Theorem 1.5.53. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ∎

Proposition 1.5.54. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(CYC_n) \geq 3$. Then*

$$\mathcal{M}(CYC_n) : x_1, x_2, \cdots, x_{\mathcal{O}(CYC_n)-1}, x_{\mathcal{O}(CYC_n)}, x_1.$$

Proof. Suppose $CYC_n : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC_n)}, x_1$ be a sequence of consecutive vertices of $CYC_n : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(CYC_n) - 1, \ x_{\mathcal{O}(CYC_n)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a longest crisp cycle with length $\mathcal{O}(CYC_n)$. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(CYC_n)$. Thus the crisp cardinality of set of vertices forming longest crisp cycle is $\mathcal{O}(CYC_n)$. By Theorem (2.5.57),

$$\mathcal{M}(CYC_n) : x_1, x_2, \cdots, x_{\mathcal{O}(CYC_n)-1}, x_{\mathcal{O}(CYC_n)}, x_1.$$

∎

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.55. There are two sections for clarifications.

(*a*) In Figure (2.31), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all vertices once. Finding longest cycle containing all vertices once has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all vertices once. Finding longest cycle containing all vertices once has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding longest cycle containing all vertices once. Finding longest cycle containing all vertices once has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5, n_6, n_1$$

is corresponded to both of hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ and n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(v) $\mathcal{M}(CYC_n)$: $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is hamiltonian neutrosophic cycle;

(vi) $\mathcal{N}(CYC_n) = 1$ is n-hamiltonian neutrosophic cycle.

(b) In Figure (2.32), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

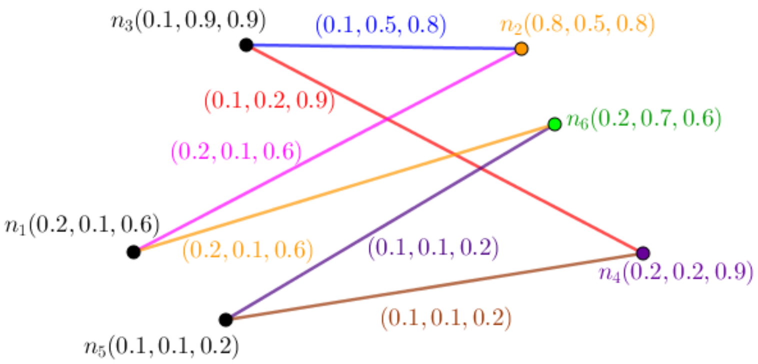

Figure 1.31: A Neutrosophic Graph in the Viewpoint of its hamiltonian neutrosophic cycle.

66NTG5

is corresponded neither to hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding longest cycle containing all vertices once. Finding longest cycle containing all vertices once has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5, n_1$$

is corresponded to neither hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(v) $\mathcal{M}(CYC_n)$: Not Existed is hamiltonian neutrosophic cycle;

(vi) $\mathcal{N}(CYC_n) = 0$.

Definition 1.5.56. (Eulerian Neutrosophic Cycle).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Eulerian neutrosophic cycle** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive edges $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}, x_1$ which is neutrosophic cycle;

(ii) **n-Eulerian neutrosophic cycle** $\mathcal{N}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive edges $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}, x_1$ which are neutrosophic cycles.

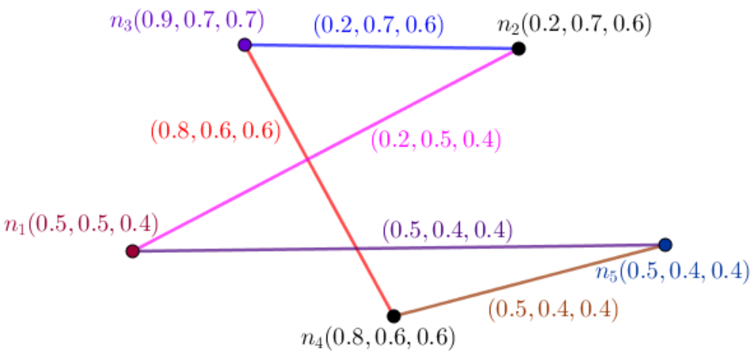

Figure 1.32: A Neutrosophic Graph in the Viewpoint of its hamiltonian neutrosophic cycle. 66NTG6

If we use the notion of neutrosophic cardinality in strong type of neutrosophic graphs, then the next result holds. If not, the situation is complicated since it's possible to have all edges in the way that, there's no value of a vertex for an edge.

66thm

Theorem 1.5.57. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 1.5.58. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{M}(CYC) : x_1, x_2, \cdots, x_{\mathcal{S}(CYC)-1}, x_{\mathcal{S}(CYC)}, x_1.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{S}(CYC)}, x_1$ be a sequence of consecutive edges of $CYC : (V, E, \sigma, \mu)$ such that

$$x_i, x_{i+1} \text{ have common vertex, } i = 1, 2, \cdots, \mathcal{S}(CYC) - 1,$$

$$x_{\mathcal{S}(CYC)}, x_1 \text{ have common vertex.}$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts lead to one crisp cycle for finding a longest crisp cycle with length $\mathcal{S}(CYC)$. For a given vertex x_i, the sequence of consecutive edges

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{S}(CYC)$. Thus the crisp cardinality of set of edges forming longest crisp cycle is $\mathcal{S}(CYC)$. By Theorem (2.5.57),

$$\mathcal{M}(CYC) : x_1, x_2, \cdots, x_{\mathcal{S}(CYC)-1}, x_{\mathcal{S}(CYC)}, x_1.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.59. There are two sections for clarifications.

(a) In Figure (2.33), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's only a path and there are only two edges but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive edges is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1n_2, n_2n_3$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(ii) if n_1n_2, n_2n_3, n_3n_4 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2, n_2n_3, and n_3n_4 according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has no result. Since there's one cycle but it isn't about all edges. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor a crisp cycle. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4$$

is corresponded neither to Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(iii) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3, n_3n_4 and n_4n_5 according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor a crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_5$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(iv) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$ is a sequence of consecutive edges, then it's obvious that there's one crisp cycle. It's also a crisp path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1 according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. Hence this neutrosophic path is both of a neutrosophic cycle and a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$$

is corresponded to both of Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ and n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(v) $\mathcal{M}(CYC) : n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$ is Eulerian neutrosophic cycle;

(vi) $\mathcal{N}(CYC) = 1$ is n-Eulerian neutrosophic cycle.

(b) In Figure (2.34), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's only a path and there are only two edges but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive edges is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1n_2, n_2n_3$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(ii) if n_1n_2, n_2n_3, n_3n_4 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2, n_2n_3, and n_3n_4 according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has no result. Since there's one cycle but it isn't about all edges. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor a crisp cycle. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4$$

is corresponded neither to Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(iii) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3, n_3n_4 and n_4n_5 according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor a crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_5$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(iv) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_1$ is a sequence of consecutive edges, then it's obvious that there's one crisp cycle. It's also a crisp path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1 according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_1$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(v) $\mathcal{M}(CYC)$: Not Existed. There is no Eulerian neutrosophic cycle and there are no corresponded sets and sequences;

(vi) $\mathcal{N}(CYC) = 0$ is n-Eulerian neutrosophic cycle and there are no corresponded sets and sequences.

Definition 1.5.60. (Eulerian(Hamiltonian) Neutrosophic Path).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Eulerian(Hamiltonian) neutrosophic path** $\mathcal{M}_e(NTG)(\mathcal{M}_h(NTG))$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}(x_1, x_2, \cdots, x_{\mathcal{O}(NTG)})$ which is neutrosophic path;

(ii) **n-Eulerian(Hamiltonian) neutrosophic path** $\mathcal{N}_e(NTG)(\mathcal{N}_h(NTG))$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}(x_1, x_2, \cdots, x_{\mathcal{O}(NTG)})$ which is neutrosophic path.

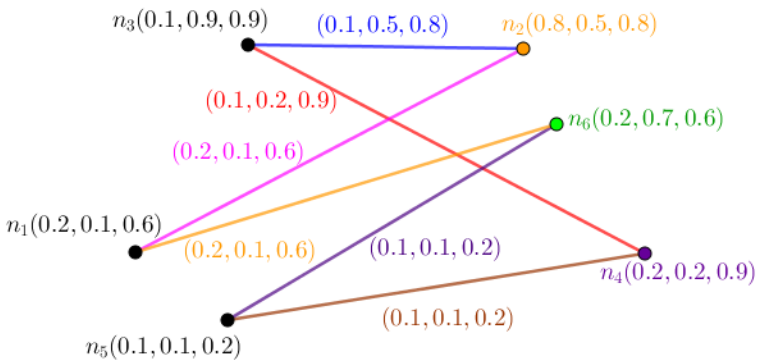

Figure 1.33: A Neutrosophic Graph in the Viewpoint of its Eulerian neutrosophic cycle. 67NTG5

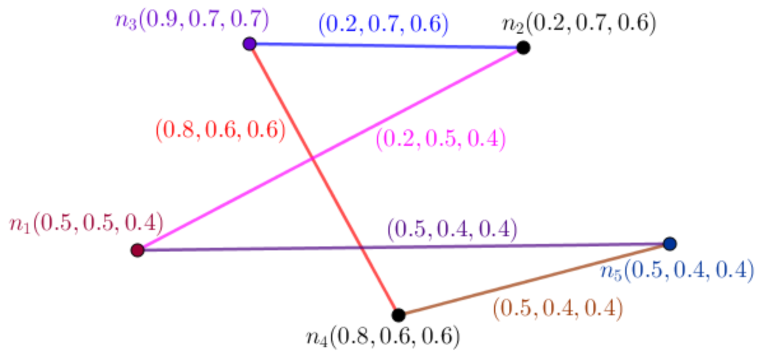

Figure 1.34: A Neutrosophic Graph in the Viewpoint of its Eulerian neutrosophic cycle. 67NTG6

Proposition 1.5.61. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

$$\mathcal{M}_e(CYC) : \textit{Not Existed};$$

$$\mathcal{M}_h(CYC) : x_i, x_{i+1}, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, \cdots, x_{i-1}.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{S}(CYC)}(x_1, x_2, \cdots, x_{\mathcal{O}(CYC)})$ be a sequence of consecutive edges (vertices) of $CYC : (V, E, \sigma, \mu)$ such that

$$x_i, x_{i+1} \text{ have common vertex, } i = 1, 2, \cdots, \mathcal{S}(CYC) - 1(\mathcal{O}(CYC) - 1),$$

$$x_{\mathcal{S}(CYC)}(x_{\mathcal{O}(CYC)}), x_1 \text{ have common vertex.}$$

There are two paths amid two given vertices. The degree of every vertex is two. There are $\mathcal{S}(CYC)(\mathcal{O}(CYC))$ paths. So the efforts lead to $\mathcal{S}(CYC)(\mathcal{O}(CYC))$ for finding a longest paths with length $\mathcal{S}(CYC)(\mathcal{O}(CYC))$. For a given vertex x_i, the sequence of consecutive edges (vertices)

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}$$

is a corresponded longest path for given vertex (edge) x_i. Every path has same length. The length is $\mathcal{S}(CYC)(\mathcal{O}(CYC))$. Thus the crisp cardinality of set of edges (vertices) forming longest path is $\mathcal{S}(CYC)(\mathcal{O}(CYC))$. $x_i, x_{i+1}, \cdots, x_{\mathcal{S}(CYC)}, \cdots, x_{i-1}$ is a sequence of consecutive edges, there's no repetition of edge in this sequence and all edges are used. Eulerian neutrosophic path is corresponded to longest path with length $\mathcal{S}(CYC)$. $x_i, x_{i+1}, \cdots, x_{\mathcal{O}(CYC)}, \cdots, x_{i-1}$ is a sequence of consecutive vertices, there's no repetition of vertex in this sequence and all vertices are used. Hamiltonian neutrosophic path is corresponded to longest path with length $\mathcal{O}(CYC)$. Thus

$$\mathcal{M}_e(CYC) : \text{Not Existed};$$

$$\mathcal{M}_h(CYC) : x_i, x_{i+1}, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, \cdots, x_{i-1}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.62. There are two sections for clarifications.

(*a*) In Figure (2.35), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (*i*) If n_1n_3, n_3n_4 is a sequence of consecutive pairs of vertices, then it isn't neutrosophic path since $\mu(n_1n_3) \not> 0$. The number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_3, n_4 and n_1n_3, n_3n_4;

  (*ii*) if n_1n_2, n_3n_4 is a sequence of edges, then it isn't neutrosophic path since $\mu(n_2n_3) \not> 0$. The number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_2, n_3, n_4 and n_1n_2, n_3n_4;

  (*iii*) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$ is a sequence of consecutive edges, then it isn't neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_6n_1) > 0$. And more, it's crisp cycle. The number of edges is greater than $\mathcal{S}(CYC)$ and the number of vertices is $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ and $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$;

  (*iv*) if n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_2n_3) > 0$. But the number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_2, n_3 and n_1n_2, n_2n_3;

(v) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$, $\mu(n_2n_3) > 0$, $\mu(n_3n_4) > 0$, $\mu(n_4n_5) > 0$ and $\mu(n_5n_6) > 0$. The number of edges is $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian neutrosophic path $\mathcal{M}_e(CYC)$ is $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and Hamiltonian neutrosophic path $\mathcal{M}_h(CYC)$ is $n_1, n_2, n_3, n_4, n_5, n_6$. Also, n-Eulerian neutrosophic path $\mathcal{N}_e(CYC)$ and n-Hamiltonian neutrosophic path $\mathcal{N}_h(CYC)$ are corresponded to these sequences $n_1, n_2, n_3, n_4, n_5, n_6$ and $n_1n_2, n_2n_3, n_3n_4, n_4, n_5, n_5n_6$;

(vi) n-Hamiltonian neutrosophic path $\mathcal{N}_h(CYC)$ equals one and corresponded sequence of consecutive edges is $n_1n_2, n_2n_3, n_3n_4, n_4, n_5, n_5n_6$. n-Eulerian neutrosophic path $\mathcal{N}_e(CYC)$ equals one and corresponded sequence of consecutive vertices is $n_1, n_2, n_3, n_4, n_5, n_6$.

(b) In Figure (2.36), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1n_3, n_3n_4 is a sequence of consecutive pairs of vertices, then it isn't neutrosophic path since $\mu(n_1n_3) \not> 0$. The number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_3, n_4 and n_1n_3, n_3n_4;

(ii) if n_1n_2, n_3n_4 is a sequence of edges, then it isn't neutrosophic path since $\mu(n_2n_3) \not> 0$. The number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_2, n_3, n_4 and n_1n_2, n_3n_4;

(iii) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_1$ is a sequence of consecutive edges, then it isn't neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_5n_1) > 0$. And more, it's crisp cycle. The number of edges is greater than $\mathcal{S}(CYC)$ and the number of vertices is $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences $n_1, n_2, n_3, n_4, n_5, n_1$ and $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_1$;

(iv) if n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_2n_3) > 0$. But the number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_2, n_3 and n_1n_2, n_2n_3;

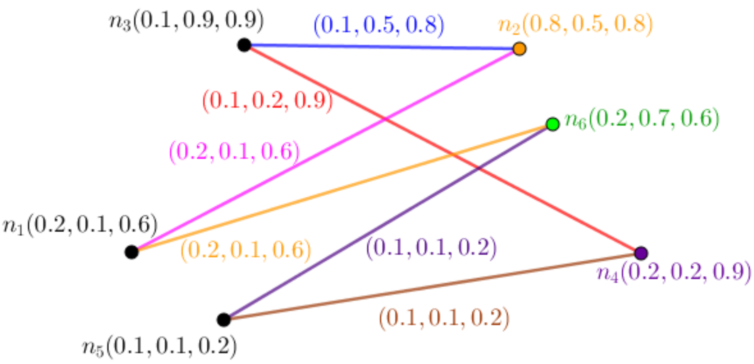

Figure 1.35: A Neutrosophic Graph in the Viewpoint of its Eulerian(Hamiltonian) neutrosophic path. 68NTG5

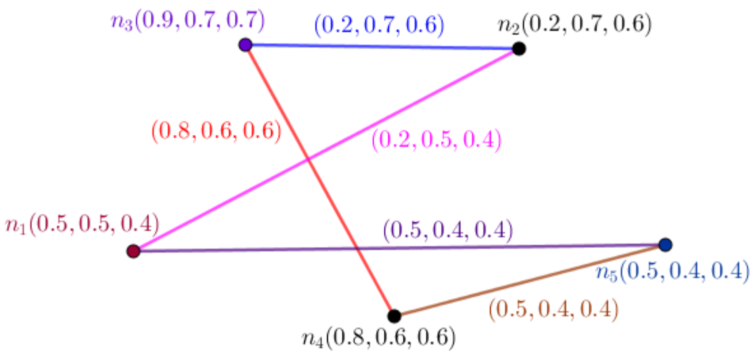

Figure 1.36: A Neutrosophic Graph in the Viewpoint of its Eulerian(Hamiltonian) neutrosophic path. 68NTG6

(v) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$, $\mu(n_2n_3) > 0$, $\mu(n_3n_4) > 0$ and $\mu(n_4n_5) > 0$. The number of edges is $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian neutrosophic path $\mathcal{M}_e(CYC)$ is $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and Hamiltonian neutrosophic path $\mathcal{M}_h(CYC)$ is n_1, n_2, n_3, n_4. Also, n-Eulerian neutrosophic path $\mathcal{N}_e(CYC)$ and n-Hamiltonian neutrosophic path $\mathcal{N}_h(CYC)$ are corresponded to these sequences n_1, n_2, n_3, n_4, n_5 and $n_1n_2, n_2n_3, n_3n_4, n_4, n_5$;

(vi) n-Hamiltonian neutrosophic path $\mathcal{N}_h(CYC)$ equals one and corresponded sequence of consecutive edges is $n_1n_2, n_2n_3, n_3n_4, n_4, n_5$. n-Eulerian neutrosophic path $\mathcal{N}_e(CYC)$ equals one and corresponded sequence of consecutive vertices is n_1, n_2, n_3, n_4, n_5.

Definition 1.5.63. (Neutrosophic Path Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a path from x to y is called **weakest path** if its length is maximum. This length is called **weakest number** amid x and y. The maximum number amid all vertices is called **weakest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{W}(NTG)$;

(ii) a path from x to y is called **neutrosophic weakest path** if its strength is $\mu(uv)$ which is less than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called **neutrosophic weakest number** amid x and y. The maximum number amid all vertices is called **neutrosophic weakest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{W}_n(NTG)$.

Proposition 1.5.64. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{W}(CYC) = \mathcal{O}(CYC) - 1 = \mathcal{S}(CYC) - 1.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. There are some neutrosophic paths. The biggest length of a path is weakest number. The biggest length of path is either size minus one or order minus one. It means the length of this path is either $\mathcal{S}(CYC) - 1$ or $\mathcal{O}(CYC) - 1$. Thus

$$\mathcal{W}(CYC) = \mathcal{O}(CYC) - 1 = \mathcal{S}(CYC) - 1.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.65. There are two sections for clarifications.

(a) In Figure (2.37), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $n_1, n_2, n_3, n_4, n_5, n_6$ is a neutrosophic path from n_1 to n_6, then it's weakest path and weakest number amid n_1 and n_6 is five. Also, $\mathcal{W}(CYC) = 5$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't weakest path and weakest number amid n_1 and n_3 is four corresponded to n_1, n_6, n_5, n_4, n_3. Also, $\mathcal{W}(CYC) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't weakest path but weakest number amid n_1 and n_4 is three corresponded to n_1, n_2, n_3, n_4. Also, $\mathcal{W}(CYC) \neq 3$. For every given couple of vertices x and y, weakest path isn't existed but weakest number is five and $\mathcal{W}(CYC) = 5$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic weakest path since neutrosophic weakest number amid n_2 and n_3 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{W}_n(CYC) = (0.1, 0.5, 0.8)$;

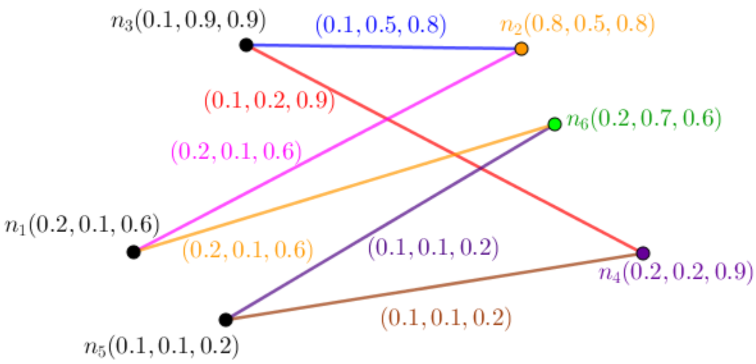

Figure 1.37: A Neutrosophic Graph in the Viewpoint of its Weakest Number and its Neutrosophic Weakest Number. 70NTG5

(v) if n_2, n_3 is a neutrosophic path from n_2 to n_3, then it's a neutrosophic weakest path and neutrosophic weakest number amid n_2 and n_3 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{W}_n(CYC) = (0.1, 0.5, 0.8)$;

(vi) for every given couple of vertices x and y, neutrosophic weakest path isn't existed, neutrosophic weakest number is $(0.1, 0.5, 0.8)$ and $\mathcal{W}_n(CYC) = (0.1, 0.5, 0.8)$.

(b) In Figure (2.38), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_3, n_4, n_5 is a neutrosophic path from n_1 to n_5, then it's weakest path and weakest number amid n_1 and n_5 is four. Also, $\mathcal{W}(CYC) = 4$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't weakest path and weakest number amid n_1 and n_3 is three corresponded to n_1, n_5, n_4, n_3. Also, $\mathcal{W}(CYC) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't weakest path but weakest number amid n_1 and n_4 is three corresponded to n_1, n_2, n_3, n_4. Also, $\mathcal{W}(CYC) \neq 3$. For every given couple of vertices x and y, weakest path isn't existed but weakest number is four and $\mathcal{W}(CYC) = 4$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic weakest path since neutrosophic weakest number amid n_3 and n_4 is $(0.8, 0.6, 0.6)$. Also, $\mathcal{W}_n(CYC) = (0.8, 0.6, 0.6)$;

(v) if n_3, n_4 is a neutrosophic path from n_3 to n_4, then it's a neutrosophic weakest path and neutrosophic weakest number amid n_3 and n_4 is $(0.8, 0.6, 0.6)$. Also, $\mathcal{W}_n(CYC) = (0.8, 0.6, 0.6)$;

(vi) for every given couple of vertices x and y, neutrosophic weakest path isn't existed, neutrosophic weakest number is $(0.8, 0.6, 0.6)$ and $\mathcal{W}_n(CYC) = (0.8, 0.6, 0.6)$.

Definition 1.5.66. (Neutrosophic Path Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

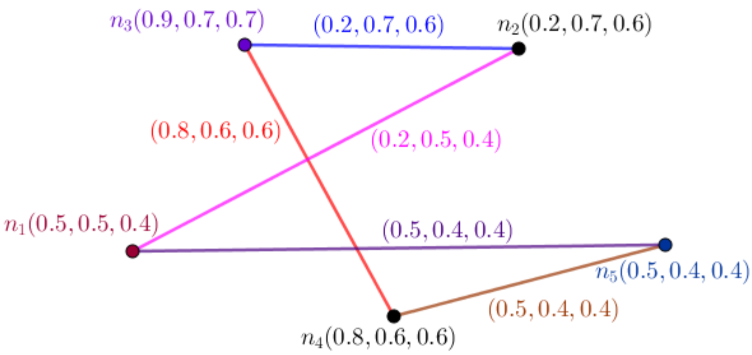

Figure 1.38: A Neutrosophic Graph in the Viewpoint of its Weakest Number and its Neutrosophic Weakest Number. 70NTG6

(i) a path from x to y is called **strongest path** if its length is minimum. This length is called **strongest number** amid x and y. The maximum number amid all vertices is called **strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}(NTG)$;

(ii) a path from x to y is called **neutrosophic strongest path** if its strength is $\mu(uv)$ which is greater than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called **neutrosophic strongest number** amid x and y. The minimum number amid all vertices is called **neutrosophic strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}_n(NTG)$.

Proposition 1.5.67. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{S}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. There are some neutrosophic paths. The biggest length of a path is strongest number. For every given couple of vertices, there are two neutrosophic paths concerning two lengths s and $\mathcal{O}(CYC) - s$. If $s < \mathcal{O}(CYC) - s$, then s is intended length; otherwise, $\mathcal{O}(CYC) - s$ is intended length. Since minimum length amid two vertices are on demand. In next step, amid all lengths, the biggest number is strongest number. The biggest length of path is either order half or order half minus one. It means the length of this path is either $\frac{\mathcal{O}(CYC)}{2}$ or $\frac{\mathcal{O}(CYC)}{2} - 1$. Thus

$$\mathcal{S}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.68. There are two sections for clarifications.

(a) In Figure (2.39), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $n_1, n_2, n_3, n_4, n_5, n_6$ is a neutrosophic path from n_1 to n_6, then it isn't strongest path and strongest number amid n_1 and n_6 is one. Also, $\mathcal{S}(CYC) = 3$.

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 is two corresponded to n_1, n_2, n_3. Also, $\mathcal{S}(CYC) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it is strongest path and strongest number amid n_1 and n_4 is three corresponded to n_1, n_2, n_3, n_4 and n_1, n_6, n_5, n_4 Also, $\mathcal{S}(CYC) = 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is three and $\mathcal{S}(CYC) = 3$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.1, 0.1, 0.2)$ but neutrosophic strongest number amid n_1 and n_4 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$;

(v) if n_2, n_3 is a neutrosophic path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.1, 0.1, 0.2)$ but neutrosophic strongest number amid n_2 and n_3 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.1, 0.1, 0.2)$ and $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$.

(b) In Figure (2.40), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_3, n_4, n_5 is a neutrosophic path from n_1 to n_5, then it isn't strongest path and strongest number amid n_1 and n_5 is one. Also, $\mathcal{S}(CYC) = 2$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it's strongest path and strongest number amid n_1 and n_3 is two. Also, $\mathcal{S}(CYC) = 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is two corresponded to n_1, n_5, n_4. Also, $\mathcal{S}(CYC) \neq 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is two and $\mathcal{S}(CYC) = 2$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path [strength is $(0.2, 0.5, 0.4)$] from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_4 is $(0.5, 0.4, 0.4)$ but neutrosophic strongest number amid n_1 and n_2 is $(0.2, 0.7, 0.6)$;

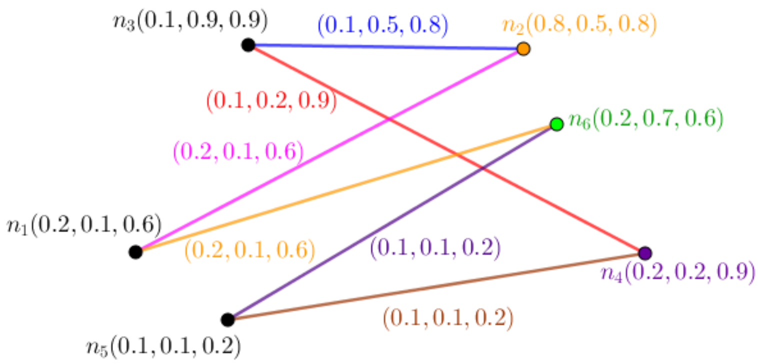

Figure 1.39: A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number. 71NTG5

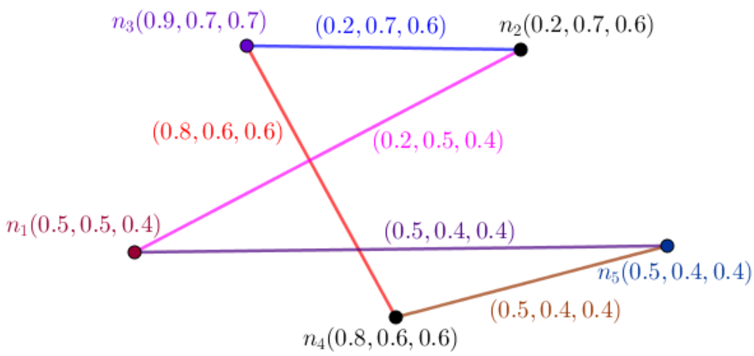

Figure 1.40: A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number. 71NTG6

neutrosophic strongest number amid n_2 and n_3 is $(0.2, 0.7, 0.6)$. Also, $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$;

(v) if n_3, n_4 is a neutrosophic path [strength is $(0.8, 0.6, 0.6)$] from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_3 and n_4 is $(0.8, 0.6, 0.6)$. Also, $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.2, 0.7, 0.6)$ and $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$.

Definition 1.5.69. (Neutrosophic Cycle Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a cycle based on x is called **cyclic connectivity** if its length is minimum. This length is called **connectivity number** based on x. The maximum number amid all vertices is called **connectivity number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{C}(NTG)$;

(*ii*) a cycle based on x is called **neutrosophic cyclic connectivity** if its strength is is greater than all strengths of all cycles based on x. This strength is called **neutrosophic connectivity number** based on x. The minimum number amid all vertices is called **neutrosophic connectivity number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{C}_n(NTG)$.

Proposition 1.5.70. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{C}(CYC) = \mathcal{O}(CYC).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. There are some neutrosophic paths. The biggest length of a cycle is connectivity number. For every given vertex, there's only one cycle concerning length $\mathcal{O}(CYC)$. Since minimum length based on one vertex is on demand, in next step, amid all lengths, the biggest number is connectivity number. The biggest length of cycle is order. It means the length of this cycle is $\mathcal{O}(CYC)$. Thus

$$\mathcal{C}(CYC) = \mathcal{O}(CYC).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.71. There are two sections for clarifications.

(*a*) In Figure (2.41), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (*i*) If $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a neutrosophic cycle based on n_1, then it's cyclic connectivity and connectivity number based on n_1 is 6. Also, $\mathcal{C}(CYC) = 6$;

  (*ii*) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't cyclic connectivity but connectivity number based on any given vertex is existed. There's only one cycle. Hence there's one cycle related to connectivity number of this cycle-neutrosophic graph. Also, $\mathcal{C}(CYC) = 6$ and $\mathcal{C}(CYC) \neq 2$;

  (*iii*) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't cyclic connectivity but connectivity number based on some sequence of consecutive vertices is existed. There's one cycle. Hence there's one cycle related to connectivity number of this cycle-neutrosophic graph. Also, $\mathcal{C}(CYC) = 6$. Also, $\mathcal{C}(CYC) \neq 3$. For every given vertex x, cyclic connectivity is existed and connectivity number is six and $\mathcal{C}(CYC) = 6$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic cyclic connectivity but neutrosophic connectivity number based on any given vertex is existed. There's one cycle so there's one cycle related to neutrosophic connectivity number which is $(0.1, 0.1, 0.2)$. Also, $\mathcal{C}_n(CYC) = (0.1, 0.1, 0.2)$;

(v) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a neutrosophic cycle based on n_1, then it's a neutrosophic cyclic connectivity since there's one cycle and there's one cycle based on n_1 and neutrosophic connectivity number based on n_1 is $(0.1, 0.1, 0.2)$. Also, $\mathcal{C}_n(CYC) = (0.1, 0.1, 0.2)$;

(vi) if $n_2, n_1, n_6, n_5, n_4, n_3, n_2$ is a neutrosophic cycle based on n_2, then it's a neutrosophic cyclic connectivity since there's one cycle and there's one cycle based on n_2 and neutrosophic connectivity number based on n_2 is $(0.1, 0.1, 0.2)$. Also, $\mathcal{C}_n(CYC) = (0.1, 0.1, 0.2)$.

(b) In Figure (2.42), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $n_1, n_2, n_3, n_4, n_5, n_1$ is a neutrosophic cycle based on n_1, then it's cyclic connectivity and connectivity number based on n_1 is 5. Also, $\mathcal{C}(CYC) = 5$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't cyclic connectivity but connectivity number based on any given vertex is existed. There's only one cycle. Hence there's one cycle related to connectivity number of this cycle-neutrosophic graph. Also, $\mathcal{C}(CYC) = 5$ and $\mathcal{C}(CYC) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't cyclic connectivity but connectivity number based on some sequence of consecutive vertices is existed. There's one cycle. Hence there's one cycle related to connectivity number of this cycle-neutrosophic graph. Also, $\mathcal{C}(CYC) = 5$. Also, $\mathcal{C}(CYC) \neq 3$. For every given vertex x, cyclic connectivity is existed and connectivity number is five and $\mathcal{C}(CYC) = 5$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic cyclic connectivity but neutrosophic connectivity number based on any given vertex is existed. There's one cycle so there's one cycle related to neutrosophic connectivity number which is $(0.2, 0.5, 0.4)$. Also, $\mathcal{C}_n(CYC) = (0.2, 0.5, 0.4)$;

(v) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a neutrosophic cycle based on n_1, then it's a neutrosophic cyclic connectivity since there's one cycle and there's one cycle based on n_1 and neutrosophic connectivity number based on n_1 is $(0.2, 0.5, 0.4)$. Also, $\mathcal{C}_n(CYC) = (0.2, 0.5, 0.4)$;

(vi) if $n_2, n_1, n_5, n_4, n_3, n_2$ is a neutrosophic cycle based on n_2, then it's a neutrosophic cyclic connectivity since there's one cycle and there's one cycle based on n_2 and neutrosophic connectivity number based on n_2 is $(0.2, 0.5, 0.4)$. Also, $\mathcal{C}_n(CYC) = (0.2, 0.5, 0.4)$.

Definition 1.5.72. (Dense Numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

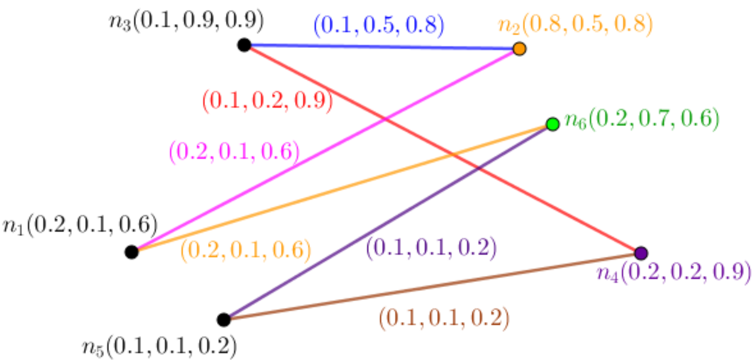

Figure 1.41: A Neutrosophic Graph in the Viewpoint of its connectivity number and its neutrosophic connectivity number. 72NTG5

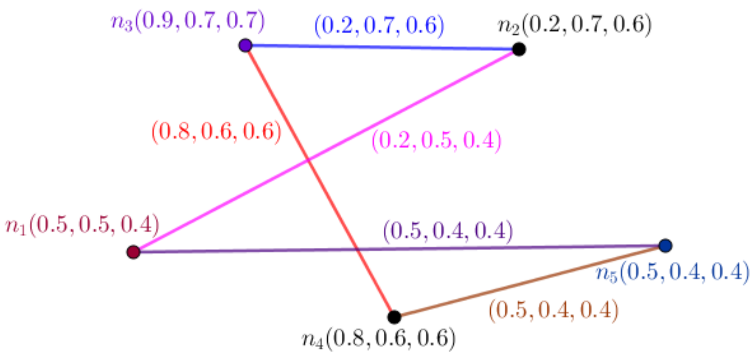

Figure 1.42: A Neutrosophic Graph in the Viewpoint of its connectivity number and its neutrosophic connectivity number. 72NTG6

(i) a set of vertices is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum cardinality between all dense sets is called **dense number** and it's denoted by $\mathcal{D}(NTG)$;

(ii) a set of vertices S is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{s \in S} \sum_{i=1}^{3} \sigma_i(s)$ between all dense sets is called **neutrosophic dense number** and it's denoted by $\mathcal{D}_n(NTG)$.

Proposition 1.5.73. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{D}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. Every vertex has two neighbors. So these vertices have same positions and by the minimum number of vertices is on demand, the result is obtained. Thus

$$\mathcal{D}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.74. There are two sections for clarifications.

(a) In Figure (2.43), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

(iii) $S_1 = \{n_1, n_4\}, S_2 = \{n_2, n_5\}, S_3 = \{n_3, n_6\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only three dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CYC) = 2$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

(vi) $S_1 = \{n_1, n_4\}, S_2 = \{n_2, n_5\}, S_3 = \{n_3, n_6\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only three dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CYC) = 2.2$ corresponded to S_1;

(b) In Figure (2.44), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_4 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, and n_4 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

(iii) $S_1 = \{n_1, n_3\}, S_2 = \{n_1, n_4\}, S_3 = \{n_2, n_4\}, S_4 = \{n_2, n_5\}, S_5 = \{n_3, n_5\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only five dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CYC) = 2$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_4 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, and n_4 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

(vi)

$$\begin{aligned} S_1 = \{n_1, n_3\} &\to 2.8 \\ S_2 = \{n_1, n_4\} &\to 2.2 \\ S_3 = \{n_2, n_4\} &\to 3.4 \\ S_4 = \{n_2, n_5\} &\to 2.5 \\ S_5 = \{n_3, n_5\} &\to 2.3 \\ \text{Minimum number is } 2.2 \end{aligned}$$

are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only five dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CYC) = 2$ corresponded to S_2.

Definition 1.5.75. (bulky numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

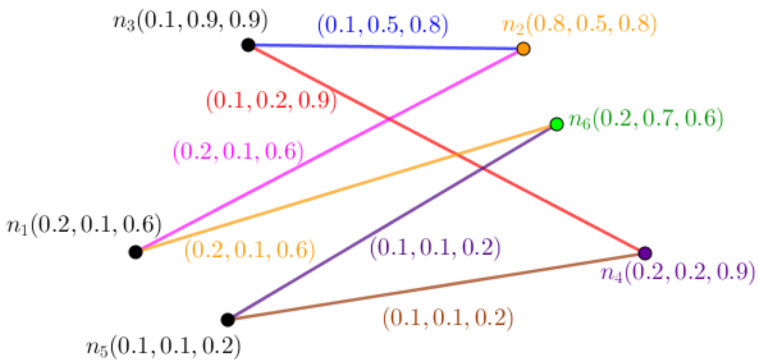

Figure 1.43: A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number. 73NTG5

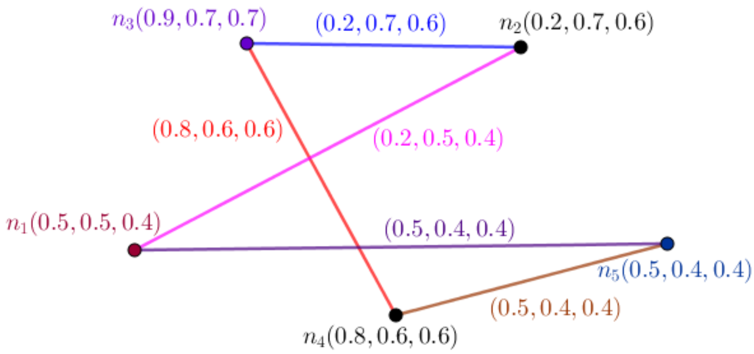

Figure 1.44: A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number. 73NTG6

(i) a set of edges S is called **bulky set** if for every edge e' outside, there's at least one edge e inside such that they've common vertex and the number of edges such that they've common vertex with e is greater than the number of edges such that they've common vertex with e'. The minimum cardinality between all bulky sets is called **bulky number** and it's denoted by $\mathcal{B}(NTG)$;

(ii) a set of edges S is called **bulky set** if for every edge e' outside, there's at least one edge e inside such that they've common vertex and the number of edges such that they've common vertex with e is greater than the number of edges such that they've common vertex with e'. The minimum neutrosophic cardinality $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(s)$ between all bulky sets is called **neutrosophic bulky number** and it's denoted by $\mathcal{B}_n(NTG)$.

Proposition 1.5.76. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{B}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. Every vertex has two neighbors. So all vertices have same positions. It implies finding edges have common endpoint. By minimum number of edges is on demand, the result is obtained. Thus

$$\mathcal{B}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.77. There are two sections for clarifications.

(a) In Figure (2.45), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it isn't a bulky set since an edge n_4n_5, outside, there's no edge inside such that they've common vertex;
- (ii) if $S = \{n_1n_2, n_4n_5\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with vertices of S is two which is equal to [greater than] two which is the number of edges such that they've common vertex with vertices of $V \setminus S$;
- (iii) All sets [2-sets] of edges containing two edges aren't bulky sets. The sets of edges $\{n_1n_2, n_4n_5\}, \{n_2n_3, n_5n_6\}, \{n_3n_4, n_6n_1\}$ are only minimal bulky sets. Since for every edge n_in_j, outside, there's at least one edge n_tn_s inside such that they've common vertex and the number of edges such that they've common vertex with n_tn_s is two which is equal to [greater than] two which is the number of edges such that they've common vertex with n_in_j. Thus $\mathcal{B}(CYC) = 2$;
- (iv) if $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it isn't a bulky set since an edge n_4n_5, outside, there's no edge n_2n_4 inside such that they've common vertex;
- (v) if $S = \{n_1n_2, n_4n_5\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with vertices of S is two which is equal to [greater than] two which is the number of edges such that they've common vertex with vertices of $V \setminus S$;
- (vi) All sets [2-sets] of edges containing two edges aren't bulky sets. The sets of edges $S_1 = \{n_1n_2, n_4n_5\}, S_2 = \{n_2n_3, n_5n_6\}$, and $S_3 = \{n_3n_4, n_6n_1\}$ are only minimal bulky sets. Since for every edge n_in_j, outside, there's at least one edge n_tn_s inside such that

they've common vertex and the number of edges such that they've common vertex with $n_t n_s$ is two which is equal to [greater than] two which is the number of edges such that they've common vertex with $n_i n_j$. Thus

$$S_1 = \{n_1 n_2, n_4 n_5\} \to 1.3$$
$$S_2 = \{n_2 n_3, n_5 n_6\} \to 1.8$$
$$S_3 = \{n_3 n_4, n_6 n_1\} \to 2.1$$
$$\text{Minimum number is } 1.3$$

It implies $\mathcal{B}_n(CYC) = 1.3$ and corresponded set of edges is $S_1 = \{n_1 n_2, n_4 n_5\}$.

(b) In Figure (2.46), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1 n_2, n_2 n_3\}$ is a set of edges, then it isn't a bulky set since an edge $n_4 n_5$, outside, there's no edge inside such that they've common vertex;

(ii) if $S = \{n_1 n_2, n_4 n_5\}$ is a set of edges, then it's bulky set since for every edge $n_i n_j$, outside, there's at least one edge $n_1 n_2$ inside such that they've common vertex and the number of edges such that they've common vertex with vertices of S is two which is equal to [greater than] two which is the number of edges such that they've common vertex with vertices of $V \setminus S$;

(iii) All sets [2-sets] of edges containing two edges aren't bulky sets. The sets of edges $S_1 = \{n_1 n_2, n_4 n_5\}, S_2 = \{n_2 n_3, n_5 n_1\}, S_3 = \{n_2 n_3, n_4 n_5\}, S_4 = \{n_3 n_4, n_5 n_1\}$, and $S_5 = \{n_3 n_4, n_1 n_2\}$ are only minimal bulky sets. Since for every edge $n_i n_j$, outside, there's at least one edge $n_t n_s$ inside such that they've common vertex and the number of edges such that they've common vertex with $n_t n_s$ is two which is equal to [greater than] two which is the number of edges such that they've common vertex with $n_i n_j$. Thus $\mathcal{B}(CYC) = 2$;

(iv) if $S = \{n_1 n_2, n_2 n_3\}$ is a set of edges, then it isn't a bulky set since an edge $n_4 n_5$, outside, there's no edge $n_2 n_4$ inside such that they've common vertex;

(v) if $S = \{n_1 n_2, n_4 n_5\}$ is a set of edges, then it's bulky set since for every edge $n_i n_j$, outside, there's at least one edge $n_1 n_2$ inside such that they've common vertex and the number of edges such that they've common vertex with vertices of S is two which is equal to [greater than] two which is the number of edges such that they've common vertex with vertices of $V \setminus S$;

(vi) All sets [2-sets] of edges containing two edges aren't bulky sets. The sets of edges $S_1 = \{n_1 n_2, n_4 n_5\}, S_2 = \{n_2 n_3, n_5 n_1\}, S_3 = \{n_2 n_3, n_4 n_5\}, S_4 = \{n_3 n_4, n_5 n_1\}$, and $S_5 = \{n_3 n_4, n_1 n_2\}$ are only minimal bulky sets. Since for every edge $n_i n_j$, outside, there's at least one edge $n_t n_s$ inside such that they've common vertex and the number of edges such that they've common vertex with $n_t n_s$ is two

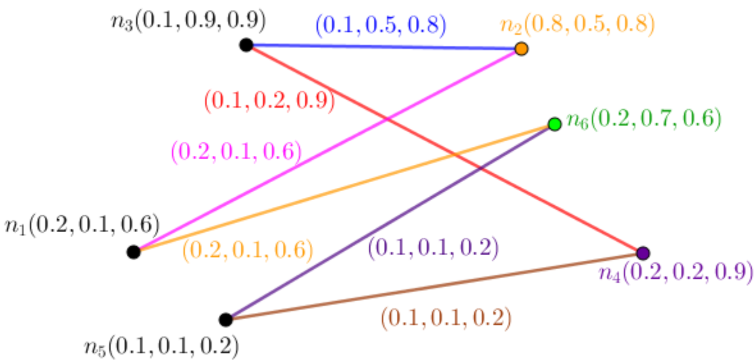

Figure 1.45: A Neutrosophic Graph in the Viewpoint of its bulky number and its neutrosophic bulky number.

74NTG5

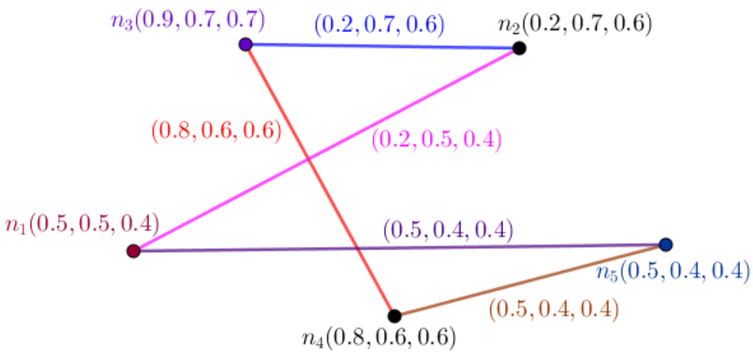

Figure 1.46: A Neutrosophic Graph in the Viewpoint of its bulky number and its neutrosophic bulky number.

74NTG6

which is equal to [greater than] two which is the number of edges such that they've common vertex with $n_i n_j$. Thus

$$S_1 = \{n_1n_2, n_4n_5\} \to 2.4$$
$$S_2 = \{n_2n_3, n_5n_1\} \to 2.8$$
$$S_3 = \{n_2n_3, n_4n_5\} \to 2.8$$
$$S_4 = \{n_3n_4, n_5n_1\} \to 3.3$$
$$S_5 = \{n_3n_4, n_1n_2\} \to 3.1$$

Minimum number is 2.4

It implies $\mathcal{B}_n(CYC) = 2.4$ and corresponded set of edges is $S_1 = \{n_1n_2, n_4n_5\}$.

Definition 1.5.78. (collapsed numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set of vertices S is called **collapsed set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$

and the number of neighbors of x is less than [equal to] the number of neighbors of y. The minimum cardinality between all collapsed sets is called **collapsed number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) a set of vertices S is called **collapsed set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is less than [equal to] the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x)$ between all collapsed sets is called **neutrosophic collapsed number** and it's denoted by $\mathcal{P}_n(NTG)$.

Proposition 1.5.79. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{P}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. Every vertex has two neighbors. So all vertices have same positions. The set

$$\{x_s, x_{s+3}, x_{s+6}, \cdots, x_i\}_{i+2 > \mathcal{O}(CYC)}$$

of vertices is called collapsed set since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum cardinality $|S|$, $\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor$, between all collapsed sets is called collapsed number and it's denoted by $\mathcal{P}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor$. Thus

$$\mathcal{P}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.80. There are two sections for clarifications.

(a) In Figure (2.47), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If $S = \{n_1, n_3\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_5 outside, such that there's no vertex inside such that they're endpoints either $n_1 n_5 \in E$ or $n_3 n_5 \in E$;

  (ii) if $S = \{n_1, n_5\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_3 outside, such that there's no vertex inside such that they're endpoints either $n_1 n_3 \in E$ or $n_5 n_3 \in E$;

(iii) all sets [2-sets] of vertices containing two vertices, aren't called collapsed sets. Sets [2-sets] of vertices $S_1 = \{n_1, n_4\}$, $S_2 = \{n_2, n_5\}$, and $S_3 = \{n_3, n_6\}$ are called minimal collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum cardinality $|S|$, 2, between all collapsed sets

$$S_1 = \{n_1, n_4\} \to 2$$
$$S_2 = \{n_2, n_5\} \to 2$$
$$S_3 = \{n_3, n_6\} \to 2$$
$$\text{The minimum is } 2$$

is called collapsed number and it's denoted by $\mathcal{P}(CYC) = 2$; $S_1 = \{n_1, n_4\}$, $S_2 = \{n_2, n_5\}$, and $S_3 = \{n_3, n_6\}$ are corresponded sets;

(iv) if $S = \{n_1, n_3\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_5 outside, such that there's no vertex inside such that they're endpoints either $n_1n_5 \in E$ or $n_3n_5 \in E$;

(v) if $S = \{n_1, n_5\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_3 outside, such that there's no vertex inside such that they're endpoints either $n_1n_3 \in E$ or $n_5n_3 \in E$;

(vi) all sets [2-sets] of vertices containing two vertices, aren't called collapsed sets. Sets [2-sets] of vertices $S_1 = \{n_1, n_4\}$, $S_2 = \{n_2, n_5\}$, and $S_3 = \{n_3, n_6\}$ are called minimal collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum neutrosophic cardinality, $\sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x)$, 2.2, between all collapsed sets

$$S_1 = \{n_1, n_4\} \to 2.2$$
$$S_2 = \{n_2, n_5\} \to 4.5$$
$$S_3 = \{n_3, n_6\} \to 3.4$$
$$\text{The minimum is } 2.2$$

is called neutrosophic collapsed number and it's denoted by $\mathcal{P}_n(CYC) = 2.2$ and corresponded set is $S_1 = \{n_1, n_4\}$.

(b) In Figure (2.48), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_4 outside, such that there's no vertex inside such that they're endpoints either $n_1n_4 \in E$ or $n_2n_4 \in E$;

(ii) if $S = \{n_4, n_5\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_2 outside, such that there's no vertex inside such that they're endpoints either $n_4n_2 \in E$ or $n_5n_2 \in E$;

(iii) all sets [2-sets] of vertices containing two vertices, aren't called collapsed sets. Sets [2-sets] of vertices $S_1 = \{n_1, n_4\}$, $S_2 = \{n_1, n_3\}$, $S_3 = \{n_2, n_5\}$, $S_4 = \{n_2, n_4\}$, and $S_5 = \{n_3, n_5\}$ are called minimal collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum cardinality $|S|$, 2, between all collapsed sets

$$\begin{aligned}
&S_1 = \{n_1, n_4\} \to 2\\
&S_2 = \{n_1, n_3\} \to 2\\
&S_3 = \{n_2, n_5\} \to 2\\
&S_4 = \{n_2, n_4\} \to 2\\
&S_5 = \{n_3, n_5\} \to 2\\
&\text{The minimum is } 2
\end{aligned}$$

is called collapsed number and it's denoted by $\mathcal{P}(CYC) = 2$; corresponded sets are $S_1 = \{n_1, n_4\}$, $S_2 = \{n_1, n_3\}$, $S_3 = \{n_2, n_5\}$, $S_4 = \{n_2, n_4\}$, and $S_5 = \{n_3, n_5\}$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_4 outside, such that there's no vertex inside such that they're endpoints either $n_1n_4 \in E$ or $n_2n_4 \in E$;

(v) if $S = \{n_4, n_5\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_2 outside, such that there's no vertex inside such that they're endpoints either $n_4n_2 \in E$ or $n_5n_2 \in E$;

(vi) all sets [2-sets] of vertices containing two vertices, aren't called collapsed sets. Sets [2-sets] of vertices $S_1 = \{n_1, n_4\}$, $S_2 = \{n_1, n_3\}$, $S_3 = \{n_2, n_5\}$, $S_4 = \{n_2, n_4\}$, and $S_5 = \{n_3, n_5\}$ are called minimal collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum neutrosophic cardinality, $\sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x)$, 2.8, between all collapsed sets

$$\begin{aligned}
&S_1 = \{n_1, n_4\} \to 3.4\\
&S_2 = \{n_1, n_3\} \to 3.7\\
&S_3 = \{n_2, n_5\} \to 2.8\\
&S_4 = \{n_2, n_4\} \to 3.6\\
&S_5 = \{n_3, n_5\} \to 3.6\\
&\text{The minimum is } 2.8
\end{aligned}$$

is called neutrosophic collapsed number and it's denoted by $\mathcal{P}_n(CYC) = 2.8$ and corresponded set is $S_3 = \{n_2, n_5\}$.

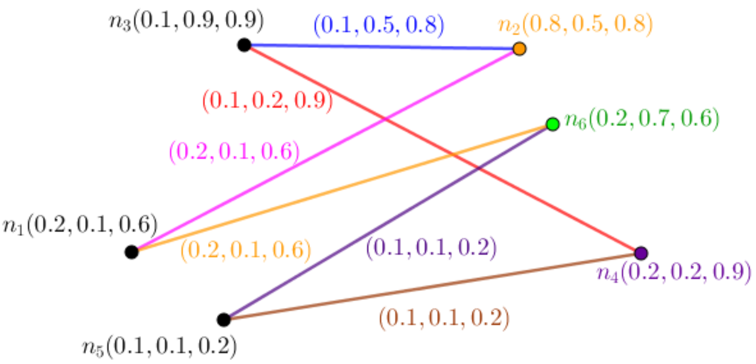

Figure 1.47: A Neutrosophic Graph in the Viewpoint of its collapsed number and its neutrosophic collapsed number.

75NTG5

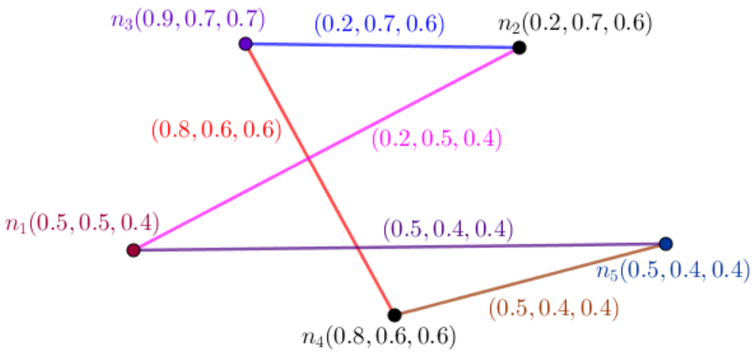

Figure 1.48: A Neutrosophic Graph in the Viewpoint of its collapsed number and its neutrosophic collapsed number.

75NTG6

Definition 1.5.81. (path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors in this process is called **path-coloring set** from x to y. The minimum cardinality between all path-coloring sets from two given vertices is called **path-coloring number** and it's denoted by $\mathcal{L}(NTG)$;

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set S of shared edges in this process is called **path-coloring set** from x to y. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all path-coloring sets, Ss, is called **neutrosophic path-coloring number** and it's denoted by $\mathcal{L}_n(NTG)$.

Proposition 1.5.82. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{L}(CYC) = 1.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors but these two paths don't share one edge. The set of colors, {red}, in this process is called path-coloring set from x to y. The minimum cardinality between all path-coloring sets from two given vertices, 1, is called path-coloring number and it's denoted by $\mathcal{L}(CYC)$. Thus

$$\mathcal{L}(CYC) = 1.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.83. There are two sections for clarifications.

(a) In Figure (2.49), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) All paths are as follows.

$$\begin{aligned}
&P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 \to \text{red}\\
&P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3 \to \text{red}\\
&P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1, n_6, n_5, n_4 \to \text{red}\\
&P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_6, n_5 \to \text{red}\\
&P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \& \ P_2 : n_1, n_6 \to \text{red}\\
&\text{The number is } 1;
\end{aligned}$$

(ii) 1-paths have same color;

(iii) $\mathcal{L}(CYC) = 1$;

(iv) the position of given vertices could be different in the terms of creating path and the behaviors in path;

(v) there are only two paths but there's no shared edge;

(vi) all paths are as follows.

$$\begin{aligned}
&P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} \to \ 0\\
&P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} \to \ 0\\
&P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1, n_6, n_5, n_4 \to \text{red} \to \text{no shared edge} \to \ 0\\
&P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_6, n_5 \to \text{red} \to \text{no shared edge} \to \ 0\\
&P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \& \ P_2 : n_1, n_6 \to \text{red} \to \text{no shared edge} \to \ 0\\
&\mathcal{L}_n(CYC) \text{ is } 0.
\end{aligned}$$

(b) In Figure (2.50), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

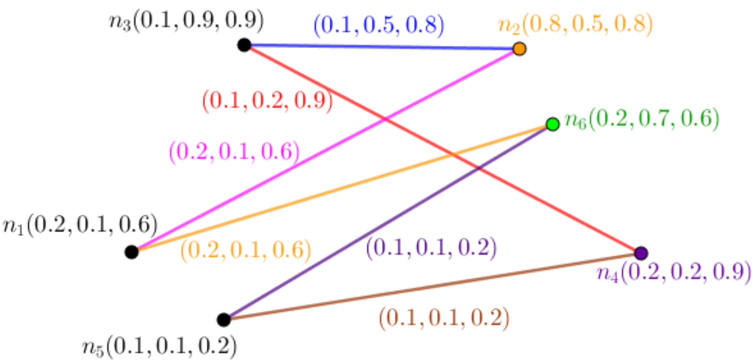

Figure 1.49: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

76NTG5

(i) All paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_5, n_4, n_3, n_2 \to \text{red}$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_5, n_4, n_3 \to \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1 n_5, n_4 \to \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_5 \to \text{red}$$
$$\text{The number is 1;}$$

(ii) 1-paths have same color;

(iii) $\mathcal{L}(CYC) = 1$;

(iv) the position of given vertices could be different in the terms of creating path and the behaviors in path;

(v) there are only two paths but there's no shared edge;

(vi) all paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1, n_5, n_4 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_5 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$\mathcal{L}_n(CYC) \text{ is } 0.$$

Definition 1.5.84. (dominating path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of different colors, S, in this process is called **dominating path-coloring set** from x to y if for every edge outside there's at least one edge inside which they've common vertex. The minimum cardinality between all dominating path-coloring sets from two given vertices is called **dominating path-coloring number** and it's denoted by $\mathcal{Q}(NTG)$;

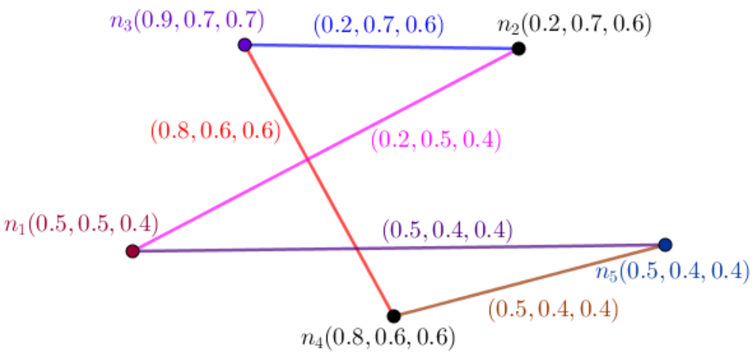

Figure 1.50: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number. 76NTG6

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set S of different colors in this process is called **dominating path-coloring set** from x to y if for every edge outside there's at least one edge inside which they've common vertex. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all dominating path-coloring sets, Ss, is called **neutrosophic dominating path-coloring number** and it's denoted by $\mathcal{Q}_n(NTG)$.

Proposition 1.5.85. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{Q}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors but these two paths don't share one edge. The set of colors, $\{\text{red}\}$, in this process is called dominating path-coloring set from x to y. The minimum cardinality between all dominating path-coloring sets from two given vertices, 1, is called dominating path-coloring number and it's denoted by $\mathcal{Q}(CYC)$. Thus

$$\mathcal{Q}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.86. There are two sections for clarifications.

(a) In Figure (2.51), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) All paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 \rightarrow \text{red}$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3 \rightarrow \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1, n_6, n_5, n_4 \rightarrow \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_6, n_5 \rightarrow \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \& \ P_2 : n_1, n_6 \rightarrow \text{red}$$
$$\text{The number is 1;}$$

(ii) 1-paths have same color;

(iii) $\mathcal{Q}(CYC) = 1$;

(iv) the position of given vertices could be different in the terms of creating path and the behaviors in path;

(v) there are only two paths but there's no shared edge;

(vi) all paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 \rightarrow \text{red} \rightarrow \text{no shared edge} \rightarrow \ 0$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3 \rightarrow \text{red} \rightarrow \text{no shared edge} \rightarrow \ 0$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1, n_6, n_5, n_4 \rightarrow \text{red} \rightarrow \text{no shared edge} \rightarrow \ 0$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_6, n_5 \rightarrow \text{red} \rightarrow \text{no shared edge} \rightarrow \ 0$$
$$P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \& \ P_2 : n_1, n_6 \rightarrow \text{red} \rightarrow \text{no shared edge} \rightarrow \ 0$$
$$\mathcal{Q}_n(CYC) \text{ is } 0.$$

(b) In Figure (2.52), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) All paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_5, n_4, n_3, n_2 \rightarrow \text{red}$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_5, n_4, n_3 \rightarrow \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1 n_5, n_4 \rightarrow \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_5 \rightarrow \text{red}$$
$$\text{The number is 1;}$$

(ii) 1-paths have same color;

(iii) $\mathcal{Q}(CYC) = 1$;

(iv) the position of given vertices could be different in the terms of creating path and the behaviors in path;

(v) there are only two paths but there's no shared edge;

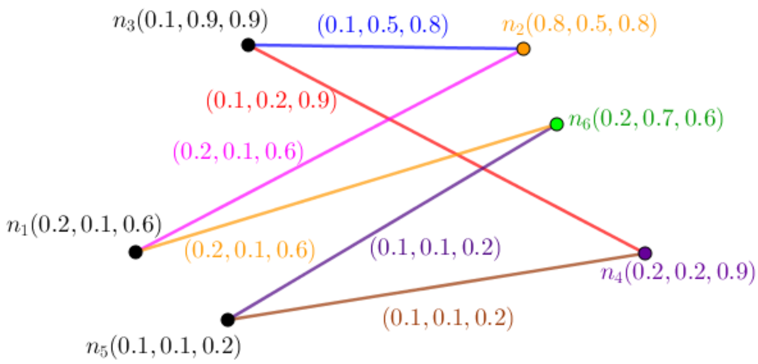

Figure 1.51: A Neutrosophic Graph in the Viewpoint of its dominating path-coloring number and its neutrosophic dominating path-coloring number. 77NTG5

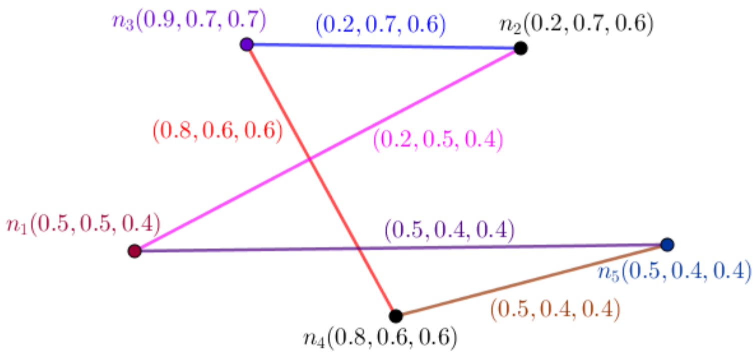

Figure 1.52: A Neutrosophic Graph in the Viewpoint of its dominating path-coloring number and its neutrosophic dominating path-coloring number. 77NTG6

(vi) all paths are as follows.

$$P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} \to 0$$
$$P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} \to 0$$
$$P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1, n_5, n_4 \to \text{red} \to \text{no shared edge} \to 0$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_5 \to \text{red} \to \text{no shared edge} \to 0$$
$$\mathcal{Q}_n(CYC) \text{ is } 0.$$

Definition 1.5.87. (path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, S, in this process is called **path-coloring set** from x to y. The minimum cardinality between all path-coloring sets from two given vertices is called **path-coloring number** and it's denoted by $\mathcal{V}(NTG)$;

(*ii*) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set S of different colors in this process is called **path-coloring set** from x to y. The minimum neutrosophic cardinality, $\sum_{x\in Z}\sum_{i=1}^{3}\sigma_i(x)$, between all sets Zs including the latter endpoints corresponded to path-coloring set Ss, is called **neutrosophic path-coloring number** and it's denoted by $\mathcal{V}_n(NTG)$.

Proposition 1.5.88. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

$$\mathcal{V}(CYC) = 2 \times (\mathcal{O}(CYC) - 1).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. If two paths from x to y share one endpoint, then they're assigned to different colors but there are only $2 \times (\mathcal{O}(CYC) - 1)$ paths for every given vertex. In the terms of number of paths, all vertices behave the same and they've same positions. The set of colors is

$$S = \{\text{red}_1, \text{red}_2, \cdots, \text{red}_{2\times(\mathcal{O}(CYC)-1)}\},$$

in this process. For given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, $S = \{\text{red}_1, \text{red}_2, \cdots, \text{red}_{2\times(\mathcal{O}(CYC)-1)}\}$, in this process is called path-coloring set from x to y. The minimum cardinality,

$$|S| = |\{\text{red}_1, \text{red}_2, \cdots, \text{red}_{2\times(\mathcal{O}(CYC)-1)}\}| = 2 \times (\mathcal{O}(CYC) - 1),$$

between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{V}(CYC)$. Thus

$$\mathcal{V}(CYC) = 2 \times (\mathcal{O}(CYC) - 1).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.89. There are two sections for clarifications.

(*a*) In Figure (2.53), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) Consider the vertex n_1. All paths with endpoint n_1 are as follow:

$$\begin{aligned}
P_1 &: n_1, n_2 \to \text{red}\\
P_2 &: n_1, n_2, n_3 \to \text{blue}\\
P_3 &: n_1, n_2, n_3, n_4 \to \text{yellow}\\
P_4 &: n_1, n_2, n_3, n_4, n_5 \to \text{white}\\
P_5 &: n_1, n_2, n_3, n_4, n_5, n_6 \to \text{black}\\
P_6 &: n_1, n_6, n_5, n_4, n_3, n_2 \to \text{pink}\\
P_7 &: n_1, n_6, n_5, n_4, n_3 \to \text{purple}\\
P_8 &: n_1, n_6, n_5, n_4 \to \text{brown}\\
P_9 &: n_1, n_6, n_5 \to \text{orange}\\
P_{10} &: n_1, n_6 \to \text{green}
\end{aligned}$$

Thus $S = \{\text{red}, \text{blue}, \text{yellow}, \text{white}, \text{black}, \text{pink}, \text{purple}, \text{brown}, \text{orange}, \text{green}\}$, is path-coloring set and its cardinality, 10, is path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors,

$$S = \{\text{red}, \text{blue}, \text{yellow}, \text{white}, \text{black}, \text{pink}, \text{purple}, \text{brown}, \text{orange}, \text{green}\},$$

in this process is called path-coloring set from x to y. The minimum cardinality, 10, between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{V}(CYC) = 10$;

(*ii*) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(*iii*) there are some different paths which have no shared endpoints. So they could been assigned to same color;

(*iv*) shared endpoints form a set of representatives of colors. Each color is corresponded to a vertex which has minimum neutrosophic cardinality;

(*v*) every color in S is corresponded to a vertex has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared endpoints to form S and after that minimum neutrosophic cardinality is optimal;

(*vi*) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors,

$$S = \{\text{red}, \text{blue}, \text{yellow}, \text{white}, \text{black}, \text{pink}, \text{purple}, \text{brown}, \text{orange}, \text{green}\},$$

in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality,

$$\sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3} \sigma_i(n_2) = 6,$$

between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by

$$\mathcal{V}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3} \sigma_i(n_2) = 6.$$

(*b*) In Figure (2.54), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) Consider the vertex n_1. All paths with endpoint n_1 are as follow:

$$\begin{aligned} P_1 : n_1, n_2 &\to \text{red} \\ P_2 : n_1, n_2, n_3 &\to \text{blue} \\ P_3 : n_1, n_2, n_3, n_4 &\to \text{yellow} \\ P_4 : n_1, n_2, n_3, n_4, n_5 &\to \text{white} \\ P_5 :: n_1, n_5, n_4, n_3, n_2 &\to \text{black} \\ P_6 : n_1, n_5, n_4, n_3 &\to \text{pink} \\ P_7 : n_1, n_5, n_4 &\to \text{purple} \\ P_8 : n_1, n_5 &\to \text{brown} \end{aligned}$$

Thus $S = \{\text{red, blue, yellow, white, black, pink, purple, brown}\}$ is path-coloring set and its cardinality, 8, is path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors,

$$S = \{\text{red, blue, yellow, white, black, pink, purple, brown}\},$$

in this process is called path-coloring set from x to y. The minimum cardinality, 8, between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{V}(CYC) = 8$;

(*ii*) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(*iii*) there are some different paths which have no shared endpoints. So they could been assigned to same color;

(*iv*) shared endpoints form a set of representatives of colors. Each color is corresponded to a vertex which has minimum neutrosophic cardinality;

(*v*) every color in S is corresponded to a vertex has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared endpoints to form S and after that minimum neutrosophic cardinality is optimal;

(*vi*) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors,

$$S = \{\text{red, blue, yellow, white, black, pink, purple, brown}\},$$

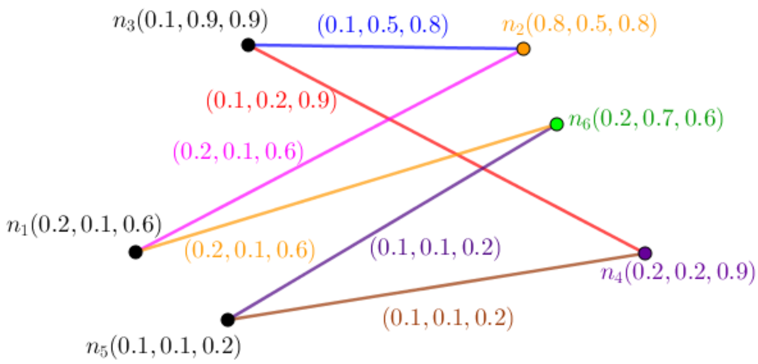

Figure 1.53: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number. 78NTG5

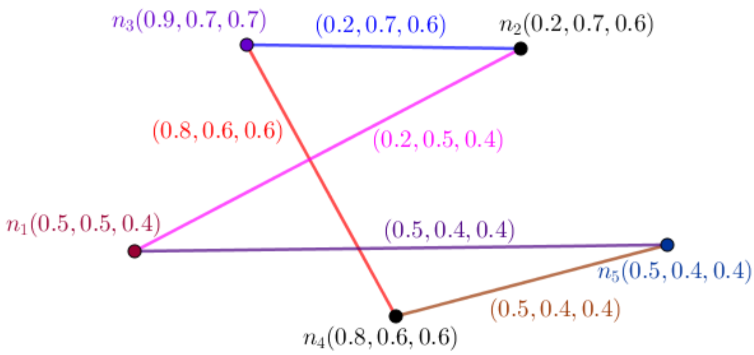

Figure 1.54: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number. 78NTG6

in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality,

$$\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}\sigma_i(n_3) = 6.2,$$

between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by

$$\mathcal{V}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}\sigma_i(n_3) = 6.2.$$

Definition 1.5.90. (Dual-Dominating Numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum cardinality between all dual-dominating sets is called **dual-dominating number** and it's denoted by $\mathcal{D}(NTG)$;

(ii) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum neutrosophic cardinality between all dual-dominating sets is called **neutrosophic dual-dominating number** and it's denoted by $\mathcal{D}_n(NTG)$.

Proposition 1.5.91. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{D}(CYC) = \lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. Two consecutive vertices could belong to S which is dual-dominating set related to dual-dominating number. Since these two vertices could be dominated by previous vertex and upcoming vertex despite them. If there are no vertices which are consecutive, then it contradicts with maximality of set S and maximum cardinality of S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor}, x_1\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus (S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor}, x_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor}, x_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}(CYC) = \lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor.$$

Thus

$$\mathcal{D}(CYC) = \lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.92. There are two sections for clarifications.

(a) In Figure (2.55), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(ii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(iii) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(iv) let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_5, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(v) let $S = \{n_1, n_2, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(vi) let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_5, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.9$.

(b) In Figure (2.56), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(iii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(iv) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(v) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. As if it, 5.1, contradicts with the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.7$;

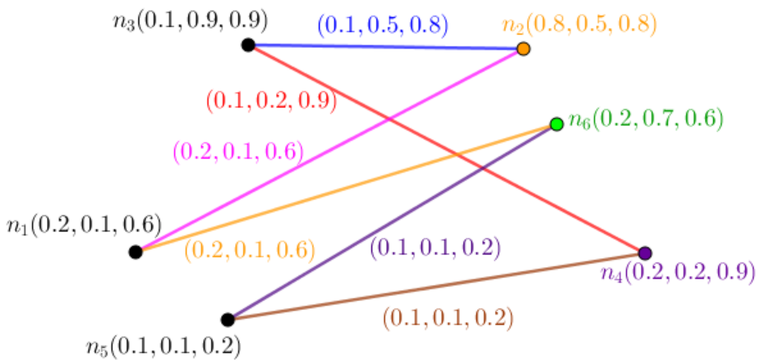

Figure 1.55: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number. 79NTG5

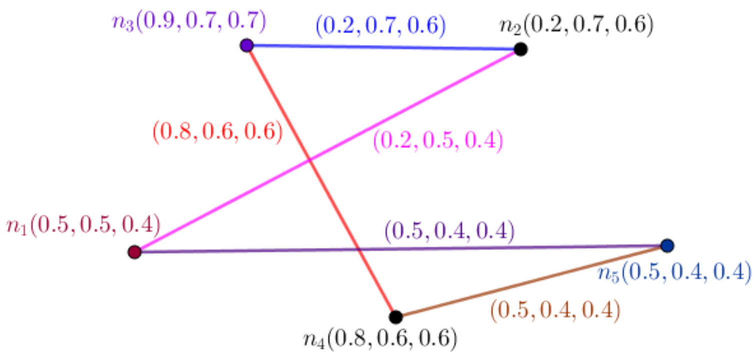

Figure 1.56: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number. 79NTG6

(vi) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.7$.

Definition 1.5.93. (dual-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, s and s' if $d(s, n) \neq d(s', n)$, then n resolves s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices s, s' in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, S is called **dual-**

resolving set. The maximum cardinality between all dual-resolving sets is called **dual-resolving number** and it's denoted by $\mathcal{R}(NTG)$;

(ii) for given two vertices, s and s' if $d(s,n) \neq d(s',n)$, then n resolves s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices s, s' in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, S is called **dual-resolving set**. The maximum neutrosophic cardinality between all dual-resolving sets is called **dual-resolving number** and it's denoted by $\mathcal{R}_n(NTG)$.

Proposition 1.5.94. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

$$\mathcal{R}(CYC) = \mathcal{O}(CYC) - 2.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. $\mathcal{O}(CYC) - 2$ consecutive vertices could belong to S which is dual-resolving set related to dual-resolving number where two neutrosophic vertices outside are consecutive. Since these two vertices could resolve all vertices. If there are no neutrosophic vertices which are consecutive, then it contradicts with maximality of set S and maximum cardinality of S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there's at least one neutrosophic vertex n in $V \setminus (S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\})$ such that n resolves s and s' then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by

$$\mathcal{R}(CYC) = \mathcal{O}(CYC) - 2.$$

Thus

$$\mathcal{R}(CYC) = \mathcal{O}(CYC) - 2.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.95. There are two sections for clarifications.

(*a*) In Figure (2.57), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices n_2 and n_3 in S, there's neutrosophic vertex n_1 in $V \setminus (S = \{n_3, n_2\})$ such that n_1 resolves n_2 and n_3, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-resolving set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(*ii*) $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices n_2 and n_4 in S, there's neutrosophic vertex n_1 in $V \setminus (S = \{n_4, n_2\})$ such that n_1 resolves n_2 and n_4, then the set of neutrosophic vertices, $S = \{n_4, n_2\}$ is called dual-resolving set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(*iii*) let $S = \{n_3, n_4, n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either neutrosophic vertex n_6 or neutrosophic vertex n_5 in $V \setminus (S = \{n_3, n_4, n_1, n_2\})$ such that either n_6 resolves s and s', or n_5 resolves s and s', then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_2\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(*iv*) let $S = \{n_3, n_4, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either neutrosophic vertex n_1 or neutrosophic vertex n_2 in $V \setminus (S = \{n_3, n_4, n_5, n_6\})$ such that either n_1 resolves s and s', or n_2 resolves s and s', then the set of neutrosophic vertices, $S = \{n_3, n_4, n_5, n_6\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(*v*) let $S = \{n_2, n_5, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either neutrosophic vertex n_3 or neutrosophic vertex n_4 in $V \setminus (S = \{n_2, n_5, n_1, n_6\})$ such that either n_3 resolves s and s', or n_4 resolves s and s', then the set of neutrosophic vertices, $S = \{n_2, n_5, n_1, n_6\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(*vi*) let $S = \{n_3, n_1, n_6, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either neutrosophic vertex n_5 or neutrosophic vertex n_4 in $V \setminus (S = \{n_3, n_1, n_6, n_2\})$ such that either n_5 resolves s and s', or n_4 resolves s and s', then the set of neutrosophic vertices, $S = \{n_3, n_1, n_6, n_2\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}_n(CYC) = 6.4$.

(b) In Figure (2.58), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices n_2 and n_3 in S, there's neutrosophic vertex n_4 in $V \setminus (S = \{n_3, n_2\})$ such that n_4 resolves n_2 and n_3, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-resolving set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(ii) $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices n_2 and n_4 in S, there's neutrosophic vertex n_5 in $V \setminus (S = \{n_4, n_2\})$ such that n_5 resolves n_2 and n_4, then the set of neutrosophic vertices, $S = \{n_4, n_2\}$ is called dual-resolving set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(iii) let $S = \{n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either a neutrosophic vertex n_1 or neutrosophic vertex n_2 in $V \setminus (S = \{n_3, n_4, n_5\})$ such that either n_1 resolves s and s' or n_2 resolves s and s', then the set of neutrosophic vertices, $S = \{n_3, n_4, n_5\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(iv) let $S = \{n_1, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either a neutrosophic vertex n_3 or neutrosophic vertex n_4 in $V \setminus (S = \{n_1, n_2, n_5\})$ such that either n_3 resolves s and s' or n_4 resolves s and s', then the set of neutrosophic vertices, $S = \{n_1, n_2, n_5\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(v) let $S = \{n_1, n_2, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either a neutrosophic vertex n_4 or neutrosophic vertex n_5 in $V \setminus (S = \{n_1, n_2, n_3\})$ such that either n_4 resolves s and s' or n_5 resolves s

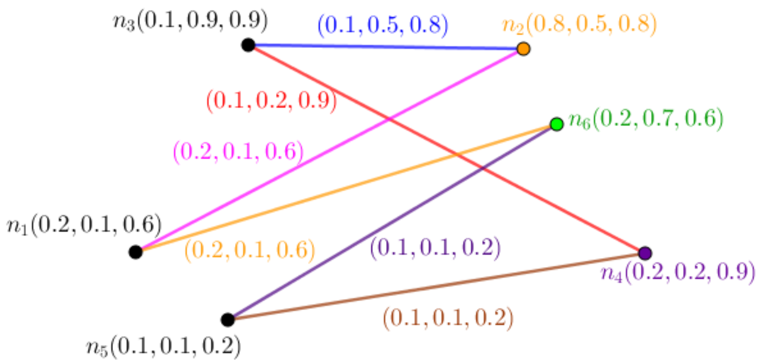

Figure 1.57: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number. 80NTG5

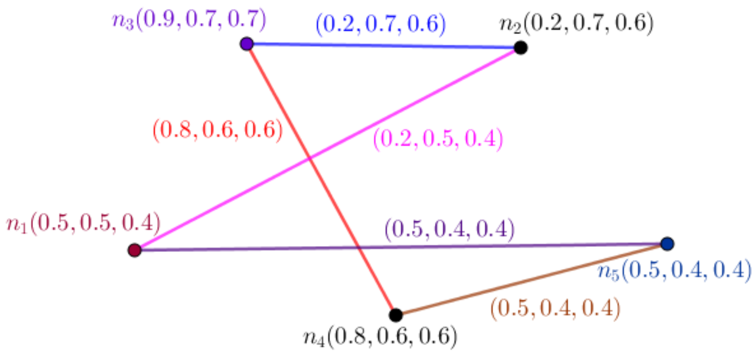

Figure 1.58: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number. 80NTG6

and s', then the set of neutrosophic vertices, $S = \{n_1, n_2, n_3\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(vi) let $S = \{n_2, n_3, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either a neutrosophic vertex n_1 or neutrosophic vertex n_5 in $V \setminus (S = \{n_2, n_3, n_4\})$ such that either n_1 resolves s and s' or n_5 resolves s and s', then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4\}$ is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}_n(CYC) = 5.8$.

Definition 1.5.96. (joint-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called **joint-dominating number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called **neutrosophic joint-dominating number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

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Proposition 1.5.97. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S dominates if and only if it joint-dominates.*

Proposition 1.5.98. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S is corresponded to joint-dominating number. Then $V \setminus D$ is S-like.*

Proposition 1.5.99. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-dominating number if and only if for all s in S, there's a vertex n in $V \setminus S$, such that $\{n' \mid n'n \in E\} \cap S = \{s\}$.*

Proposition 1.5.100. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. $\mathcal{O}(CYC) - 2$ consecutive vertices could belong to S which is joint-dominating set related to joint-dominating number where two neutrosophic vertices outside are "consecutive". Since it's possible to have a path amid every two of vertices in S and two vertices outside could be joint-dominated by their neighbors in S. If there are no neutrosophic vertices which are consecutive, then it contradicts with the term joint-dominating set for S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For given vertex n if $sn \in E$, then s joint-dominates

n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in

$$V \setminus (S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}),$$

there's only one neutrosophic vertex s in

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

such that s joint-dominates n, then the set of neutrosophic vertices,

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

is called joint-dominating set where for every two vertices in

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\},$$

there's only one path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2.$$

Thus

$$\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2.$$

■

Proposition 1.5.101. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $3 \times \mathcal{O}(CYC) + 1$ joint-dominating sets.*

Proposition 1.5.102. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $\mathcal{O}(CYC)$ joint-dominating set corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.103. There are two sections for clarifications.

(a) In Figure (2.59), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) For given two neutrosophic vertices, s and s', there are only two paths between them;

  (ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of cycle;

  (iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$

$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\}$.

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2 = 4$;

(iv) there are nineteen joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_5, n_1, n_2, n_3, n_4\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6, n_2, n_3, n_4, n_5\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_4, n_5, n_6, n_1\}, \{n_2, n_4, n_5, n_6, n_1\}, \{n_3, n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_3, n_5, n_6, n_1, n_2\}, \{n_4, n_5, n_6, n_1, n_2\},$$
$$\{n_6, n_1, n_2, n_3\}, \{n_4, n_6, n_1, n_2, n_3\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_3, n_4\},$$

as if it's possible to have six of them

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\}$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are nineteen joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_5, n_1, n_2, n_3, n_4\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6, n_2, n_3, n_4, n_5\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_4, n_5, n_6, n_1\}, \{n_2, n_4, n_5, n_6, n_1\}, \{n_3, n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_3, n_5, n_6, n_1, n_2\}, \{n_4, n_5, n_6, n_1, n_2\},$$
$$\{n_6, n_1, n_2, n_3\}, \{n_4, n_6, n_1, n_2, n_3\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_3, n_4\},$$

as if there is six joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4, n_5, n_6, n_1\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}(\sigma(n_2) + \sigma(n_3)) = 4.1$.

(b) In Figure (2.60), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there are only two paths between them;

(ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of cycle;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2 = 3$;

(iv) there are sixteen joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_4, n_1, n_2, n_3\}, \{n_5, n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_5, n_2, n_3, n_4\},$$
$$\{n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_4, n_5, n_1\}, \{n_2, n_4, n_5, n_1\}, \{n_3, n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\}, \{n_3, n_5, n_1, n_2\}, \{n_4, n_5, n_1, n_2\},$$
$$\{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have five of them

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

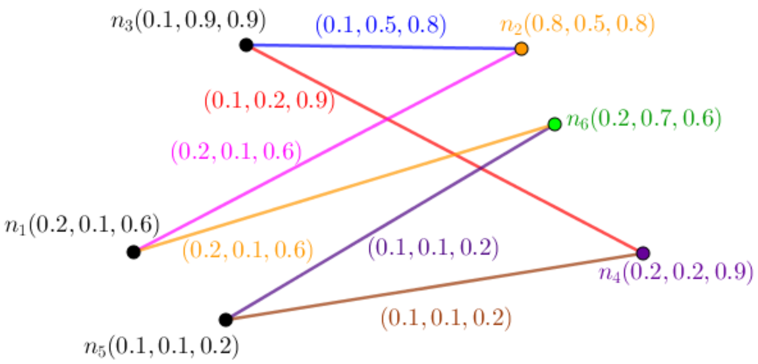

Figure 1.59: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number.

81NTG5

(v) there are sixteen joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_4, n_1, n_2, n_3\}, \{n_5, n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_5, n_2, n_3, n_4\},$$
$$\{n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_4, n_5, n_1\}, \{n_2, n_4, n_5, n_1\}, \{n_3, n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\}, \{n_3, n_5, n_1, n_2\}, \{n_4, n_5, n_1, n_2\},$$
$$\{n_1, n_2, n_3, n_4, n_5\},$$

as if there is five joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_5, n_1, n_2\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}(\sigma(n_3) + \sigma(n_4)) = 4.2$.

Definition 1.5.104. (joint-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from

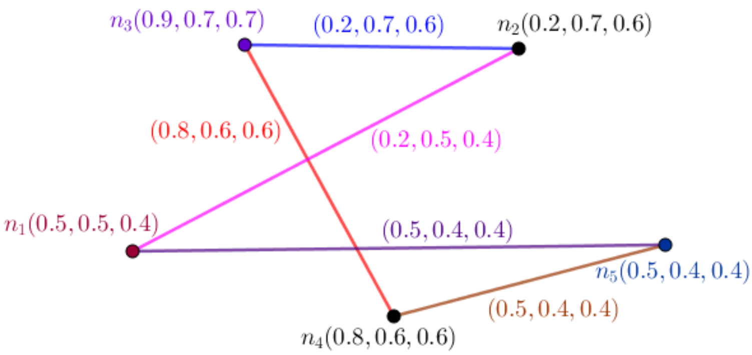

Figure 1.60: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG6

the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called **joint-resolving number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called **neutrosophic joint-resolving number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

82prp9 **Proposition 1.5.105.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S resolves if and only if it joint-resolves.*

Proposition 1.5.106. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-resolving number if and only if for all s in S, either there are vertices n and n' in $V \setminus S$, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$ or there's vertex s' in S, such that are s and s' twin vertices.*

Proposition 1.5.107. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{J}(CYC) = 2.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. 2 consecutive vertices could belong to S which is joint-resolving set related to joint-resolving number. If there are no neutrosophic vertices which are consecutive, then it contradicts with the term joint-resolving set for S. All joint-resolving sets corresponded to joint-resolving number are

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}(CYC) = 2.$$

Thus

$$\mathcal{J}(CYC) = 2.$$

∎

Proposition 1.5.108. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $(\mathcal{O}(CYC) \times (2^{\mathcal{O}(CYC)-2} - 1)) + 1$ joint-resolving sets.*

Proposition 1.5.109. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $\mathcal{O}(CYC)$ joint-resolving set corresponded to joint-resolving number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.110. There are two sections for clarifications.

(a) In Figure (2.77), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, there are only two paths between them;

(ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$

$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$
$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$
$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$
$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$
$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$
$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$
$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$
$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$
$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$
$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$
$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$
$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$
$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$
$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$
$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$
$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$
$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$
$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$
$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$
$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$
$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$
$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$
$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are ninety-one joint-resolving sets

$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$
$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$
$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$
$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$
$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$
$\{n_1, n_2, n_3, n_4, n_5, n_6\},$
$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$
$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$
$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$
$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$
$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$
$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$
$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$

$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$
$$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$
$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 1.7.$$

S is $\{n_4, n_5\}$ corresponded to neutrosophic joint-resolving number.

(b) In Figure (2.78), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, there are only two paths between them;

(ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$
$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(*v*) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$
$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(*vi*) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 2.7.$$

S is $\{n_1, n_5\}$ corresponded to neutrosophic joint-resolving number.

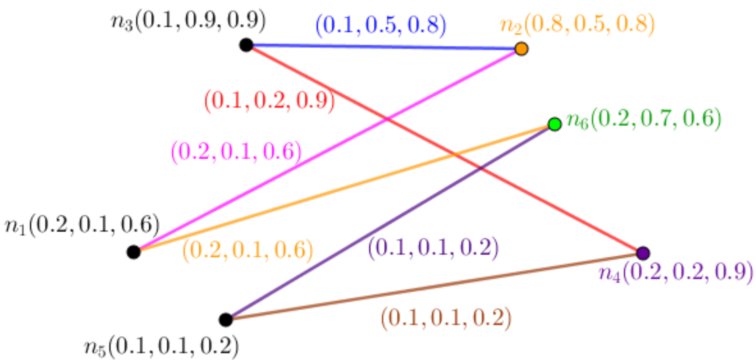

Figure 1.61: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG5

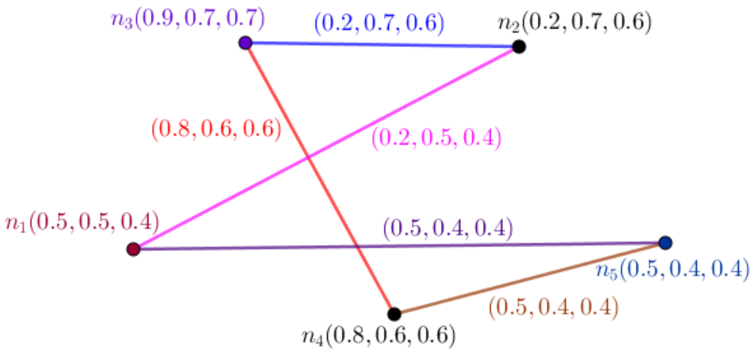

Figure 1.62: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG6

Definition 1.5.111. (perfect-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called **perfect-dominating set**. The minimum cardinality between all perfect-dominating sets is called **perfect-dominating number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called **perfect-dominating set**. The minimum neutrosophic cardinality between all perfect-dominating sets is called **neutrosophic perfect-dominating number** and it's denoted by $\mathcal{P}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

83prp9

Proposition 1.5.112. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S dominates if and only if it perfect-dominates.*

Proposition 1.5.113. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and dominating set has one member. Then a vertex of dominating set corresponded to dominating number dominates if and only if it perfect-dominates.*

Proposition 1.5.114. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to perfect-dominating number if and only if for all s in S, there's a vertex n in $V \setminus S$, such that $\{s' \mid s'n \in E\} \cap S = \{s\}$.*

Proposition 1.5.115. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{P}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. All perfect-dominating sets corresponded to perfect-dominating number are

$$\{n_1, n_4, \ldots\}_{|S|=\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}, \{n_2, n_5, \ldots\}_{|S|=\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}, \ldots,$$

where last vertices could be neighbors as if they couldn't have less than three edges amid them. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum cardinality between all perfect-dominating sets is called perfect-dominating number and it's denoted by

$$\mathcal{P}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

Thus

$$\mathcal{P}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.116. There are two sections for clarifications.

(a) In Figure (2.63), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex couldn't be dominated by more than one vertex as if the structure of dominating and perfect-dominating are the same in the terms of sets and numbers where only some sets coincide;

(iii) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1, n_4\}, \{n_2, n_5\}$, and $\{n_3, n_6\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum cardinality between all perfect-dominating sets is called perfect-dominating number and it's denoted by $\mathcal{P}(CYC) = 2$ and corresponded to perfect-dominating sets are $\{n_1, n_4\}, \{n_2, n_5\}$, and $\{n_3, n_6\}$;

(iv) there are ten perfect-dominating sets

$$\begin{aligned}
&\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\},\\
&\{n_1, n_4, n_5, n_6\}, \{n_1, n_2, n_3, n_4\}, \{n_2, n_5, n_6, n_1\},\\
&\{n_2, n_3, n_4, n_5\}, \{n_3, n_6, n_1, n_2\}, \{n_3, n_4, n_5, n_6\},\\
&\{n_1, n_2, , n_3, n_4, n_5, n_6\},
\end{aligned}$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is characteristic;

(v) there are ten perfect-dominating sets

$$\begin{aligned}
&\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\},\\
&\{n_1, n_4, n_5, n_6\}, \{n_1, n_2, n_3, n_4\}, \{n_2, n_5, n_6, n_1\},\\
&\{n_2, n_3, n_4, n_5\}, \{n_3, n_6, n_1, n_2\}, \{n_3, n_4, n_5, n_6\},\\
&\{n_1, n_2, , n_3, n_4, n_5, n_6\},
\end{aligned}$$

corresponded to perfect-dominating number as if there's one perfect-dominating set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is the determiner;

(vi) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1, n_4\}, \{n_2, n_5\}$, and $\{n_3, n_6\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum neutrosophic cardinality between all perfect-dominating sets is called neutrosophic perfect-dominating number and it's denoted by $\mathcal{P}_n(CYC) = 2.2$ and corresponded to perfect-dominating sets are $\{n_1, n_4\}$.

(b) In Figure (2.64), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex couldn't be dominated by more than one vertex as if the structure of dominating and perfect-dominating are the same in the terms of sets and numbers where only some sets coincide;

(iii) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum cardinality between all perfect-dominating sets is called perfect-dominating number and it's denoted by $\mathcal{P}(CYC) = 3$ and corresponded to perfect-dominating sets are $\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$;

(iv) there are five perfect-dominating sets

$$\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is characteristic;

(v) there are five perfect-dominating sets

$$\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\},$$

corresponded to perfect-dominating number as if there's one perfect-dominating set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is the determiner;

(vi) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only

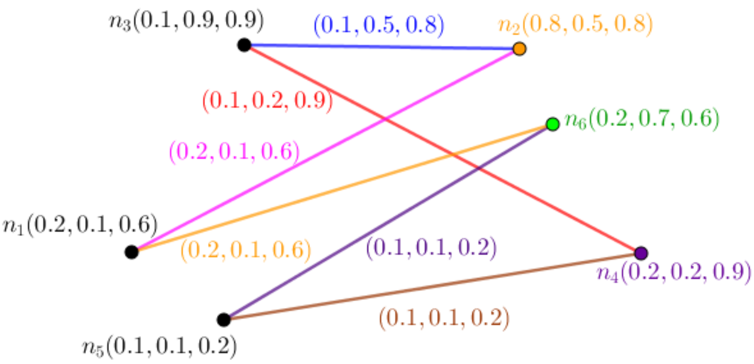

Figure 1.63: A Neutrosophic Graph in the Viewpoint of its perfect-dominating number and its neutrosophic perfect-dominating number. 83NTG5

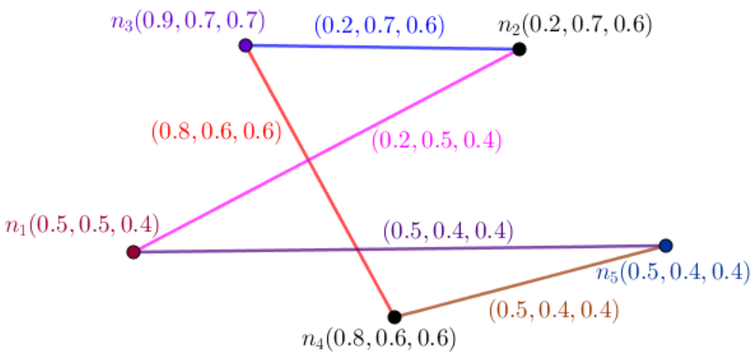

Figure 1.64: A Neutrosophic Graph in the Viewpoint of its perfect-dominating number and its neutrosophic perfect-dominating number. 83NTG6

one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum neutrosophic cardinality between all perfect-dominating sets is called neutrosophic perfect-dominating number and it's denoted by $\mathcal{P}_n(CYC) = 4.2$ and corresponded to perfect-dominating sets are $\{n_2, n_5, n_1\}$.

Definition 1.5.117. (perfect-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called **perfect-resolving set**. The minimum cardinality between all perfect-resolving sets is called **perfect-resolving number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) for given vertices n and n' if $d(s,n) \neq d(s,n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called **perfect-resolving set**. The minimum neutrosophic cardinality between all perfect-resolving sets is called **neutrosophic perfect-resolving number** and it's denoted by $\mathcal{P}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

84prp9

Proposition 1.5.118. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S resolves if and only if it perfect-resolves.*

Proposition 1.5.119. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and resolving set has one member. Then a vertex of resolving set corresponded to resolving number resolves if and only if it perfect-resolves.*

Proposition 1.5.120. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to perfect-resolving number if and only if for all s in S, there are neutrosophic vertices n and n' in $V \setminus S$, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$ and for all neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$.*

84prp12

Proposition 1.5.121. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V and $V \setminus \{x\}$ are S.*

Proposition 1.5.122. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{P}(CYC) = \mathcal{O}(CYC) - 1.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, two vertices couldn't be resolved by more than one vertex so as the structure of resolving and perfect-resolving are different in the terms of sets. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't perfect-resolve, by S has two members in settings of resolving as if these vertices aren't unique in the terms of resolving since some vertices are resolved by both of them and adding them to intended growing set is useless. Thus, by Proposition (2.5.121), S has either $\mathcal{O}(CYC)$ or $\mathcal{O}(CYC) - 1$. All perfect-resolving sets corresponded to perfect-resolving number are

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\},$$
$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\},$$

$$\dots$$
$$\{n_2, n_3, n_4, \dots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\},$$

For given vertices n and n' if $d(s,n) \neq d(s,n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum cardinality between all perfect-resolving sets is called perfect-resolving number and it's denoted by

$$\mathcal{P}(CYC) = \mathcal{O}(CYC) - 1$$

and corresponded to perfect-resolving sets are

$$\{n_1, n_2, n_3, \dots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_1, n_2, n_3, \dots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\},$$
$$\{n_1, n_2, n_3, \dots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\},$$
$$\dots$$
$$\{n_2, n_3, n_4, \dots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\}.$$

Thus

$$\mathcal{P}(CYC) = \mathcal{O}(CYC) - 1.$$

■

Proposition 1.5.123. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then perfect-resolving number isn't equal to resolving number.*

Proposition 1.5.124. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then the number of perfect-resolving sets corresponded to perfect-resolving number is equal to $\mathcal{O}(CYC)$.*

Proposition 1.5.125. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then the number of perfect-resolving sets is equal to $\mathcal{O}(CYC) + 1$.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.126. There are two sections for clarifications.

(a) In Figure (2.65), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, two vertices couldn't be resolved by more than one vertex so as the structure of resolving and perfect-resolving are different in the terms of sets. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't perfect-resolve, by S has two members in settings of resolving as if these vertices aren't unique in the terms of resolving since some vertices are resolved by both of them and adding them to intended growing set is useless. Thus, by Proposition (2.5.121), S has either $\mathcal{O}(CYC)$ or $\mathcal{O}(CYC) - 1$;

(iii) all perfect-resolving sets corresponded to perfect-resolving number are

$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\},$$
$$\{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$

For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum cardinality between all perfect-resolving sets is called perfect-resolving number and it's denoted by $\mathcal{P}(CYC) = 5$ and corresponded to perfect-resolving sets are

$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\},$$
$$\{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\};$$

(iv) there are seven perfect-resolving sets

$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\},$$
$$\{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is characteristic;

(v) there are six perfect-resolving sets

$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\},$$
$$\{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$

corresponded to perfect-resolving number as if there's one perfect-resolving set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is the determiner;

(vi) all perfect-resolving sets corresponded to perfect-resolving number are $\{n_1\}$ and $\{n_6\}$. For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and

d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum neutrosophic cardinality between all perfect-resolving sets is called neutrosophic perfect-resolving number and it's denoted by $\mathcal{P}_n(CYC) = 6$ and corresponded to perfect-resolving sets are $\{n_1, n_3, n_4, n_5, n_6\}$.

(b) In Figure (2.66), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, two vertices couldn't be resolved by more than one vertex so as the structure of resolving and perfect-resolving are different in the terms of sets. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't perfect-resolve, by S has two members in settings of resolving as if these vertices aren't unique in the terms of resolving since some vertices are resolved by both of them and adding them to intended growing set is useless. Thus, by Proposition (2.5.121), S has either $\mathcal{O}(CYC)$ or $\mathcal{O}(CYC) - 1$;

(iii) all perfect-resolving sets corresponded to perfect-resolving number are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\},$$

For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum cardinality between all perfect-resolving sets is called perfect-resolving number and it's denoted by $\mathcal{P}(CYC) = 4$ and corresponded to perfect-resolving sets are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\};$$

(iv) there are six perfect-resolving sets

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is characteristic;

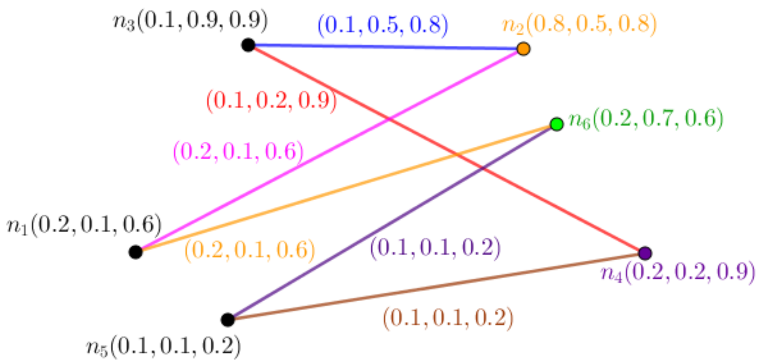

Figure 1.65: A Neutrosophic Graph in the Viewpoint of its perfect-resolving number and its neutrosophic perfect-resolving number. 84NTG5

(v) there are five perfect-resolving sets

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\},$$

corresponded to perfect-resolving number as if there's one perfect-resolving set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is the determiner;

(vi) all perfect-resolving sets corresponded to perfect-resolving number are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\},$$

For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum neutrosophic cardinality between all perfect-resolving sets is called neutrosophic perfect-resolving number and it's denoted by $\mathcal{P}_n(CYC) = 6.6$ and corresponded to perfect-resolving sets are $\{n_1, n_2, n_4, n_5\}$.

Definition 1.5.127. (total-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called **total-dominating**

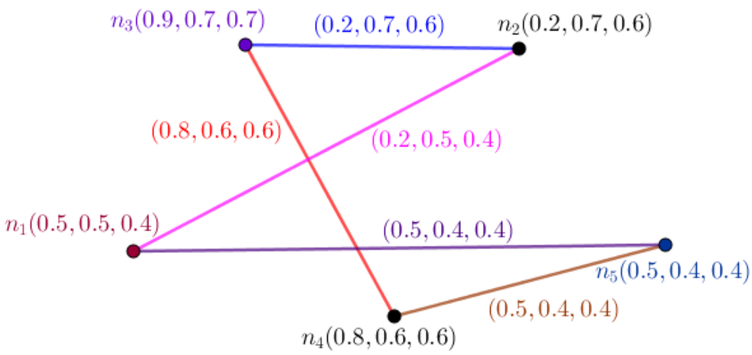

Figure 1.66: A Neutrosophic Graph in the Viewpoint of its perfect-resolving number and its neutrosophic perfect-resolving number.

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set. The minimum cardinality between all total-dominating sets is called **total-dominating number** and it's denoted by $\mathcal{T}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called **total-dominating set**. The minimum neutrosophic cardinality between all total-dominating sets is called **neutrosophic total-dominating number** and it's denoted by $\mathcal{T}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

Proposition 1.5.128. *Let* $NTG : (V, E, \sigma, \mu)$ *be a neutrosophic graph. Then* $|S| \geq 2$.

Proposition 1.5.129. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

$$\mathcal{T}(CYC) = (\lfloor)\lceil \frac{\mathcal{O}(CYC)}{2} \rceil(\rfloor)(+1).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself. Thus two neighbors are necessary in S. All total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5, n_6, n_9, n_{10} \ldots\}, \{n_2, n_3, n_6, n_7, n_{10}, n_{11} \ldots\}, \{n_2, n_3, n_4, n_7, n_8, \ldots\},$$
$$\ldots$$
$$\{\ldots, n_{\mathcal{O}(CYC)-10}, n_{\mathcal{O}(CYC)-9,\mathcal{O}(CYC)-6}, n_{\mathcal{O}(CYC)-5}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\}$$
$$\{\ldots, n_{\mathcal{O}(CYC)-9}, n_{\mathcal{O}(CYC)-8,\mathcal{O}(CYC)-5}, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\}.$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum cardinality between all total-dominating sets is called total-dominating number and it's denoted by

$$\mathcal{T}(CYC) = (\lfloor)\lceil \frac{\mathcal{O}(CYC)}{2} \rceil(\rfloor)(+1)$$

and corresponded to total-dominating sets are

$$\{n_1, n_2, n_5, n_6, n_9, n_{10} \ldots\}, \{n_2, n_3, n_6, n_7, n_{10}, n_{11} \ldots\}, \{n_2, n_3, n_4, n_7, n_8, \ldots\},$$
$$\ldots$$
$$\{\ldots, n_{\mathcal{O}(CYC)-10}, n_{\mathcal{O}(CYC)-9,\mathcal{O}(CYC)-6}, n_{\mathcal{O}(CYC)-5}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\}$$
$$\{\ldots, n_{\mathcal{O}(CYC)-9}, n_{\mathcal{O}(CYC)-8,\mathcal{O}(CYC)-5}, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\}.$$

Thus

$$\mathcal{T}(CYC) = (\lfloor)\lceil \frac{\mathcal{O}(CYC)}{2} \rceil(\rfloor)(+1).$$

■

Proposition 1.5.130. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then total-dominating number isn't equal to dominating number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.131. There are two sections for clarifications.

(a) In Figure (2.67), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) For given neutrosophic vertex, s, there are only two paths with other vertices;

  (ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself. Thus two neighbors are necessary in S;

(*iii*) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\},$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum cardinality between all total-dominating sets is called total-dominating number and it's denoted by $\mathcal{T}(CYC) = 4$ and corresponded to total-dominating sets are

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\};$$

(*iv*) there are sixteen total-dominating sets

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\},$$
$$\{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_6, n_2, n_3, n_4, n_5\}, \{n_6, n_1, n_3, n_4, n_5\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_5, n_1, n_2, n_3, n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is characteristic;

(*v*) there are nine total-dominating sets

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\},$$

corresponded to total-dominating number as if there's one total-dominating set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is the determiner;

(*vi*) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\},$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum neutrosophic cardinality between all total-dominating sets is called neutrosophic total-dominating number and it's denoted by $\mathcal{T}_n(CYC) = 4.1$ and corresponded to total-dominating sets are

$$\{n_4, n_5, n_1, n_6\}.$$

(b) In Figure (2.68), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself. Thus two neighbors are necessary in S;

(iii) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\},$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum cardinality between all total-dominating sets is called total-dominating number and it's denoted by $\mathcal{T}(CYC) = 3$ and corresponded to total-dominating sets are

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\};$$

(iv) there are eleven total-dominating sets

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is characteristic;

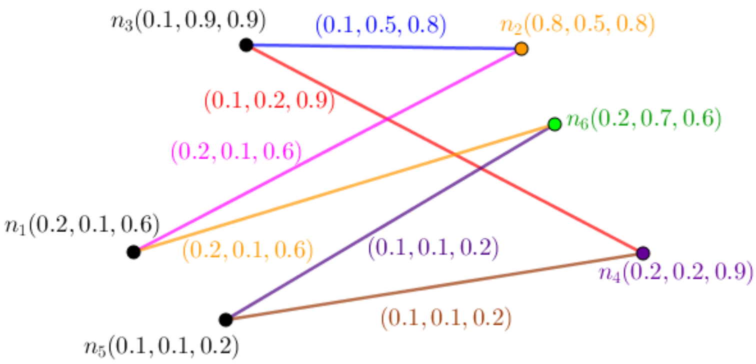

Figure 1.67: A Neutrosophic Graph in the Viewpoint of its total-dominating number and its neutrosophic total-dominating number.

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(v) there are five total-dominating sets

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\},$$

corresponded to total-dominating number as if there's one total-dominating set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is the determiner;

(vi) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\},$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum neutrosophic cardinality between all total-dominating sets is called neutrosophic total-dominating number and it's denoted by $\mathcal{T}_n(CYC) = 4.2$ and corresponded to total-dominating sets are

$$\{n_1, n_2, n_5\}.$$

Definition 1.5.132. (total-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices, $d \geq 1$ and all vertices have to be total-resolved otherwise it will be mentioned which is about $d \geq 0$ in some cases but all vertices have to be total-resolved forever. Let S be a set of neutrosophic vertices [a vertex alongside triple

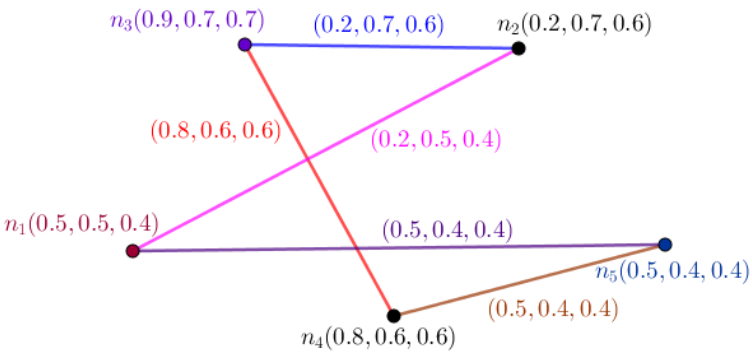

Figure 1.68: A Neutrosophic Graph in the Viewpoint of its total-dominating number and its neutrosophic total-dominating number. 85NTG6

pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called **total-resolving set**. The minimum cardinality between all total-resolving sets is called **total-resolving number** and it's denoted by $\mathcal{T}(NTG)$;

(ii) for given vertices n and n' if $d(s,n) \neq d(s,n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices, $d \geq 1$ and all vertices have to be total-resolved otherwise it will be mentioned which is about $d \geq 0$ in some cases but all vertices have to be total-resolved forever. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called **total-resolving set**. The minimum neutrosophic cardinality between all total-resolving sets is called **neutrosophic total-resolving number** and it's denoted by $\mathcal{T}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

Proposition 1.5.133. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then* $|S| \geq 2$.

86prp10 **Proposition 1.5.134.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then if there are twin vertices then total-resolving set and total-resolving number are Not Existed.*

Proposition 1.5.135. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$ and $d \geq 0$. Then*

$$\mathcal{T}(CYC) = 2.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve since a vertex couldn't resolve itself. Thus two [minus antipodal pairs] vertices are necessary in S. All total-resolving sets corresponded to total-resolving number are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(PTH)-2}\}, \{n_1, n_{\mathcal{O}(PTH)-1}\}, \{n_1, n_{\mathcal{O}(PTH)}\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_2, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(PTH)-2}\}, \{n_2, n_{\mathcal{O}(PTH)-1}\}, \{n_2, n_{\mathcal{O}(PTH)}\},$$
$$\{n_3, n_4\}, \{n_2, n_5\}, \{n_2, n_6\}, \ldots, \{n_2, n_{\mathcal{O}(PTH)-2}\}, \{n_2, n_{\mathcal{O}(PTH)-1}\}, \{n_2, n_{\mathcal{O}(PTH)}\},$$
$$\ldots,$$
$$\{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)-2}\}, \{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)-1}\}, \{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)}\},$$
$$\{n_{\mathcal{O}(PTH)-2}, n_{\mathcal{O}(PTH)-1}\}, \{n_{\mathcal{O}(PTH)-2}, n_{\mathcal{O}(PTH)}\},$$
$$\{n_{\mathcal{O}(PTH)-1}, n_{\mathcal{O}(PTH)}\}$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum cardinality between all total-resolving sets is called total-resolving number and it's denoted by $\mathcal{T}(CYC) = 2$ and corresponded to total-resolving sets are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(PTH)-2}\}, \{n_1, n_{\mathcal{O}(PTH)-1}\}, \{n_1, n_{\mathcal{O}(PTH)}\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_2, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(PTH)-2}\}, \{n_2, n_{\mathcal{O}(PTH)-1}\}, \{n_2, n_{\mathcal{O}(PTH)}\},$$
$$\{n_3, n_4\}, \{n_2, n_5\}, \{n_2, n_6\}, \ldots, \{n_2, n_{\mathcal{O}(PTH)-2}\}, \{n_2, n_{\mathcal{O}(PTH)-1}\}, \{n_2, n_{\mathcal{O}(PTH)}\},$$
$$\ldots,$$
$$\{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)-2}\}, \{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)-1}\}, \{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)}\},$$
$$\{n_{\mathcal{O}(PTH)-2}, n_{\mathcal{O}(PTH)-1}\}, \{n_{\mathcal{O}(PTH)-2}, n_{\mathcal{O}(PTH)}\},$$
$$\{n_{\mathcal{O}(PTH)-1}, n_{\mathcal{O}(PTH)}\}$$

Thus
$$\mathcal{T}(CYC) = 2.$$

■

Proposition 1.5.136. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $d \geq 0$. Then total-resolving number is equal to resolving number.*

Antipodal vertices in even-cycle-neutrosophic graph differ the number in cycle-neutrosophic graph.

Proposition 1.5.137. *Let $NTG : (V, E, \sigma, \mu)$ be an odd-cycle-neutrosophic graph where $d \geq 0$. Then the number of total-resolving sets corresponded to total-resolving number is equal to $\mathcal{O}(CYC)$ choose two.*

Proposition 1.5.138. *Let $NTG : (V, E, \sigma, \mu)$ be an odd-cycle-neutrosophic graph where $d \geq 0$. Then the number of total-resolving sets is equal to $2^{\mathcal{O}(CYC)} - \mathcal{O}(CYC) - 1$.*

We've to eliminate antipodal vertices due to total-resolving implies complete resolving.

Proposition 1.5.139. *Let $NTG : (V, E, \sigma, \mu)$ be an even-cycle-neutrosophic graph where $d \geq 0$. Then the number of total-resolving sets corresponded to total-resolving number is equal to $\mathcal{O}(CYC)$ choose two after that minus $\mathcal{O}(CYC)$.*

Proposition 1.5.140. *Let $NTG : (V, E, \sigma, \mu)$ be an even-cycle-neutrosophic graph where $d \geq 0$. Then the number of total-resolving sets is equal to $2^{\mathcal{O}(CYC)} - 2\mathcal{O}(CYC) - 1$.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.141. There are two sections for clarifications.

(a) In Figure (2.69), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve since a vertex couldn't resolve itself. Thus two [minus antipodal pairs] vertices are necessary in S. Antipodal pairs are

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\};$$

(iii) all total-resolving sets corresponded to total-resolving number are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \ldots.$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum cardinality between all total-resolving sets is called total-resolving number and it's denoted by $\mathcal{T}(CYC) = 2$ and corresponded to total-resolving sets are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \ldots;$$

(*iv*) there are fifty-seven [minus antipodal pairs] total-resolving sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},$$
$$\{n_1, n_4, n_5\}, \{n_2, n_3, n_4\}, \{n_2, n_3, n_5\},$$
$$\{n_2, n_4, n_5\}, \{n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \ldots$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is characteristic;

(*v*) there are fifteen [minus antipodal pairs] total-resolving sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \ldots,$$

corresponded to total-resolving number as if there's one total-resolving set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is the determiner;

(*vi*) all total-resolving sets corresponded to total-resolving number are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \ldots.$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum neutrosophic cardinality between all total-resolving sets is called neutrosophic total-resolving number and it's denoted by $\mathcal{T}_n(CYC) = 1.3$ and corresponded to total-resolving sets are

$$\{n_1, n_5\}.$$

(*b*) In Figure (2.70), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) For given neutrosophic vertex, s, there are only two paths with other vertices;

(*ii*) in the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve since a vertex couldn't resolve itself. Thus two vertices are necessary in S;

(*iii*) all total-resolving sets corresponded to total-resolving number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}.$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum cardinality between all total-resolving sets is called total-resolving number and it's denoted by $\mathcal{T}(CYC) = 2$ and corresponded to total-resolving sets are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\};$$

(*iv*) there are twenty-six total-resolving sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},$$
$$\{n_1, n_4, n_5\}, \{n_2, n_3, n_4\}, \{n_2, n_3, n_5\},$$
$$\{n_2, n_4, n_5\}, \{n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is characteristic;

(*v*) there are ten total-resolving sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\},$$

corresponded to total-resolving number as if there's one total-resolving set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is the determiner;

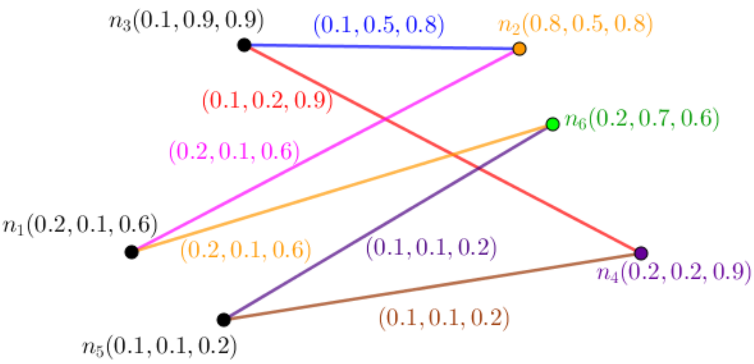

Figure 1.69: A Neutrosophic Graph in the Viewpoint of its total-resolving number and its neutrosophic total-resolving number.

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(vi) all total-resolving sets corresponded to total-resolving number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}.$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum neutrosophic cardinality between all total-resolving sets is called neutrosophic total-resolving number and it's denoted by $\mathcal{T}_n(CYC) = 2.7$ and corresponded to total-resolving sets are

$$\{n_1, n_5\}.$$

Definition 1.5.142. (stable-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-dominating set**. The minimum cardinality between all stable-dominating sets is called **stable-dominating number** and it's denoted by $\mathcal{S}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where

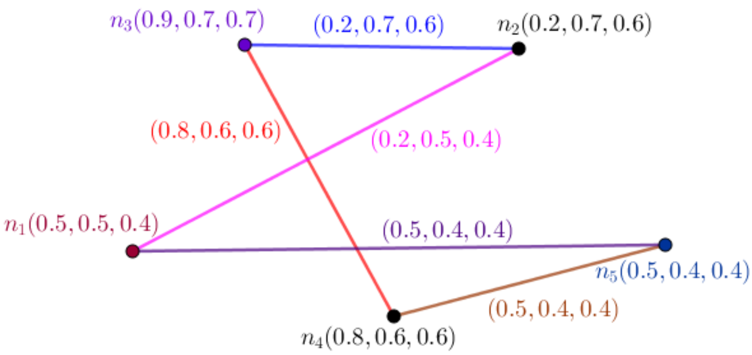

Figure 1.70: A Neutrosophic Graph in the Viewpoint of its total-resolving number and its neutrosophic total-resolving number. 86NTG6

for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-dominating set**. The minimum neutrosophic cardinality between all stable-dominating sets is called **neutrosophic stable-dominating number** and it's denoted by $\mathcal{S}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

86prp9 **Proposition 1.5.143.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Assume $|S|$ has one member. Then*

(i) a vertex dominates if and only if it stable-dominates;

(ii) S is dominating set if and only if it's stable-dominating set;

(iii) a number is dominating number if and only if it's stable-dominating number.

Proposition 1.5.144. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is stable-dominating set corresponded to stable-dominating number if and only if for every neutrosophic vertex s in S, there's at least a neutrosophic vertex n in $V \setminus S$ such that $\{s' \in S \mid s'n \in E\} = \{s\}$.*

Proposition 1.5.145. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V isn't S.*

Proposition 1.5.146. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then stable-dominating number is between one and $\mathcal{O}(NTG) - 1$.*

Proposition 1.5.147. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then stable-dominating number is between one and $\mathcal{O}_n(NTG) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x)$.*

Proposition 1.5.148. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{S}(CYC) = \lceil \frac{\mathcal{O}(CYC)}{3} \rceil.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates since a vertex dominates neighbors thus in S, there aren't any neighbors and all vertices are neighborless in S. All stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4, n_7, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_2, n_5, n_8, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}\},$$
$$\ldots.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum cardinality between all stable-dominating sets is called stable-dominating number and it's denoted by

$$\mathcal{S}(CYC) = \lceil \frac{\mathcal{O}(CYC)}{3} \rceil$$

and corresponded to stable-dominating sets are

$$\{n_1, n_4, n_7, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_2, n_5, n_8, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}\},$$
$$\ldots.$$

Thus

$$\mathcal{S}(CYC) = \lceil \frac{\mathcal{O}(CYC)}{3} \rceil.$$

■

Proposition 1.5.149. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then stable-dominating number is equal to dominating number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.150. There are two sections for clarifications.

(a) In Figure (2.71), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates since a vertex dominates neighbors thus in S, there aren't any neighbors and all vertices are neighborless in S;

(iii) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum cardinality between all stable-dominating sets is called stable-dominating number and it's denoted by $\mathcal{S}(CYC) = 2$; and corresponded to stable-dominating sets are

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\};$$

(iv) there are five stable-dominating sets

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\},$$
$$\{n_1, n_3, n_5\}, \{n_2, n_4, n_6\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is characteristic;

(v) there are three stable-dominating setsc

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\},$$

corresponded to stable-dominating number as if there's one stable-dominating set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is the determiner;

(vi) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum neutrosophic cardinality between all stable-dominating sets is called neutrosophic stable-dominating number and it's denoted by $\mathcal{S}_n(CYC) = 2.2$; and corresponded to stable-dominating sets are

$$\{n_1, n_4\}.$$

(b) In Figure (2.72), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't stable-dominate since a vertex couldn't dominate itself. Thus two vertices are necessary in S;

(iii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates since a vertex dominates neighbors thus in S, there aren't any neighbors and all vertices are neighborless in S;

(iii) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\},$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum cardinality between all stable-dominating sets is called stable-dominating number and it's denoted by $\mathcal{S}(CYC) = 2$; and corresponded to stable-dominating sets are

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\};$$

(iv) there are five stable-dominating sets

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is characteristic;

(v) there are five stable-dominating sets

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\},$$

corresponded to stable-dominating number as if there's one stable-dominating set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is the determiner;

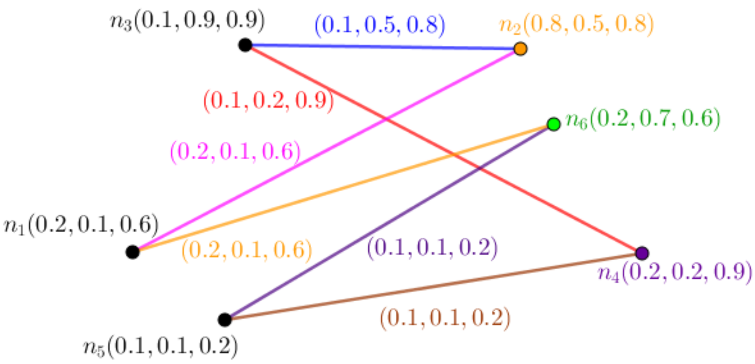

Figure 1.71: A Neutrosophic Graph in the Viewpoint of its stable-dominating number and its neutrosophic stable-dominating number. 87NTG5

(vi) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\},$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum neutrosophic cardinality between all stable-dominating sets is called neutrosophic stable-dominating number and it's denoted by $\mathcal{S}_n(CYC) = 2.8$; and corresponded to stable-dominating sets are

$$\{n_2, n_5\}.$$

Definition 1.5.151. (stable-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-resolving set**. The minimum cardinality between all stable-resolving sets is called **stable-resolving number** and it's denoted by $\mathcal{S}(NTG)$;

(ii) for given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple

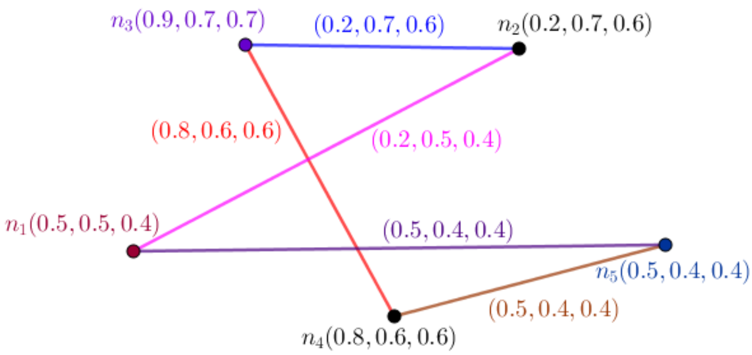

Figure 1.72: A Neutrosophic Graph in the Viewpoint of its stable-dominating number and its neutrosophic stable-dominating number. 87NTG6

pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **neutrosophic stable-resolving set**. The minimum neutrosophic cardinality between all stable-resolving sets is called **neutrosophic stable-resolving number** and it's denoted by $\mathcal{S}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

88prp9 **Proposition 1.5.152.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Assume $|S|$ has one member. Then*

(i) *a vertex resolves if and only if it stable-resolves;*

(ii) *S is resolving set if and only if it's stable-resolving set;*

(iii) *a number is resolving number if and only if it's stable-resolving number.*

Proposition 1.5.153. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is stable-resolving set corresponded to stable-resolving number if and only if for every neutrosophic vertex s in S, there are at least neutrosophic vertices n and n' in $V \setminus S$ such that $\{s' \in S \mid d(s', n) \neq d(s', n')\} = \{s\}$.*

Proposition 1.5.154. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V isn't S.*

Proposition 1.5.155. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{S}(CYC) = 2.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$n_1, n_2, \cdots, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}, n_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve since two neighbors aren't allowed in the setting of stable-resolving. All stable-resolving sets corresponded to stable-resolving number are

$$\{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(CYC)-3}\}, \{n_1, n_{\mathcal{O}(CYC)-2}\}, \{n_1, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_2, n_4\}, \{n_1, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(CYC)-2}\}, \{n_2, n_{\mathcal{O}(CYC)-1}\}, \{n_2, n_{\mathcal{O}(CYC)}\},$$
$$\{n_3, n_5\}, \{n_3, n_6\}, \ldots, \{n_3, n_{\mathcal{O}(CYC)-2}\}, \{n_3, n_{\mathcal{O}(CYC)-1}\}, \{n_3, n_{\mathcal{O}(CYC)}\},$$
$$\ldots$$
$$\{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}\}, \{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-1}\}, \{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)}\},$$
$$\{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\}, \{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\},$$
$$\{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\}.$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum cardinality between all stable-resolving sets is called stable-resolving number and it's denoted by

$$\mathcal{S}(CYC) = 2;$$

and corresponded to stable-resolving sets are

$$\{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(CYC)-3}\}, \{n_1, n_{\mathcal{O}(CYC)-2}\}, \{n_1, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_2, n_4\}, \{n_1, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(CYC)-2}\}, \{n_2, n_{\mathcal{O}(CYC)-1}\}, \{n_2, n_{\mathcal{O}(CYC)}\},$$
$$\{n_3, n_5\}, \{n_3, n_6\}, \ldots, \{n_3, n_{\mathcal{O}(CYC)-2}\}, \{n_3, n_{\mathcal{O}(CYC)-1}\}, \{n_3, n_{\mathcal{O}(CYC)}\},$$
$$\ldots$$
$$\{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}\}, \{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-1}\}, \{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)}\},$$
$$\{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\}, \{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\},$$
$$\{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\}.$$

Thus

$$\mathcal{S}(CYC) = 2.$$

■

Proposition 1.5.156. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then stable-resolving number is equal to resolving number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.157. There are two sections for clarifications.

(a) In Figure (2.73), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve since two neighbors aren't allowed in the setting of stable-resolving;

(iii) all stable-resolving sets corresponded to stable-resolving number are

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\}.$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum cardinality between all stable-resolving sets is called stable-resolving number and it's denoted by $\mathcal{S}(CYC) = 2$; and corresponded to stable-resolving sets are

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\};$$

(iv) there are six stable-resolving sets

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\}, \{n_1, n_3, n_5\}, \{n_2, n_4, n_6\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-resolving number so as neutrosophic cardinality is characteristic;

(v) there are four stable-resolving sets

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\}$$

corresponded to stable-resolving number as if there's one stable-resolving set corresponded to neutrosophic stable-resolving number so as neutrosophic cardinality is the determiner;

(vi) all stable-resolving sets corresponded to stable-resolving number are

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\}.$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum neutrosophic cardinality between all stable-resolving sets is called neutrosophic stable-resolving number and it's denoted by $\mathcal{S}_n(CYC) = 1.3$; and corresponded to stable-resolving sets are

$$\{n_1, n_5\}.$$

(*b*) In Figure (2.74), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) For given neutrosophic vertex, s, there are only two paths with other vertices;

(*ii*) in the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve since two neighbors aren't allowed in the setting of stable-resolving;

(*iii*) all stable-resolving sets corresponded to stable-resolving number are

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}.$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum cardinality between all stable-resolving sets is called stable-resolving number and it's denoted by $\mathcal{S}(CYC) = 2$; and corresponded to stable-resolving sets are

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\};$$

(*iv*) there are four stable-resolving sets

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-resolving number so as neutrosophic cardinality is characteristic;

(*v*) there are four stable-resolving sets

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\};$$

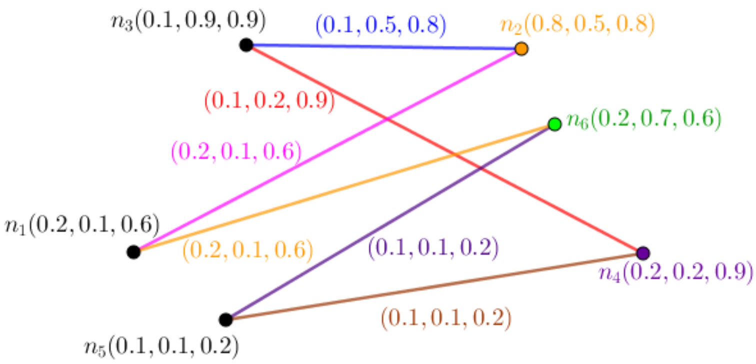

Figure 1.73: A Neutrosophic Graph in the Viewpoint of its stable-resolving number and its neutrosophic stable-resolving number.

88NTG5

corresponded to stable-resolving number as if there's one stable-resolving set corresponded to neutrosophic stable-resolving number so as neutrosophic cardinality is the determiner;

(vi) all stable-resolving sets corresponded to stable-resolving number are

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}.$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum neutrosophic cardinality between all stable-resolving sets is called neutrosophic stable-resolving number and it's denoted by $\mathcal{S}_n(CYC) = 2.8$; and corresponded to stable-resolving sets are

$$\{n_2, n_5\}.$$

1.6 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are cycle-neutrosophic graph.

1.7 Modelling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

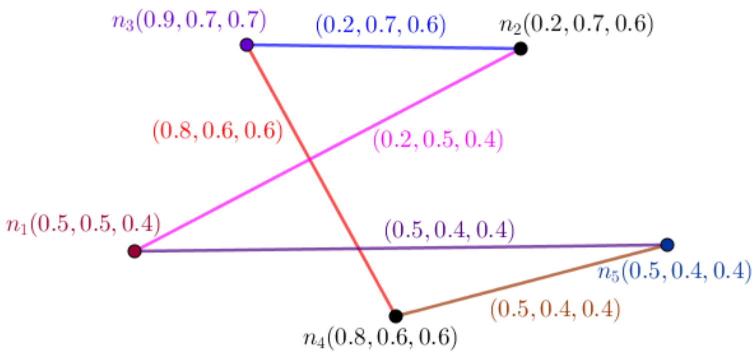

Figure 1.74: A Neutrosophic Graph in the Viewpoint of its stable-resolving number and its neutrosophic stable-resolving number. 88NTG6

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (2.1), clarifies about the assigned numbers to these situations.

Table 1.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model. 88tbl1

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

1.8 Case 1: cycle-neutrosophic Model

Step 4. (Solution) The neutrosophic graph model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application when the notion of family is applied in

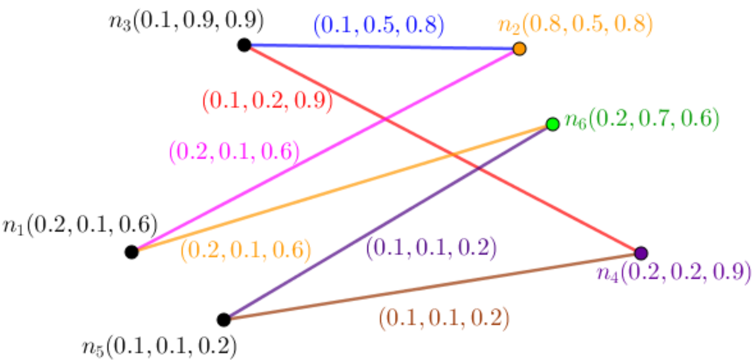

Figure 1.75: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG5

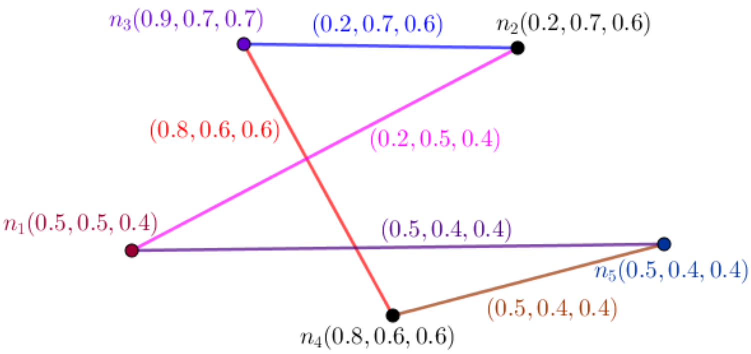

Figure 1.76: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG6

the way that all members of family are from same classes of neutrosophic graphs. As follows, there are five subjects which are represented as Figure (2.75). This model is strong and even more. And the study proposes using specific number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (2.75). In Figure (2.75), a cycle-neutrosophic graph. is illustrated. Some points are represented in follow-up items as follows.

(a) In Figure (2.75), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(*vi*) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(*b*) In Figure (2.76), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$

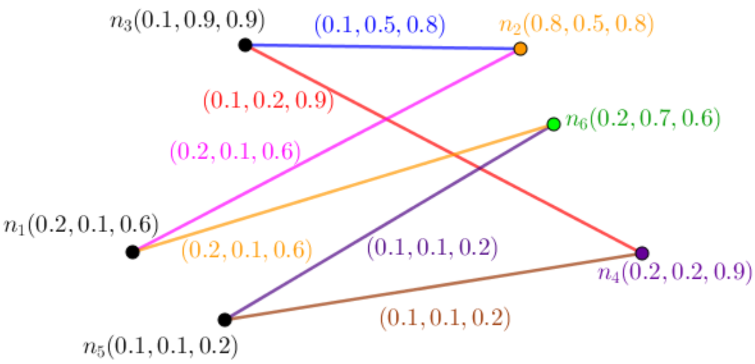

Figure 1.77: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG5

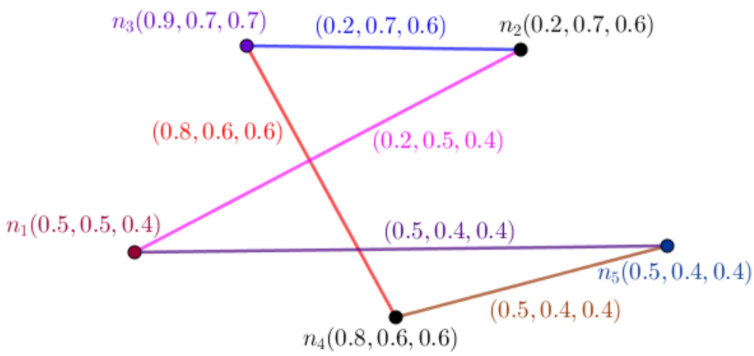

Figure 1.78: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG6

and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

1.9 Case 2: cycle-neutrosophic Model alongside its Neutrosophic Graph

Step 4. (Solution) The neutrosophic graph as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are four subjects which are represented in the formation of one model as Figure (2.77). This model is neutrosophic strong as individual and even more. And the study proposes using specific number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (2.77). There is one section for clarifications.

(a) In Figure (2.77), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices, there are only two paths between them;
- (ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;
- (iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$
$$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$$
$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$
$$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$

$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$
$$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$$
$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$
$$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$
$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 1.7.$$

S is $\{n_4, n_5\}$ corresponded to neutrosophic joint-resolving number.

(b) In Figure (2.78), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, there are only two paths between them;

(ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$
$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$

$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 2.7.$$

S is $\{n_1, n_5\}$ corresponded to neutrosophic joint-resolving number.

1.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are defined in cycle-neutrosophic graphs. Thus,

Question 1.10.1. *Is it possible to use other types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable?*

Question 1.10.2. *Are existed some connections amid different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs?*

Question 1.10.3. *Is it possible to construct some classes of cycle-neutrosophic graphs which have "nice" behavior?*

Question 1.10.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 1.10.5. *Which parameters are related to this parameter?*

Problem 1.10.6. *Which approaches do work to construct applications to create independent study?*

Problem 1.10.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

1.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.
This study uses some definitions concerning different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs. Further

Table 1.2: A Brief Overview about Advantages and Limitations of this Study 88tbl

Advantages	Limitations
1. Neutrosophic Numbers of Model	1. Connections amid Classes
2. Acting on All Edges	
3. Minimal Sets	2. Study on Families
4. Maximal Sets	
5. Acting on All Vertices	3. Same Models in Family

studies could be about changes in the settings to compare these notions amid different settings of cycle-neutrosophic graphs. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2.2), some limitations and advantages of this study are pointed out.

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CHAPTER 2

Neutrosophic Tools

2.1 Abstract

New setting is introduced to study different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs. Minimum number and maximum number of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, is a number which is representative based on those vertices or edges. Minimum or maximum neutrosophic number or polynomial of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are called neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number or polynomial. Forming sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable to figure out different types of number of vertices in the sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable sets in the terms of minimum (maximum) number of vertices to get minimum (maximum) number to assign in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs, is key type of approach to have

these notions namely different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets. As concluding results, there are some statements, remarks, examples and clarifications about cycle-neutrosophic graphs. The clarifications are also presented in both sections "Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable
AMS Subject Classification: 05C17, 05C22, 05E45

2.2 Background

Different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable are addressed in Bibliography. Specially, properties of SuperHyperGraph and neutrosophic SuperHyperGraph by Henry Garrett (2022), is studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book by Henry Garrett (2022).
In this section, I use two sections to illustrate a perspective about the background of this study.

2.3 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 2.3.1. *Is it possible to use mixed versions of ideas concerning "different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial", "neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable* *number and polynomial" and "cycle-neutrosophic graphs" to define some notions which are applied to cycle-neutrosophic graphs?*

It's motivation to find notions to use in cycle-neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. In both settings, corresponded numbers or polynomials conclude the discussion. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In section "Preliminaries", new notions of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial' in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. As concluding results, there are some statements, remarks, examples and clarifications about cycle-neutrosophic graphs. The clarifications are also presented in both sections 'Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for complete notions, namely cycle-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

2.4 Preliminaries

In this section, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 2.4.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 2.4.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v\in V}\sum_{i=1}^{3}\sigma_i(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e\in E}\sum_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 2.4.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1}\mu(x_ix_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t}\ \bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1});$$

(iv) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1}\mu(v_iv_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

Definition 2.4.4. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0,1]$, and $\mu_i : E \to [0,1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 2.4.5. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_σ if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH where $x_ix_{i+1} \in E$, $i = 0, 1, \cdots, n-1$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** and denoted by CYC where $x_ix_{i+1} \in E$, $i = 0, 1, \cdots, n-1$, $x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_iv_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1,σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1,σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1,σ_2}.

Remark 2.4.6. Using notations which is mixed with literatures, are reviewed.

2.4.6.1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$, $\mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;

2.4.6.2. $CMT_\sigma, PTH, CYC, STR_{1,\sigma_2}, CMT_{\sigma_1,\sigma_2}, CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$, and WHL_{1,σ_2}.

2.5 Setting of notion neutrosophic-number

In this section, I provide some results in the setting of neutrosophic notion number.

Definition 2.5.1. (Zero Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Zero forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(ii) **Zero forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

Proposition 2.5.2. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}_n(NTG) = \min\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y)\}_{xy\in E.}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex is a neighbor for two vertices. Two vertices which are neighbors, are only members of S is a set of black vertices. Thus the color-change rule implies all vertices are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}_n(NTG) = \min\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y)\}_{xy\in E.}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.3. There are two sections for clarifications.

(a) In Figure (2.1), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";
- (ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";
- (iii) if $S = \{n_2\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(v) 2 is zero forcing number and its corresponded sets are $\{n_1, n_2\}$,$\{n_1, n_3\}$,
$\{n_1, n_4\}$,$\{n_1, n_5\}$,$\{n_1, n_6\}$,$\{n_2, n_3\}$,
$\{n_2, n_4\}$,$\{n_2, n_5\}$,$\{n_2, n_6\}$,$\{n_3, n_4\}$,
$\{n_3, n_5\}$,
$\{n_3, n_6\}$,
$\{n_4, n_5\}$,$\{n_4, n_6\}$, and
$\{n_5, n_6\}$;

(vi) 1.3 is zero forcing neutrosophic-number and its corresponded set is $\{n_1, n_5\}$.

(b) In Figure (2.2), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(v) 2 is zero forcing number and its corresponded sets are $\{n_1, n_2\}$,$\{n_1, n_3\}$,
$\{n_1, n_4\}$,$\{n_1, n_5\}$,
$\{n_2, n_3\}$,$\{n_2, n_4\}$,
$\{n_2, n_5\}$,$\{n_3, n_4\}$,
$\{n_3, n_5\}$, and $\{n_4, n_5\}$;

(vi) 2.7 is zero forcing neutrosophic-number and its corresponded set is $\{n_1, n_5\}$.

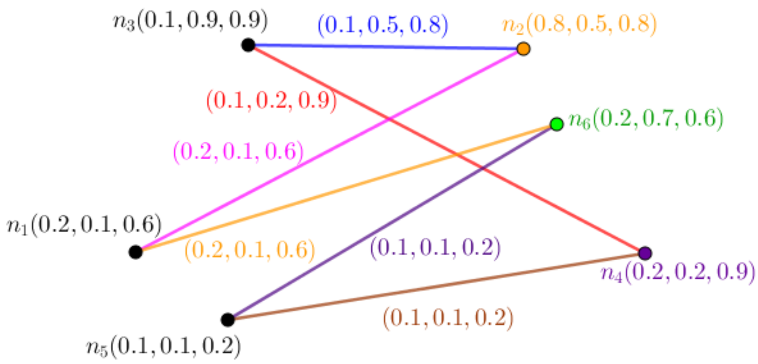

Figure 2.1: A Neutrosophic Graph in the Viewpoint of its Zero Forcing Neutrosophic-Number. 47NTG13

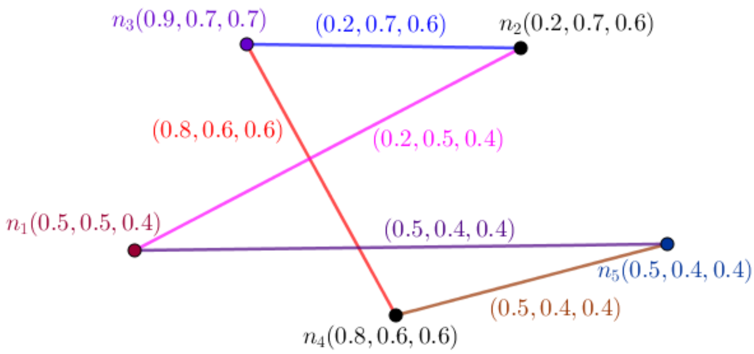

Figure 2.2: A Neutrosophic Graph in the Viewpoint of its Zero Forcing Neutrosophic-Number. 47NTG14

The main definition is presented in next section. The notions of failed zero-forcing number and failed zero-forcing neutrosophic-number facilitate the ground to introduce new results. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types. New setting is introduced to study failed zero-forcing number and failed zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex but now reverse approach is on demand which is finding biggest set which doesn't force.

Definition 2.5.4. (Failed Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Failed zero-forcing number** $\mathcal{Z}'(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(*ii*) **Failed zero-forcing neutrosophic-number** $\mathcal{Z}'_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

Proposition 2.5.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}'_n(NTG) = \max\{\Sigma_{i=1}^3 \sigma_i(x_j) + \Sigma_{i=1}^3 \sigma_i(x_{j+s}) + \cdots\}_{s \geq 2}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex is a neighbor for two vertices. Vertices with distance two, are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'_n(NTG) = \max\{\Sigma_{i=1}^3 \sigma_i(x_j) + \Sigma_{i=1}^3 \sigma_i(x_{j+s}) + \cdots\}_{s \geq 2}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.6. There are two sections for clarifications.

(*a*) In Figure (2.3), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(*v*) 3 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(*vi*) 4.9 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

(*b*) In Figure (2.4), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

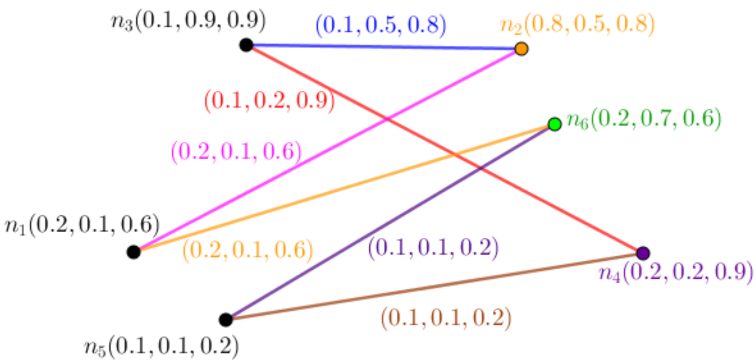

Figure 2.3: A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number. 48NTG5

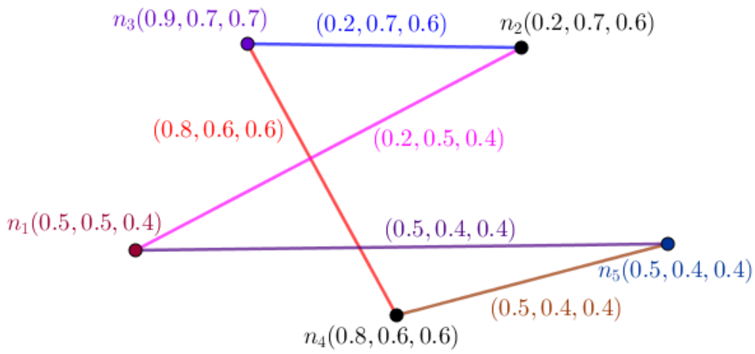

Figure 2.4: A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number. 48NTG6

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4\}$, $\{n_3, n_5\}$, $\{n_2, n_5\}$, $\{n_4, n_1\}$, and $\{n_1, n_3\}$;

(vi) 3.7 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3\}$.

The main definition is presented in next section. The notions of 1-zero-forcing number and 1-zero-forcing neutrosophic-number facilitate the ground to introduce new results. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types. New setting is introduced to study 1-zero-forcing number and 1-zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex and now approach is on demand which is finding smallest set which forces.

Definition 2.5.7. (1-Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-zero-forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

(ii) **1-zero-forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

Proposition 2.5.8. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}_n(NTG) = \min\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x \textit{ is a vertex}}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex is a neighbor for two vertices. Two vertices which are neighbors, are only members of S is a set of black vertices through color-change rule. Thus the color-change rule implies all vertices are black vertices but extra condition implies every given vertex is member of S is a set of black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition. So

$$\mathcal{Z}_n(NTG) = \min\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x \text{ is a vertex}}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.9. There are two sections for clarifications.

(a) In Figure (2.5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices but extra condition implies n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices but extra condition implies n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

(v) 1 is 1-zero-forcing number and its corresponded sets are $\{n_1\}, \{n_2\}, \{n_3\}, \{n_4\}, \{n_5\}$. and $\{n_6\}$;

(vi) 0.4 is 1-zero-forcing neutrosophic-number and its corresponded set is $\{n_5\}$.

(b) In Figure (2.6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices but extra condition implies n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

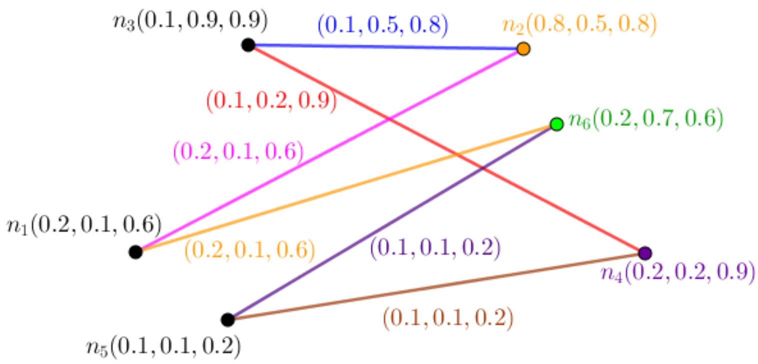

Figure 2.5: A Neutrosophic Graph in the Viewpoint of its 1-Zero-Forcing Number. 49NTG5

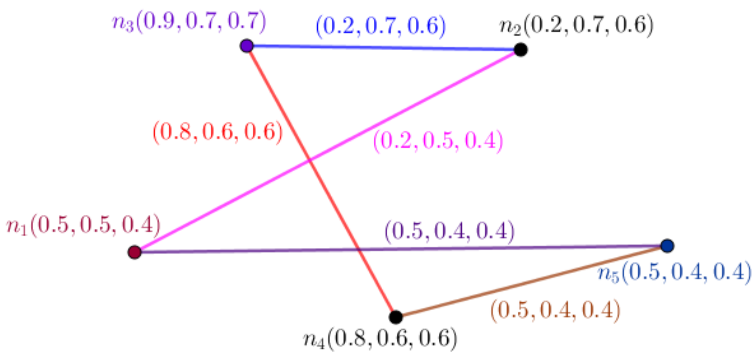

Figure 2.6: A Neutrosophic Graph in the Viewpoint of its 1-Zero-Forcing Number. 49NTG6

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices but extra condition implies n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

(v) 1 is 1-zero-forcing number and its corresponded sets are $\{n_1\}, \{n_2\}, \{n_3\}, \{n_4\}, \{n_5\}$. and $\{n_6\}$;

(vi) 1.3 is 1-zero-forcing neutrosophic-number and its corresponded set is $\{n_5\}$.

Definition 2.5.10. (Independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously;

(ii) **independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S

of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously.

Proposition 2.5.11. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_1) + \sigma_i(x_3) + \cdots + \sigma_i(x_t)),$$

$$\sum_{i=1}^{3}\sigma_i(x_2) + \sigma_i(x_4) + \cdots + \sigma_i(x'_t))\}_{x_ix_{i+1}\in E}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Assume $|S| > \lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S|>\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. In other side, for having an edge, there's a need to have two vertices. So by using the members of $S = \{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from $S = \{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$. It implies that $S = \{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$ is corresponded to independent number. Thus

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_1) + \sigma_i(x_3) + \cdots + \sigma_i(x_t)),$$

$$\sum_{i=1}^{3}\sigma_i(x_2) + \sigma_i(x_4) + \cdots + \sigma_i(x'_t))\}_{x_ix_{i+1}\in E}.$$

∎

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.12. There are two sections for clarifications.

(a) In Figure (2.7), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|\neq\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$;

(*ii*) if $S = \{n_2, n_4, n_6\}$ is a set of vertices, then there's no vertex in S but n_2, n_4 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_2, n_4, n_6\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S and It doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*iv*) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S hence it implies that $S = \{n_1, n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*v*) 3 is independent number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(*vi*) 3.2 is independent neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

(*b*) In Figure (2.8), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S and it implies that $S = \{n_2, n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*ii*) if $S = \{n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It implies that $S = \{n_3, n_5\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using

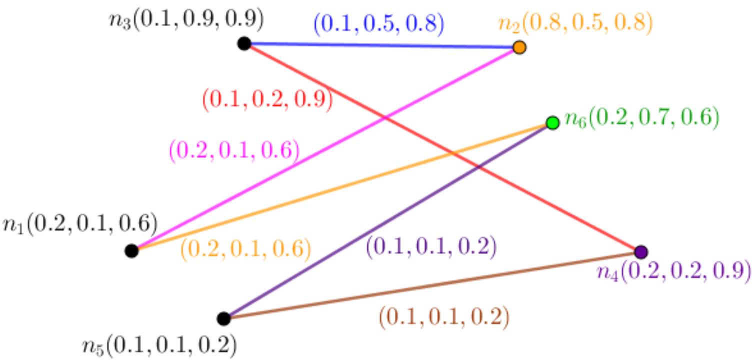

Figure 2.7: A Neutrosophic Graph in the Viewpoint of its Independent Number. 50NTG5

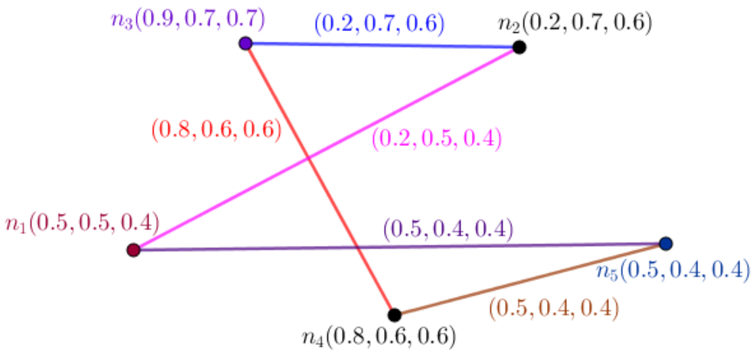

Figure 2.8: A Neutrosophic Graph in the Viewpoint of its Independent Number. 50NTG6

the members either n_3, n_4 or n_4, n_5 or n_5, n_1 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5 or n_5n_1. There are three edges to have exclusive endpoints from S and It doesn't imply that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to either independent number $\mathcal{I}(NTG)$ or independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(iv) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge n_1n_5. There's one edge n_1n_5 to have exclusive endpoints n_1 and n_5 from S hence it implies that $S = \{n_1, n_3, n_5\}$ isn't corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(v) 2 is independent number and its corresponded sets are $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, and $\{n_3, n_5\}$;

(vi) 2.8 is independent neutrosophic-number and its corresponded set is $\{n_2, n_5\}$.

The natural way proposes us to use the restriction "minimum" instead of "maximum."

Definition 2.5.13. (Failed independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(ii) **failed independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Thus we replace the term "minimum" by the term ''maximum." Hence,

Definition 2.5.14. (Failed independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(ii) **failed independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Proposition 2.5.15. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_j) + \sigma_i(x_{j+1}))\}_{x_j x_{j+1} \in E}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. If $|S| > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors. Thus there is no triangle but there's one edge. One edge has two endpoints. These endpoints are corresponded to failed independent number $\mathcal{I}(NTG)$. So

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_j) + \sigma_i(x_{j+1}))\}_{x_j x_{j+1} \in E}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.16. There are two sections for clarifications.

(a) In Figure (2.9), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either failed independent number $\mathcal{I}(NTG)$ or failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_1, n_3\}$ is corresponded to either failed independent number $\mathcal{I}(NTG)$ or failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither failed independent number $\mathcal{I}(NTG)$ nor failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_2, n_3\}$ is a set of vertices, then there's no vertex in S but n_2 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_2, n_3\}$ is corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 2 is failed independent number and its corresponded set is $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_1, n_6\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_2, n_6\}$, $\{n_3, n_4\}$, $\{n_3, n_5\}$, $\{n_3, n_6\}$, $\{n_4, n_5\}$, $\{n_4, n_6\}$, $\{n_5, n_6\}$, and $\{n_6, n_1\}$;

(vi) 4 is failed independent neutrosophic-number and its corresponded set is $\{n_2, n_3\}$.

(b) In Figure (2.10), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either failed independent number $\mathcal{I}(NTG)$ or failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

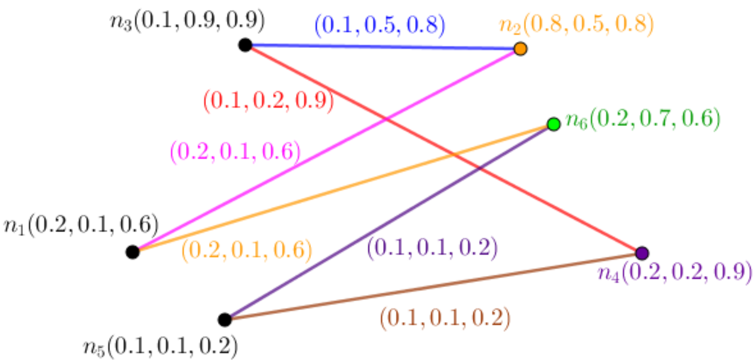

Figure 2.9: A Neutrosophic Graph in the Viewpoint of its Failed Independent Number. 51NTG5

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_1, n_3\}$ is corresponded to either failed independent number $\mathcal{I}(NTG)$ or failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither failed independent number $\mathcal{I}(NTG)$ nor failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_3, n_4\}$ is corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 2 is failed independent number and its corresponded set is $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_3, n_4\}$, $\{n_3, n_5\}$, $\{n_4, n_5\}$, and $\{n_5, n_1\}$;

(vi) 4.3 is failed independent neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Definition 2.5.17. (1-independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph NTG : (V, E, σ, μ) is maximum cardinality of a set S of vertices such that every

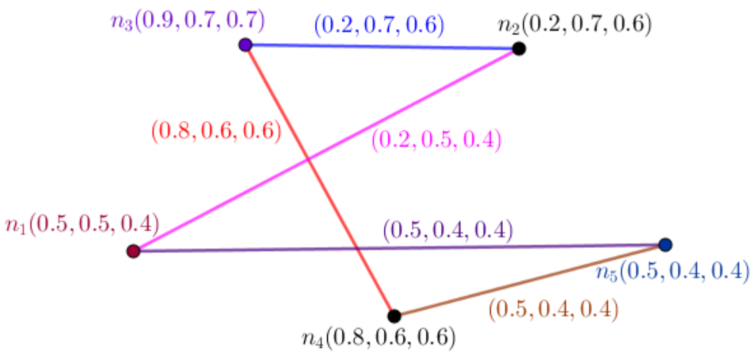

Figure 2.10: A Neutrosophic Graph in the Viewpoint of its Failed Independent Number. 51NTG6

two vertices of S aren't endpoints for an edge, simultaneously For one time, one vertex is allowed to be endpoint;

(ii) **1-independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously. For one time, one vertex is allowed to be endpoint.

Definition 2.5.18. (Failed 1-independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed 1-independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. For one time, one vertex is allowed not to be endpoint;

(ii) **failed 1-independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. For one time, one vertex is allowed not to be endpoint.

Proposition 2.5.19. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \sum_{i=1}^{3} \sigma_i(z) + \max\{\sum_{i=1}^{3}(\sigma_i(x_1) + \sigma_i(x_3) + \cdots + \sigma_i(x_t)),$$

$$\sum_{i=1}^{3} \sigma_i(x_2) + \sigma_i(x_4) + \cdots + \sigma_i(x'_t))\}_{x_i x_{i+1} \in E}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Assume $|S| > \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S|>\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$ isn't corresponded to 1-independent number $\mathcal{I}(NTG)$. In other side, for having an edge, there's a need to have two vertices. So by using the members of $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$. It implies that $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$ is corresponded to 1-independent number. But extra condition implies

$$\mathcal{I}_n(NTG) = \sum_{i=1}^{3} \sigma_i(z) + \max\{\sum_{i=1}^{3} (\sigma_i(x_1) + \sigma_i(x_3) + \cdots + \sigma_i(x_t)),$$

$$\sum_{i=1}^{3} \sigma_i(x_2) + \sigma_i(x_4) + \cdots + \sigma_i(x'_t))\}_{x_i x_{i+1} \in E}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.20. There are two sections for clarifications.

(*a*) In Figure (2.11), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but It doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either 1-independent number $\mathcal{I}(NTG)$ or 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| \neq \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1}$;

(*ii*) if $S = \{n_2, n_4, n_6\}$ is a set of vertices, then there's no vertex in S but n_2, n_4 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_2, n_4, n_6\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor + 1}$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. But extra condition implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to 1-independent number $\mathcal{I}(NTG)$ but not 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor+1}$;

(iv) if $S = \{n_1, n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_1, n_3, n_5\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}$;

(v) 4 is 1-independent number and its corresponded sets are $\{n_2, n_4, n_6, n_1\}$, $\{n_2, n_4, n_6, n_3\}$, $\{n_2, n_4, n_6, n_5\}$, $\{n_1, n_3, n_5, n_2\}$, $\{n_1, n_3, n_5, n_4\}$, and $\{n_1, n_3, n_5, n_6\}$;

(vi) 5.1 is 1-independent neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6, n_3\}$.

(b) In Figure (2.12), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_2, n_4\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor+1}$;

(ii) if $S = \{n_3, n_5\}$ is a set of vertices, then there's no vertex in S but n_3 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_3, n_5\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor+1}$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 or n_5, n_1 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5 or n_5n_1. There are three edges to have exclusive endpoints from S. But extra condition implies that $S = \{n_1, n_3, n_4, n_5\}$ isn't corresponded to 1-independent number $\mathcal{I}(NTG)$ and 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S|>\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor+1}$;

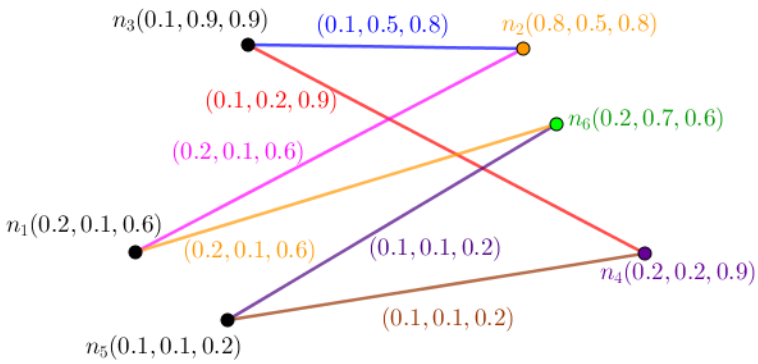

Figure 2.11: A Neutrosophic Graph in the Viewpoint of its 1-Independent Number. 52NTG5

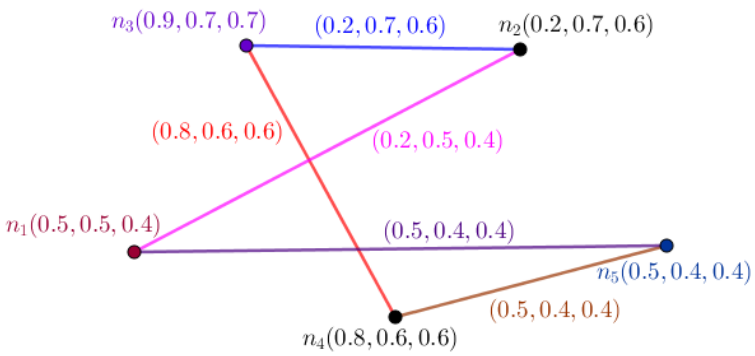

Figure 2.12: A Neutrosophic Graph in the Viewpoint of its 1-Independent Number. 52NTG6

(iv) if $S = \{n_4, n_2, n_5\}$ is a set of vertices, then there's no vertex in S but n_4, n_2 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge n_4n_5. There's one edge n_4n_5 to have exclusive endpoints n_4 and n_5 from S. But extra condition implies that $S = \{n_4, n_2, n_5\}$ is corresponded to both 1-independent number $\mathcal{I}(NTG)$ and 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$. Since $S = \{n_i\}_{|S| \neq \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor}$;

(v) 3 is 1-independent number and its corresponded sets are $\{n_1, n_3, n_2\}$, $\{n_1, n_3, n_4\}$, $\{n_1, n_3, n_5\}$, $\{n_1, n_4, n_2\}$, $\{n_1, n_4, n_3\}$, $\{n_1, n_4, n_5\}$, $\{n_2, n_4, n_1\}$, $\{n_2, n_4, n_3\}$, $\{n_2, n_4, n_5\}$, $\{n_2, n_5, n_1\}$, $\{n_2, n_5, n_3\}$, $\{n_2, n_5, n_4\}$, $\{n_3, n_5, n_2\}$, $\{n_3, n_5, n_4\}$, and $\{n_3, n_5, n_1\}$;

(vi) 5.1 is 1-independent neutrosophic-number and its corresponded set is $\{n_2, n_5, n_3\}$.

The natural way proposes us to use the restriction "maximum" instead of "minimum."

Definition 2.5.21. (Clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **clique number** $\mathcal{C}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(ii) **clique neutrosophic-number** $\mathcal{C}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Proposition 2.5.22. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{C}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_j) + \sigma_i(x_{j+1}))\}_{x_j x_{j+1} \in E}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. If $|S| > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors. Thus there is no triangle but there's one edge. One edge has two endpoints. These endpoints are corresponded to clique number $\mathcal{C}(NTG)$. So

$$\mathcal{C}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_j) + \sigma_i(x_{j+1}))\}_{x_j x_{j+1} \in E}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.23. There are two sections for clarifications.

(a) In Figure (2.13), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either clique number $\mathcal{C}(NTG)$ or clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_1, n_3\}$ is corresponded to either clique number $\mathcal{C}(NTG)$ or clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither clique number $\mathcal{C}(NTG)$ nor clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iv*) if $S = \{n_2, n_3\}$ is a set of vertices, then there's no vertex in S but n_2 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_2, n_3\}$ is corresponded to both clique number $\mathcal{C}(NTG)$ and clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*v*) 2 is clique number and its corresponded set is $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_1, n_6\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_2, n_6\}$, $\{n_3, n_4\}$, $\{n_3, n_5\}$, $\{n_3, n_6\}$, $\{n_4, n_5\}$, $\{n_4, n_6\}$, $\{n_5, n_6\}$, and $\{n_6, n_1\}$;

(*vi*) 4 is clique neutrosophic-number and its corresponded set is $\{n_2, n_3\}$.

(*b*) In Figure (2.14), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_2, n_4\}$ is corresponded to either clique number $\mathcal{C}(NTG)$ or clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*ii*) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ but it doesn't imply that $S = \{n_1, n_3\}$ is corresponded to either clique number $\mathcal{C}(NTG)$ or clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither clique number $\mathcal{C}(NTG)$ nor clique neutrosophic-number $\mathcal{C}_n(NTG)$;

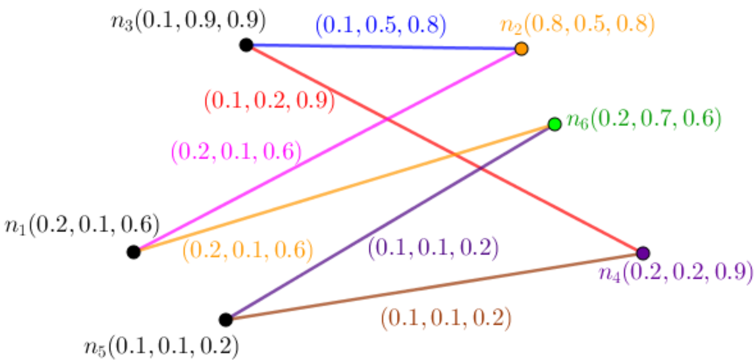

Figure 2.13: A Neutrosophic Graph in the Viewpoint of its clique Number. 53NTG5

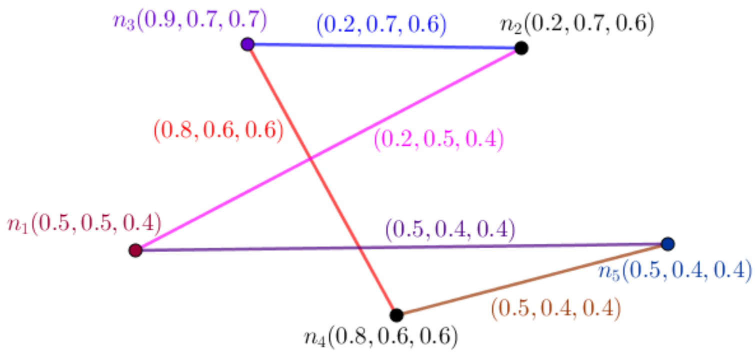

Figure 2.14: A Neutrosophic Graph in the Viewpoint of its clique Number. 53NTG6

(iv) if $S = \{n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 2}$ thus it implies that $S = \{n_3, n_4\}$ is corresponded to both clique number $\mathcal{C}(NTG)$ and clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 2 is clique number and its corresponded set is $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_3, n_4\}$, $\{n_3, n_5\}$, $\{n_4, n_5\}$, and $\{n_5, n_1\}$;

(vi) 4.3 is clique neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

The natural way proposes us to use the restriction "minimum" instead of "maximum."

Definition 2.5.24. (Failed Clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed clique number** $\mathcal{C}^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously;

(*ii*) **failed clique neutrosophic-number** $\mathcal{C}_n^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously.

Proposition 2.5.25. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

(*i*) *if* $\mathcal{O}(NTG) = 0$, *then*
$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0;$$

(*ii*) *if* $\mathcal{O}(NTG) = 1$, *then*
$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0;$$

(*iii*) *if* $\mathcal{O}(NTG) = 2$, *then*
$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0;$$

(*iv*) *if* $\mathcal{O}(NTG) = 3$, *then*
$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0;$$

(*v*) *if* $\mathcal{O}(NTG) \geq 4$, *then*
$$\mathcal{C}_n^{\mathcal{F}}(NTG) = \min\{\sum_{i=1}^{3} \sigma_i(x) + \sum_{i=1}^{3} \sigma_i(y)\}_{xy \notin E}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex.

(*i*) If $\mathcal{O}(NTG) = 0$, then there's no vertex to be considered. So minimum cardinality of a set is zero. It implies
$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0;$$

(*ii*) if $\mathcal{O}(NTG) = 1$, then by using Definition, there aren't two vertices. Thus it implies
$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0;$$

(*iii*) if $\mathcal{O}(NTG) = 2$, then there are two vertices. By it's cycle-neutrosophic graph, it's contradiction. Since if it's cycle-neutrosophic graph, then $\mathcal{O}(NTG) \neq 2$. In other words, it's cycle-neutrosophic graph, then $\mathcal{O}(NTG) \geq 3$. At least two vertices are needed to have new notion but at least three vertices are needed to have cycle-neutrosophic graph. Thus
$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0;$$

(*iv*) if $\mathcal{O}(NTG) = 3$, then, by it's cycle-neutrosophic graph, there aren't two vertices x and y such that x and y aren't endpoints of an edge. It implies
$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0;$$

(v) if $\mathcal{O}(NTG) \geq 4$, then, by it's cycle-neutrosophic graph, there are two vertices x and y such that x and y aren't endpoints of an edge. Thus lower bound is achieved for failed clique number. It implies

$$\mathcal{C}_n^{\mathcal{F}}(NTG) = \min\{\sum_{i=1}^{3} \sigma_i(x) + \sum_{i=1}^{3} \sigma_i(y)\}_{xy \notin E}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.26. There are two sections for clarifications.

(a) In Figure (2.15), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ implies that $S = \{n_2, n_4\}$ is corresponded to failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ but not failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

  (ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ implies that $S = \{n_1, n_3\}$ is corresponded to failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ but not failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

  (iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. But n_1 and n_3 aren't endpoints for any given edge. $S = \{n_i\}_{|S| \neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ nor failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

  (iv) if $S = \{n_1, n_5\}$ is a set of vertices, then there's no vertex in S but n_1 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ thus it implies that $S = \{n_1, n_5\}$ is corresponded to both failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(v) 2 is failed clique number and its corresponded set is $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_1, n_5\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, $\{n_2, n_6\}$, $\{n_3, n_5\}$, $\{n_3, n_6\}$, and $\{n_4, n_6\}$;

(vi) 1.3 is failed clique neutrosophic-number and its corresponded set is $\{n_1, n_5\}$.

(b) In Figure (2.16), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ implies that $S = \{n_2, n_4\}$ is corresponded to failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ but not failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ implies that $S = \{n_1, n_3\}$ is corresponded to failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ but not failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iii) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. But n_1 and n_3 aren't endpoints for every given edge. $S = \{n_i\}_{|S|\neq 2}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ nor failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iv) if $S = \{n_2, n_5\}$ is a set of vertices, then there's no vertex in S but n_2 and n_5. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|=2}$ thus it implies that $S = \{n_2, n_5\}$ is corresponded to both failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(v) 2 is failed clique number and its corresponded set is $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_4\}$, $\{n_2, n_5\}$, and $\{n_3, n_5\}$;

(vi) 2.8 is failed clique neutrosophic-number and its corresponded set is $\{n_2, n_5\}$.

Definition 2.5.27. (1-clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-clique number** $\mathcal{C}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices

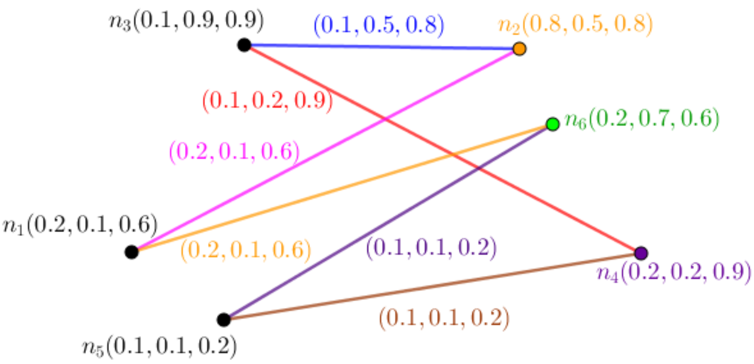

Figure 2.15: A Neutrosophic Graph in the Viewpoint of its Failed Clique Number. 54NTG5

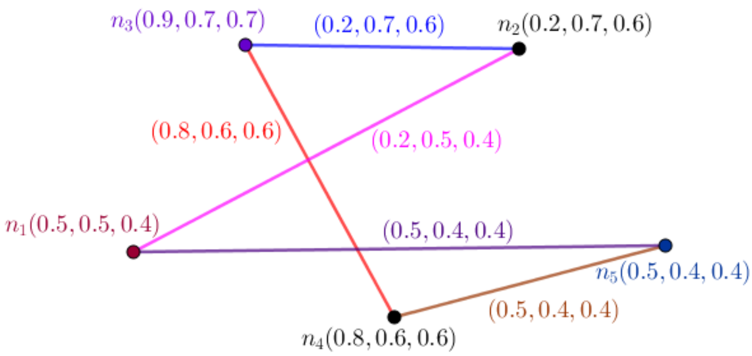

Figure 2.16: A Neutrosophic Graph in the Viewpoint of its Failed Clique Number. 54NTG6

of S are endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time;

(ii) **1-clique neutrosophic-number** $\mathcal{C}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time.

Definition 2.5.28. (Failed 1-clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed 1-clique number** $\mathcal{C}^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time;

(ii) **failed 1-clique neutrosophic-number** $\mathcal{C}_n^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time.

Proposition 2.5.29. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{C}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_j) + \sigma_i(x_{j+1}) + \sigma_i(x_{j+2}))\}_{x_jx_{j+1},x_{j+1}x_{j+2}\in E}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. If $|S| > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors. Thus there is no triangle but there's one edge. One edge has two endpoints. These endpoints are corresponded to 1-clique number $\mathcal{C}(NTG)$. Two vertices could be satisfied in extra condition. So

$$\mathcal{C}_n(NTG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_j) + \sigma_i(x_{j+1}) + \sigma_i(x_{j+2}))\}_{x_jx_{j+1},x_{j+1}x_{j+2}\in E}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.30. There are two sections for clarifications.

(a) In Figure (2.17), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 3}$ implies that $S = \{n_2, n_4\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 3}$ implies that $S = \{n_1, n_3\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iii*) if $S = \{n_1, n_3, n_4, n_5\}$ is a set of vertices, then there's no vertex in S but n_1, n_3, n_4 and n_5. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_4, n_5 of S, it's possible to have endpoints of an edge either n_3n_4 or n_4n_5. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ thus it implies that $S = \{n_1, n_3, n_4, n_5\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iv*) if $S = \{n_5, n_6\}$ is a set of vertices, then there's no vertex in S but n_5 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ thus it implies that $S = \{n_5, n_6\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*v*) 3 is 1-clique number and its corresponded sets are like $\{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$ which contain two edges;

(*vi*) 4.9 is 1-clique neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3\}$.

(*b*) In Figure (2.18), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ implies that $S = \{n_2, n_4\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*ii*) if $S = \{n_1, n_3\}$ is a set of vertices, then there's no vertex in S but n_1 and n_3. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 3}$ implies that $S = \{n_1, n_3\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iii*) if $S = \{n_3, n_4, n_2\}$ is a set of vertices, then there's no vertex in S but n_3, n_4 and n_2. In other side, for having an edge, there's a need to have two vertices which are consecutive. So by using the members either n_3, n_4 or n_2, n_3 of S, it's possible to have endpoints of an edge either n_2n_3 or n_3n_4. There are two edges to have exclusive endpoints from S. $S = \{n_i\}_{|S| = 3}$ thus it implies that $S = \{n_3, n_4, n_2\}$ is corresponded to both 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iv*) if $S = \{n_5, n_6\}$ is a set of vertices, then there's no vertex in S but n_5 and n_6. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S.

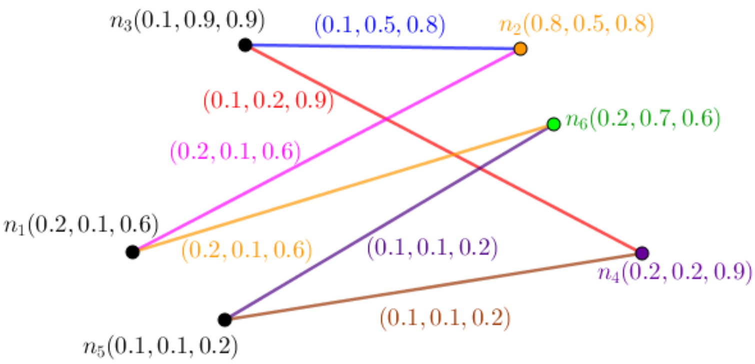

Figure 2.17: A Neutrosophic Graph in the Viewpoint of its 1-Clique Number. 55NTG5

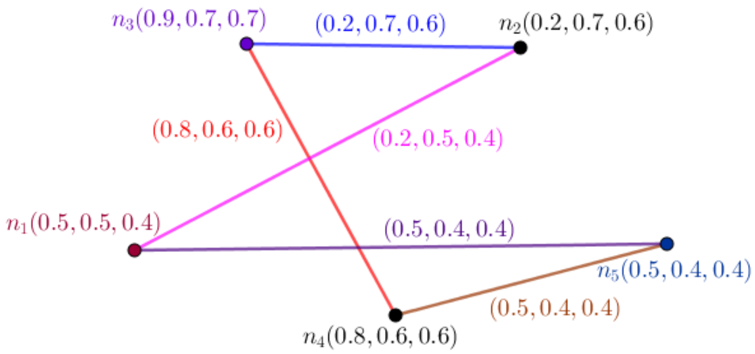

Figure 2.18: A Neutrosophic Graph in the Viewpoint of its 1-Clique Number. 55NTG6

$S = \{n_i\}_{|S| \neq 4}$ thus it implies that $S = \{n_5, n_6\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 3 is 1-clique number and its corresponded sets are like $\{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$ which contain two edges;

(vi) 6.3 is 1-clique neutrosophic-number and its corresponded set is $\{n_3, n_4, n_2\}$.

Definition 2.5.31. (Matching Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **matching number** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of edges such that every two edges of S don't have any vertex in common;

(ii) **matching neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of edges such that every two edges of S don't have any vertex in common.

Proposition 2.5.32. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{|S|=\lfloor \frac{n}{2} \rfloor}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}}$ be consecutive arrangements of vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_ix_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O} - 1.$$

Define

$$S = \{x_1x_2, x_3x_4, \cdots, x_ix_{i+1}\}_{i=1}^{\mathcal{O}-1}.$$

In S, there aren't two edges which have common endpoints. S is matching set and it has maximum cardinality amid such these sets which are matching set which is a set in that, there aren't two edges which have common endpoints. So

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{|S|=\lfloor \frac{n}{2} \rfloor}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.33. There are two sections for clarifications.

(a) In Figure (2.19), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iii*) if $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is a set of edges, then there are three edges in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges in S. Cardinality and structure of S implies that $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.5, of S implies $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iv*) if $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is a set of edges, then there are three edges in S In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.7, of S implies $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*v*) 3 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4, n_5n_6\}$, and $\{n_2n_3, n_4n_5, n_6n_1\}$;

(*vi*) 2.5 is matching neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4, n_5n_6\}$.

(*b*) In Figure (2.20), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*ii*) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching number $\mathcal{M}(NTG)$ nor matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iii*) if $S = \{n_2n_3, n_4n_5\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_4n_5. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.8, of S implies $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

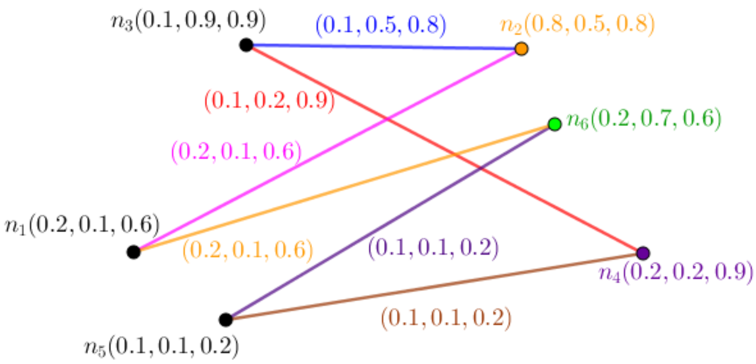

Figure 2.19: A Neutrosophic Graph in the Viewpoint of its Matching Number. 58NTG5

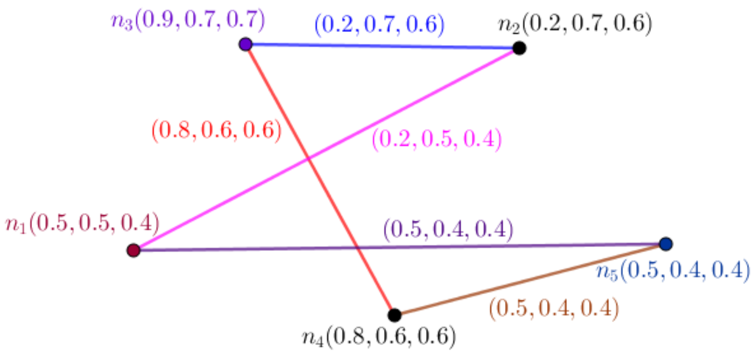

Figure 2.20: A Neutrosophic Graph in the Viewpoint of its Matching Number. 58NTG6

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 3.1, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, and $\{n_2n_3, n_4n_5\}$;

(vi) 2.8 is matching neutrosophic-number and its corresponded set is $\{n_2n_3, n_4n_5\}$.

Definition 2.5.34. (Matching Polynomial).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **matching polynomial** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the matching polynomial represent the number of sets of independent edges of various cardinalities in G.

(*ii*) **matching polynomial neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the matching polynomial represent the number of sets of independent edges of various neutrosophic cardinalities in G.

Proposition 2.5.35. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = cx^{\max\{\sum_{s\in S}\sum_{i=1}^{3}\mu_i(s)\}\}_{|S|=\lfloor \frac{S(NTG)}{2}\rfloor}} + \cdots + c'x^{\min\{\sum_{s\in E}\sum_{i=1}^{3}\mu_i(s)\}}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}}$ be consecutive arrangements of vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E,\ i = 1, 2, \cdots, \mathcal{O} - 1.$$

Define

$$S = \{x_1x_2, x_3x_4, \cdots, x_ix_{i+1}\}_{i=1}^{\mathcal{O}-1}.$$

In S, there aren't two edges which have common endpoints. S is matching polynomial set and it has maximum cardinality amid such these sets which are matching polynomial set which is a set in that, there aren't two edges which have common endpoints. So

$$\mathcal{M}_n(NTG) = cx^{\max\{\sum_{s\in S}\sum_{i=1}^{3}\mu_i(s)\}\}_{|S|=\lfloor \frac{S(NTG)}{2}\rfloor}} + \cdots + c'x^{\min\{\sum_{s\in E}\sum_{i=1}^{3}\mu_i(s)\}}.$$

∎

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.36. There are two sections for clarifications.

(*a*) In Figure (2.21), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_5, n_4n_6\}$ is corresponded to neither matching polynomial $\mathcal{M}(NTG)$ nor matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(*ii*) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching polynomial $\mathcal{M}(NTG)$ nor matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is a set of edges, then there are three edges in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges in S. Cardinality and structure of S implies that $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.5, of S implies $S = \{n_1n_2, n_3n_4, n_5n_6\}$ is corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is a set of edges, then there are three edges in S In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.7, of S implies $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $2x^3 + 9x^2 + 6x + 1$ is matching polynomial and its corresponded sets are $\{n_1n_2, n_3n_4, n_5n_6\}$, and $\{n_2n_3, n_4n_5, n_6n_1\}$ for coefficient of biggest term;

(vi) $x^{2.5} + x^{2.4} + x^{1.4}$ is matching polynomial neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4, n_5n_6\}$.

(b) In Figure (2.22), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to neither matching polynomial $\mathcal{M}(NTG)$ nor matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to neither matching polynomial $\mathcal{M}(NTG)$ nor matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_2n_3, n_4n_5\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_4n_5. In other side, for having a common vertex,

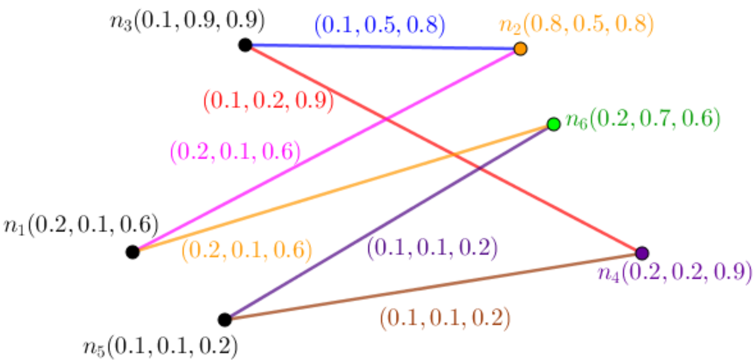

Figure 2.21: A Neutrosophic Graph in the Viewpoint of its Matching Polynomial. 60NTG5

there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.8, of S implies $S = \{n_2n_3, n_4n_5\}$ is corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ but neutrosophic cardinality, 3.1, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $2x^2 + 5x + 1$ is matching polynomial and its corresponded sets are $\{n_1n_2, n_3n_4\}$, and $\{n_2n_3, n_4n_5\}$ for coefficient of biggest term;

(vi) $x^{2.8} + x^2$ is matching polynomial neutrosophic-number and its corresponded set is $\{n_2n_3, n_4n_5\}$ for coefficient of biggest term.

Definition 2.5.37. (e-Matching Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **e-matching number** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S containing endpoints of edges such that every two edges of S don't have any vertex in common;

(ii) **e-matching neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S containing endpoints of edges such that every two edges of S don't have any vertex in common.

Definition 2.5.38. (e-Matching Polynomial).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

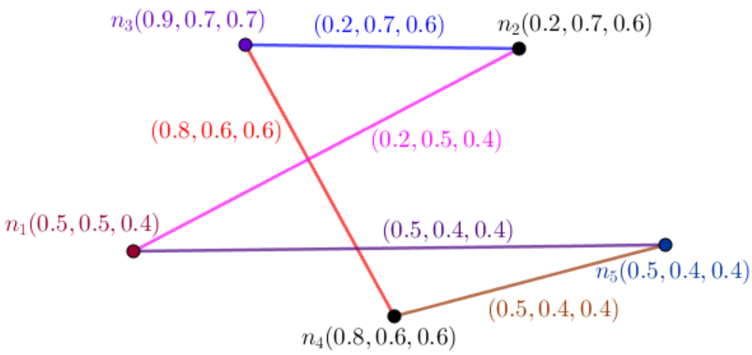

Figure 2.22: A Neutrosophic Graph in the Viewpoint of its Matching Polynomial. 60NTG6

(i) **e-matching polynomial** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the e-matching polynomial represent the number of sets of endpoints of independent edges of various cardinalities in G.

(ii) **e-matching polynomial neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the e-matching polynomial represent the number of sets of endpoints of independent edges of various neutrosophic cardinalities in G.

Proposition 2.5.39. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = \mathcal{O}_n(NTG)$$

where the parity of $\mathcal{O}(NTG)$ is even. And

$$\mathcal{M}_n(NTG) = \mathcal{O}_n(NTG) - \sum_{i=1}^{3} \sigma_i(x)$$

where the parity of $\mathcal{O}(NTG)$ is odd and $x \in \{y \in V \mid \sigma(y) = \min_{z \in V} \sigma(z)\}$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}}$ be consecutive arrangements of vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O} - 1.$$

Define

$$S = \{x_1x_2, x_3x_4, \cdots, x_ix_{i+1}\}_{i=1}^{\mathcal{O}-1}.$$

In S, there aren't two edges which have common endpoints. S is corresponded to e-matching neutrosophic-number and it has maximum cardinality amid such these sets which are corresponded to e-matching neutrosophic-number which is a set in that, there aren't two edges which have common endpoints. So

$$\mathcal{M}_n(NTG) = \mathcal{O}_n(NTG)$$

where the parity of $\mathcal{O}(NTG)$ is even. And

$$\mathcal{M}_n(NTG) = \mathcal{O}_n(NTG) - \sum_{i=1}^{3} \sigma_i(x)$$

where the parity of $\mathcal{O}(NTG)$ is odd and $x \in \{y \in V \mid \sigma(y) = \min_{z \in V} \sigma(z)\}$. ■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.40. There are two sections for clarifications.

(a) In Figure (2.23), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $\{n_1n_3, n_2n_5, n_4n_6\}$ is a set of edges, then there's no edge from S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1, n_3, n_2, n_5, n_4, n_6\}$ is corresponded to neither e-matching number $\mathcal{M}(NTG)$ nor e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge from S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to neither e-matching number $\mathcal{M}(NTG)$ nor e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $\{n_1, n_2, n_3, n_4, n_5, n_6\}$ is a set of edges, then there are three edges from S. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality, $\mathcal{O}(NTG) = 6$, and structure of S implies that

$$S = \{n_1, n_2, n_3, n_4, n_5, n_6\} = V$$

is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, $10.1 = \mathcal{O}_n(NTG)$, of S implies

$$S = \{n_1, n_2, n_3, n_4, n_5, n_6\} = V$$

is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_2n_3, n_4n_5, n_6n_1\}$ is a set of edges, then there are three edges from S In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are three edges from S. Cardinality of S, $\mathcal{O}(NTG) = 6$, implies that

$$S = \{n_2, n_3, n_4, n_5, n_6, n_1\} = V$$

is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, $10.1 = \mathcal{O}_n(NTG)$, of S implies

$$S = \{n_2, n_3, n_4, n_5, n_6, n_1\} = V$$

is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $6 = \mathcal{O}(NTG)$ is e-matching number and its corresponded set is $S = \{n_1, n_2, n_3, n_4, n_5, n_6\} = V$;

(vi) $10.1 = \mathcal{O}_n(NTG)$ is e-matching neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4, n_5, n_6\}$.

(b) In Figure (2.24), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $\{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is no edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to neither e-matching number $\mathcal{M}(NTG)$ nor e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have at least one endpoint for two edges. There is one edge from S. Cardinality of S implies but the structure of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to neither e-matching number $\mathcal{M}(NTG)$ nor e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $\{n_2n_3, n_4n_5\}$ is a set of edges, then there's no edge from S but n_2n_3 and n_4n_5. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that $S = \{n_2, n_3, n_4, n_5\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 7.1, of S implies $S = \{n_2, n_3, n_4, n_5\}$ isn't corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $\{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint for two edges which is impossible. So

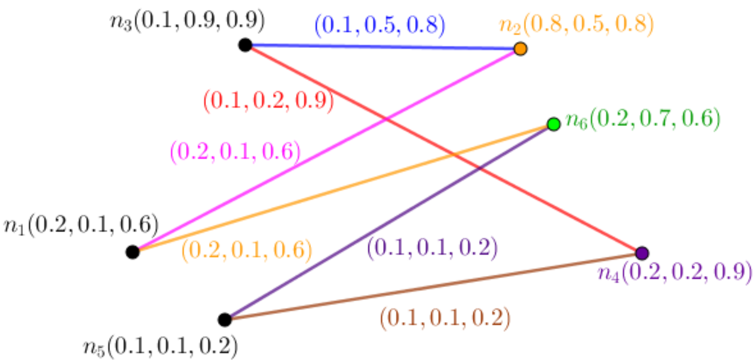

Figure 2.23: A Neutrosophic Graph in the Viewpoint of its e-Matching Number. 61NTG6

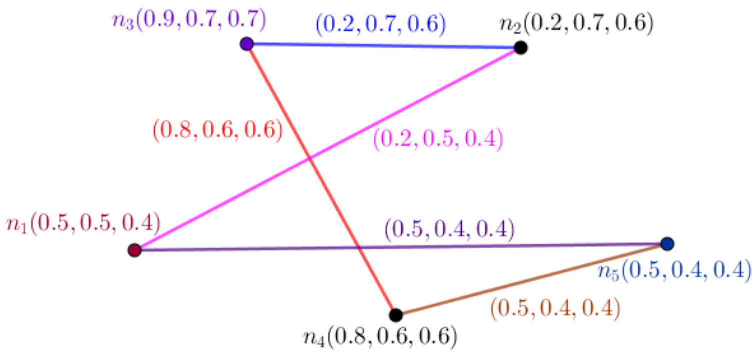

Figure 2.24: A Neutrosophic Graph in the Viewpoint of its e-matching Number. 61NTG7

by using the members of S, it's impossible to have endpoints for two edges. There are two edges from S. Cardinality of S implies that

$$S = \{n_1, n_2, n_3, n_4\} = V - \{n_5\} \neq V$$

is corresponded to e-matching $\mathcal{M}(NTG)$ and neutrosophic cardinality, 7.2, of S implies

$$S = \{n_1, n_2, n_3, n_4\} = V - \{n_5\} \neq V$$

is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $4 = \mathcal{O}(NTG) - 1 \neq \mathcal{O}(NTG)$ is e-matching number and its corresponded set is $\{n_1, n_2, n_3, n_4\} = V - \{n_5\} \neq V$;

(vi) $7.2 = \mathcal{O}_n(NTG) - \sum_{i=1}^{3} \sigma_i(n_5)$ is e-matching neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4\} = V - \{n_5\} \neq V$;

Definition 2.5.41. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞;

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞.

Proposition 2.5.42. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(NTG) \geq 3$. *Then*

$$\mathcal{G}_n(NTG) = \mathcal{O}_n(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E,\ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. It implies

$$\mathcal{G}_n(NTG) = \mathcal{O}_n(NTG).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.43. There are two sections for clarifications.

(a) In Figure (2.75), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(*v*) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(*vi*) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(*b*) In Figure (2.76), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

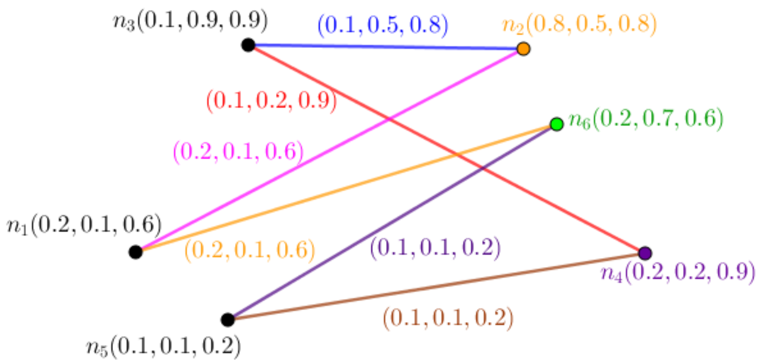

Figure 2.25: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG5

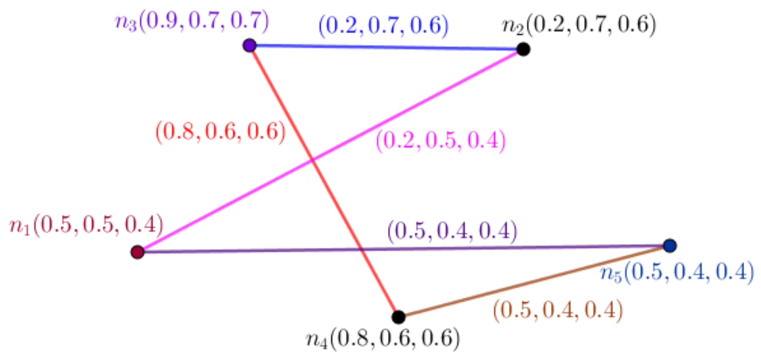

Figure 2.26: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG6

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Definition 2.5.44. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞;

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞.

63thm **Theorem 2.5.45.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 2.5.46. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(NTG) \geq 3$. Then*

$$\mathcal{G}_n(NTG) = \mathcal{O}_n(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. By Theorem (2.5.49),

$$\mathcal{G}_n(NTG) = \mathcal{O}_n(NTG).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.47. There are two sections for clarifications.

(a) In Figure (2.27), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, $n_1 n_2$ and $n_2 n_3$, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(*v*) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(*vi*) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(*b*) In Figure (2.28), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Definition 2.5.48. (Girth Polynomial and Neutrosophic Girth Polynomial). Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

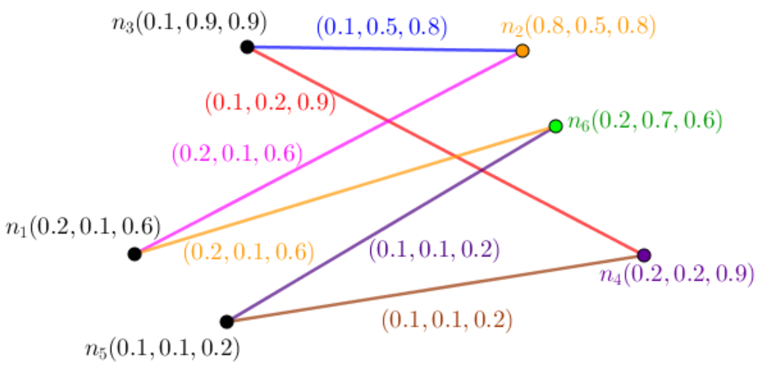

Figure 2.27: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG5

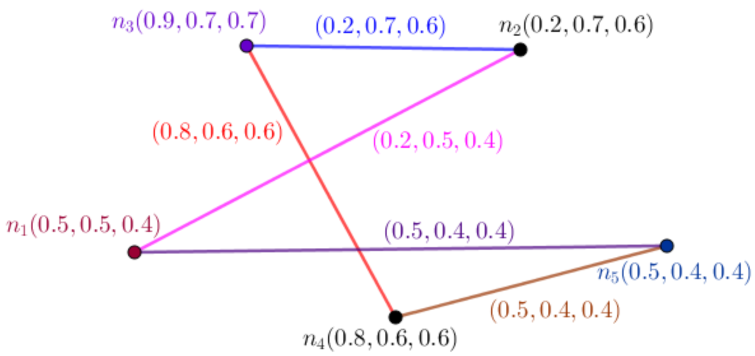

Figure 2.28: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG6

(i) **girth polynomial** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1 x^{m_1} + n_2 x^{m_2} + \cdots + n_s x^3$ where n_i is the number of cycle with m_i as its crisp cardinality of the set of vertices of cycle;

(ii) **neutrosophic girth polynomial** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1 x^{m_1} + n_2 x^{m_2} + \cdots + n_s x^{m_s}$ where n_i is the number of cycle with m_i as its neutrosophic cardinality of the set of vertices of cycle.

63thm **Theorem 2.5.49.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 2.5.50. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(NTG) \geq 3$. Then*

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. By Theorem (2.5.49),

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.51. There are two sections for clarifications.

(*a*) In Figure (2.29), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, $n_1 n_2$ and $n_2 n_3$, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth polynomial $\mathcal{G}(NTG)$ and neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^{6=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $x^{8.1=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (2.30), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*iv*) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth polynomial $\mathcal{G}(NTG)$ and neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*v*) $x^{5=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(*vi*) $x^{8.5=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Definition 2.5.52. (Hamiltonian Neutrosophic Cycle).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **hamiltonian neutrosophic cycle** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive vertices $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ which is neutrosophic cycle;

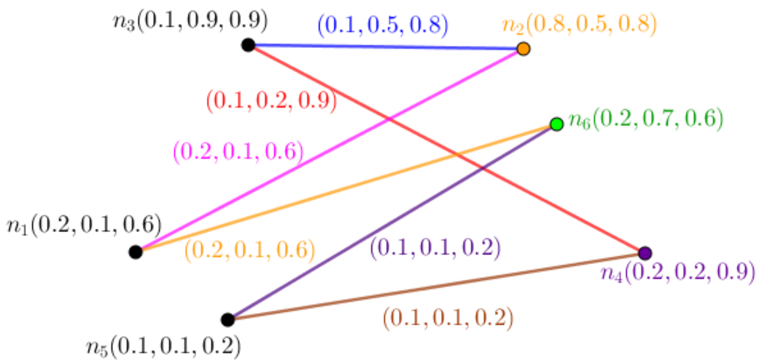

Figure 2.29: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG5

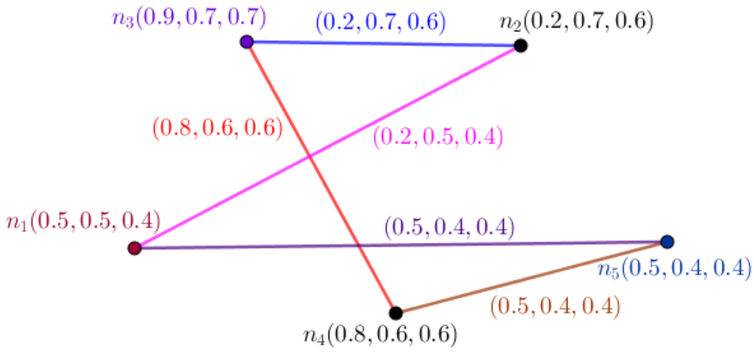

Figure 2.30: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG6

(ii) **n-hamiltonian neutrosophic cycle** $\mathcal{N}(HNC)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive vertices $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ which are neutrosophic cycles.

If we use the notion of neutrosophic cardinality in strong type of neutrosophic graphs, then the next result holds. If not, the situation is complicated since it's possible to have all edges in the way that, there's no value of a vertex for an edge.

66thm **Theorem 2.5.53.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 2.5.54. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(CYC_n) \geq 3$. Then*

$$\mathcal{N}(CYC_n) = 1.$$

Proof. Suppose $CYC_n : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC_n)}, x_1$ be a sequence of consecutive vertices of $CYC_n : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E,\ i = 1, 2, \cdots, \mathcal{O}(CYC_n) - 1,\ x_{\mathcal{O}(CYC_n)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a longest crisp cycle with length $\mathcal{O}(CYC_n)$. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(CYC_n)$. Thus the crisp cardinality of set of vertices forming longest crisp cycle is $\mathcal{O}(CYC_n)$. By Theorem (2.5.57),

$$\mathcal{N}(CYC_n) = 1.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.55. There are two sections for clarifications.

(a) In Figure (2.31), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all vertices once. Finding longest cycle containing all vertices once has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges.

So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all vertices once. Finding longest cycle containing all vertices once has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding longest cycle containing all vertices once. Finding longest cycle containing all vertices once has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5, n_6, n_1$$

is corresponded to both of hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ and n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(v) $\mathcal{M}(CYC_n) : n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is hamiltonian neutrosophic cycle;

(vi) $\mathcal{N}(CYC_n) = 1$ is n-hamiltonian neutrosophic cycle.

(b) In Figure (2.32), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle.

So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding longest cycle containing all vertices once. Finding longest cycle containing all vertices once has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5, n_1$$

is corresponded to neither hamiltonian neutrosophic cycle $\mathcal{M}(CYC_n)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CYC_n)$;

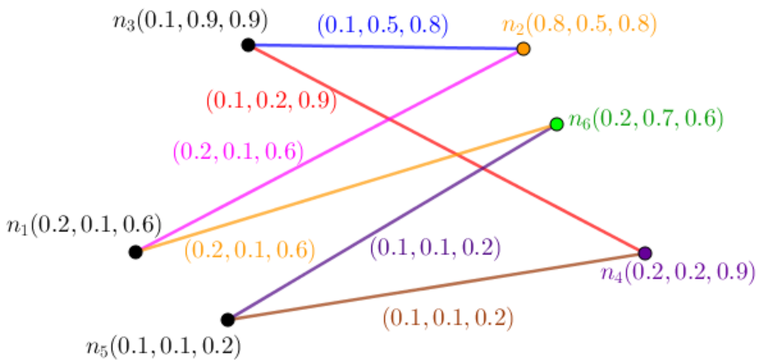

Figure 2.31: A Neutrosophic Graph in the Viewpoint of its hamiltonian neutrosophic cycle. 66NTG5

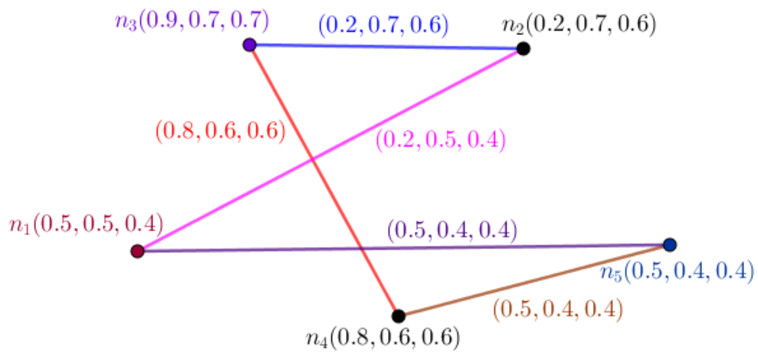

Figure 2.32: A Neutrosophic Graph in the Viewpoint of its hamiltonian neutrosophic cycle. 66NTG6

(v) $\mathcal{M}(CYC_n)$: Not Existed is hamiltonian neutrosophic cycle;

(vi) $\mathcal{N}(CYC_n) = 0$.

Definition 2.5.56. (Eulerian Neutrosophic Cycle).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Eulerian neutrosophic cycle** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive edges $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}, x_1$ which is neutrosophic cycle;

(ii) **n-Eulerian neutrosophic cycle** $\mathcal{N}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive edges $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}, x_1$ which are neutrosophic cycles.

If we use the notion of neutrosophic cardinality in strong type of neutrosophic graphs, then the next result holds. If not, the situation is complicated since it's possible to have all edges in the way that, there's no value of a vertex for an edge.

66thm

Theorem 2.5.57. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 2.5.58. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{N}(CYC) = 1.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{S}(CYC)}, x_1$ be a sequence of consecutive edges of $CYC : (V, E, \sigma, \mu)$ such that

$$x_i, x_{i+1} \text{ have common vertex}, \ i = 1, 2, \cdots, \mathcal{S}(CYC) - 1,$$

$$x_{\mathcal{S}(CYC)}, x_1 \text{ have common vertex.}$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts lead to one crisp cycle for finding a longest crisp cycle with length $\mathcal{S}(CYC)$. For a given vertex x_i, the sequence of consecutive edges

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{S}(CYC)$. Thus the crisp cardinality of set of edges forming longest crisp cycle is $\mathcal{S}(CYC)$. By Theorem (2.5.57),

$$\mathcal{N}(CYC) = 1.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.59. There are two sections for clarifications.

(*a*) In Figure (2.33), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's only a path and there are only two edges but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive edges is at most 2, then it's impossible to have cycle.

So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1n_2, n_2n_3$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(ii) if n_1n_2, n_2n_3, n_3n_4 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2, n_2n_3, and n_3n_4 according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has no result. Since there's one cycle but it isn't about all edges. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor a crisp cycle. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4$$

is corresponded neither to Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(iii) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3, n_3n_4 and n_4n_5 according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor a crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_5$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(iv) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$ is a sequence of consecutive edges, then it's obvious that there's one crisp cycle. It's also a crisp path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1 according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. Hence this neutrosophic path is both of a neutrosophic cycle and a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$$

is corresponded to both of Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ and n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(v) $\mathcal{M}(CYC)$: $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$ is Eulerian neutrosophic cycle;

(vi) $\mathcal{N}(CYC) = 1$ is n-Eulerian neutrosophic cycle.

(b) In Figure (2.34), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's only a path and there are only two edges but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive edges is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1n_2, n_2n_3$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(ii) if n_1n_2, n_2n_3, n_3n_4 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2, n_2n_3, and n_3n_4 according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has no result. Since there's one cycle but it isn't about all edges. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor a crisp cycle. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4$$

is corresponded neither to Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(iii) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3, n_3n_4 and n_4n_5 according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor a crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_5$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

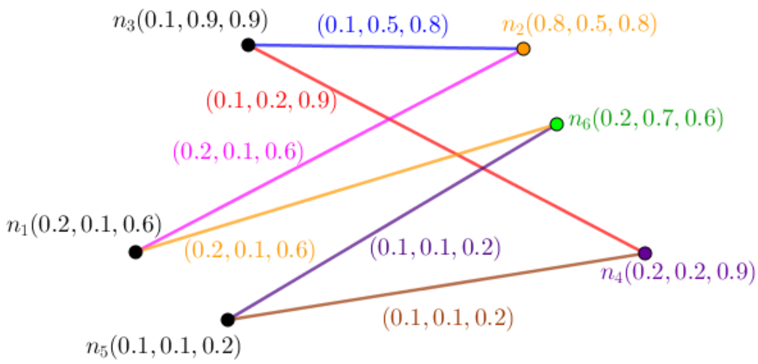

Figure 2.33: A Neutrosophic Graph in the Viewpoint of its Eulerian neutrosophic cycle.

67NTG5

(*iv*) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_1$ is a sequence of consecutive edges, then it's obvious that there's one crisp cycle. It's also a crisp path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1 according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding longest cycle containing all edges once. Finding longest cycle containing all edges once has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_1$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CYC)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CYC)$;

(*v*) $\mathcal{M}(CYC)$: Not Existed. There is no Eulerian neutrosophic cycle and there are no corresponded sets and sequences;

(*vi*) $\mathcal{N}(CYC) = 0$ is n-Eulerian neutrosophic cycle and there are no corresponded sets and sequences.

Definition 2.5.60. (Eulerian(Hamiltonian) Neutrosophic Path).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **Eulerian(Hamiltonian) neutrosophic path** $\mathcal{M}_e(NTG)(\mathcal{M}_h(NTG))$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}(x_1, x_2, \cdots, x_{\mathcal{O}(NTG)})$ which is neutrosophic path;

(*ii*) **n-Eulerian(Hamiltonian) neutrosophic path** $\mathcal{N}_e(NTG)(\mathcal{N}_h(NTG))$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}(x_1, x_2, \cdots, x_{\mathcal{O}(NTG)})$ which is neutrosophic path.

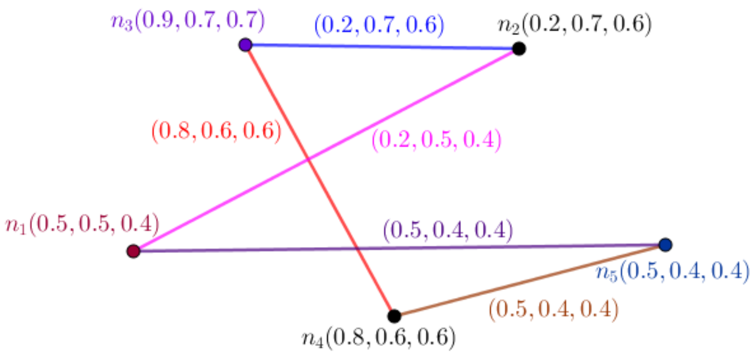

Figure 2.34: A Neutrosophic Graph in the Viewpoint of its Eulerian neutrosophic cycle. 67NTG6

Proposition 2.5.61. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{N}_e(CYC) = 0;$$
$$\mathcal{N}_h(CYC) = \mathcal{O}(CYC).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{S}(CYC)}(x_1, x_2, \cdots, x_{\mathcal{O}(CYC)})$ be a sequence of consecutive edges (vertices) of $CYC : (V, E, \sigma, \mu)$ such that

$$x_i, x_{i+1} \text{ have common vertex, } i = 1, 2, \cdots, \mathcal{S}(CYC) - 1(\mathcal{O}(CYC) - 1),$$

$$x_{\mathcal{S}(CYC)}(x_{\mathcal{O}(CYC)}), x_1 \text{ have common vertex.}$$

There are two paths amid two given vertices. The degree of every vertex is two. There are $\mathcal{S}(CYC)(\mathcal{O}(CYC))$ paths. So the efforts lead to $\mathcal{S}(CYC)(\mathcal{O}(CYC))$ for finding a longest paths with length $\mathcal{S}(CYC)(\mathcal{O}(CYC))$. For a given vertex x_i, the sequence of consecutive edges (vertices)

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}$$

is a corresponded longest path for given vertex (edge) x_i. Every path has same length. The length is $\mathcal{S}(CYC)(\mathcal{O}(CYC))$. Thus the crisp cardinality of set of edges (vertices) forming longest path is $\mathcal{S}(CYC)(\mathcal{O}(CYC))$. $x_i, x_{i+1}, \cdots, x_{\mathcal{S}(CYC)}, \cdots, x_{i-1}$ is a sequence of consecutive edges, there's no repetition of edge in this sequence and all edges are used. Eulerian neutrosophic path is corresponded to longest path with length $\mathcal{S}(CYC)$. $x_i, x_{i+1}, \cdots, x_{\mathcal{O}(CYC)}, \cdots, x_{i-1}$ is a sequence of consecutive vertices, there's no repetition of vertex in this sequence and all vertices are used. Hamiltonian neutrosophic path is corresponded to longest path with length $\mathcal{O}(CYC)$. Thus

$$\mathcal{N}_e(CYC) = 0;$$
$$\mathcal{N}_h(CYC) = \mathcal{O}(CYC).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.62. There are two sections for clarifications.

(a) In Figure (2.35), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1n_3, n_3n_4 is a sequence of consecutive pairs of vertices, then it isn't neutrosophic path since $\mu(n_1n_3) \not> 0$. The number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_3, n_4 and n_1n_3, n_3n_4;

(ii) if n_1n_2, n_3n_4 is a sequence of edges, then it isn't neutrosophic path since $\mu(n_2n_3) \not> 0$. The number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_2, n_3, n_4 and n_1n_2, n_3n_4;

(iii) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$ is a sequence of consecutive edges, then it isn't neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_6n_1) > 0$. And more, it's crisp cycle. The number of edges is greater than $\mathcal{S}(CYC)$ and the number of vertices is $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ and $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6, n_6n_1$;

(iv) if n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_2n_3) > 0$. But the number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_2, n_3 and n_1n_2, n_2n_3;

(v) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$, $\mu(n_2n_3) > 0$, $\mu(n_3n_4) > 0$, $\mu(n_4n_5) > 0$ and $\mu(n_5n_6) > 0$. The number of edges is $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian neutrosophic path $\mathcal{M}_e(CYC)$ is $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and Hamiltonian neutrosophic path $\mathcal{M}_h(CYC)$ is $n_1, n_2, n_3, n_4, n_5, n_6$. Also, n-Eulerian neutrosophic path $\mathcal{N}_e(CYC)$ and n-Hamiltonian neutrosophic path $\mathcal{N}_h(CYC)$ are corresponded to these sequences $n_1, n_2, n_3, n_4, n_5, n_6$ and $n_1n_2, n_2n_3, n_3n_4, n_4, n_5, n_5n_6$;

(*vi*) n-Hamiltonian neutrosophic path $\mathcal{N}_h(CYC)$ equals one and corresponded sequence of consecutive edges is $n_1n_2, n_2n_3, n_3n_4, n_4, n_5, n_5n_6$. n-Eulerian neutrosophic path $\mathcal{N}_e(CYC)$ equals one and corresponded sequence of consecutive vertices is $n_1, n_2, n_3, n_4, n_5, n_6$.

(*b*) In Figure (2.36), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1n_3, n_3n_4 is a sequence of consecutive pairs of vertices, then it isn't neutrosophic path since $\mu(n_1n_3) \not> 0$. The number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_3, n_4 and n_1n_3, n_3n_4;

(*ii*) if n_1n_2, n_3n_4 is a sequence of edges, then it isn't neutrosophic path since $\mu(n_2n_3) \not> 0$. The number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_2, n_3, n_4 and n_1n_2, n_3n_4;

(*iii*) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_1$ is a sequence of consecutive edges, then it isn't neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_5n_1) > 0$. And more, it's crisp cycle. The number of edges is greater than $\mathcal{S}(CYC)$ and the number of vertices is $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences $n_1, n_2, n_3, n_4, n_5, n_1$ and $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_1$;

(*iv*) if n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_2n_3) > 0$. But the number of edges isn't $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CYC)(\mathcal{M}_h(CYC))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CYC)(\mathcal{N}_h(CYC))$ isn't corresponded to these sequences n_1, n_2, n_3 and n_1n_2, n_2n_3;

(*v*) if $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0,\ \mu(n_2n_3) > 0,\ \mu(n_3n_4) > 0$ and $\mu(n_4n_5) > 0$. The number of edges is $\mathcal{S}(CYC)$ and the number of vertices isn't $\mathcal{O}(CYC)$. Thus Eulerian neutrosophic path $\mathcal{M}_e(CYC)$ is $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and Hamiltonian neutrosophic path $\mathcal{M}_h(CYC)$ is n_1, n_2, n_3, n_4. Also, n-Eulerian neutrosophic path $\mathcal{N}_e(CYC)$ and n-Hamiltonian neutrosophic path $\mathcal{N}_h(CYC)$ are corresponded to these sequences n_1, n_2, n_3, n_4, n_5 and $n_1n_2, n_2n_3, n_3n_4, n_4, n_5$;

(*vi*) n-Hamiltonian neutrosophic path $\mathcal{N}_h(CYC)$ equals one and corresponded sequence of consecutive edges is $n_1n_2, n_2n_3, n_3n_4, n_4, n_5$. n-Eulerian neutrosophic path $\mathcal{N}_e(CYC)$ equals one and corresponded sequence of consecutive vertices is n_1, n_2, n_3, n_4, n_5.

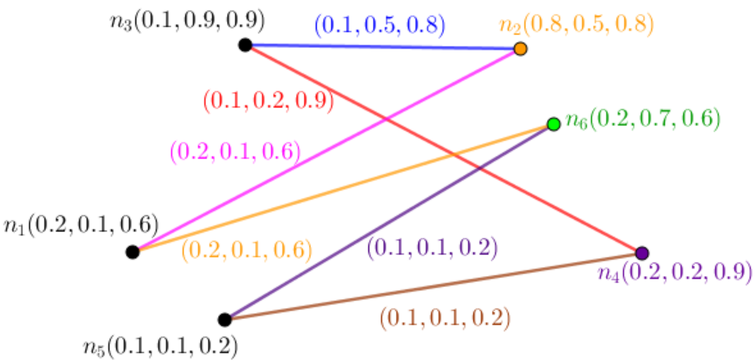

Figure 2.35: A Neutrosophic Graph in the Viewpoint of its Eulerian(Hamiltonian) neutrosophic path. 68NTG5

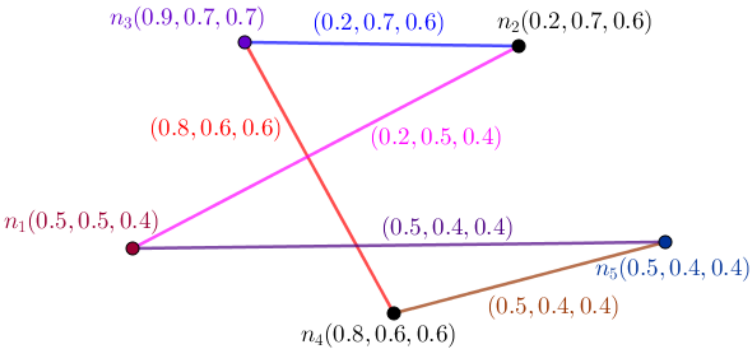

Figure 2.36: A Neutrosophic Graph in the Viewpoint of its Eulerian(Hamiltonian) neutrosophic path. 68NTG6

Definition 2.5.63. (Neutrosophic Path Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a path from x to y is called **weakest path** if its length is maximum. This length is called **weakest number** amid x and y. The maximum number amid all vertices is called **weakest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{W}(NTG)$;

(ii) a path from x to y is called **neutrosophic weakest path** if its strength is $\mu(uv)$ which is less than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called **neutrosophic weakest number** amid x and y. The maximum number amid all vertices is called **neutrosophic weakest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{W}_n(NTG)$.

Proposition 2.5.64. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{W}_n(CYC) = \max\{\mu(xy) \mid \mu(xy) = \bigwedge_{i=1,2,\cdots,s-1} \mu(v_i v_{i+1}),\ P : v_1, v_2, \cdots, v_s\}.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. There are some neutrosophic paths. The biggest length of a path is weakest number. The biggest length of path is either size minus one or order minus one. It means the length of this path is either $\mathcal{S}(CYC) - 1$ or $\mathcal{O}(CYC) - 1$. Thus

$$\mathcal{W}_n(CYC) = \max\{\mu(xy) \mid \mu(xy) = \bigwedge_{i=1,2,\cdots,s-1} \mu(v_i v_{i+1}),\ P : v_1, v_2, \cdots, v_s\}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.65. There are two sections for clarifications.

(*a*) In Figure (2.37), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (*i*) If $n_1, n_2, n_3, n_4, n_5, n_6$ is a neutrosophic path from n_1 to n_6, then it's weakest path and weakest number amid n_1 and n_6 is five. Also, $\mathcal{W}(CYC) = 5$;
- (*ii*) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't weakest path and weakest number amid n_1 and n_3 is four corresponded to n_1, n_6, n_5, n_4, n_3. Also, $\mathcal{W}(CYC) \neq 2$;
- (*iii*) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't weakest path but weakest number amid n_1 and n_4 is three corresponded to n_1, n_2, n_3, n_4. Also, $\mathcal{W}(CYC) \neq 3$. For every given couple of vertices x and y, weakest path isn't existed but weakest number is five and $\mathcal{W}(CYC) = 5$;
- (*iv*) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic weakest path since neutrosophic weakest number amid n_2 and n_3 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{W}_n(CYC) = (0.1, 0.5, 0.8)$;
- (*v*) if n_2, n_3 is a neutrosophic path from n_2 to n_3, then it's a neutrosophic weakest path and neutrosophic weakest number amid n_2 and n_3 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{W}_n(CYC) = (0.1, 0.5, 0.8)$;
- (*vi*) for every given couple of vertices x and y, neutrosophic weakest path isn't existed, neutrosophic weakest number is $(0.1, 0.5, 0.8)$ and $\mathcal{W}_n(CYC) = (0.1, 0.5, 0.8)$.

(*b*) In Figure (2.38), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (*i*) If n_1, n_2, n_3, n_4, n_5 is a neutrosophic path from n_1 to n_5, then it's weakest path and weakest number amid n_1 and n_5 is four. Also, $\mathcal{W}(CYC) = 4$;

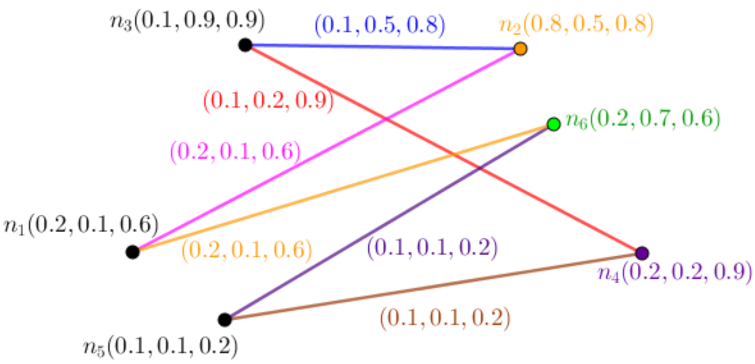

Figure 2.37: A Neutrosophic Graph in the Viewpoint of its Weakest Number and its Neutrosophic Weakest Number. 70NTG5

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't weakest path and weakest number amid n_1 and n_3 is three corresponded to n_1, n_5, n_4, n_3. Also, $\mathcal{W}(CYC) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't weakest path but weakest number amid n_1 and n_4 is three corresponded to n_1, n_2, n_3, n_4. Also, $\mathcal{W}(CYC) \neq 3$. For every given couple of vertices x and y, weakest path isn't existed but weakest number is four and $\mathcal{W}(CYC) = 4$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic weakest path since neutrosophic weakest number amid n_3 and n_4 is $(0.8, 0.6, 0.6)$. Also, $\mathcal{W}_n(CYC) = (0.8, 0.6, 0.6)$;

(v) if n_3, n_4 is a neutrosophic path from n_3 to n_4, then it's a neutrosophic weakest path and neutrosophic weakest number amid n_3 and n_4 is $(0.8, 0.6, 0.6)$. Also, $\mathcal{W}_n(CYC) = (0.8, 0.6, 0.6)$;

(vi) for every given couple of vertices x and y, neutrosophic weakest path isn't existed, neutrosophic weakest number is $(0.8, 0.6, 0.6)$ and $\mathcal{W}_n(CYC) = (0.8, 0.6, 0.6)$.

Definition 2.5.66. (Neutrosophic Path Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a path from x to y is called **strongest path** if its length is minimum. This length is called **strongest number** amid x and y. The maximum number amid all vertices is called **strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}(NTG)$;

(ii) a path from x to y is called **neutrosophic strongest path** if its strength is $\mu(uv)$ which is greater than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called **neutrosophic strongest number** amid x and y. The minimum number amid all vertices is called **neutrosophic strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}_n(NTG)$.

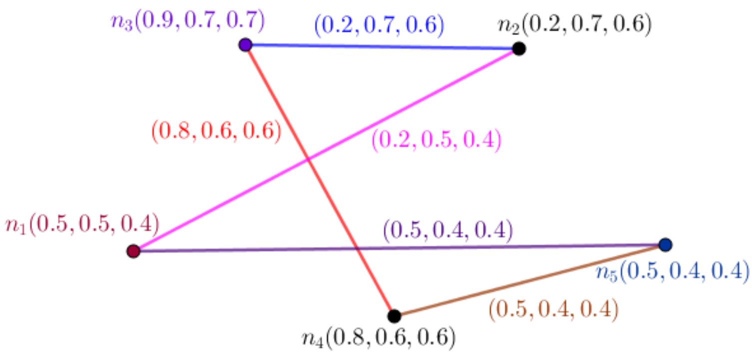

Figure 2.38: A Neutrosophic Graph in the Viewpoint of its Weakest Number and its Neutrosophic Weakest Number. 70NTG6

Proposition 2.5.67. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{S}_n(CYC) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. There are some neutrosophic paths. The biggest length of a path is strongest number. For every given couple of vertices, there are two neutrosophic paths concerning two lengths s and $\mathcal{O}(CYC) - s$. If $s < \mathcal{O}(CYC) - s$, then s is intended length; otherwise, $\mathcal{O}(CYC) - s$ is intended length. Since minimum length amid two vertices are on demand. In next step, amid all lengths, the biggest number is strongest number. The biggest length of path is either order half or order half minus one. It means the length of this path is either $\frac{\mathcal{O}(CYC)}{2}$ or $\frac{\mathcal{O}(CYC)}{2} - 1$. There are only two paths amid given couple of vertices. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All paths involving s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. Consider the maximum number assigning to couple of vertices arising from their paths as the start and the end. Thus the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. It implies the minimum number amid these intended numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{S}_n(CYC) = \min_{v \in V} \sigma(v).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.68. There are two sections for clarifications.

(a) In Figure (2.39), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $n_1, n_2, n_3, n_4, n_5, n_6$ is a neutrosophic path from n_1 to n_6, then it isn't strongest path and strongest number amid n_1 and n_6 is one. Also, $\mathcal{S}(CYC) = 3$.

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 is two corresponded to n_1, n_2, n_3. Also, $\mathcal{S}(CYC) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it is strongest path and strongest number amid n_1 and n_4 is three corresponded to n_1, n_2, n_3, n_4 and n_1, n_6, n_5, n_4 Also, $\mathcal{S}(CYC) = 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is three and $\mathcal{S}(CYC) = 3$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.1, 0.1, 0.2)$ but neutrosophic strongest number amid n_1 and n_4 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$;

(v) if n_2, n_3 is a neutrosophic path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_4 and n_5 is $(0.1, 0.1, 0.2)$ but neutrosophic strongest number amid n_2 and n_3 is $(0.1, 0.5, 0.8)$. Also, $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.1, 0.1, 0.2)$ and $\mathcal{S}_n(CYC) = (0.1, 0.1, 0.2)$.

(b) In Figure (2.40), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_3, n_4, n_5 is a neutrosophic path from n_1 to n_5, then it isn't strongest path and strongest number amid n_1 and n_5 is one. Also, $\mathcal{S}(CYC) = 2$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it's strongest path and strongest number amid n_1 and n_3 is two. Also, $\mathcal{S}(CYC) = 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is two corresponded to n_1, n_5, n_4. Also, $\mathcal{S}(CYC) \neq 3$. For every given couple of vertices x and y, strongest path isn't existed but strongest number is two and $\mathcal{S}(CYC) = 2$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path [strength is $(0.2, 0.5, 0.4)$] from n_1 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_4 is $(0.5, 0.4, 0.4)$ but neutrosophic strongest number amid n_1 and n_2 is $(0.2, 0.7, 0.6)$; neutrosophic strongest number amid n_2 and n_3 is $(0.2, 0.7, 0.6)$. Also, $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$;

(v) if n_3, n_4 is a neutrosophic path [strength is $(0.8, 0.6, 0.6)$] from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic

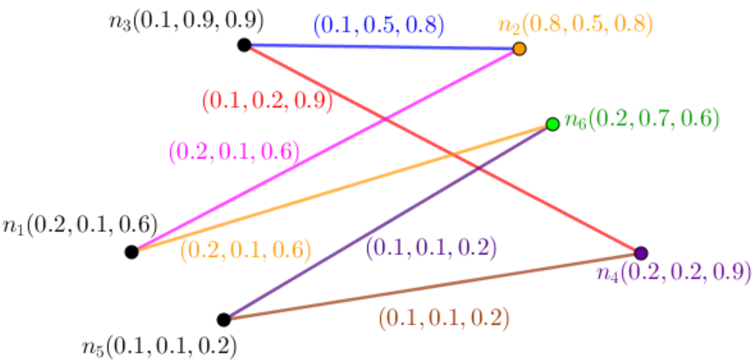

Figure 2.39: A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number. 71NTG5

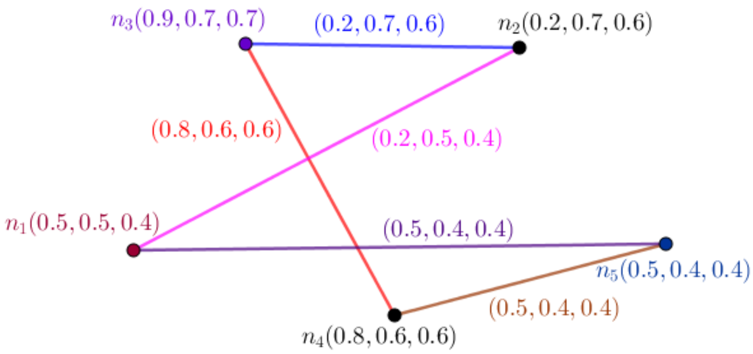

Figure 2.40: A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number. 71NTG6

strongest number amid n_3 and n_4 is $(0.8, 0.6, 0.6)$. Also, $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$;

(vi) for every given couple of vertices x and y, neutrosophic strongest path isn't existed, neutrosophic strongest number is $(0.2, 0.7, 0.6)$ and $\mathcal{S}_n(CYC) = (0.2, 0.7, 0.6)$.

Definition 2.5.69. (Neutrosophic Cycle Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a cycle based on x is called **cyclic connectivity** if its length is minimum. This length is called **connectivity number** based on x. The maximum number amid all vertices is called **connectivity number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{C}(NTG)$;

(ii) a cycle based on x is called **neutrosophic cyclic connectivity** if its strength is is greater than all strengths of all cycles based on x. This strength is called **neutrosophic connectivity number** based on x. The minimum number amid all vertices is called **neutrosophic connectivity number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{C}_n(NTG)$.

Proposition 2.5.70. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{C}_n(CYC) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. There's only one cycle for all given vertices so all vertices are only based on one cycle which is common for all of them. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All cycles based on s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of all cycles based on s is $\sigma(s) = \min_{v \in V} \sigma(v)$ which is representative strength based on s. It implies the minimum number amid all representative numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{C}_n(CYC) = \min_{v \in V} \sigma(v).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.71. There are two sections for clarifications.

(a) In Figure (2.41), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a neutrosophic cycle based on n_1, then it's cyclic connectivity and connectivity number based on n_1 is 6. Also, $\mathcal{C}(CYC) = 6$;
- (ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't cyclic connectivity but connectivity number based on any given vertex is existed. There's only one cycle. Hence there's one cycle related to connectivity number of this cycle-neutrosophic graph. Also, $\mathcal{C}(CYC) = 6$ and $\mathcal{C}(CYC) \neq 2$;
- (iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't cyclic connectivity but connectivity number based on some sequence of consecutive vertices is existed. There's one cycle. Hence there's one cycle related to connectivity number of this cycle-neutrosophic graph. Also, $\mathcal{C}(CYC) = 6$. Also, $\mathcal{C}(CYC) \neq 3$. For every given vertex x, cyclic connectivity is existed and connectivity number is six and $\mathcal{C}(CYC) = 6$;
- (iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic cyclic connectivity but neutrosophic connectivity number based on any given vertex is existed. There's one cycle so there's one cycle related to neutrosophic connectivity number which is $(0.1, 0.1, 0.2)$. Also, $\mathcal{C}_n(CYC) = (0.1, 0.1, 0.2)$;

(v) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a neutrosophic cycle based on n_1, then it's a neutrosophic cyclic connectivity since there's one cycle and there's one cycle based on n_1 and neutrosophic connectivity number based on n_1 is $(0.1, 0.1, 0.2)$. Also, $\mathcal{C}_n(CYC) = (0.1, 0.1, 0.2)$;

(vi) if $n_2, n_1, n_6, n_5, n_4, n_3, n_2$ is a neutrosophic cycle based on n_2, then it's a neutrosophic cyclic connectivity since there's one cycle and there's one cycle based on n_2 and neutrosophic connectivity number based on n_2 is $(0.1, 0.1, 0.2)$. Also, $\mathcal{C}_n(CYC) = (0.1, 0.1, 0.2)$.

(b) In Figure (2.42), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $n_1, n_2, n_3, n_4, n_5, n_1$ is a neutrosophic cycle based on n_1, then it's cyclic connectivity and connectivity number based on n_1 is 5. Also, $\mathcal{C}(CYC) = 5$;

(ii) if n_1, n_2, n_3 is a neutrosophic path from n_1 to n_3, then it isn't cyclic connectivity but connectivity number based on any given vertex is existed. There's only one cycle. Hence there's one cycle related to connectivity number of this cycle-neutrosophic graph. Also, $\mathcal{C}(CYC) = 5$ and $\mathcal{C}(CYC) \neq 2$;

(iii) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't cyclic connectivity but connectivity number based on some sequence of consecutive vertices is existed. There's one cycle. Hence there's one cycle related to connectivity number of this cycle-neutrosophic graph. Also, $\mathcal{C}(CYC) = 5$. Also, $\mathcal{C}(CYC) \neq 3$. For every given vertex x, cyclic connectivity is existed and connectivity number is five and $\mathcal{C}(CYC) = 5$;

(iv) if n_1, n_2, n_3, n_4 is a neutrosophic path from n_1 to n_4, then it isn't a neutrosophic cyclic connectivity but neutrosophic connectivity number based on any given vertex is existed. There's one cycle so there's one cycle related to neutrosophic connectivity number which is $(0.2, 0.5, 0.4)$. Also, $\mathcal{C}_n(CYC) = (0.2, 0.5, 0.4)$;

(v) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a neutrosophic cycle based on n_1, then it's a neutrosophic cyclic connectivity since there's one cycle and there's one cycle based on n_1 and neutrosophic connectivity number based on n_1 is $(0.2, 0.5, 0.4)$. Also, $\mathcal{C}_n(CYC) = (0.2, 0.5, 0.4)$;

(vi) if $n_2, n_1, n_5, n_4, n_3, n_2$ is a neutrosophic cycle based on n_2, then it's a neutrosophic cyclic connectivity since there's one cycle and there's one cycle based on n_2 and neutrosophic connectivity number based on n_2 is $(0.2, 0.5, 0.4)$. Also, $\mathcal{C}_n(CYC) = (0.2, 0.5, 0.4)$.

Definition 2.5.72. (Dense Numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set of vertices is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum cardinality between all dense sets is called **dense number** and it's denoted by $\mathcal{D}(NTG)$;

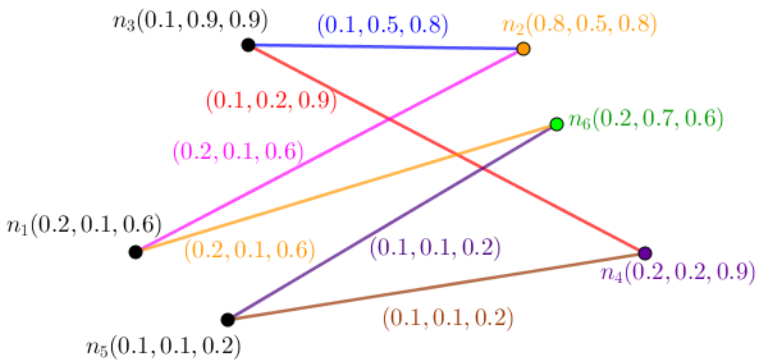

Figure 2.41: A Neutrosophic Graph in the Viewpoint of its connectivity number and its neutrosophic connectivity number. 72NTG5

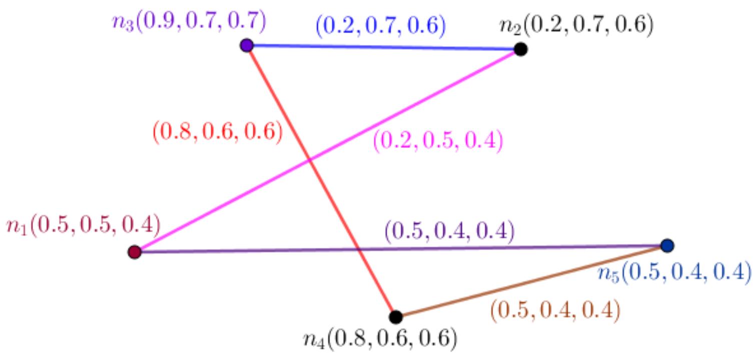

Figure 2.42: A Neutrosophic Graph in the Viewpoint of its connectivity number and its neutrosophic connectivity number. 72NTG6

(ii) a set of vertices S is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{s\in S}\sum_{i=1}^{3}\sigma_i(s)$ between all dense sets is called **neutrosophic dense number** and it's denoted by $\mathcal{D}_n(NTG)$.

Proposition 2.5.73. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{D}_n(CYC) = \min\{\sum_{i=-2}^{\mathcal{O}(CYC)-3} \sigma(x_{i+3})\}$$

where rearrangements of indexes are possible in any arbitrary ways.

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. Every vertex has two neighbors. So these vertices have same positions and by the minimum number of vertices is on demand, the result is obtained. Thus

$$\mathcal{D}_n(CYC) = \min\{ \sum_{i=-2}^{\mathcal{O}(CYC)-3} \sigma(x_{i+3})\}$$

where rearrangements of indexes are possible in any arbitrary ways. ■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.74. There are two sections for clarifications.

(a) In Figure (2.43), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

  (ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

  (iii) $S_1 = \{n_1, n_4\}, S_2 = \{n_2, n_5\}, S_3 = \{n_3, n_6\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only three dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CYC) = 2$;

  (iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

  (v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

  (vi) $S_1 = \{n_1, n_4\}, S_2 = \{n_2, n_5\}, S_3 = \{n_3, n_6\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only three dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CYC) = 2.2$ corresponded to S_1;

(b) In Figure (2.44), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_4 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, and n_4 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

(iii) $S_1 = \{n_1, n_3\}, S_2 = \{n_1, n_4\}, S_3 = \{n_2, n_4\}, S_4 = \{n_2, n_5\}, S_5 = \{n_3, n_5\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only five dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CYC) = 2$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_4 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, and n_4 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

(vi)

$$\begin{aligned} S_1 = \{n_1, n_3\} &\to 2.8 \\ S_2 = \{n_1, n_4\} &\to 2.2 \\ S_3 = \{n_2, n_4\} &\to 3.4 \\ S_4 = \{n_2, n_5\} &\to 2.5 \\ S_5 = \{n_3, n_5\} &\to 2.3 \\ \text{Minimum number is } 2.2 \end{aligned}$$

are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only five dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CYC) = 2$ corresponded to S_2.

Definition 2.5.75. (bulky numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

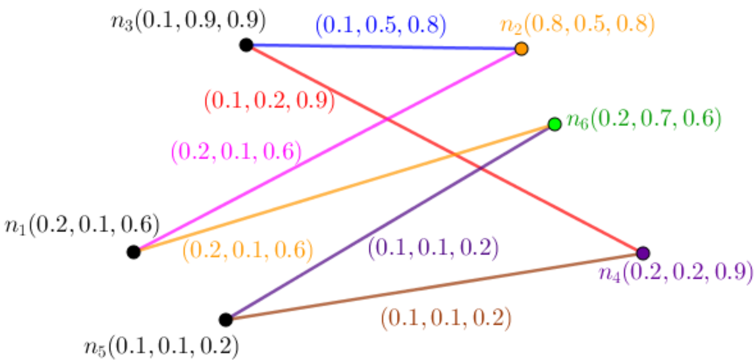

Figure 2.43: A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

73NTG5

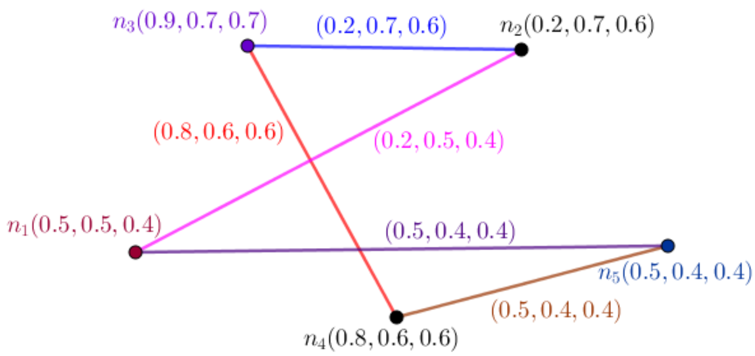

Figure 2.44: A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

73NTG6

(i) a set of edges S is called **bulky set** if for every edge e' outside, there's at least one edge e inside such that they've common vertex and the number of edges such that they've common vertex with e is greater than the number of edges such that they've common vertex with e'. The minimum cardinality between all bulky sets is called **bulky number** and it's denoted by $\mathcal{B}(NTG)$;

(ii) a set of edges S is called **bulky set** if for every edge e' outside, there's at least one edge e inside such that they've common vertex and the number of edges such that they've common vertex with e is greater than the number of edges such that they've common vertex with e'. The minimum neutrosophic cardinality $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(s)$ between all bulky sets is called **neutrosophic bulky number** and it's denoted by $\mathcal{B}_n(NTG)$.

Proposition 2.5.76. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{B}_n(CYC) = \min\{\sum_{i=-2}^{\mathcal{O}(CYC)-3} \mu(e_{i+3})\}.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. Every vertex has two neighbors. So all vertices have same positions. It implies finding edges have common endpoint. By minimum number of edges is on demand, the result is obtained. Thus

$$\mathcal{B}_n(CYC) = \min\{\sum_{i=-2}^{\mathcal{O}(CYC)-3} \mu(e_{i+3})\}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.77. There are two sections for clarifications.

(a) In Figure (2.45), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it isn't a bulky set since an edge n_4n_5, outside, there's no edge inside such that they've common vertex;
- (ii) if $S = \{n_1n_2, n_4n_5\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with vertices of S is two which is equal to [greater than] two which is the number of edges such that they've common vertex with vertices of $V \setminus S$;
- (iii) All sets [2-sets] of edges containing two edges aren't bulky sets. The sets of edges $\{n_1n_2, n_4n_5\}, \{n_2n_3, n_5n_6\}, \{n_3n_4, n_6n_1\}$ are only minimal bulky sets. Since for every edge n_in_j, outside, there's at least one edge n_tn_s inside such that they've common vertex and the number of edges such that they've common vertex with n_tn_s is two which is equal to [greater than] two which is the number of edges such that they've common vertex with n_in_j. Thus $\mathcal{B}(CYC) = 2$;
- (iv) if $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it isn't a bulky set since an edge n_4n_5, outside, there's no edge n_2n_4 inside such that they've common vertex;
- (v) if $S = \{n_1n_2, n_4n_5\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with vertices of S is two which is equal to [greater than] two which is the number of edges such that they've common vertex with vertices of $V \setminus S$;
- (vi) All sets [2-sets] of edges containing two edges aren't bulky sets. The sets of edges $S_1 = \{n_1n_2, n_4n_5\}, S_2 = \{n_2n_3, n_5n_6\}$, and $S_3 = \{n_3n_4, n_6n_1\}$ are only minimal bulky sets. Since for every

edge n_in_j, outside, there's at least one edge n_tn_s inside such that they've common vertex and the number of edges such that they've common vertex with n_tn_s is two which is equal to [greater than] two which is the number of edges such that they've common vertex with n_in_j. Thus

$$S_1 = \{n_1n_2, n_4n_5\} \to 1.3$$
$$S_2 = \{n_2n_3, n_5n_6\} \to 1.8$$
$$S_3 = \{n_3n_4, n_6n_1\} \to 2.1$$
$$\text{Minimum number is } 1.3$$

It implies $\mathcal{B}_n(CYC) = 1.3$ and corresponded set of edges is $S_1 = \{n_1n_2, n_4n_5\}$.

(b) In Figure (2.46), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it isn't a bulky set since an edge n_4n_5, outside, there's no edge inside such that they've common vertex;

(ii) if $S = \{n_1n_2, n_4n_5\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with vertices of S is two which is equal to [greater than] two which is the number of edges such that they've common vertex with vertices of $V \setminus S$;

(iii) All sets [2-sets] of edges containing two edges aren't bulky sets. The sets of edges $S_1 = \{n_1n_2, n_4n_5\}, S_2 = \{n_2n_3, n_5n_1\}, S_3 = \{n_2n_3, n_4n_5\}, S_4 = \{n_3n_4, n_5n_1\}$, and $S_5 = \{n_3n_4, n_1n_2\}$ are only minimal bulky sets. Since for every edge n_in_j, outside, there's at least one edge n_tn_s inside such that they've common vertex and the number of edges such that they've common vertex with n_tn_s is two which is equal to [greater than] two which is the number of edges such that they've common vertex with n_in_j. Thus $\mathcal{B}(CYC) = 2$;

(iv) if $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it isn't a bulky set since an edge n_4n_5, outside, there's no edge n_2n_4 inside such that they've common vertex;

(v) if $S = \{n_1n_2, n_4n_5\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with vertices of S is two which is equal to [greater than] two which is the number of edges such that they've common vertex with vertices of $V \setminus S$;

(vi) All sets [2-sets] of edges containing two edges aren't bulky sets. The sets of edges $S_1 = \{n_1n_2, n_4n_5\}, S_2 = \{n_2n_3, n_5n_1\}, S_3 = \{n_2n_3, n_4n_5\}, S_4 = \{n_3n_4, n_5n_1\}$, and $S_5 = \{n_3n_4, n_1n_2\}$ are only minimal bulky sets. Since for every edge n_in_j, outside, there's at least one edge n_tn_s inside such that they've common vertex and the number of edges such that they've common vertex with n_tn_s is two

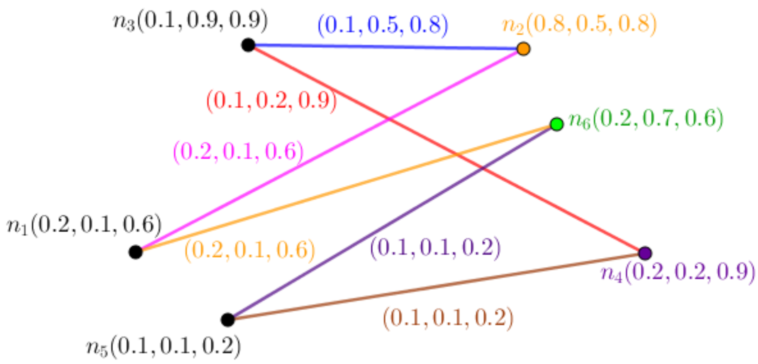

Figure 2.45: A Neutrosophic Graph in the Viewpoint of its bulky number and its neutrosophic bulky number. 74NTG5

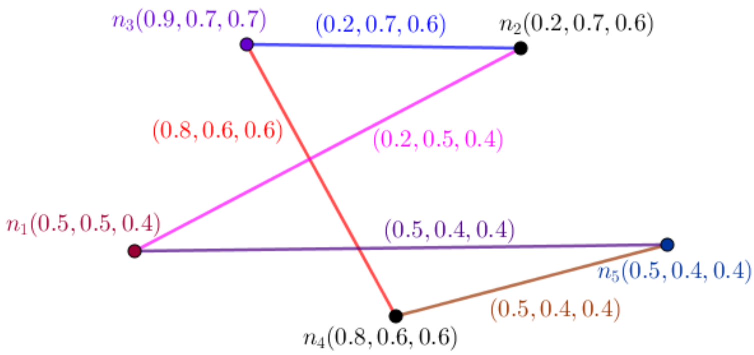

Figure 2.46: A Neutrosophic Graph in the Viewpoint of its bulky number and its neutrosophic bulky number. 74NTG6

which is equal to [greater than] two which is the number of edges such that they've common vertex with n_in_j. Thus

$$\begin{aligned}
S_1 &= \{n_1n_2, n_4n_5\} \to 2.4\\
S_2 &= \{n_2n_3, n_5n_1\} \to 2.8\\
S_3 &= \{n_2n_3, n_4n_5\} \to 2.8\\
S_4 &= \{n_3n_4, n_5n_1\} \to 3.3\\
S_5 &= \{n_3n_4, n_1n_2\} \to 3.1\\
&\text{Minimum number is } 2.4
\end{aligned}$$

It implies $\mathcal{B}_n(CYC) = 2.4$ and corresponded set of edges is $S_1 = \{n_1n_2, n_4n_5\}$.

Definition 2.5.78. (collapsed numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set of vertices S is called **collapsed set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$

and the number of neighbors of x is less than [equal to] the number of neighbors of y. The minimum cardinality between all collapsed sets is called **collapsed number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) a set of vertices S is called **collapsed set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is less than [equal to] the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x)$ between all collapsed sets is called **neutrosophic collapsed number** and it's denoted by $\mathcal{P}_n(NTG)$.

Proposition 2.5.79. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{P}_n(CYC) = \min \sum_{x_j \in \{x_s, x_{s+3}, x_{s+6}, \cdots, x_i\}_{i+2>\mathcal{O}(CYC)}} \sigma(x_j).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. Every vertex has two neighbors. So all vertices have same positions. The set

$$\{x_s, x_{s+3}, x_{s+6}, \cdots, x_i\}_{i+2>\mathcal{O}(CYC)}$$

of vertices is called collapsed set since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum neutrosophic cardinality, $\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x)$, $\min \sum_{x_j \in \{x_s, x_{s+3}, x_{s+6}, \cdots, x_i\}_{i+2>\mathcal{O}(CYC)}} \sigma(x_j)$, between all collapsed sets is called neutrosophic collapsed number and it's denoted by $\mathcal{P}_n(CYC) = \min \sum_{x_j \in \{x_s, x_{s+3}, x_{s+6}, \cdots, x_i\}_{i+2>\mathcal{O}(CYC)}} \sigma(x_j)$. Thus

$$\mathcal{P}_n(CYC) = \min \sum_{x_j \in \{x_s, x_{s+3}, x_{s+6}, \cdots, x_i\}_{i+2>\mathcal{O}(CYC)}} \sigma(x_j).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.80. There are two sections for clarifications.

(a) In Figure (2.47), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_3\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_5 outside, such that there's no vertex inside such that they're endpoints either $n_1n_5 \in E$ or $n_3n_5 \in E$;

(*ii*) if $S = \{n_1, n_5\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_3 outside, such that there's no vertex inside such that they're endpoints either $n_1n_3 \in E$ or $n_5n_3 \in E$;

(*iii*) all sets [2-sets] of vertices containing two vertices, aren't called collapsed sets. Sets [2-sets] of vertices $S_1 = \{n_1, n_4\},\ S_2 = \{n_2, n_5\}$, and $S_3 = \{n_3, n_6\}$ are called minimal collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum cardinality $|S|,\ 2$, between all collapsed sets

$$S_1 = \{n_1, n_4\} \to 2$$
$$S_2 = \{n_2, n_5\} \to 2$$
$$S_3 = \{n_3, n_6\} \to 2$$
$$\text{The minimum is } 2$$

is called collapsed number and it's denoted by $\mathcal{P}(CYC) = 2$; $S_1 = \{n_1, n_4\},\ S_2 = \{n_2, n_5\}$, and $S_3 = \{n_3, n_6\}$ are corresponded sets;

(*iv*) if $S = \{n_1, n_3\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_5 outside, such that there's no vertex inside such that they're endpoints either $n_1n_5 \in E$ or $n_3n_5 \in E$;

(*v*) if $S = \{n_1, n_5\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_3 outside, such that there's no vertex inside such that they're endpoints either $n_1n_3 \in E$ or $n_5n_3 \in E$;

(*vi*) all sets [2-sets] of vertices containing two vertices, aren't called collapsed sets. Sets [2-sets] of vertices $S_1 = \{n_1, n_4\},\ S_2 = \{n_2, n_5\}$, and $S_3 = \{n_3, n_6\}$ are called minimal collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum neutrosophic cardinality, $\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x),\ 2.2$, between all collapsed sets

$$S_1 = \{n_1, n_4\} \to 2.2$$
$$S_2 = \{n_2, n_5\} \to 4.5$$
$$S_3 = \{n_3, n_6\} \to 3.4$$
$$\text{The minimum is } 2.2$$

is called neutrosophic collapsed number and it's denoted by $\mathcal{P}_n(CYC) = 2.2$ and corresponded set is $S_1 = \{n_1, n_4\}$.

(*b*) In Figure (2.48), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1, n_2\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_4 outside, such that there's no vertex inside such that they're endpoints either $n_1n_4 \in E$ or $n_2n_4 \in E$;

(*ii*) if $S = \{n_4, n_5\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_2 outside, such that there's no vertex inside such that they're endpoints either $n_4n_2 \in E$ or $n_5n_2 \in E$;

(*iii*) all sets [2-sets] of vertices containing two vertices, aren't called collapsed sets. Sets [2-sets] of vertices $S_1 = \{n_1, n_4\}$, $S_2 = \{n_1, n_3\}$, $S_3 = \{n_2, n_5\}$, $S_4 = \{n_2, n_4\}$, and $S_5 = \{n_3, n_5\}$ are called minimal collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum cardinality $|S|$, 2, between all collapsed sets

$$S_1 = \{n_1, n_4\} \to 2$$
$$S_2 = \{n_1, n_3\} \to 2$$
$$S_3 = \{n_2, n_5\} \to 2$$
$$S_4 = \{n_2, n_4\} \to 2$$
$$S_5 = \{n_3, n_5\} \to 2$$
The minimum is 2

is called collapsed number and it's denoted by $\mathcal{P}(CYC) = 2$; corresponded sets are $S_1 = \{n_1, n_4\}$, $S_2 = \{n_1, n_3\}$, $S_3 = \{n_2, n_5\}$, $S_4 = \{n_2, n_4\}$, and $S_5 = \{n_3, n_5\}$;

(*iv*) if $S = \{n_1, n_2\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_4 outside, such that there's no vertex inside such that they're endpoints either $n_1n_4 \in E$ or $n_2n_4 \in E$;

(*v*) if $S = \{n_4, n_5\}$ is a set of vertices, then a set of vertices S isn't called collapsed set since there's a vertex n_2 outside, such that there's no vertex inside such that they're endpoints either $n_4n_2 \in E$ or $n_5n_2 \in E$;

(*vi*) all sets [2-sets] of vertices containing two vertices, aren't called collapsed sets. Sets [2-sets] of vertices $S_1 = \{n_1, n_4\}$, $S_2 = \{n_1, n_3\}$, $S_3 = \{n_2, n_5\}$, $S_4 = \{n_2, n_4\}$, and $S_5 = \{n_3, n_5\}$ are called minimal collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum neutrosophic cardinality, $\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x)$, 2.8, between all collapsed sets

$$S_1 = \{n_1, n_4\} \to 3.4$$
$$S_2 = \{n_1, n_3\} \to 3.7$$
$$S_3 = \{n_2, n_5\} \to 2.8$$
$$S_4 = \{n_2, n_4\} \to 3.6$$
$$S_5 = \{n_3, n_5\} \to 3.6$$
The minimum is 2.8

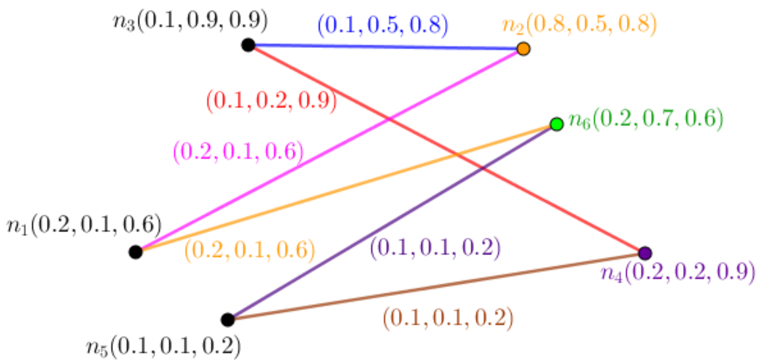

Figure 2.47: A Neutrosophic Graph in the Viewpoint of its collapsed number and its neutrosophic collapsed number. 75NTG5

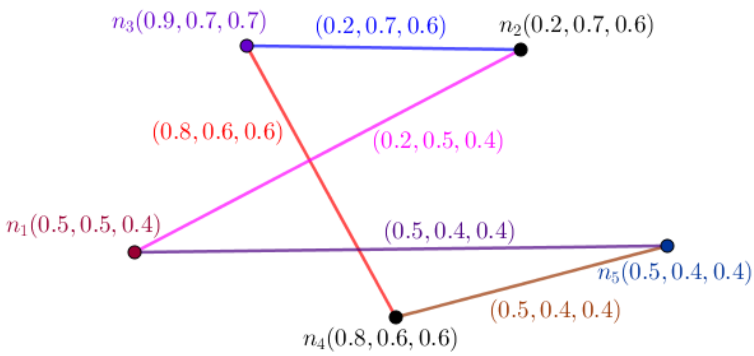

Figure 2.48: A Neutrosophic Graph in the Viewpoint of its collapsed number and its neutrosophic collapsed number. 75NTG6

is called neutrosophic collapsed number and it's denoted by $\mathcal{P}_n(CYC) = 2.8$ and corresponded set is $S_3 = \{n_2, n_5\}$.

Definition 2.5.81. (path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors in this process is called **path-coloring set** from x to y. The minimum cardinality between all path-coloring sets from two given vertices is called **path-coloring number** and it's denoted by $\mathcal{L}(NTG)$;

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set S of shared edges in this process is called **path-coloring set** from x to y. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, Ss, is called **neutrosophic path-coloring number** and it's denoted by $\mathcal{L}_n(NTG)$.

Proposition 2.5.82. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{L}_n(CYC) = \min_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors but these two paths don't share one edge. The set S of shared edges in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by

$$\mathcal{L}_n(CYC) = \min_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.83. There are two sections for clarifications.

(a) In Figure (2.49), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) All paths are as follows.

$$\begin{aligned} P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 &\to \text{red} \\ P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3 &\to \text{red} \\ P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1, n_6, n_5, n_4 &\to \text{red} \\ P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_6, n_5 &\to \text{red} \\ P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \&\ P_2 : n_1, n_6 &\to \text{red} \\ &\text{The number is 1;} \end{aligned}$$

  (ii) 1-paths have same color;

  (iii) $\mathcal{L}(CYC) = 1$;

  (iv) the position of given vertices could be different in the terms of creating path and the behaviors in path;

  (v) there are only two paths but there's no shared edge;

(vi) all paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_6, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1, n_6, n_5, n_4 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_6, n_5 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \& \ P_2 : n_1, n_6 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$\mathcal{L}_n(CYC) \text{ is } 0.$$

(b) In Figure (2.50), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) All paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_5, n_4, n_3, n_2 \to \text{red}$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_5, n_4, n_3 \to \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1 n_5, n_4 \to \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_5 \to \text{red}$$
$$\text{The number is 1;}$$

(ii) 1-paths have same color;

(iii) $\mathcal{L}(CYC) = 1$;

(iv) the position of given vertices could be different in the terms of creating path and the behaviors in path;

(v) there are only two paths but there's no shared edge;

(vi) all paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1, n_5, n_4 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_5 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$\mathcal{L}_n(CYC) \text{ is } 0.$$

Definition 2.5.84. (dominating path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of different colors, S, in this process is called **dominating path-coloring set** from x to y if for every edge outside there's at least one edge inside which they've common vertex. The minimum cardinality between all dominating path-coloring sets from two given vertices is called **dominating path-coloring number** and it's denoted by $\mathcal{Q}(NTG)$;

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set S of different colors in this process is called **dominating**

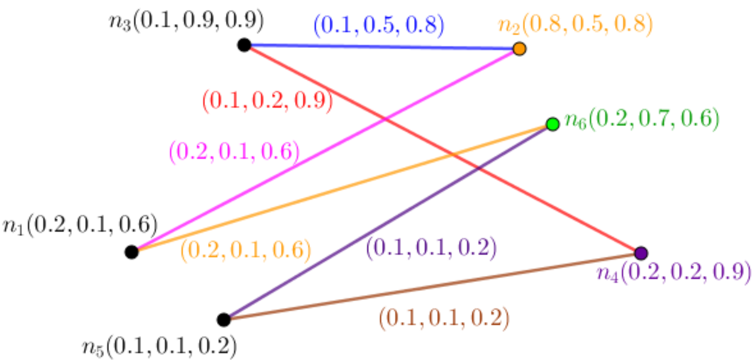

Figure 2.49: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number. 76NTG5

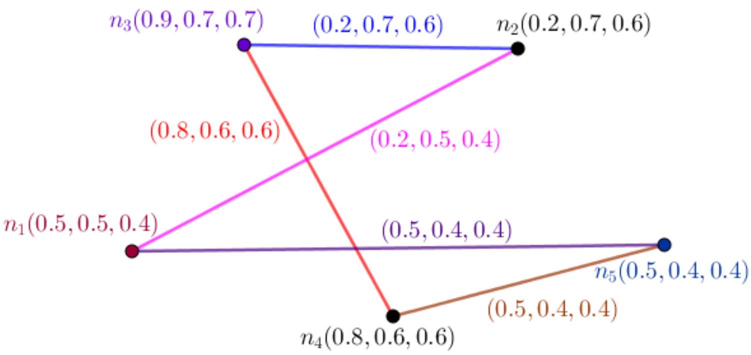

Figure 2.50: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number. 76NTG6

path-coloring set from x to y if for every edge outside there's at least one edge inside which they've common vertex. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all dominating path-coloring sets, Ss, is called **neutrosophic dominating path-coloring number** and it's denoted by $\mathcal{Q}_n(NTG)$.

Proposition 2.5.85. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{Q}_n(CYC) = \min_{S,\ |S|=\lceil\frac{\mathcal{O}(CYC)}{3}\rceil} \sum_{e\in S}\sum_{i=1}^{3}\mu_i(e).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors but these two paths don't share one edge. The set S of shared edges in this process is called dominating path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all dominating path-coloring sets, Ss, is called neutrosophic dominating path-coloring number and it's denoted by

$$\mathcal{Q}_n(CYC) = \min_{S,\ |S|=\lceil \frac{\mathcal{O}(CYC)}{3} \rceil} \sum_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.86. There are two sections for clarifications.

(a) In Figure (2.51), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) All paths are as follows.

$$\begin{aligned}
P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 &\to \text{red}\\
P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3 &\to \text{red}\\
P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1, n_6, n_5, n_4 &\to \text{red}\\
P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_6, n_5 &\to \text{red}\\
P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \&\ P_2 : n_1, n_6 &\to \text{red}\\
&\text{The number is 1;}
\end{aligned}$$

(ii) 1-paths have same color;

(iii) $\mathcal{Q}(CYC) = 1$;

(iv) the position of given vertices could be different in the terms of creating path and the behaviors in path;

(v) there are only two paths but there's no shared edge;

(vi) all paths are as follows.

$$\begin{aligned}
P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} &\to\ 0\\
P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} &\to\ 0\\
P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1, n_6, n_5, n_4 \to \text{red} \to \text{no shared edge} &\to\ 0\\
P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_6, n_5 \to \text{red} \to \text{no shared edge} &\to\ 0\\
P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \&\ P_2 : n_1, n_6 \to \text{red} \to \text{no shared edge} &\to\ 0\\
&\mathcal{Q}_n(CYC) \text{ is } 0.
\end{aligned}$$

(b) In Figure (2.52), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) All paths are as follows.

$$P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_5, n_4, n_3, n_2 \to \text{red}$$

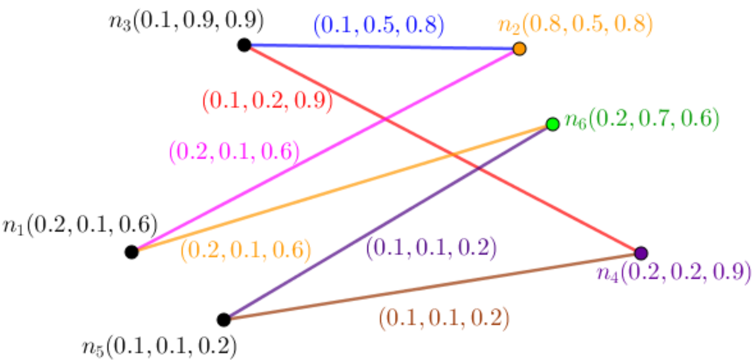

Figure 2.51: A Neutrosophic Graph in the Viewpoint of its dominating path-coloring number and its neutrosophic dominating path-coloring number. 77NTG5

$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_5, n_4, n_3 \to \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1 n_5, n_4 \to \text{red}$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_5 \to \text{red}$$
$$\text{The number is } 1;$$

(ii) 1-paths have same color;

(iii) $\mathcal{Q}(CYC) = 1$;

(iv) the position of given vertices could be different in the terms of creating path and the behaviors in path;

(v) there are only two paths but there's no shared edge;

(vi) all paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1, n_5, n_4 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_5 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$\mathcal{Q}_n(CYC) \text{ is } 0.$$

Definition 2.5.87. (path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, S, in this process is called **path-coloring set** from x to y. The minimum cardinality between all path-coloring sets from two given vertices is called **path-coloring number** and it's denoted by $\mathcal{V}(NTG)$;

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set S of different colors in this process is called **path-coloring**

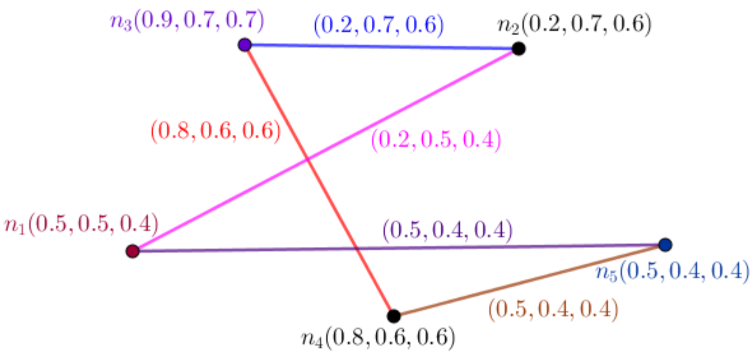

Figure 2.52: A Neutrosophic Graph in the Viewpoint of its dominating path-coloring number and its neutrosophic dominating path-coloring number. 77NTG6

set from x to y. The minimum neutrosophic cardinality, $\sum_{x\in Z}\sum_{i=1}^{3}\sigma_i(x)$, between all sets Zs including the latter endpoints corresponded to path-coloring set Ss, is called **neutrosophic path-coloring number** and it's denoted by $\mathcal{V}_n(NTG)$.

Proposition 2.5.88. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{V}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x\in S}\sum_{i=1}^{3}\sigma_i(x).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. If two paths from x to y share one endpoint, then they're assigned to different colors but there are only $2 \times (\mathcal{O}(CYC) - 1)$ paths for every given vertex. In the terms of number of paths, all vertices behave the same and they've same positions. The set of colors is

$$S = \{\text{red}_1, \text{red}_2, \cdots, \text{red}_{2\times(\mathcal{O}(CYC)-1)}\},$$

in this process. For given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, $S = \{\text{red}_1, \text{red}_2, \cdots, \text{red}_{2\times(\mathcal{O}(CYC)-1)}\}$, in this process is called path-coloring set from x to y. The minimum cardinality,

$$|S| = |\{\text{red}_1, \text{red}_2, \cdots, \text{red}_{2\times(\mathcal{O}(CYC)-1)}\}| = 2 \times (\mathcal{O}(CYC) - 1),$$

between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{V}(CYC)$. Thus

$$\mathcal{V}(CYC) = 2 \times (\mathcal{O}(CYC) - 1).$$

For given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set S of different colors in this process is called path-coloring set from x to y.

The minimum neutrosophic cardinality, $\sum_{x\in Z}\sum_{i=1}^{3}\sigma_i(x)$, between all sets Zs including the latter endpoints corresponded to path-coloring set Ss, is called neutrosophic path-coloring number and it's denoted by $\mathcal{V}_n(CYC)$. Thus

$$\mathcal{V}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x\in S}\sum_{i=1}^{3}\sigma_i(x).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.89. There are two sections for clarifications.

(a) In Figure (2.53), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Consider the vertex n_1. All paths with endpoint n_1 are as follow:

$$\begin{aligned}
P_1 &: n_1, n_2 \to \text{red}\\
P_2 &: n_1, n_2, n_3 \to \text{blue}\\
P_3 &: n_1, n_2, n_3, n_4 \to \text{yellow}\\
P_4 &: n_1, n_2, n_3, n_4, n_5 \to \text{white}\\
P_5 &: n_1, n_2, n_3, n_4, n_5, n_6 \to \text{black}\\
P_6 &: n_1, n_6, n_5, n_4, n_3, n_2 \to \text{pink}\\
P_7 &: n_1, n_6, n_5, n_4, n_3 \to \text{purple}\\
P_8 &: n_1, n_6, n_5, n_4 \to \text{brown}\\
P_9 &: n_1, n_6, n_5 \to \text{orange}\\
P_{10} &: n_1, n_6 \to \text{green}
\end{aligned}$$

Thus $S = \{\text{red, blue, yellow, white, black, pink, purple, brown, orange, green}\}$, is path-coloring set and its cardinality, 10, is path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors,

$$S = \{\text{red, blue, yellow, white, black, pink, purple, brown, orange, green}\},$$

in this process is called path-coloring set from x to y. The minimum cardinality, 10, between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{V}(CYC) = 10$;

(ii) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(iii) there are some different paths which have no shared endpoints. So they could been assigned to same color;

(*iv*) shared endpoints form a set of representatives of colors. Each color is corresponded to a vertex which has minimum neutrosophic cardinality;

(*v*) every color in S is corresponded to a vertex has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared endpoints to form S and after that minimum neutrosophic cardinality is optimal;

(*vi*) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors,

$$S = \{\text{red}, \text{blue}, \text{yellow}, \text{white}, \text{black}, \text{pink}, \text{purple}, \text{brown}, \text{orange}, \text{green}\},$$

in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality,

$$\sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3} \sigma_i(n_2) = 6,$$

between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by

$$\mathcal{V}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3} \sigma_i(n_2) = 6.$$

(*b*) In Figure (2.54), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) Consider the vertex n_1. All paths with endpoint n_1 are as follow:

$$\begin{aligned}
P_1 : n_1, n_2 &\to \text{red} \\
P_2 : n_1, n_2, n_3 &\to \text{blue} \\
P_3 : n_1, n_2, n_3, n_4 &\to \text{yellow} \\
P_4 : n_1, n_2, n_3, n_4, n_5 &\to \text{white} \\
P_5 :: n_1, n_5, n_4, n_3, n_2 &\to \text{black} \\
P_6 : n_1, n_5, n_4, n_3 &\to \text{pink} \\
P_7 : n_1, n_5, n_4 &\to \text{purple} \\
P_8 : n_1, n_5 &\to \text{brown}
\end{aligned}$$

Thus $S = \{\text{red}, \text{blue}, \text{yellow}, \text{white}, \text{black}, \text{pink}, \text{purple}, \text{brown}\}$ is path-coloring set and its cardinality, 8, is path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors,

$$S = \{\text{red}, \text{blue}, \text{yellow}, \text{white}, \text{black}, \text{pink}, \text{purple}, \text{brown}\},$$

in this process is called path-coloring set from x to y. The minimum cardinality, 8, between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{V}(CYC) = 8$;

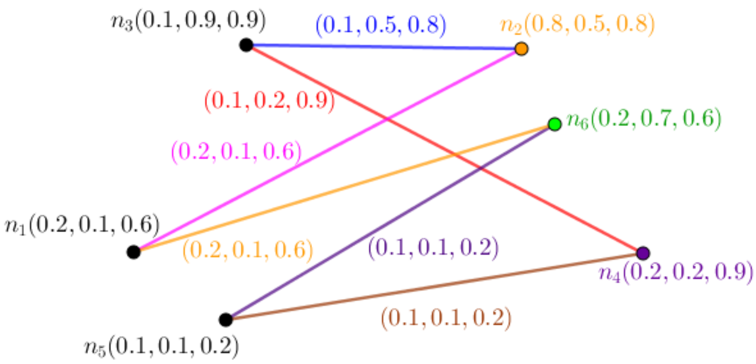

Figure 2.53: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

78NTG5

(ii) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(iii) there are some different paths which have no shared endpoints. So they could been assigned to same color;

(iv) shared endpoints form a set of representatives of colors. Each color is corresponded to a vertex which has minimum neutrosophic cardinality;

(v) every color in S is corresponded to a vertex has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared endpoints to form S and after that minimum neutrosophic cardinality is optimal;

(vi) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors,

$$S = \{\text{red}, \text{blue}, \text{yellow}, \text{white}, \text{black}, \text{pink}, \text{purple}, \text{brown}\},$$

in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality,

$$\sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3} \sigma_i(n_3) = 6.2,$$

between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by

$$\mathcal{V}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3} \sigma_i(n_3) = 6.2.$$

Definition 2.5.90. (Dual-Dominating Numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

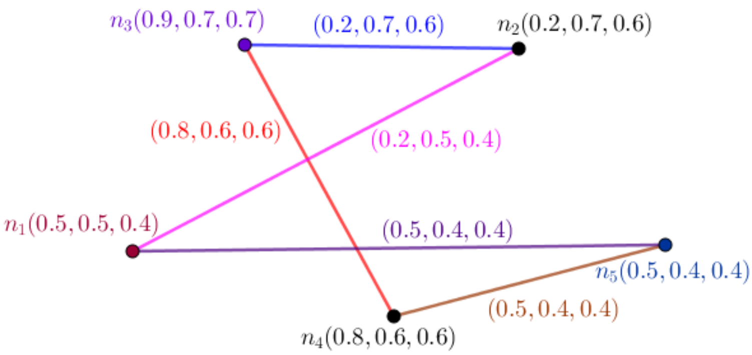

Figure 2.54: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number. 78NTG6

(i) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum cardinality between all dual-dominating sets is called **dual-dominating number** and it's denoted by $\mathcal{D}(NTG)$;

(ii) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum neutrosophic cardinality between all dual-dominating sets is called **neutrosophic dual-dominating number** and it's denoted by $\mathcal{D}_n(NTG)$.

Proposition 2.5.91. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{D}_n(CYC) = \max_{x \in S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor}\}} \sum_{i=1}^{3} \sigma_i(x)$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. Two consecutive vertices could belong to S which is dual-dominating set related to dual-dominating number. Since these two vertices could be dominated by previous vertex and upcoming vertex despite them. If there are no vertices which are consecutive, then it contradicts with maximality of set S and maximum cardinality of S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor}, x_1\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus (S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}, x_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}, x_1\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}_n(CYC) = \max_{x \in S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}\}} \sum_{i=1}^{3} \sigma_i(x)$$

Thus

$$\mathcal{D}_n(CYC) = \max_{x \in S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}\}} \sum_{i=1}^{3} \sigma_i(x)$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.92. There are two sections for clarifications.

(a) In Figure (2.55), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(ii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(iii) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V\backslash(S = \{n_3, n_4, n_1, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(iv) let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V\backslash(S = \{n_2, n_3, n_5, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(v) let $S = \{n_1, n_2, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V\backslash(S = \{n_1, n_2, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(vi) let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V\backslash(S = \{n_2, n_3, n_5, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.9$.

(b) In Figure (2.56), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V\backslash(S = \{n_3, n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(iii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For

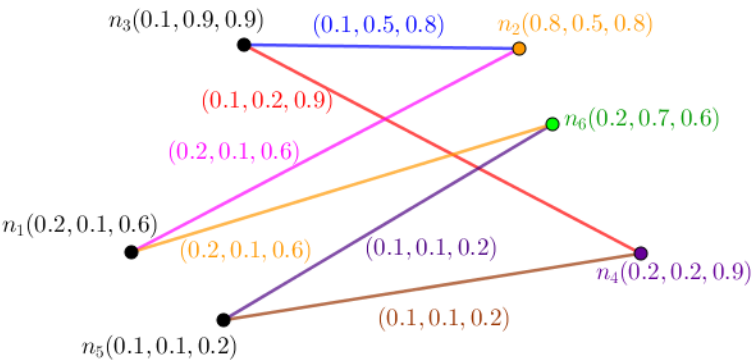

Figure 2.55: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

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every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(iv) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(v) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. As if it, 5.1, contradicts with the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.7$;

(vi) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.7$.

Definition 2.5.93. (dual-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

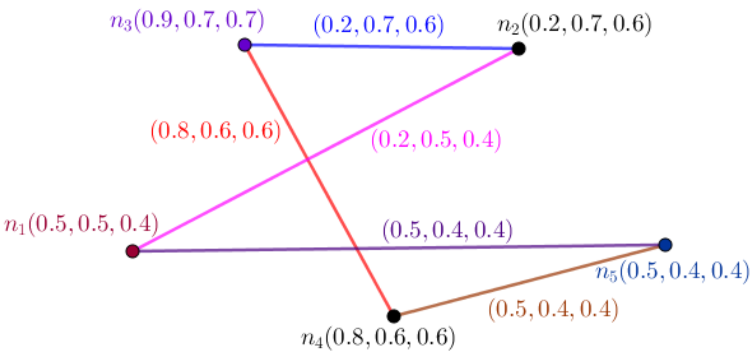

Figure 2.56: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

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(i) for given two vertices, s and s' if $d(s,n) \neq d(s',n)$, then n resolves s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices s, s' in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, S is called **dual-resolving set**. The maximum cardinality between all dual-resolving sets is called **dual-resolving number** and it's denoted by $\mathcal{R}(NTG)$;

(ii) for given two vertices, s and s' if $d(s,n) \neq d(s',n)$, then n resolves s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices s, s' in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, S is called **dual-resolving set**. The maximum neutrosophic cardinality between all dual-resolving sets is called **dual-resolving number** and it's denoted by $\mathcal{R}_n(NTG)$.

Proposition 2.5.94. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{R}_n(CYC) = \mathcal{O}_n(CYC) - \min_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. $\mathcal{O}(CYC) - 2$ consecutive vertices could belong to S which is dual-resolving set related to dual-resolving number where two neutrosophic vertices outside are consecutive. Since these two vertices could resolve all vertices. If there are no neutrosophic vertices which are consecutive, then it contradicts with maximality of set S and maximum cardinality of S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there's at least one neutrosophic vertex n in $V \setminus (S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\})$ such that n resolves s and s' then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$ is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by

$$\mathcal{R}_n(CYC) = \mathcal{O}_n(CYC) - \min_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Thus

$$\mathcal{R}_n(CYC) = \mathcal{O}_n(CYC) - \min_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.95. There are two sections for clarifications.

(a) In Figure (2.57), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices n_2 and n_3 in S, there's neutrosophic vertex n_1 in $V \setminus (S = \{n_3, n_2\})$ such that n_1 resolves n_2 and n_3, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-resolving set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(ii) $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices n_2 and n_4 in S, there's neutrosophic vertex n_1 in $V \setminus (S = \{n_4, n_2\})$ such that n_1 resolves n_2 and n_4, then the set of neutrosophic vertices, $S = \{n_4, n_2\}$ is called dual-resolving set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(*iii*) let $S = \{n_3, n_4, n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either neutrosophic vertex n_6 or neutrosophic vertex n_5 in $V \setminus (S = \{n_3, n_4, n_1, n_2\})$ such that either n_6 resolves s and s', or n_5 resolves s and s', then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_2\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(*iv*) let $S = \{n_3, n_4, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either neutrosophic vertex n_1 or neutrosophic vertex n_2 in $V \setminus (S = \{n_3, n_4, n_5, n_6\})$ such that either n_1 resolves s and s', or n_2 resolves s and s', then the set of neutrosophic vertices, $S = \{n_3, n_4, n_5, n_6\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(*v*) let $S = \{n_2, n_5, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either neutrosophic vertex n_3 or neutrosophic vertex n_4 in $V \setminus (S = \{n_2, n_5, n_1, n_6\})$ such that either n_3 resolves s and s', or n_4 resolves s and s', then the set of neutrosophic vertices, $S = \{n_2, n_5, n_1, n_6\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 4$;

(*vi*) let $S = \{n_3, n_1, n_6, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either neutrosophic vertex n_5 or neutrosophic vertex n_4 in $V \setminus (S = \{n_3, n_1, n_6, n_2\})$ such that either n_5 resolves s and s', or n_4 resolves s and s', then the set of neutrosophic vertices, $S = \{n_3, n_1, n_6, n_2\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}_n(CYC) = 6.4$.

(*b*) In Figure (2.58), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices n_2 and n_3 in S, there's neutrosophic vertex n_4 in $V \setminus (S = \{n_3, n_2\})$ such that n_4 resolves n_2 and n_3, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-resolving set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(*ii*) $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices n_2 and n_4 in S, there's neutrosophic vertex n_5 in $V \setminus (S = \{n_4, n_2\})$ such that n_5 resolves n_2 and n_4, then the set of neutrosophic vertices, $S = \{n_4, n_2\}$ is called dual-resolving set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(iii) let $S = \{n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either a neutrosophic vertex n_1 or neutrosophic vertex n_2 in $V \setminus (S = \{n_3, n_4, n_5\})$ such that either n_1 resolves s and s' or n_2 resolves s and s', then the set of neutrosophic vertices, $S = \{n_3, n_4, n_5\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(iv) let $S = \{n_1, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either a neutrosophic vertex n_3 or neutrosophic vertex n_4 in $V \setminus (S = \{n_1, n_2, n_5\})$ such that either n_3 resolves s and s' or n_4 resolves s and s', then the set of neutrosophic vertices, $S = \{n_1, n_2, n_5\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(v) let $S = \{n_1, n_2, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either a neutrosophic vertex n_4 or neutrosophic vertex n_5 in $V \setminus (S = \{n_1, n_2, n_3\})$ such that either n_4 resolves s and s' or n_5 resolves s and s', then the set of neutrosophic vertices, $S = \{n_1, n_2, n_3\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(CYC) = 3$;

(vi) let $S = \{n_2, n_3, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices s and s' in S, there are either a neutrosophic vertex n_1 or neutrosophic vertex n_5 in $V \setminus (S = \{n_2, n_3, n_4\})$ such that either n_1 resolves s and s' or n_5 resolves s and s', then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4\}$ is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}_n(CYC) = 5.8$.

Definition 2.5.96. (joint-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then

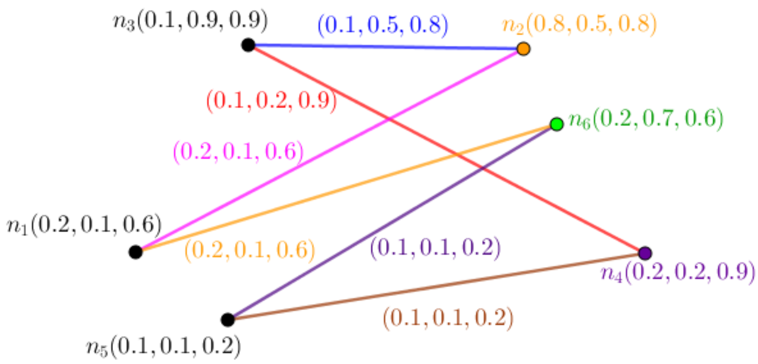

Figure 2.57: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number. 80NTG5

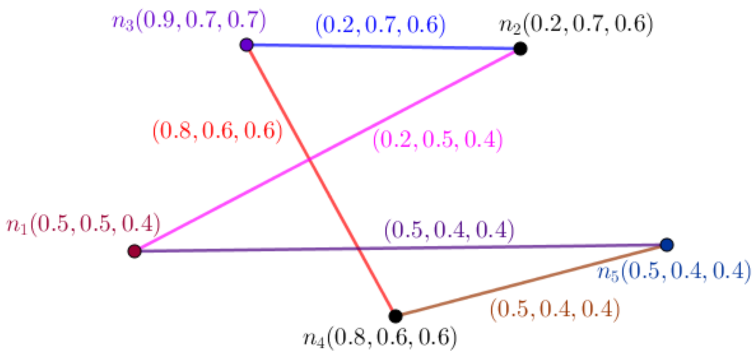

Figure 2.58: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number. 80NTG6

the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called **joint-dominating number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called **neutrosophic joint-dominating number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

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Proposition 2.5.97. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S dominates if and only if it joint-dominates.*

Proposition 2.5.98. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S is corresponded to joint-dominating number. Then $V \setminus D$ is S-like.*

Proposition 2.5.99. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-dominating number if and only if for all s in S, there's a vertex n in $V \setminus S$, such that $\{n' \mid n'n \in E\} \cap S = \{s\}$.*

Proposition 2.5.100. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. $\mathcal{O}(CYC) - 2$ consecutive vertices could belong to S which is joint-dominating set related to joint-dominating number where two neutrosophic vertices outside are "consecutive". Since it's possible to have a path amid every two of vertices in S and two vertices outside could be joint-dominated by their neighbors in S. If there are no neutrosophic vertices which are consecutive, then it contradicts with the term joint-dominating set for S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in

$$V \setminus (S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}),$$

there's only one neutrosophic vertex s in

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

such that s joint-dominates n, then the set of neutrosophic vertices,

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\}$$

is called joint-dominating set where for every two vertices in

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-3}, x_{\mathcal{O}(CYC)-2}\},$$

there's only one path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Thus

$$\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

■

Proposition 2.5.101. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $3 \times \mathcal{O}(CYC) + 1$ joint-dominating sets.*

Proposition 2.5.102. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $\mathcal{O}(CYC)$ joint-dominating set corresponded to joint-dominating number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.103. There are two sections for clarifications.

(a) In Figure (2.59), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there are only two paths between them;

(ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of cycle;

(iii) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\}.$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2 = 4$;

(iv) there are nineteen joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_5, n_1, n_2, n_3, n_4\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6, n_2, n_3, n_4, n_5\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_4, n_5, n_6, n_1\}, \{n_2, n_4, n_5, n_6, n_1\}, \{n_3, n_4, n_5, n_6, n_1\},$$

$$\{n_5, n_6, n_1, n_2\}, \{n_3, n_5, n_6, n_1, n_2\}, \{n_4, n_5, n_6, n_1, n_2\},$$
$$\{n_6, n_1, n_2, n_3\}, \{n_4, n_6, n_1, n_2, n_3\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_3, n_4\},$$

as if it's possible to have six of them

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\}$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are nineteen joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_5, n_1, n_2, n_3, n_4\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \{n_6, n_2, n_3, n_4, n_5\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_4, n_5, n_6, n_1\}, \{n_2, n_4, n_5, n_6, n_1\}, \{n_3, n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_3, n_5, n_6, n_1, n_2\}, \{n_4, n_5, n_6, n_1, n_2\},$$
$$\{n_6, n_1, n_2, n_3\}, \{n_4, n_6, n_1, n_2, n_3\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_3, n_4\},$$

as if there is six joint-dominating sets

$$\{n_1, n_2, n_3, n_4\}, \{n_2, n_3, n_4, n_5\}, \{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_6, n_1\},$$
$$\{n_5, n_6, n_1, n_2\}, \{n_6, n_1, n_2, n_3\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4, n_5, n_6, n_1\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}(\sigma(n_2) + \sigma(n_3)) = 4.1$.

(b) In Figure (2.60), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there are only two paths between them;

(ii) one vertex only dominates two vertices, then it only dominates its two neighbors thus it implies the vertex joint-dominates is different from the vertex dominates vertices in the setting of cycle;

(*iii*) all joint-dominating sets corresponded to joint-dominating number are

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CYC) = \mathcal{O}(CYC) - 2 = 3$;

(*iv*) there are sixteen joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_4, n_1, n_2, n_3\}, \{n_5, n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_5, n_2, n_3, n_4\},$$
$$\{n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_4, n_5, n_1\}, \{n_2, n_4, n_5, n_1\}, \{n_3, n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\}, \{n_3, n_5, n_1, n_2\}, \{n_4, n_5, n_1, n_2\},$$
$$\{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have five of them

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(*v*) there are sixteen joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_4, n_1, n_2, n_3\}, \{n_5, n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\}, \{n_5, n_2, n_3, n_4\},$$
$$\{n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_4, n_5, n_1\}, \{n_2, n_4, n_5, n_1\}, \{n_3, n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\}, \{n_3, n_5, n_1, n_2\}, \{n_4, n_5, n_1, n_2\},$$
$$\{n_1, n_2, n_3, n_4, n_5\},$$

as if there is five joint-dominating sets

$$\{n_1, n_2, n_3\}, \{n_2, n_3, n_4, \}, \{n_3, n_4, n_5\}, \{n_4, n_5, n_1\},$$
$$\{n_5, n_1, n_2\},$$

corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

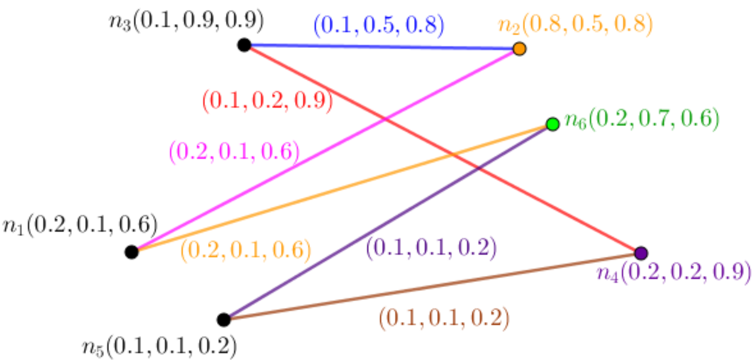

Figure 2.59: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG5

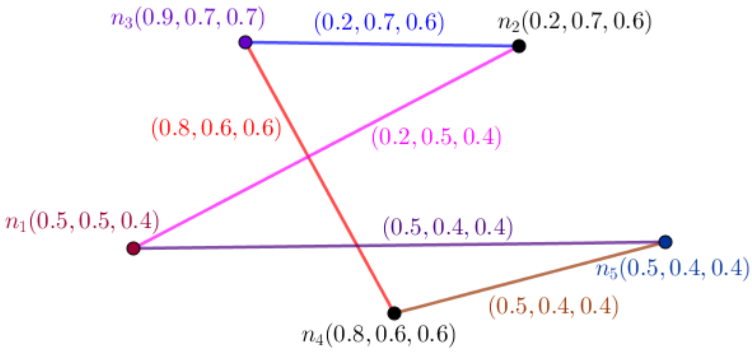

Figure 2.60: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG6

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_5, n_1, n_2\}$. For given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CYC) = \mathcal{O}_n(CYC) - \sum_{i=1}^{3}(\sigma(n_3) + \sigma(n_4)) = 4.2$.

Definition 2.5.104. (joint-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called **joint-resolving number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called **neutrosophic joint-resolving number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

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Proposition 2.5.105. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S resolves if and only if it joint-resolves.*

Proposition 2.5.106. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-resolving number if and only if for all s in S, either there are vertices n and n' in $V \setminus S$, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$ or there's vertex s' in S, such that are s and s' twin vertices.*

Proposition 2.5.107. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{J}_n(CYC) = \min\{\sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y))\}_{x \text{ and } y \text{ are consecutive vertices.}}.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. 2 consecutive vertices could belong to S which is joint-resolving set related to joint-resolving number. If there are no neutrosophic vertices which are consecutive, then it contradicts with the term joint-resolving set for S. All joint-resolving sets corresponded to joint-resolving number are

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}.$$

For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \ldots,$$
$$\{x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}\}, \{x_{\mathcal{O}(CYC)}, x_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = \min\{\sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y))\}_{x \text{ and } y \text{ are consecutive vertices.}}.$$

Thus

$$\mathcal{J}_n(CYC) = \min\{\sum_{i=1}^{3}(\sigma_i(x) + \sigma_i(y))\}_{x \text{ and } y \text{ are consecutive vertices.}}.$$

■

Proposition 2.5.108. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $(\mathcal{O}(CYC) \times (2^{\mathcal{O}(CYC)-2} - 1)) + 1$ joint-resolving sets.*

Proposition 2.5.109. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then there are $\mathcal{O}(CYC)$ joint-resolving set corresponded to joint-resolving number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.110. There are two sections for clarifications.

(a) In Figure (2.77), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) For given two neutrosophic vertices, there are only two paths between them;

  (ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(*iii*) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(*iv*) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$
$$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$$
$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$

$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$
$$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$
$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$
$$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$$
$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$

$$\{n_5, n_6, n_2, n_4\}, \{n_5, n_6, n_3, n_4\}, \{n_5, n_6, n_1, n_2, n_3\},$$
$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$
$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 1.7.$$

S is $\{n_4, n_5\}$ corresponded to neutrosophic joint-resolving number.

(b) In Figure (2.78), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, there are only two paths between them;

(ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$
$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(*v*) there are thirty-six joint-resolving sets

$$\{n_1,n_2\},\{n_1,n_2,n_3\},\{n_1,n_2,n_4\},$$
$$\{n_1,n_2,n_5\},\{n_1,n_2,n_3,n_4\}\{n_1,n_2,n_3,n_5\}$$
$$\{n_1,n_2,n_4,n_5\},\{n_3,n_2\},\{n_3,n_2,n_1\},\{n_3,n_2,n_4\},$$
$$\{n_3,n_2,n_5\},\{n_3,n_2,n_1,n_4\}\{n_3,n_2,n_1,n_5\},$$
$$\{n_3,n_2,n_4,n_5\},\{n_3,n_4\},\{n_3,n_4,n_1\},$$
$$\{n_3,n_4,n_2\},\{n_3,n_4,n_5\},\{n_3,n_4,n_1,n_2\},$$
$$\{n_3,n_4,n_1,n_5\},\{n_3,n_4,n_2,n_5\},\{n_5,n_4\},$$
$$\{n_5,n_4,n_1\},\{n_5,n_4,n_2\},\{n_5,n_4,n_3\},$$
$$\{n_5,n_4,n_1,n_2\}\{n_5,n_4,n_1,n_3\},\{n_5,n_4,n_2,n_3\},$$
$$\{n_5,n_1\},\{n_5,n_1,n_4\},\{n_5,n_1,n_2\},$$
$$\{n_5,n_1,n_3\},\{n_5,n_1,n_4,n_2\}\{n_5,n_1,n_4,n_3\},$$
$$\{n_5,n_1,n_2,n_3\},\{n_5,n_1,n_4,n_2,n_3\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(*vi*) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1,n_2\},\{n_2,n_3\},\{n_3,n_4\},$$
$$\{n_4,n_5\},\{n_5,n_1\}.$$

For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1,n_2\},\{n_2,n_3\},\{n_3,n_4\},$$
$$\{n_4,n_5\},\{n_5,n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1,n_2\},\{n_2,n_3\},\{n_3,n_4\},$$
$$\{n_4,n_5\},\{n_5,n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 2.7.$$

S is $\{n_1,n_5\}$ corresponded to neutrosophic joint-resolving number.

Definition 2.5.111. (perfect-dominating numbers).
Let $NTG : (V,E,\sigma,\mu)$ be a neutrosophic graph. Then

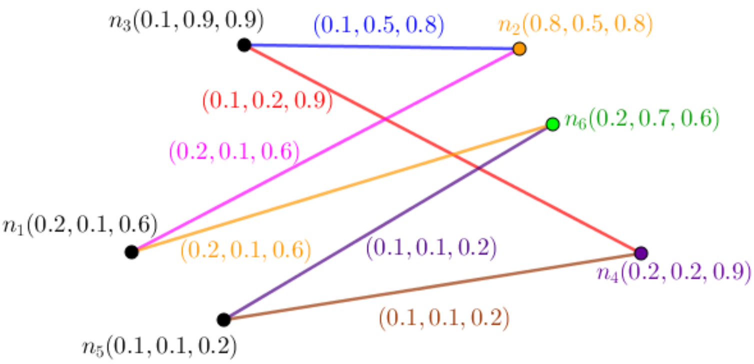

Figure 2.61: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG5

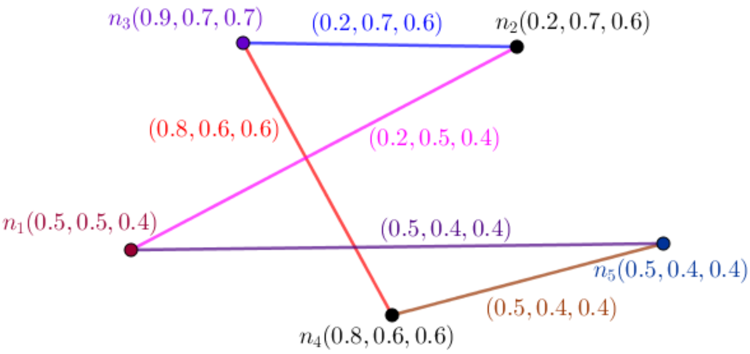

Figure 2.62: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG6

(i) for given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called **perfect-dominating set**. The minimum cardinality between all perfect-dominating sets is called **perfect-dominating number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called **perfect-dominating set**. The minimum neutrosophic cardinality between all perfect-dominating sets is called **neutrosophic perfect-dominating number** and it's denoted by $\mathcal{P}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

83prp9

Proposition 2.5.112. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S dominates if and only if it perfect-dominates.*

Proposition 2.5.113. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and dominating set has one member. Then a vertex of dominating set corresponded to dominating number dominates if and only if it perfect-dominates.*

Proposition 2.5.114. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to perfect-dominating number if and only if for all s in S, there's a vertex n in $V \setminus S$, such that $\{s' \mid s'n \in E\} \cap S = \{s\}$.*

Proposition 2.5.115. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{P}_n(CYC) = \min_{|S|=\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor} \sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. All perfect-dominating sets corresponded to perfect-dominating number are

$$\{n_1, n_4, \ldots\}_{|S|=\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}, \{n_2, n_5, \ldots\}_{|S|=\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}, \ldots,$$

where last vertices could be neighbors as if they couldn't have less than three edges amid them. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum neutrosophic cardinality between all perfect-dominating sets is called neutrosophic perfect-dominating number and it's denoted by

$$\mathcal{P}_n(CYC) = \min_{|S|=\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor} \sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x).$$

Thus

$$\mathcal{P}_n(CYC) = \min_{|S|=\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor} \sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.116. There are two sections for clarifications.

(a) In Figure (2.63), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex couldn't be dominated by more than one vertex as if the structure of dominating and perfect-dominating are the same in the terms of sets and numbers where only some sets coincide;

(iii) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1, n_4\}, \{n_2, n_5\}$, and $\{n_3, n_6\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum cardinality between all perfect-dominating sets is called perfect-dominating number and it's denoted by $\mathcal{P}(CYC) = 2$ and corresponded to perfect-dominating sets are $\{n_1, n_4\}, \{n_2, n_5\}$, and $\{n_3, n_6\}$;

(iv) there are ten perfect-dominating sets

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\},$$
$$\{n_1, n_4, n_5, n_6\}, \{n_1, n_2, n_3, n_4\}, \{n_2, n_5, n_6, n_1\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_3, n_6, n_1, n_2\}, \{n_3, n_4, n_5, n_6\},$$
$$\{n_1, n_2, , n_3, n_4, n_5, n_6\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is characteristic;

(v) there are ten perfect-dominating sets

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\},$$
$$\{n_1, n_4, n_5, n_6\}, \{n_1, n_2, n_3, n_4\}, \{n_2, n_5, n_6, n_1\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_3, n_6, n_1, n_2\}, \{n_3, n_4, n_5, n_6\},$$
$$\{n_1, n_2, , n_3, n_4, n_5, n_6\},$$

corresponded to perfect-dominating number as if there's one perfect-dominating set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is the determiner;

(vi) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1, n_4\}, \{n_2, n_5\}$, and $\{n_3, n_6\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum neutrosophic cardinality between all perfect-dominating sets is called neutrosophic perfect-dominating number and it's denoted by $\mathcal{P}_n(CYC) = 2.2$ and corresponded to perfect-dominating sets are $\{n_1, n_4\}$.

(b) In Figure (2.64), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex couldn't be dominated by more than one vertex as if the structure of dominating and perfect-dominating are the same in the terms of sets and numbers where only some sets coincide;

(iii) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum cardinality between all perfect-dominating sets is called perfect-dominating number and it's denoted by $\mathcal{P}(CYC) = 3$ and corresponded to perfect-dominating sets are $\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$;

(iv) there are five perfect-dominating sets

$$\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is characteristic;

(v) there are five perfect-dominating sets

$$\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4, n_5\},$$

corresponded to perfect-dominating number as if there's one perfect-dominating set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is the determiner;

(vi) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1, n_4, n_5\}, \{n_2, n_5, n_1\}, \{n_1, n_2, n_3\}$, and $\{n_2, n_3, n_4\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only

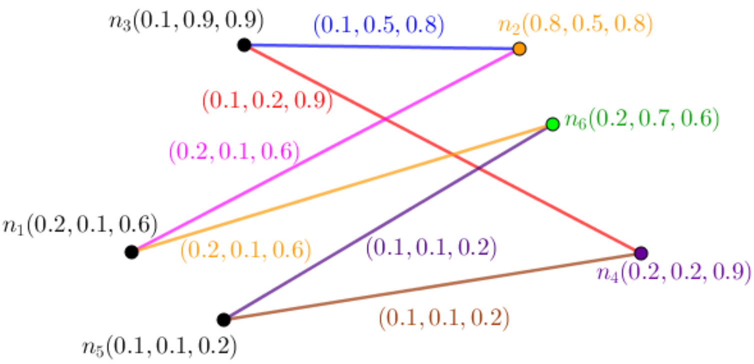

Figure 2.63: A Neutrosophic Graph in the Viewpoint of its perfect-dominating number and its neutrosophic perfect-dominating number.

83NTG5

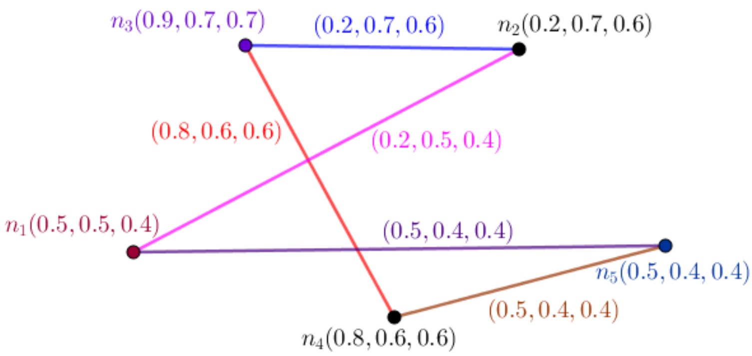

Figure 2.64: A Neutrosophic Graph in the Viewpoint of its perfect-dominating number and its neutrosophic perfect-dominating number.

83NTG6

one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum neutrosophic cardinality between all perfect-dominating sets is called neutrosophic perfect-dominating number and it's denoted by $\mathcal{P}_n(CYC) = 4.2$ and corresponded to perfect-dominating sets are $\{n_2, n_5, n_1\}$.

Definition 2.5.117. (perfect-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called **perfect-resolving set**. The minimum cardinality between all perfect-resolving sets is called **perfect-resolving number** and it's denoted by $\mathcal{P}(NTG)$;

(*ii*) for given vertices n and n' if $d(s,n) \neq d(s,n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called **perfect-resolving set**. The minimum neutrosophic cardinality between all perfect-resolving sets is called **neutrosophic perfect-resolving number** and it's denoted by $\mathcal{P}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

84prp9

Proposition 2.5.118. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S resolves if and only if it perfect-resolves.*

Proposition 2.5.119. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and resolving set has one member. Then a vertex of resolving set corresponded to resolving number resolves if and only if it perfect-resolves.*

Proposition 2.5.120. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to perfect-resolving number if and only if for all s in S, there are neutrosophic vertices n and n' in $V \setminus S$, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$ and for all neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$.*

84prp12

Proposition 2.5.121. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V and $V \setminus \{x\}$ are S.*

Proposition 2.5.122. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{P}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, two vertices couldn't be resolved by more than one vertex so as the structure of resolving and perfect-resolving are different in the terms of sets. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't perfect-resolve, by S has two members in settings of resolving as if these vertices aren't unique in the terms of resolving since some vertices are resolved by both of them and adding them to intended growing set is useless. Thus, by Proposition (2.5.121), S has either $\mathcal{O}(CYC)$ or $\mathcal{O}(CYC) - 1$. All perfect-resolving sets corresponded to perfect-resolving number are

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\},$$

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\},$$
$$\ldots$$
$$\{n_2, n_3, n_4, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\},$$

For given vertices n and n' if $d(s,n) \neq d(s,n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum cardinality between all perfect-resolving sets is called perfect-resolving number and it's denoted by

$$\mathcal{P}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x)$$

and corresponded to perfect-resolving sets are

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\},$$
$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\},$$
$$\ldots$$
$$\{n_2, n_3, n_4, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\}.$$

Thus

$$\mathcal{P}_n(CYC) = \mathcal{O}_n(CYC) - \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

■

Proposition 2.5.123. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then perfect-resolving number isn't equal to resolving number.*

Proposition 2.5.124. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then the number of perfect-resolving sets corresponded to perfect-resolving number is equal to $\mathcal{O}(CYC)$.*

Proposition 2.5.125. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then the number of perfect-resolving sets is equal to $\mathcal{O}(CYC) + 1$.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.126. There are two sections for clarifications.

(a) In Figure (2.65), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, two vertices couldn't be resolved by more than one vertex so as the structure of resolving and perfect-resolving are different in the terms of sets. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't perfect-resolve, by S has two members in settings of resolving as if these vertices aren't unique in the terms of resolving since some vertices are resolved by both of them and adding them to intended growing set is useless. Thus, by Proposition (2.5.121), S has either $\mathcal{O}(CYC)$ or $\mathcal{O}(CYC) - 1$;

(iii) all perfect-resolving sets corresponded to perfect-resolving number are

$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\},$$
$$\{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$

For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum cardinality between all perfect-resolving sets is called perfect-resolving number and it's denoted by $\mathcal{P}(CYC) = 5$ and corresponded to perfect-resolving sets are

$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\},$$
$$\{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\};$$

(iv) there are seven perfect-resolving sets

$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\},$$
$$\{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is characteristic;

(v) there are six perfect-resolving sets

$$\{n_1, n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\},$$
$$\{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_3, n_4, n_5, n_6\}, \{n_2, n_3, n_4, n_5, n_6\},$$

corresponded to perfect-resolving number as if there's one perfect-resolving set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is the determiner;

(*vi*) all perfect-resolving sets corresponded to perfect-resolving number are $\{n_1\}$ and $\{n_6\}$. For given vertices n and n' if $d(s,n) \neq d(s,n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum neutrosophic cardinality between all perfect-resolving sets is called neutrosophic perfect-resolving number and it's denoted by $\mathcal{P}_n(CYC) = 6$ and corresponded to perfect-resolving sets are $\{n_1, n_3, n_4, n_5, n_6\}$.

(*b*) In Figure (2.66), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) For given neutrosophic vertex, s, there are only two paths with other vertices;

(*ii*) in the setting of cycle, two vertices couldn't be resolved by more than one vertex so as the structure of resolving and perfect-resolving are different in the terms of sets. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't perfect-resolve, by S has two members in settings of resolving as if these vertices aren't unique in the terms of resolving since some vertices are resolved by both of them and adding them to intended growing set is useless. Thus, by Proposition (2.5.121), S has either $\mathcal{O}(CYC)$ or $\mathcal{O}(CYC) - 1$;

(*iii*) all perfect-resolving sets corresponded to perfect-resolving number are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\},$$

For given vertices n and n' if $d(s,n) \neq d(s,n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum cardinality between all perfect-resolving sets is called perfect-resolving number and it's denoted by $\mathcal{P}(CYC) = 4$ and corresponded to perfect-resolving sets are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\};$$

(*iv*) there are six perfect-resolving sets

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\},$$

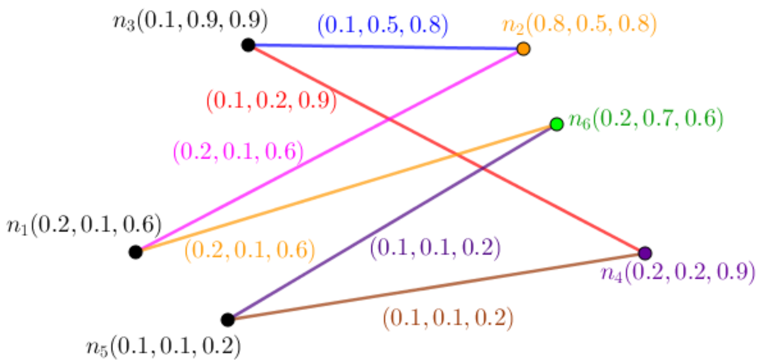

Figure 2.65: A Neutrosophic Graph in the Viewpoint of its perfect-resolving number and its neutrosophic perfect-resolving number. 84NTG5

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is characteristic;

(v) there are five perfect-resolving sets

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\},$$

corresponded to perfect-resolving number as if there's one perfect-resolving set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is the determiner;

(vi) all perfect-resolving sets corresponded to perfect-resolving number are

$$\{n_1, n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_3, n_4, n_5\}, \{n_2, n_3, n_4, n_5\},$$

For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum neutrosophic cardinality between all perfect-resolving sets is called neutrosophic perfect-resolving number and it's denoted by $\mathcal{P}_n(CYC) = 6.6$ and corresponded to perfect-resolving sets are $\{n_1, n_2, n_4, n_5\}$.

Definition 2.5.127. (total-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is

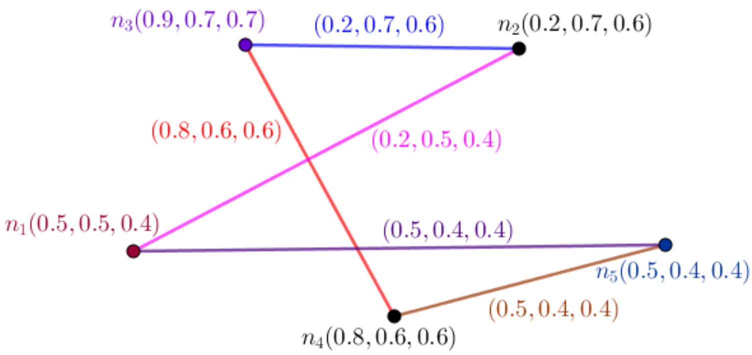

Figure 2.66: A Neutrosophic Graph in the Viewpoint of its perfect-resolving number and its neutrosophic perfect-resolving number. 84NTG6

called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called **total-dominating set**. The minimum cardinality between all total-dominating sets is called **total-dominating number** and it's denoted by $\mathcal{T}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called **total-dominating set**. The minimum neutrosophic cardinality between all total-dominating sets is called **neutrosophic total-dominating number** and it's denoted by $\mathcal{T}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

Proposition 2.5.128. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then* $|S| \geq 2$.

Proposition 2.5.129. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{T}_n(CYC) = \min_{|S|=(\lfloor)\lceil \frac{\mathcal{O}(CYC)}{2} \rceil(\rfloor)(+1)} \sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself. Thus two neighbors are necessary in S. All total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5, n_6, n_9, n_{10} \ldots\}, \{n_2, n_3, n_6, n_7, n_{10}, n_{11} \ldots\}, \{n_2, n_3, n_4, n_7, n_8, \ldots\},$$
$$\ldots$$
$$\{\ldots, n_{\mathcal{O}(CYC)-10}, n_{\mathcal{O}(CYC)-9,\mathcal{O}(CYC)-6}, n_{\mathcal{O}(CYC)-5}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\}$$
$$\{\ldots, n_{\mathcal{O}(CYC)-9}, n_{\mathcal{O}(CYC)-8,\mathcal{O}(CYC)-5}, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\}.$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum neutrosophic cardinality between all total-dominating sets is called neutrosophic total-dominating number and it's denoted by

$$\mathcal{T}_n(CYC) = \min_{|S|=(\lfloor)\lceil \frac{\mathcal{O}(CYC)}{2} \rceil(\rfloor)(+1)} \sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x)$$

and corresponded to total-dominating sets are

$$\{n_1, n_2, n_5, n_6, n_9, n_{10} \ldots\}, \{n_2, n_3, n_6, n_7, n_{10}, n_{11} \ldots\}, \{n_2, n_3, n_4, n_7, n_8, \ldots\},$$
$$\ldots$$
$$\{\ldots, n_{\mathcal{O}(CYC)-10}, n_{\mathcal{O}(CYC)-9,\mathcal{O}(CYC)-6}, n_{\mathcal{O}(CYC)-5}, n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\}$$
$$\{\ldots, n_{\mathcal{O}(CYC)-9}, n_{\mathcal{O}(CYC)-8,\mathcal{O}(CYC)-5}, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}\}.$$

Thus

$$\mathcal{T}_n(CYC) = \min_{|S|=(\lfloor)\lceil \frac{\mathcal{O}(CYC)}{2} \rceil(\rfloor)(+1)} \sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x).$$

■

Proposition 2.5.130. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then total-dominating number isn't equal to dominating number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.131. There are two sections for clarifications.

(*a*) In Figure (2.67), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself. Thus two neighbors are necessary in S;

(iii) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\},$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum cardinality between all total-dominating sets is called total-dominating number and it's denoted by $\mathcal{T}(CYC) = 4$ and corresponded to total-dominating sets are

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\};$$

(iv) there are sixteen total-dominating sets

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\},$$
$$\{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_6, n_2, n_3, n_4, n_5\}, \{n_6, n_1, n_3, n_4, n_5\}, \{n_6, n_1, n_2, n_3, n_4\},$$
$$\{n_5, n_1, n_2, n_3, n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is characteristic;

(v) there are nine total-dominating sets

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\},$$

corresponded to total-dominating number as if there's one total-dominating set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is the determiner;

(vi) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5, n_6\}, \{n_2, n_3, n_6, n_1\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_5, n_6\}, \{n_4, n_5, n_2, n_3\}, \{n_4, n_5, n_1, n_6\},$$
$$\{n_1, n_2, n_4, n_5\}, \{n_2, n_3, n_5, n_6\}, \{n_3, n_4, n_6, n_1\},$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum neutrosophic cardinality between all total-dominating sets is called neutrosophic total-dominating number and it's denoted by $\mathcal{T}_n(CYC) = 4.1$ and corresponded to total-dominating sets are

$$\{n_4, n_5, n_1, n_6\}.$$

(*b*) In Figure (2.68), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) For given neutrosophic vertex, s, there are only two paths with other vertices;

(*ii*) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself. Thus two neighbors are necessary in S;

(*iii*) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\},$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum cardinality between all total-dominating sets is called total-dominating number and it's denoted by $\mathcal{T}(CYC) = 3$ and corresponded to total-dominating sets are

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\};$$

(*iv*) there are eleven total-dominating sets

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is characteristic;

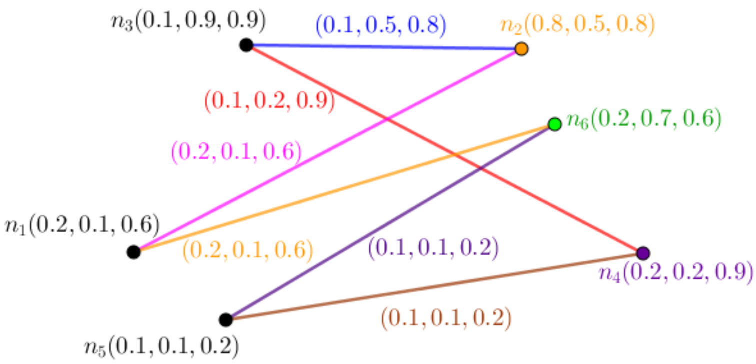

Figure 2.67: A Neutrosophic Graph in the Viewpoint of its total-dominating number and its neutrosophic total-dominating number.

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(v) there are five total-dominating sets

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\},$$

corresponded to total-dominating number as if there's one total-dominating set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is the determiner;

(vi) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2, n_5\}, \{n_2, n_3, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_4, n_5, n_3\}, \{n_5, n_1, n_4\},$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum neutrosophic cardinality between all total-dominating sets is called neutrosophic total-dominating number and it's denoted by $\mathcal{T}_n(CYC) = 4.2$ and corresponded to total-dominating sets are

$$\{n_1, n_2, n_5\}.$$

Definition 2.5.132. (total-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices, $d \geq 1$ and all vertices have to be total-resolved otherwise it will be mentioned which is about $d \geq 0$ in some cases but all vertices have to be total-resolved forever. Let S be a set of neutrosophic vertices [a vertex alongside triple

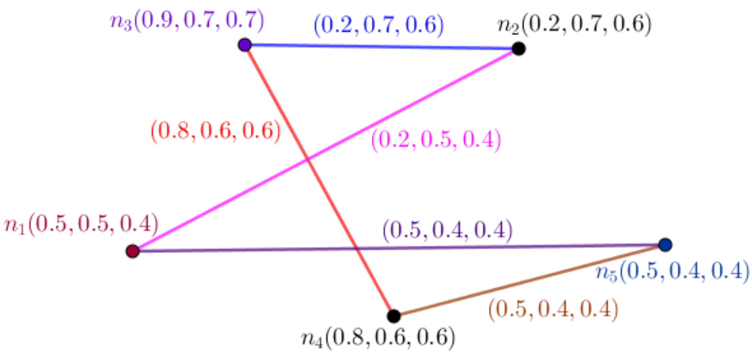

Figure 2.68: A Neutrosophic Graph in the Viewpoint of its total-dominating number and its neutrosophic total-dominating number. 85NTG6

pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called **total-resolving set**. The minimum cardinality between all total-resolving sets is called **total-resolving number** and it's denoted by $\mathcal{T}(NTG)$;

(ii) for given vertices n and n' if $d(s,n) \neq d(s,n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices, $d \geq 1$ and all vertices have to be total-resolved otherwise it will be mentioned which is about $d \geq 0$ in some cases but all vertices have to be total-resolved forever. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called **total-resolving set**. The minimum neutrosophic cardinality between all total-resolving sets is called **neutrosophic total-resolving number** and it's denoted by $\mathcal{T}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

Proposition 2.5.133. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then* $|S| \geq 2$.

86prp10 **Proposition 2.5.134.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then if there are twin vertices then total-resolving set and total-resolving number are Not Existed.*

Proposition 2.5.135. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$ and $d \geq 0$. Then*

$$\mathcal{T}(CYC) = \min_{x,y \in V,\ x,y \text{ aren't antipodal.}} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve since a vertex couldn't resolve itself. Thus two [minus antipodal pairs] vertices are necessary in S. All total-resolving sets corresponded to total-resolving number are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(PTH)-2}\}, \{n_1, n_{\mathcal{O}(PTH)-1}\}, \{n_1, n_{\mathcal{O}(PTH)}\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_2, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(PTH)-2}\}, \{n_2, n_{\mathcal{O}(PTH)-1}\}, \{n_2, n_{\mathcal{O}(PTH)}\},$$
$$\{n_3, n_4\}, \{n_2, n_5\}, \{n_2, n_6\}, \ldots, \{n_2, n_{\mathcal{O}(PTH)-2}\}, \{n_2, n_{\mathcal{O}(PTH)-1}\}, \{n_2, n_{\mathcal{O}(PTH)}\},$$
$$\ldots,$$
$$\{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)-2}\}, \{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)-1}\}, \{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)}\},$$
$$\{n_{\mathcal{O}(PTH)-2}, n_{\mathcal{O}(PTH)-1}\}, \{n_{\mathcal{O}(PTH)-2}, n_{\mathcal{O}(PTH)}\},$$
$$\{n_{\mathcal{O}(PTH)-1}, n_{\mathcal{O}(PTH)}\}$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum cardinality between all total-resolving sets is called total-resolving number and it's denoted by $\mathcal{T}(CYC) = 2$ and corresponded to total-resolving sets are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(PTH)-2}\}, \{n_1, n_{\mathcal{O}(PTH)-1}\}, \{n_1, n_{\mathcal{O}(PTH)}\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_2, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(PTH)-2}\}, \{n_2, n_{\mathcal{O}(PTH)-1}\}, \{n_2, n_{\mathcal{O}(PTH)}\},$$
$$\{n_3, n_4\}, \{n_2, n_5\}, \{n_2, n_6\}, \ldots, \{n_2, n_{\mathcal{O}(PTH)-2}\}, \{n_2, n_{\mathcal{O}(PTH)-1}\}, \{n_2, n_{\mathcal{O}(PTH)}\},$$
$$\ldots,$$
$$\{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)-2}\}, \{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)-1}\}, \{n_{\mathcal{O}(PTH)-3}, n_{\mathcal{O}(PTH)}\},$$
$$\{n_{\mathcal{O}(PTH)-2}, n_{\mathcal{O}(PTH)-1}\}, \{n_{\mathcal{O}(PTH)-2}, n_{\mathcal{O}(PTH)}\},$$
$$\{n_{\mathcal{O}(PTH)-1}, n_{\mathcal{O}(PTH)}\}$$

Thus

$$\mathcal{T}(CYC) = 2.$$

■

Proposition 2.5.136. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $d \geq 0$. Then total-resolving number is equal to resolving number.*

Antipodal vertices in even-cycle-neutrosophic graph differ the number in cycle-neutrosophic graph.

Proposition 2.5.137. *Let $NTG : (V, E, \sigma, \mu)$ be an odd-cycle-neutrosophic graph where $d \geq 0$. Then the number of total-resolving sets corresponded to total-resolving number is equal to $\mathcal{O}(CYC)$ choose two.*

Proposition 2.5.138. *Let $NTG : (V, E, \sigma, \mu)$ be an odd-cycle-neutrosophic graph where $d \geq 0$. Then the number of total-resolving sets is equal to $2^{\mathcal{O}(CYC)} - \mathcal{O}(CYC) - 1$.*

We've to eliminate antipodal vertices due to total-resolving implies complete resolving.

Proposition 2.5.139. *Let $NTG : (V, E, \sigma, \mu)$ be an even-cycle-neutrosophic graph where $d \geq 0$. Then the number of total-resolving sets corresponded to total-resolving number is equal to $\mathcal{O}(CYC)$ choose two after that minus $\mathcal{O}(CYC)$.*

Proposition 2.5.140. *Let $NTG : (V, E, \sigma, \mu)$ be an even-cycle-neutrosophic graph where $d \geq 0$. Then the number of total-resolving sets is equal to $2^{\mathcal{O}(CYC)} - 2\mathcal{O}(CYC) - 1$.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.141. There are two sections for clarifications.

(a) In Figure (2.69), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex, s, there are only two paths with other vertices;
- (ii) in the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve since a vertex couldn't resolve itself. Thus two [minus antipodal pairs] vertices are necessary in S. Antipodal pairs are

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\};$$

- (iii) all total-resolving sets corresponded to total-resolving number are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \ldots.$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum cardinality between all total-resolving sets is called total-resolving number and it's denoted by $\mathcal{T}(CYC) = 2$ and corresponded to total-resolving sets are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$

$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \ldots;$$

(iv) there are fifty-seven [minus antipodal pairs] total-resolving sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},$$
$$\{n_1, n_4, n_5\}, \{n_2, n_3, n_4\}, \{n_2, n_3, n_5\},$$
$$\{n_2, n_4, n_5\}, \{n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\}, \ldots$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is characteristic;

(v) there are fifteen [minus antipodal pairs] total-resolving sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \ldots,$$

corresponded to total-resolving number as if there's one total-resolving set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is the determiner;

(vi) all total-resolving sets corresponded to total-resolving number are [minus antipodal pairs]

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \ldots.$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum neutrosophic cardinality between all total-resolving sets is called neutrosophic total-resolving number and it's denoted by $\mathcal{T}_n(CYC) = 1.3$ and corresponded to total-resolving sets are

$$\{n_1, n_5\}.$$

(b) In Figure (2.70), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve since a vertex couldn't resolve itself. Thus two vertices are necessary in S;

(iii) all total-resolving sets corresponded to total-resolving number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}.$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum cardinality between all total-resolving sets is called total-resolving number and it's denoted by $\mathcal{T}(CYC) = 2$ and corresponded to total-resolving sets are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\};$$

(iv) there are twenty-six total-resolving sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_3, n_4\}, \{n_1, n_3, n_5\},$$
$$\{n_1, n_4, n_5\}, \{n_2, n_3, n_4\}, \{n_2, n_3, n_5\},$$
$$\{n_2, n_4, n_5\}, \{n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4\},$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_4, n_5\}, \{n_1, n_3, n_4, n_5\},$$
$$\{n_2, n_3, n_4, n_5\}, \{n_1, n_2, n_3, n_4, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is characteristic;

(v) there are ten total-resolving sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$

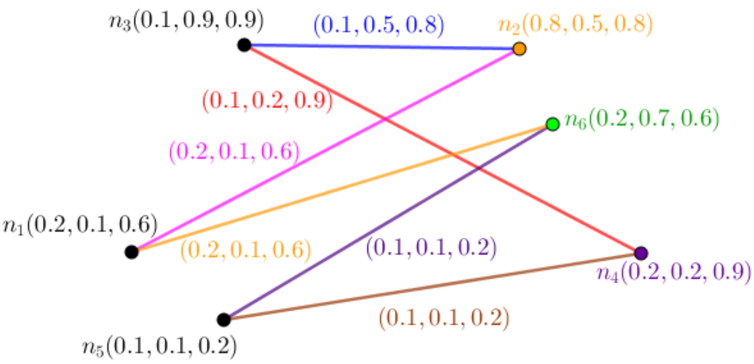

Figure 2.69: A Neutrosophic Graph in the Viewpoint of its total-resolving number and its neutrosophic total-resolving number.

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$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\},$$

corresponded to total-resolving number as if there's one total-resolving set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is the determiner;

(vi) all total-resolving sets corresponded to total-resolving number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_1, n_5\}, \{n_2, n_3\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}, \{n_3, n_4\}, \{n_3, n_5\},$$
$$\{n_4, n_5\}.$$

For given vertex n, if $sn \in E$, then s total-resolves n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-resolves n, then the set of neutrosophic vertices, S is called total-resolving set. The minimum neutrosophic cardinality between all total-resolving sets is called neutrosophic total-resolving number and it's denoted by $\mathcal{T}_n(CYC) = 2.7$ and corresponded to total-resolving sets are

$$\{n_1, n_5\}.$$

Definition 2.5.142. (stable-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them,

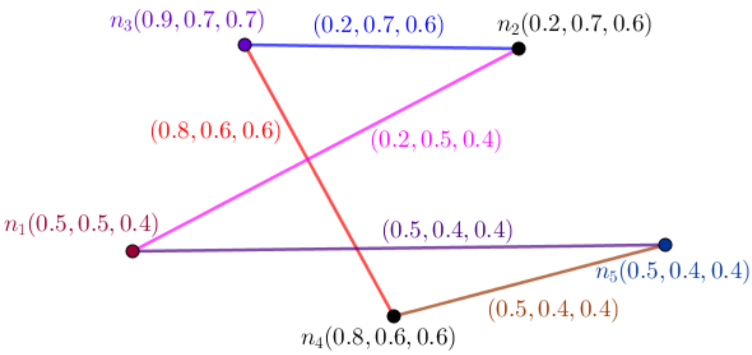

Figure 2.70: A Neutrosophic Graph in the Viewpoint of its total-resolving number and its neutrosophic total-resolving number. 86NTG6

then the set of neutrosophic vertices, S is called **stable-dominating set**. The minimum cardinality between all stable-dominating sets is called **stable-dominating number** and it's denoted by $\mathcal{S}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-dominating set**. The minimum neutrosophic cardinality between all stable-dominating sets is called **neutrosophic stable-dominating number** and it's denoted by $\mathcal{S}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

86prp9 **Proposition 2.5.143.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Assume $|S|$ has one member. Then*

(i) *a vertex dominates if and only if it stable-dominates;*

(ii) *S is dominating set if and only if it's stable-dominating set;*

(iii) *a number is dominating number if and only if it's stable-dominating number.*

Proposition 2.5.144. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is stable-dominating set corresponded to stable-dominating number if and only if for every neutrosophic vertex s in S, there's at least a neutrosophic vertex n in $V \setminus S$ such that $\{s' \in S \mid s'n \in E\} = \{s\}$.*

Proposition 2.5.145. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V isn't S.*

Proposition 2.5.146. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then stable-dominating number is between one and $\mathcal{O}(NTG) - 1$.*

Proposition 2.5.147. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then stable-dominating number is between one and $\mathcal{O}_n(NTG) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x)$.*

Proposition 2.5.148. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{S}_n(CYC) = \min_{|S| = \lceil \frac{\mathcal{O}(CYC)}{3} \rceil} \sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates since a vertex dominates neighbors thus in S, there aren't any neighbors and all vertices are neighborless in S. All stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4, n_7, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_2, n_5, n_8, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}\},$$
$$\ldots.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum neutrosophic cardinality between all stable-dominating sets is called neutrosophic stable-dominating number and it's denoted by

$$\mathcal{S}_n(CYC) = \min_{|S| = \lceil \frac{\mathcal{O}(CYC)}{3} \rceil} \sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x)$$

and corresponded to stable-dominating sets are

$$\{n_1, n_4, n_7, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}\},$$
$$\{n_2, n_5, n_8, \ldots, n_{\mathcal{O}(CYC)-4}, n_{\mathcal{O}(CYC)-1}\},$$
$$\ldots.$$

Thus

$$\mathcal{S}_n(CYC) = \min_{|S| = \lceil \frac{\mathcal{O}(CYC)}{3} \rceil} \sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x).$$

■

Proposition 2.5.149. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then stable-dominating number is equal to dominating number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.150. There are two sections for clarifications.

(a) In Figure (2.71), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates since a vertex dominates neighbors thus in S, there aren't any neighbors and all vertices are neighborless in S;

(iii) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum cardinality between all stable-dominating sets is called stable-dominating number and it's denoted by $\mathcal{S}(CYC) = 2$; and corresponded to stable-dominating sets are

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\};$$

(iv) there are five stable-dominating sets

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\},$$
$$\{n_1, n_3, n_5\}, \{n_2, n_4, n_6\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is characteristic;

(v) there are three stable-dominating setsc

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\},$$

corresponded to stable-dominating number as if there's one stable-dominating set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is the determiner;

(vi) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4\}, \{n_2, n_5\}, \{n_3, n_6\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum neutrosophic cardinality between all stable-dominating sets is called neutrosophic stable-dominating number and it's denoted by $\mathcal{S}_n(CYC) = 2.2$; and corresponded to stable-dominating sets are

$$\{n_1, n_4\}.$$

(b) In Figure (2.72), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there are only two paths with other vertices;

(ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't stable-dominate since a vertex couldn't dominate itself. Thus two vertices are necessary in S;

(iii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates since a vertex dominates neighbors thus in S, there aren't any neighbors and all vertices are neighborless in S;

(iii) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\},$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum cardinality between all stable-dominating sets is called stable-dominating number and it's denoted by $\mathcal{S}(CYC) = 2$; and corresponded to stable-dominating sets are

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\};$$

(*iv*) there are five stable-dominating sets

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is characteristic;

(*v*) there are five stable-dominating sets

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\},$$

corresponded to stable-dominating number as if there's one stable-dominating set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is the determiner;

(*vi*) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1, n_4\}, \{n_2, n_4\}, \{n_2, n_5\},$$
$$\{n_1, n_3\}, \{n_5, n_3\},$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum neutrosophic cardinality between all stable-dominating sets is called neutrosophic stable-dominating number and it's denoted by $\mathcal{S}_n(CYC) = 2.8$; and corresponded to stable-dominating sets are

$$\{n_2, n_5\}.$$

Definition 2.5.151. (stable-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) for given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-resolving set**. The minimum cardinality between all stable-resolving sets is called **stable-resolving number** and it's denoted by $\mathcal{S}(NTG)$;

(*ii*) for given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple

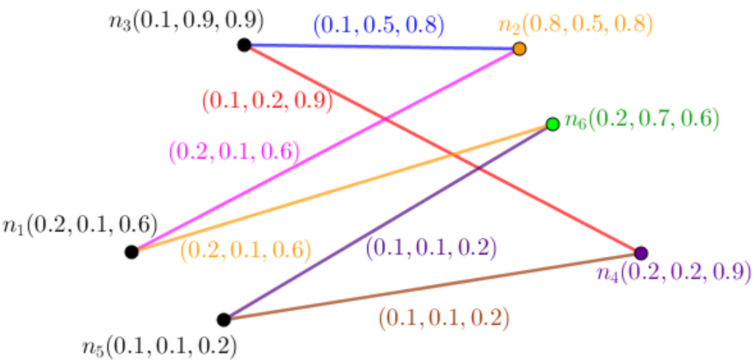

Figure 2.71: A Neutrosophic Graph in the Viewpoint of its stable-dominating number and its neutrosophic stable-dominating number. 87NTG5

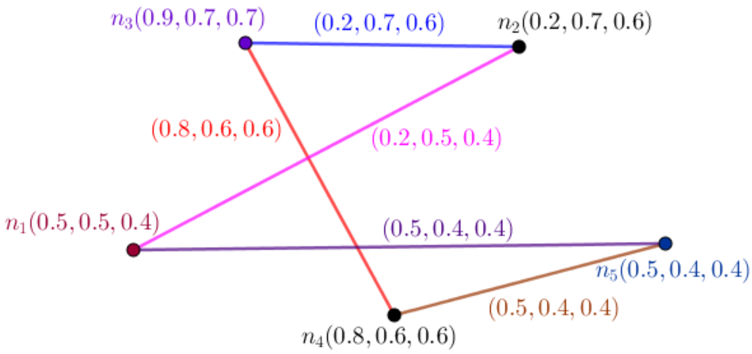

Figure 2.72: A Neutrosophic Graph in the Viewpoint of its stable-dominating number and its neutrosophic stable-dominating number. 87NTG6

pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **neutrosophic stable-resolving set**. The minimum neutrosophic cardinality between all stable-resolving sets is called **neutrosophic stable-resolving number** and it's denoted by $\mathcal{S}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

88prp9 **Proposition 2.5.152.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Assume $|S|$ has one member. Then*

(i) *a vertex resolves if and only if it stable-resolves;*

(ii) *S is resolving set if and only if it's stable-resolving set;*

(iii) *a number is resolving number if and only if it's stable-resolving number.*

Proposition 2.5.153. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is stable-resolving set corresponded to stable-resolving number if and only if for every neutrosophic vertex s in S, there are at least neutrosophic vertices n and n' in $V \setminus S$ such that $\{s' \in S \mid d(s', n) \neq d(s', n')\} = \{s\}$.*

Proposition 2.5.154. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V isn't S.*

Proposition 2.5.155. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{S}_n(CYC) = \min\{\sum_{i=1}^{3} \sigma_i(n_i) + \sum_{i=1}^{3} \sigma_i(n_j)\}_{n_i \text{ and } n_j \text{ are neither antipodal nor neighbor}}.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$n_1, n_2, \cdots, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}, n_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. In the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve since two neighbors aren't allowed in the setting of stable-resolving. All stable-resolving sets corresponded to stable-resolving number are

$$\begin{aligned}
&\{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(CYC)-3}\}, \{n_1, n_{\mathcal{O}(CYC)-2}\}, \{n_1, n_{\mathcal{O}(CYC)-1}\},\\
&\{n_2, n_4\}, \{n_1, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(CYC)-2}\}, \{n_2, n_{\mathcal{O}(CYC)-1}\}, \{n_2, n_{\mathcal{O}(CYC)}\},\\
&\{n_3, n_5\}, \{n_3, n_6\}, \ldots, \{n_3, n_{\mathcal{O}(CYC)-2}\}, \{n_3, n_{\mathcal{O}(CYC)-1}\}, \{n_3, n_{\mathcal{O}(CYC)}\},\\
&\ldots\\
&\{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}\}, \{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-1}\}, \{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)}\},\\
&\{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\}, \{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\},\\
&\{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\}.
\end{aligned}$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum neutrosophic cardinality between all stable-resolving sets is called neutrosophic stable-resolving number and it's denoted by

$$\mathcal{S}_n(CYC) = \min\{\sum_{i=1}^{3} \sigma_i(n_i) + \sum_{i=1}^{3} \sigma_i(n_j)\}_{n_i \text{ and } n_j \text{ are neither antipodal nor neighbor}};$$

and corresponded to stable-resolving sets are

$$\begin{aligned}
&\{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(CYC)-3}\}, \{n_1, n_{\mathcal{O}(CYC)-2}\}, \{n_1, n_{\mathcal{O}(CYC)-1}\},\\
&\{n_2, n_4\}, \{n_1, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(CYC)-2}\}, \{n_2, n_{\mathcal{O}(CYC)-1}\}, \{n_2, n_{\mathcal{O}(CYC)}\},\\
&\{n_3, n_5\}, \{n_3, n_6\}, \ldots, \{n_3, n_{\mathcal{O}(CYC)-2}\}, \{n_3, n_{\mathcal{O}(CYC)-1}\}, \{n_3, n_{\mathcal{O}(CYC)}\},\\
&\ldots
\end{aligned}$$

$\{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-2}\}, \{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)-1}\}, \{n_{\mathcal{O}(CYC)-3}, n_{\mathcal{O}(CYC)}\},$
$\{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)-1}\}, \{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\},$
$\{n_{\mathcal{O}(CYC)-2}, n_{\mathcal{O}(CYC)}\}.$

Thus

$$\mathcal{S}_n(CYC) = \min\{\sum_{i=1}^{3} \sigma_i(n_i) + \sum_{i=1}^{3} \sigma_i(n_j)\}_{n_i \text{ and } n_j \text{ are neither antipodal nor neighbor}}.$$

■

Proposition 2.5.156. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then stable-resolving number is equal to resolving number.*

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.157. There are two sections for clarifications.

(a) In Figure (2.73), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex, s, there are only two paths with other vertices;
- (ii) in the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve since two neighbors aren't allowed in the setting of stable-resolving;
- (iii) all stable-resolving sets corresponded to stable-resolving number are

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\}.$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum cardinality between all stable-resolving sets is called stable-resolving number and it's denoted by $\mathcal{S}(CYC) = 2$; and corresponded to stable-resolving sets are

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\};$$

(*iv*) there are six stable-resolving sets

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\}, \{n_1, n_3, n_5\}, \{n_2, n_4, n_6\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-resolving number so as neutrosophic cardinality is characteristic;

(*v*) there are four stable-resolving sets

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\}$$

corresponded to stable-resolving number as if there's one stable-resolving set corresponded to neutrosophic stable-resolving number so as neutrosophic cardinality is the determiner;

(*vi*) all stable-resolving sets corresponded to stable-resolving number are

$$\{n_1, n_3\}, \{n_1, n_5\}, \{n_2, n_4\},$$
$$\{n_2, n_6\}.$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum neutrosophic cardinality between all stable-resolving sets is called neutrosophic stable-resolving number and it's denoted by $\mathcal{S}_n(CYC) = 1.3$; and corresponded to stable-resolving sets are

$$\{n_1, n_5\}.$$

(*b*) In Figure (2.74), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) For given neutrosophic vertex, s, there are only two paths with other vertices;

(*ii*) in the setting of cycle, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve since two neighbors aren't allowed in the setting of stable-resolving;

(*iii*) all stable-resolving sets corresponded to stable-resolving number are

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}.$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum cardinality between all stable-resolving sets is called stable-resolving number and it's denoted by $\mathcal{S}(CYC) = 2$; and corresponded to stable-resolving sets are

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\};$$

(iv) there are four stable-resolving sets

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-resolving number so as neutrosophic cardinality is characteristic;

(v) there are four stable-resolving sets

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\};$$

corresponded to stable-resolving number as if there's one stable-resolving set corresponded to neutrosophic stable-resolving number so as neutrosophic cardinality is the determiner;

(vi) all stable-resolving sets corresponded to stable-resolving number are

$$\{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_4\},$$
$$\{n_2, n_5\}.$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum neutrosophic cardinality between all stable-resolving sets is called neutrosophic stable-resolving number and it's denoted by $\mathcal{S}_n(CYC) = 2.8$; and corresponded to stable-resolving sets are

$$\{n_2, n_5\}.$$

2.6 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are cycle-neutrosophic graph.

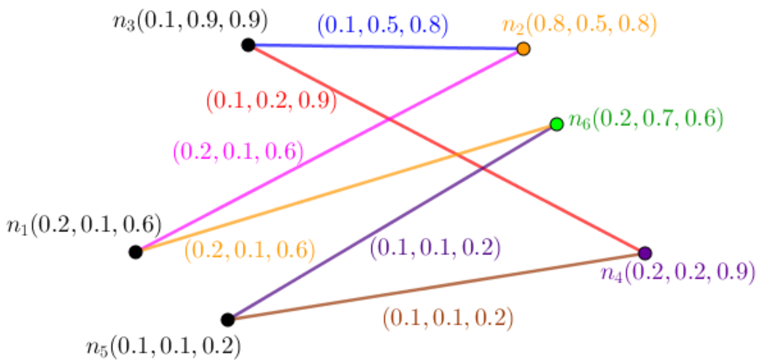

Figure 2.73: A Neutrosophic Graph in the Viewpoint of its stable-resolving number and its neutrosophic stable-resolving number. 88NTG5

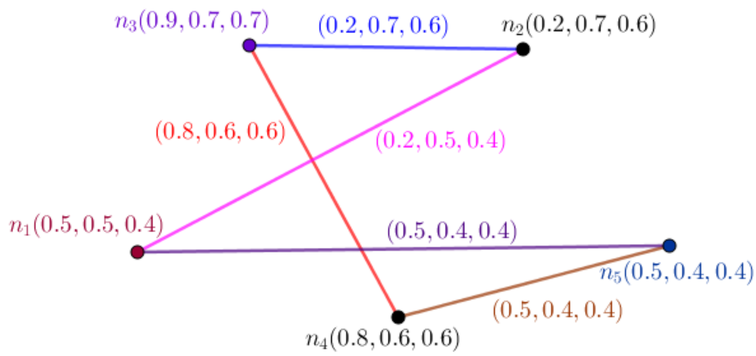

Figure 2.74: A Neutrosophic Graph in the Viewpoint of its stable-resolving number and its neutrosophic stable-resolving number. 88NTG6

2.7 Modelling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (2.1), clarifies about the assigned numbers to these situations.

Table 2.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model. 88tbl1

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

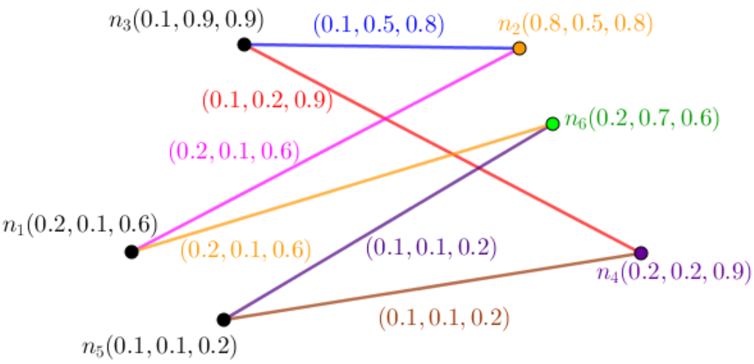

Figure 2.75: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG5

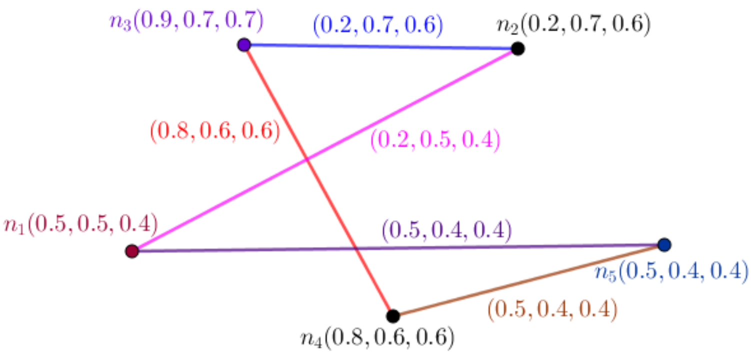

Figure 2.76: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG6

2.8 Case 1: cycle-neutrosophic Model

Step 4. (Solution) The neutrosophic graph model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are five subjects which are represented as Figure (2.75). This model is strong and even more. And the study proposes using specific number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (2.75). In Figure (2.75), a cycle-neutrosophic graph. is illustrated. Some points are represented in follow-up items as follows.

(a) In Figure (2.75), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (2.76), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

2.9 Case 2: cycle-neutrosophic Model alongside its Neutrosophic Graph

Step 4. (Solution) The neutrosophic graph as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are four subjects which are represented in the formation of one model as Figure (2.77). This model is neutrosophic strong as individual and even more. And the study proposes using specific number

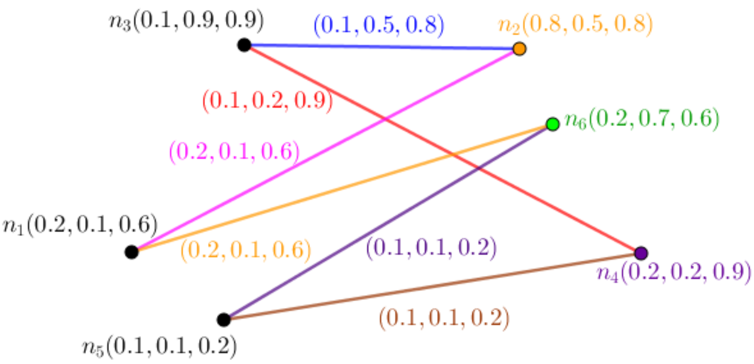

Figure 2.77: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG5

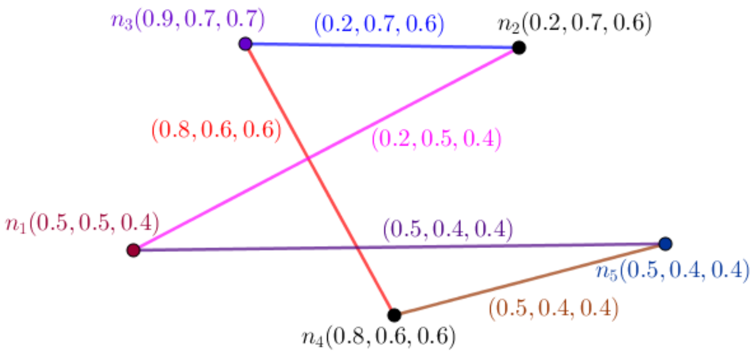

Figure 2.78: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG6

for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (2.77). There is one section for clarifications.

(a) In Figure (2.77), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices, there are only two paths between them;
- (ii) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(iii) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(iv) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
$$\{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_3, n_2, n_1, n_4\}$$
$$\{n_3, n_2, n_1, n_5\}, \{n_3, n_2, n_1, n_6\}, \{n_3, n_2, n_4, n_5\},$$
$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$

$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
$$\{n_5, n_6, n_3\}, \{n_1, n_6, n_4\}, \{n_5, n_6, n_1, n_2\}$$
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$$\{n_5, n_6, n_1, n_2, n_4\}, \{n_5, n_6, n_1, n_3, n_4\}, \{n_5, n_6, n_2, n_3, n_4\},$$
$$\{n_1, n_6\}, \{n_1, n_6, n_3\}, \{n_1, n_6, n_4\},$$
$$\{n_1, n_6, n_5\}, \{n_1, n_6, n_2\}, \{n_1, n_6, n_3, n_4\}$$
$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$
$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are ninety-one joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_6\}, \{n_1, n_2, n_3, n_4\}$$
$$\{n_1, n_2, n_3, n_5\}, \{n_1, n_2, n_3, n_6\}, \{n_1, n_2, n_4, n_5\},$$
$$\{n_1, n_2, n_4, n_6\}, \{n_1, n_2, n_5, n_6\}, \{n_1, n_2, n_3, n_4, n_5\},$$
$$\{n_1, n_2, n_3, n_4, n_6\}, \{n_1, n_2, n_3, n_5, n_6\}, \{n_1, n_2, n_4, n_5, n_6\},$$
$$\{n_1, n_2, n_3, n_4, n_5, n_6\},$$
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$$\{n_3, n_2, n_4, n_6\}, \{n_3, n_2, n_5, n_6\}, \{n_3, n_2, n_1, n_4, n_5\},$$
$$\{n_3, n_2, n_1, n_4, n_6\}, \{n_3, n_2, n_1, n_5, n_6\}, \{n_3, n_2, n_4, n_5, n_6\},$$
$$\{n_3, n_4\}, \{n_3, n_4, n_1\}, \{n_3, n_4, n_2\},$$
$$\{n_3, n_4, n_5\}, \{n_1, n_4, n_6\}, \{n_3, n_4, n_1, n_2\}$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_1, n_6\}, \{n_3, n_4, n_2, n_5\},$$
$$\{n_3, n_4, n_2, n_6\}, \{n_3, n_4, n_5, n_6\}, \{n_3, n_4, n_1, n_2, n_5\},$$
$$\{n_3, n_4, n_1, n_2, n_6\}, \{n_3, n_4, n_1, n_5, n_6\}, \{n_3, n_4, n_2, n_5, n_6\},$$
$$\{n_5, n_4\}, \{n_5, n_4, n_1\}, \{n_5, n_4, n_2\},$$
$$\{n_5, n_4, n_3\}, \{n_1, n_4, n_6\}, \{n_5, n_4, n_1, n_2\}$$
$$\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_1, n_6\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_4, n_2, n_6\}, \{n_5, n_4, n_3, n_6\}, \{n_5, n_4, n_1, n_2, n_3\},$$
$$\{n_5, n_4, n_1, n_2, n_6\}, \{n_5, n_4, n_1, n_3, n_6\}, \{n_5, n_4, n_2, n_3, n_6\},$$
$$\{n_5, n_6\}, \{n_5, n_6, n_1\}, \{n_5, n_6, n_2\},$$
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$$\{n_5, n_6, n_1, n_3\}, \{n_5, n_6, n_1, n_4\}, \{n_5, n_6, n_2, n_3\},$$
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$$\{n_1, n_6, n_3, n_5\}, \{n_1, n_6, n_3, n_2\}, \{n_1, n_6, n_4, n_5\},$$
$$\{n_1, n_6, n_4, n_2\}, \{n_1, n_6, n_5, n_2\}, \{n_1, n_6, n_3, n_4, n_5\},$$
$$\{n_1, n_6, n_3, n_4, n_2\}, \{n_1, n_6, n_3, n_5, n_2\}, \{n_1, n_6, n_4, n_5, n_2\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_6\}, \{n_6, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 1.7.$$

S is $\{n_4, n_5\}$ corresponded to neutrosophic joint-resolving number.

(b) In Figure (2.78), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, there are only two paths between them;

(*ii*) one vertex only resolves some vertices as if not all if they aren't two neighbor vertices, then it only resolves some of all vertices and if they aren't two neighbor vertices, then they resolves all vertices thus it implies the vertex joint-resolves as same as the vertex resolves vertices in the setting of cycle, by joint-resolving set corresponded to joint-resolving number has two neighbor vertices;

(*iii*) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CYC) = 2$;

(*iv*) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$
$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\}$$

as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are thirty-six joint-resolving sets

$$\{n_1, n_2\}, \{n_1, n_2, n_3\}, \{n_1, n_2, n_4\},$$
$$\{n_1, n_2, n_5\}, \{n_1, n_2, n_3, n_4\}\{n_1, n_2, n_3, n_5\}$$
$$\{n_1, n_2, n_4, n_5\}, \{n_3, n_2\}, \{n_3, n_2, n_1\}, \{n_3, n_2, n_4\},$$
$$\{n_3, n_2, n_5\}, \{n_3, n_2, n_1, n_4\}\{n_3, n_2, n_1, n_5\},$$
$$\{n_3, n_2, n_4, n_5\}, \{n_3, n_4\}, \{n_3, n_4, n_1\},$$
$$\{n_3, n_4, n_2\}, \{n_3, n_4, n_5\}, \{n_3, n_4, n_1, n_2\},$$
$$\{n_3, n_4, n_1, n_5\}, \{n_3, n_4, n_2, n_5\}, \{n_5, n_4\},$$
$$\{n_5, n_4, n_1\}, \{n_5, n_4, n_2\}, \{n_5, n_4, n_3\},$$
$$\{n_5, n_4, n_1, n_2\}\{n_5, n_4, n_1, n_3\}, \{n_5, n_4, n_2, n_3\},$$
$$\{n_5, n_1\}, \{n_5, n_1, n_4\}, \{n_5, n_1, n_2\},$$
$$\{n_5, n_1, n_3\}, \{n_5, n_1, n_4, n_2\}\{n_5, n_1, n_4, n_3\},$$
$$\{n_5, n_1, n_2, n_3\}, \{n_5, n_1, n_4, n_2, n_3\},$$

as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex in S such that joint-resolves n and n', then the set of neutrosophic vertices, S is either of

$$\{n_1, n_2\}, \{n_2, n_3\}, \{n_3, n_4\},$$
$$\{n_4, n_5\}, \{n_5, n_1\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CYC) = 2.7.$$

S is $\{n_1, n_5\}$ corresponded to neutrosophic joint-resolving number.

2.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are defined in cycle-neutrosophic graphs. Thus,

Question 2.10.1. *Is it possible to use other types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable?*

Question 2.10.2. *Are existed some connections amid different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs?*

Question 2.10.3. *Is it possible to construct some classes of cycle-neutrosophic graphs which have "nice" behavior?*

Question 2.10.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 2.10.5. *Which parameters are related to this parameter?*

Problem 2.10.6. *Which approaches do work to construct applications to create independent study?*

Problem 2.10.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

2.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses some definitions concerning different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in cycle-neutrosophic graphs assigned to cycle-neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of cycle-neutrosophic graphs. One way is finding some relations

Table 2.2: A Brief Overview about Advantages and Limitations of this Study 88tbl

Advantages	Limitations
1. Neutrosophic Numbers of Model	1. Connections amid Classes
2. Acting on All Edges	
3. Minimal Sets	2. Study on Families
4. Maximal Sets	
5. Acting on All Vertices	3. Same Models in Family

amid all definitions of notions to make sensible definitions. In Table (2.2), some limitations and advantages of this study are pointed out.

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