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Neutrosophic Complete

Ideas | Approaches | Accessibility | Availability

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Report | Exposition | References | Research #22 2022

Abstract

In this book, some notions are introduced about "Neutrosophic Complete". Some frameworks are devised as "Different Types" of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs.

New setting is introduced to study different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs. Minimum number and maximum number of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, is a number which is representative based on those vertices or edges. Minimum or maximum neutrosophic number or polynomial of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are called neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number or polynomial. Forming sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable to figure out different types of number of vertices in the sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable sets in the terms of minimum (maximum) number of vertices to get minimum (maximum) number to assign in complete-neutrosophic graphs assigned to complete-neutrosophic graphs, is key type of approach to have these notions namely different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets. As concluding results, there are some statements, remarks, examples and clarifications about complete-neutrosophic graphs. The clarifications are also presented in both sections "Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable are addressed in Bibliography. Specially, properties of SuperHyperGraph and neutrosophic SuperHyperGraph by Henry Garrett (2022), is studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book by Henry Garrett (2022).

In this study, there's an idea which could be considered as a motivation.

Question 0.0.1. *Is it possible to use mixed versions of ideas concerning "different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial", "neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial" and "complete-neutrosophic graphs" to define some notions which are applied to complete-neutrosophic graphs?*

It's motivation to find notions to use in complete-neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. In both settings, corresponded numbers or polynomials conclude the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In section “Preliminaries”, new notions of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial' in complete-neutrosophic graphs assigned to complete-neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. As concluding results, there are some statements, remarks, examples and clarifications about complete-neutrosophic graphs. The clarifications are also presented in both sections ‘Setting of neutrosophic notion number,” and “ Setting of notion neutrosophic-number,” for introduced results and used classes. In section “Applications in Time Table and Scheduling”, two applications are posed for complete notions, namely complete-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section “Open Problems”, some problems and questions for further studies are proposed. In section “Conclusion and Closing Remarks”, gentle discussion about results and applications is featured. In section “Conclusion and Closing Remarks”, a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

Some frameworks are devised as “Different Types” of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs.

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The words of mind and the minds of words, are too eligible to be in the stage of acknowledgements

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CHAPTER 1

Neutrosophic Notions

1.1 Abstract

New setting is introduced to study different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs. Minimum number and maximum number of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, is a number which is representative based on those vertices or edges. Minimum or maximum neutrosophic number or polynomial of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are called neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number or polynomial. Forming sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable to figure out different types of number of vertices in the sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable sets in the terms of minimum (maximum) number of vertices to get minimum (maximum) number to assign in complete-neutrosophic graphs assigned to complete-neutrosophic

graphs, is key type of approach to have these notions namely different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets. As concluding results, there are some statements, remarks, examples and clarifications about complete-neutrosophic graphs. The clarifications are also presented in both sections "Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable
AMS Subject Classification: 05C17, 05C22, 05E45

1.2 Background

Different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable are addressed in Bibliography. Specially, properties of SuperHyperGraph and neutrosophic SuperHyperGraph by Henry Garrett (2022), is studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book by Henry Garrett (2022).
In this section, I use two sections to illustrate a perspective about the background of this study.

1.3 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.3.1. *Is it possible to use mixed versions of ideas concerning "different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial", "neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable* *number and polynomial" and "complete-neutrosophic graphs" to define some notions which are applied to complete-neutrosophic graphs?*

It's motivation to find notions to use in complete-neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. In both settings, corresponded numbers or polynomials conclude the discussion. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In section "Preliminaries", new notions of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial' in complete-neutrosophic graphs assigned to complete-neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. As concluding results, there are some statements, remarks, examples and clarifications about complete-neutrosophic graphs. The clarifications are also presented in both sections 'Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for complete notions, namely complete-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.4 Preliminaries

In this section, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.4.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.4.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v\in V}\sum_{i=1}^{3}\sigma_i(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e\in E}\sum_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1}\mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t}\ \bigwedge_{i=0,\cdots,t-1}\mu(x_i x_{i+1});$$

(iv) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1}\mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

Definition 1.4.4. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 1.4.5. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_σ if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** and denoted by CYC where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1,\ x_{\mathcal{O}(NTG)} x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1, σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1, σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1, σ_2}.

Remark 1.4.6. Using notations which is mixed with literatures, are reviewed.

1.4.6.1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$, $\mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;

1.4.6.2. $CMT_\sigma, PTH, CYC, STR_{1,\sigma_2}, CMT_{\sigma_1,\sigma_2}, CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$, and WHL_{1,σ_2}.

1.5 Setting of neutrosophic notion number

In this section, I provide some results in the setting of stable-resolving number.

Definition 1.5.1. (Zero Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **Zero forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(*ii*) **Zero forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

In next result, a complete-neutrosophic graph is considered in the way that, its neutrosophic zero forcing number and its zero forcing neutrosophic-number this model are computed. A complete-neutrosophic graph has specific attribute which implies every vertex is neighbor to all other vertices in the way that, two given vertices have edge is incident to these endpoints.

Proposition 1.5.2. *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-neutrosophic graph. Then*

$$\mathcal{Z}(NTG) = \mathcal{O}(NTG) - 1.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. If S is a set of black vertices and $S < \mathcal{O}(NTG) - 1$, then there are x and y such that they've more than one neighbor in S. Thus the color-change rule doesn't imply these vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}(NTG) = \mathcal{O}(NTG) - 1.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.3. In Figure (2.1), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_2 is black vertex. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex. Thus n_1 and n_2 aren't black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_2, n_3, n_4\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_1, n_2, n_4\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. Thus n_3 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iv) if $S = \{n_1, n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_2 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

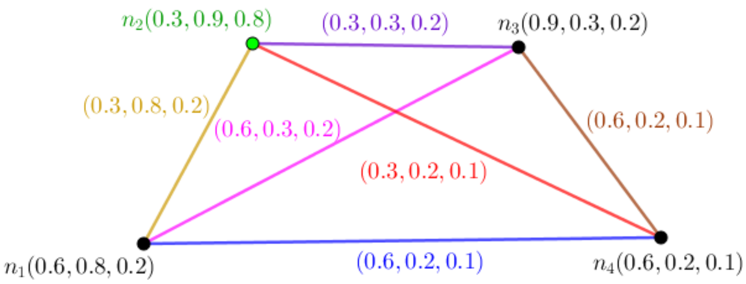

Figure 1.1: A Neutrosophic Graph in the Viewpoint of its Zero Forcing Number. 47NTG2

(v) 3 is zero forcing number and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is zero forcing neutrosophic-number and its corresponded set is $\{n_1, n_3, n_4\}$.

The main definition is presented in next section. The notions of failed zero-forcing number and failed zero-forcing neutrosophic-number facilitate the ground to introduce new results. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types. New setting is introduced to study failed zero-forcing number and failed zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex but now reverse approach is on demand which is finding biggest set which doesn't force.

Definition 1.5.4. (Failed Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Failed zero-forcing number** $\mathcal{Z}'(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(ii) **Failed zero-forcing neutrosophic-number** $\mathcal{Z}'_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

In next result, a complete-neutrosophic graph is considered in the way that, its neutrosophic failed zero-forcing number and its failed zero-forcing neutrosophic-number this model are computed. A complete-neutrosophic graph has specific attribute which implies every vertex is neighbor to all other vertices in the way that, two given vertices have edge is incident to these endpoints.

Proposition 1.5.5. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. If S is a set of black vertices and $S < \mathcal{O}(NTG) - 1$, then there are x and y such that they've more than one neighbor in S. Thus the color-change rule doesn't imply these vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.6. In Figure (2.2), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_2 is black vertex. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex. Thus n_1 and n_2 aren't black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_3, n_4\}$ could form failed zero-forcing number;

(ii) if $S = \{n_2, n_3, n_4\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_1, n_2, n_4\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. Thus n_3 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iv) if $S = \{n_1, n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_2 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_1, n_2\}, \{n_1, n_3\}$,
$\{n_1, n_4\}, \{n_2, n_3\}, \{n_2, n_4\}$, and $\{n_3, n_4\}$;

(vi) 3.6 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_2\}$.

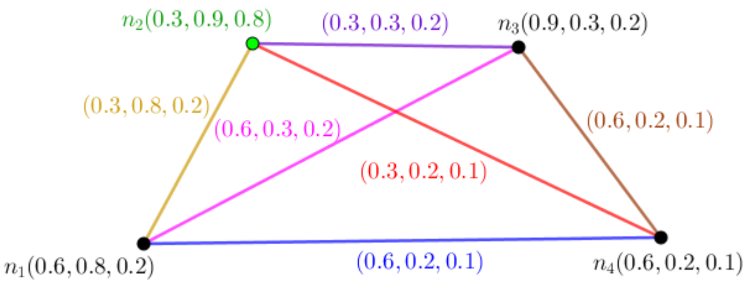

Figure 1.2: A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

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The main definition is presented in next section. The notions of 1-zero-forcing number and 1-zero-forcing neutrosophic-number facilitate the ground to introduce new results. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types. New setting is introduced to study 1-zero-forcing number and 1-zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex and now approach is on demand which is finding smallest set which forces.

Definition 1.5.7. (1-Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-zero-forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

(ii) **1-zero-forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

In next result, a complete-neutrosophic graph is considered in the way that, its neutrosophic 1-zero-forcing number and its 1-zero-forcing neutrosophic-number these models are computed. A complete-neutrosophic graph has specific attribute which implies every vertex is neighbor to all other vertices in the way that, two given vertices have edge is incident to these endpoints.

Proposition 1.5.8. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{Z}(NTG) = \mathcal{O}(NTG) - 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. If S is a set of black vertices and $|S| < \mathcal{O}(NTG) - 1$, then there are x and y such that they've more than one neighbor in S. Thus the color-change rule doesn't imply these vertices are black vertices but extra condition implies where $|S| = \mathcal{O}(NTG) - 2$. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition. So

$$\mathcal{Z}(NTG) = \mathcal{O}(NTG) - 2.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.9. In Figure (2.3), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_1, n_4\}$ is a set of black vertices, then n_2 and n_3 are white neighbors of n_1 and n_4. Thus the color-change rule doesn't imply n_2 is black vertex but extra condition implies. n_2 is white neighbor of n_1 and n_4. Thus the color-change rule implies n_3 is black vertex. Thus n_2 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

(ii) if $S = \{n_2, n_4\}$ is a set of black vertices, then n_1 and n_3 are white neighbors of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex but extra condition implies. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule implies n_3 is black vertex. Thus n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

(iii) if $S = \{n_1\}$ is a set of black vertices, then n_2, n_3 and n_4 are white neighbors of n_2. Thus the color-change rule doesn't imply neither of n_2, n_3 and n_4 are black vertices and extra condition doesn't imply, too. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule" and extra condition;

(iv) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_1 and n_2 are white neighbors of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex but extra condition implies. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule implies n_2 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

(v) 3 is 1-zero-forcing number and its corresponded sets are $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, and $\{n_3, n_4\}$;

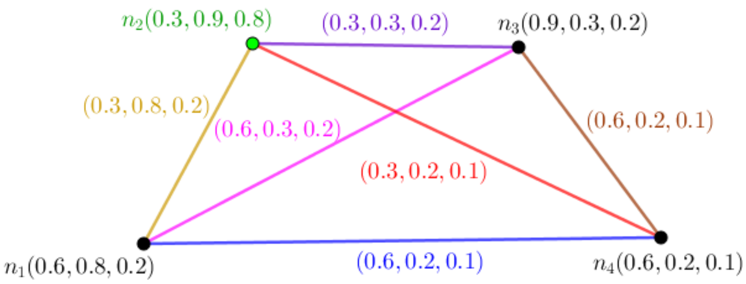

Figure 1.3: A Neutrosophic Graph in the Viewpoint of its 1-Zero-Forcing Number.

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(vi) 2.3 is 1-zero-forcing neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Definition 1.5.10. (Independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously;

(ii) **independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously.

Proposition 1.5.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{I}(NTG) = 1.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. It induces if $S = \{n\}$ is a set of vertices, then there's no vertex in S but n. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n\}$ is corresponded to independent number. Thus

$$\mathcal{I}(NTG) = 1.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.12. In Figure (2.4), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_2\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to both independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 1 is independent number and its corresponded sets are $\{n_1\}, \{n_2\}$, $\{n_3\}$, and $\{n_4\}$;

(vi) 0.9 is independent neutrosophic-number and its corresponded set is $\{n_4\}$.

The natural way proposes us to use the restriction "minimum" instead of "maximum."

Definition 1.5.13. (Failed independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

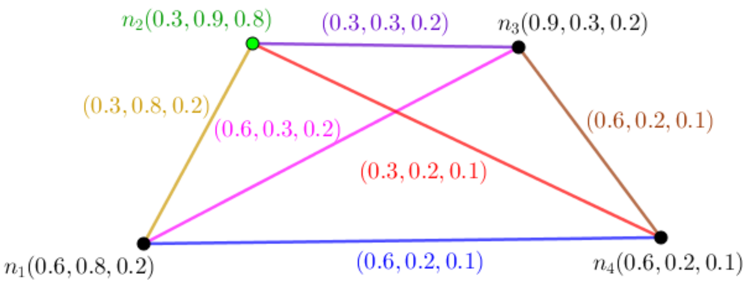

Figure 1.4: A Neutrosophic Graph in the Viewpoint of its Independent Number. 50NTG2

(*ii*) **failed independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Example 1.5.14. In Figure (2.5), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1, n_2\}$ is corresponded to failed independent number $\mathcal{I}(NTG)$ but not failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*ii*) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. It implies that $S = \{n_2, n_4\}$ is corresponded to failed independent number $\mathcal{I}(NTG)$ but not failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iii*) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. But it implies that $S = \{n_1\}$ isn't corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iv*) if $S = \{n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. It implies that $S = \{n_2, n_4\}$ is corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*v*) 2 is failed independent number and its corresponded sets are $\{n_1, n_2\}, \{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, and $\{n_3 n_4\}$;

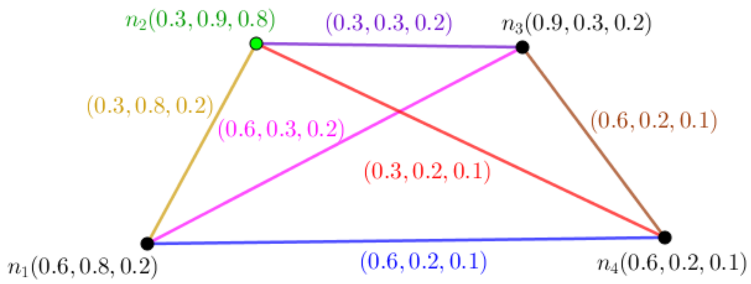

Figure 1.5: A Neutrosophic Graph in the Viewpoint of its Failed independent Number and its Failed Independent Neutrosophic-Number. 51NTG1

(vi) 2.3 is failed independent neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

But the results are always about the number two where connected model is used. For example,

Proposition 1.5.15. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{I}(NTG) = 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| = 2$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1, n_2\}$ is corresponded to failed independent number $\mathcal{I}(NTG)$. It induces by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S|=2}$ is corresponded to failed independent number. Thus

$$\mathcal{I}(NTG) = 2.$$

∎

Thus we replace the term "minimum" by the term ''maximum." Hence,

Definition 1.5.16. (Failed independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(ii) **failed independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.5.17. In Figure (2.5), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_2, n_4\}$ isn't corresponded to both of failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. But it implies that $S = \{n_1\}$ isn't corresponded to both of failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 4 is failed independent number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(vi) $\mathcal{O}_n(NTG) = 5.9$ is failed independent neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Proposition 1.5.18. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{I}(NTG) = \mathcal{O}(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x, y and z in S such that they're endpoints of an edge, simultaneously, and they

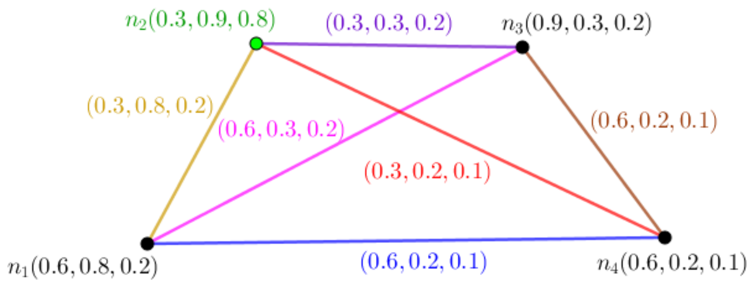

Figure 1.6: A Neutrosophic Graph in the Viewpoint of its Failed independent Number and its Failed Independent Neutrosophic-Number. 51NTG1

form a triangle. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are all possible edges to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S|=\mathcal{O}(NTG)}$ is corresponded to failed independent number. Thus

$$\mathcal{I}(NTG) = \mathcal{O}(NTG).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.19. In Figure (2.6), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq\mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq\mathcal{O}(NTG)}$. Thus it implies that $S = \{n_2, n_4\}$ is corresponded to neither failed independent number $\mathcal{I}(NTG)$ nor failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an

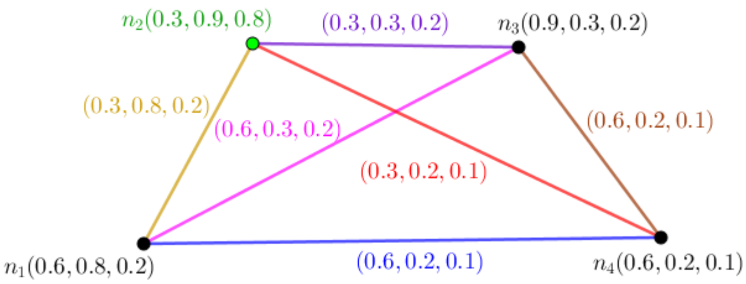

Figure 1.7: A Neutrosophic Graph in the Viewpoint of its Failed Independent Number. 51NTG2

edge. Furthermore, There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1\}$ is corresponded to neither failed independent number $\mathcal{I}(NTG)$ nor failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. $S = \{n_i\}_{|S| = \mathcal{O}(NTG)}$. Thus there are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 4 is failed independent number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(vi) $\mathcal{O}_n(NTG) = 5.9$ is failed independent neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Definition 1.5.20. (1-independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously For one time, one vertex is allowed to be endpoint;

(ii) **1-independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously. For one time, one vertex is allowed to be endpoint.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

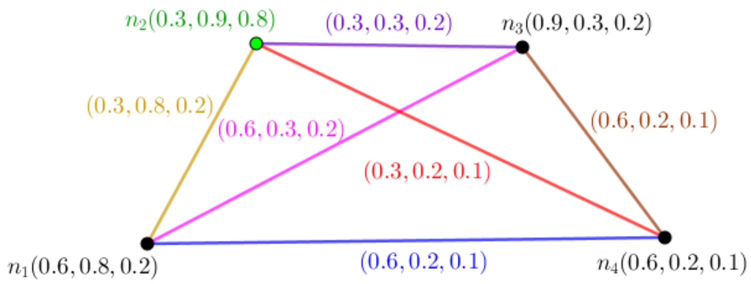

Figure 1.8: A Neutrosophic Graph in the Viewpoint of its 1-Independent Number and its 1-Independent Neutrosophic-Number. 52NTG1

Example 1.5.21. In Figure (2.8), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. Extra condition implies that $S = \{n_1\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. Extra condition implies that $S = \{n_2\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S but extra condition implies that $S = \{n_1, n_2\}$ is corresponded to both 1-independent number $\mathcal{I}(NTG)$ and 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but extra condition implies that $S = \{n_4\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 2 is 1-independent number and its corresponded sets are $\{n_1, n_2\}$,$\{n_1, n_3\}$,$\{n_1, n_4\}$,$\{n_2, n_3\}$,$\{n_2, n_4$ and $\{n_3, n_4\}$;

(vi) 3.6 is 1-independent neutrosophic-number and its corresponded set is $\{n_1, n_2\}$.

Definition 1.5.22. (Failed 1-independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed 1-independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. For one time, one vertex is allowed not to be endpoint;

(ii) **failed 1-independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. For one time, one vertex is allowed not to be endpoint.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.5.23. In Figure (2.8), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of failed 1-independent number $\mathcal{I}(NTG)$ and failed 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_2, n_4\}$ isn't corresponded to both of failed 1-independent number $\mathcal{I}(NTG)$ and failed 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. But it implies that $S = \{n_1\}$ isn't corresponded to both of failed 1-independent number $\mathcal{I}(NTG)$ and failed 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both failed 1-independent number $\mathcal{I}(NTG)$ and failed 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

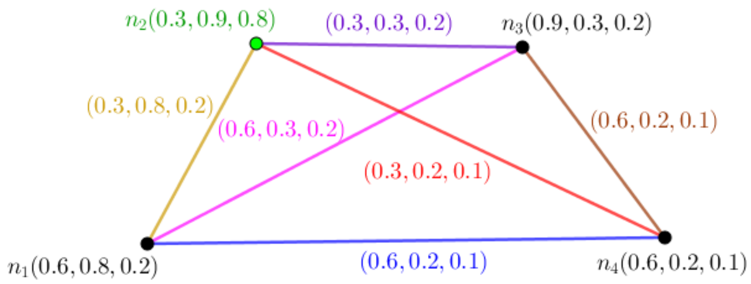

Figure 1.9: A Neutrosophic Graph in the Viewpoint of its Failed 1-Independent Number and its Failed 1-Independent Neutrosophic-Number. 52NTG1

(*v*) 4 is failed 1-independent number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(*vi*) $\mathcal{O}_n(NTG) = 5.9$ is failed 1-independent neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Proposition 1.5.24. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{I}(NTG) = 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to 1-independent number $\mathcal{I}(NTG)$. It induces if $S = \{n\}$ is a set of vertices, then there's no vertex in S but n. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n\}$ is corresponded to 1-independent number. Thus

$$\mathcal{I}(NTG) = 2.$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.25. In Figure (2.9), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

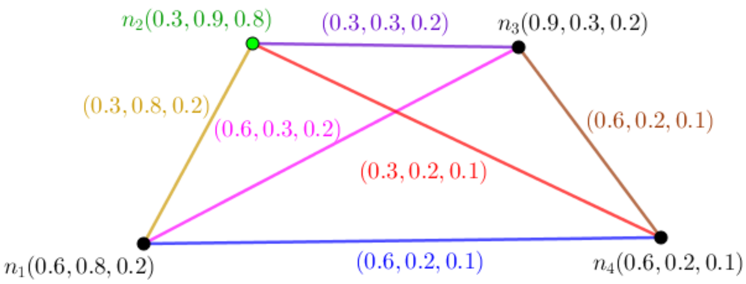

Figure 1.10: A Neutrosophic Graph in the Viewpoint of its 1-Independent Number.

52NTG2

(i) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_1\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_2\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_1, n_2\}$ is corresponded to both of 1-independent number $\mathcal{I}(NTG)$ and 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_4\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 2 is 1-independent number and its corresponded sets are $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, and $\{n_3, n_4\}$;

(vi) 3.6 is 1-independent neutrosophic-number and its corresponded set is $\{n_1, n_2\}$.

The natural way proposes us to use the restriction "maximum" instead of "minimum."

Definition 1.5.26. (Clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **clique number** $\mathcal{C}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(ii) **clique neutrosophic-number** $\mathcal{C}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Proposition 1.5.27. *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-neutrosophic graph. Then*

$$\mathcal{C}(NTG) = \mathcal{O}(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x, y and z in S such that they're endpoints of an edge, simultaneously, and they form a triangle. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are all possible edges to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S|=\mathcal{O}(NTG)}$ is corresponded to clique number. Thus

$$\mathcal{C}(NTG) = \mathcal{O}(NTG).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.28. In Figure (2.10), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq\mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of clique number $\mathcal{C}(NTG)$ and clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq\mathcal{O}(NTG)}$. Thus it implies that $S = \{n_2, n_4\}$ is corresponded to neither clique number $\mathcal{C}(NTG)$ nor clique neutrosophic-number $\mathcal{C}_n(NTG)$;

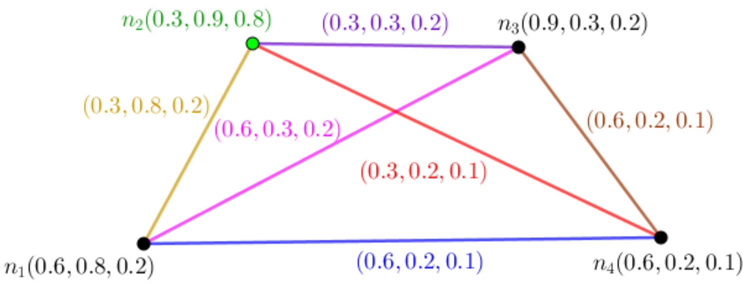

Figure 1.11: A Neutrosophic Graph in the Viewpoint of its clique Number.

53NTG2

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1\}$ is corresponded to neither clique number $\mathcal{C}(NTG)$ nor clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. $S = \{n_i\}_{|S| = \mathcal{O}(NTG)}$. Thus there are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both clique number $\mathcal{C}(NTG)$ and clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 4 is clique number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(vi) $\mathcal{O}_n(NTG) = 5.9$ is clique neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4\}$.

The natural way proposes us to use the restriction "minimum" instead of "maximum."

Definition 1.5.29. (Failed Clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed clique number** $\mathcal{C}^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously;

(ii) **failed clique neutrosophic-number** $\mathcal{C}^{\mathcal{F}}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously.

Proposition 1.5.30. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{C}^{\mathcal{F}}(NTG) = 0.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x, y and z in S such that they're endpoints of an edge, simultaneously, and they form a triangle. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are all possible edges to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S|=0}$ is corresponded to clique number. Thus

$$\mathcal{C}^{\mathcal{F}}(NTG) = 0.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.31. In Figure (2.11), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 0}$. Thus it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 0}$. Thus it implies that $S = \{n_2, n_4\}$ is corresponded to neither failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ nor failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 0}$. Thus it implies that $S = \{n_1\}$ is corresponded to neither failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ nor failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. $S = \{n_i\}_{|S|\neq 0}$. Thus there are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ isn't corresponded to both failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(v) 0 is failed clique number and its corresponded sets is $\{\}$;

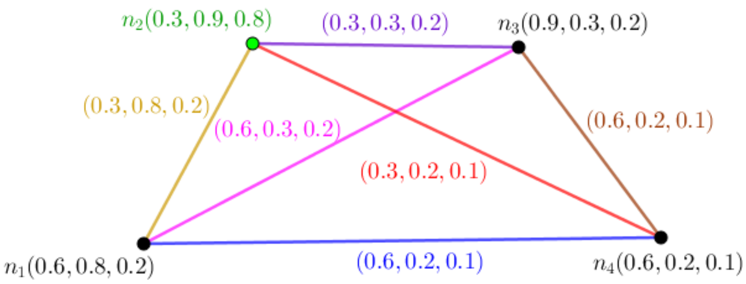

Figure 1.12: A Neutrosophic Graph in the Viewpoint of its Failed Clique Number.

54NTG2

(vi) $\mathcal{O}_n(NTG) = 0$ is failed clique neutrosophic-number and its corresponded set is $\{\}$.

Definition 1.5.32. (1-clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-clique number** $\mathcal{C}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time;

(ii) **1-clique neutrosophic-number** $\mathcal{C}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.5.33. In Figure (2.13), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices.

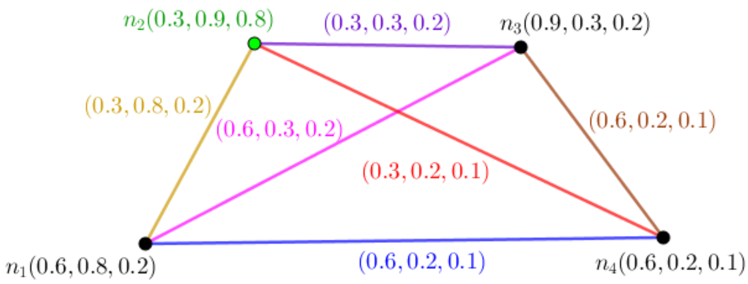

Figure 1.13: A Neutrosophic Graph in the Viewpoint of its 1-Clique Number and its 1-Clique Neutrosophic-Number. 55NTG1

So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_2, n_4\}$ isn't corresponded to both of 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. But it implies that $S = \{n_1\}$ isn't corresponded to both of 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 4 is 1-clique number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(vi) $\mathcal{O}_n(NTG) = 5.9$ is 1-clique neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4\}$.

Definition 1.5.34. (Failed 1-clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed 1-clique number** $\mathcal{C}^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time;

(ii) **failed 1-clique neutrosophic-number** $\mathcal{C}^{\mathcal{F}}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.5.35. In Figure (2.13), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of failed 1-clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed 1-clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_2, n_4\}$ isn't corresponded to both of failed 1-clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed 1-clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. But it implies that $S = \{n_1\}$ isn't corresponded to both of failed 1-clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed 1-clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ isn't corresponded to both failed 1-clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed 1-clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(v) 0 is failed 1-clique number and its corresponded sets is $\{\}$;

(vi) $\mathcal{O}_n(NTG) = 0$ is failed 1-clique neutrosophic-number and its corresponded set is $\{\}$.

Proposition 1.5.36. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{C}(NTG) = \mathcal{O}(NTG).$$

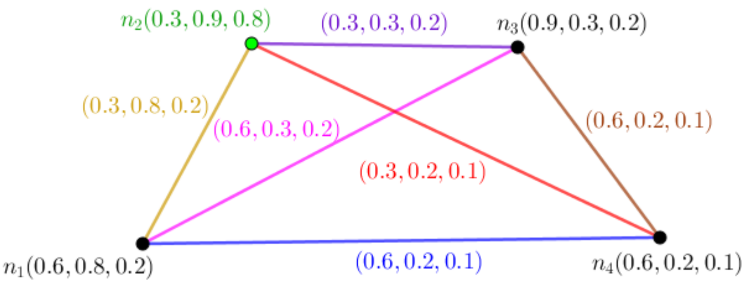

Figure 1.14: A Neutrosophic Graph in the Viewpoint of its Failed 1-Clique number and its Failed 1-Clique neutrosophic-number. 55NTG1

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x, y and z in S such that they're endpoints of an edge, simultaneously, and they form a triangle. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are all possible edges to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S|=\mathcal{O}(NTG)}$ is corresponded to 1-clique number. Thus

$$\mathcal{C}(NTG) = \mathcal{O}(NTG).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.37. In Figure (2.14), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq\mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq\mathcal{O}(NTG)}$. Thus it implies that $S = \{n_2, n_4\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices.

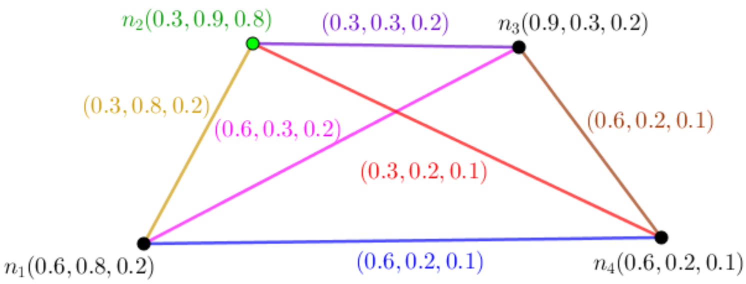

Figure 1.15: A Neutrosophic Graph in the Viewpoint of its 1-Clique Number. 55NTG2

So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. $S = \{n_i\}_{|S| = \mathcal{O}(NTG)}$. Thus there are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 4 is 1-clique number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(vi) $\mathcal{O}_n(NTG) = 5.9$ is 1-clique neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4\}$.

Definition 1.5.38. (Matching Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **matching number** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of edges such that every two edges of S don't have any vertex in common;

(ii) **matching neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of edges such that every two edges of S don't have any vertex in common.

Proposition 1.5.39. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. For every given vertex, there's one option to choose an edge. Thus a set S, referred to a set of edges with a maximal cardinality, has the cardinality $\lfloor \frac{n}{2} \rfloor$. This number is maximum so

$$\mathcal{M}(NTG) = \lfloor \frac{n}{2} \rfloor.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.40. In Figure (2.15), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_1n_3, n_2n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_2n_3, n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_4\}$ isn't corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 0.9, of S implies $S = \{n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.2, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

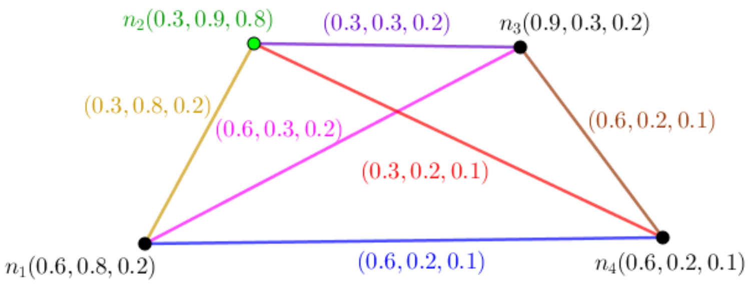

Figure 1.16: A Neutrosophic Graph in the Viewpoint of its matching Number. 58NTG2

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$;

(vi) 2.2 is matching neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$.

Definition 1.5.41. (Matching Polynomial).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **matching polynomial** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the matching polynomial represent the number of sets of independent edges of various cardinalities in G.

(ii) **matching polynomial neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the matching polynomial represent the number of sets of independent edges of various neutrosophic cardinalities in G.

Proposition 1.5.42. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = (\mathcal{O}(NTG) - 1)x^{\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor} + \cdots + \mathcal{S}(NTG)x + 1.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. For every given vertex, there's one option to choose an edge. Thus a set S, referred to a set of edges with a maximal cardinality, has the cardinality $\lfloor \frac{n}{2} \rfloor$. This number is maximum so

$$\mathcal{M}(NTG) = (\mathcal{O}(NTG) - 1)x^{\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor} + \cdots + \mathcal{S}(NTG)x + 1.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.43. In Figure (2.16), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_1n_3, n_2n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(*ii*) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_2n_3, n_1n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iii*) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_4\}$ isn't corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 0.9, of S implies $S = \{n_1n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iv*) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.2, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(*v*) $3x^2 + 6x + 1$ is matching polynomial and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$ for coefficient of biggest term;

(*vi*) $x^{2.2} + x^{1.1}$ is matching polynomial neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$ for coefficient of biggest term.

Definition 1.5.44. (e-Matching Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **e-matching number** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S containing endpoints of edges such that every two edges of S don't have any vertex in common;

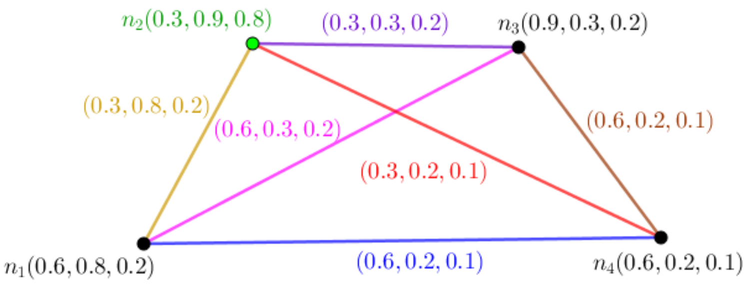

Figure 1.17: A Neutrosophic Graph in the Viewpoint of its Matching Polynomial. 60NTG2

(ii) **e-matching neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S containing endpoints of edges such that every two edges of S don't have any vertex in common.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.5.45. In Figure (2.17), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $\{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two distinct edges which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $\{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of at least two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_4\}$ isn't corresponded to e-matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 2.5, of

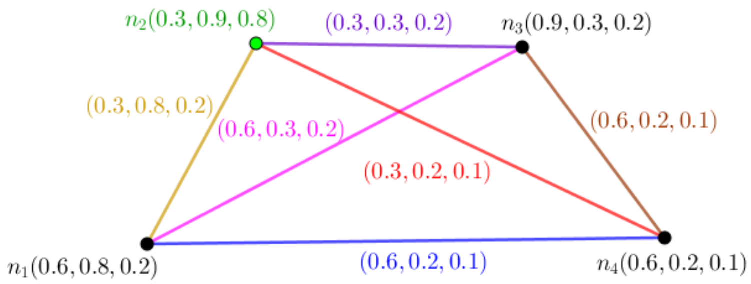

Figure 1.18: A Neutrosophic Graph in the Viewpoint of its e-Matching Number and its e-Matching Neutrosophic-Number. 61NTG1

S implies $S = \{n_1, n_4\}$ isn't corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $\{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_2, n_3, n_4\} = V$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $\{n_1, n_2, n_3, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $4 = \mathcal{O}(NTG)$ is e-matching number and its corresponded set is $S = \{n_1, n_2, n_3, n_4\} = V$;

(vi) $5.9 = \mathcal{O}_n(NTG)$ is e-matching neutrosophic-number and its corresponded set is $S = \{n_1, n_2, n_3, n_4\} = V$.

Definition 1.5.46. (e-Matching Polynomial).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **e-matching polynomial** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the e-matching polynomial represent the number of sets of endpoints of independent edges of various cardinalities in G.

(ii) **e-matching polynomial neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the e-matching polynomial represent the number of sets of endpoints of independent edges of various neutrosophic cardinalities in G.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.5.47. In Figure (2.18), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $\{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*ii*) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two distinct edges which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iii*) if $\{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of at least two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_4\}$ isn't corresponded to e-matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 2.5, of S implies $S = \{n_1, n_4\}$ isn't corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iv*) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*v*) $x^4 + 3x^2$ is e-matching polynomial and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$ for coefficient of biggest term; also $S = \{n_1, n_2, n_3, n_4\}$;

(*vi*) $x^{5.9} + x^{3.4}$ is e-matching polynomial neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$ for coefficient of biggest term; also $S = \{n_1, n_2, n_3, n_4\}$.

Proposition 1.5.48. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{M}(NTG) = \mathcal{O}(NTG).$$

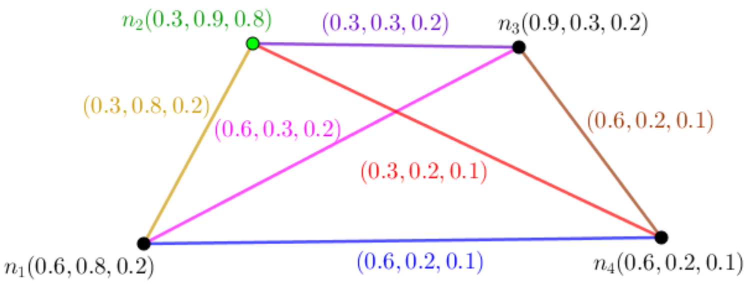

Figure 1.19: A Neutrosophic Graph in the Viewpoint of its e-Matching Polynomial and its e-Matching Polynomial Neutrosophic-Number. 61NTG2

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. For every given vertex, there's one option to choose an edge. Thus a set S, referred to a set of edges with a maximal cardinality, has the cardinality $\lfloor \frac{n}{2} \rfloor$. This number is maximum so

$$\mathcal{M}(NTG) = \mathcal{O}(NTG).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.49. In Figure (2.19), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $\{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two distinct edges which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $\{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint

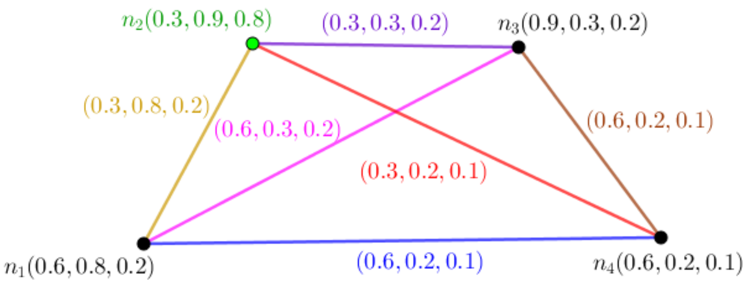

Figure 1.20: A Neutrosophic Graph in the Viewpoint of its e-Matching Number. 61NTG3

of at least two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_4\}$ isn't corresponded to e-matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 2.5, of S implies $S = \{n_1, n_4\}$ isn't corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $\{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_2, n_3, n_4\} = V$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $\{n_1, n_2, n_3, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $4 = \mathcal{O}(NTG)$ is e-matching number and its corresponded set is $S = \{n_1, n_2, n_3, n_4\} = V$;

(vi) $5.9 = \mathcal{O}_n(NTG)$ is e-matching neutrosophic-number and its corresponded set is $S = \{n_1, n_2, n_3, n_4\} = V$.

Definition 1.5.50. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞;

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞.

Proposition 1.5.51. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 3.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth is three. Thus

$$\mathcal{G}(NTG) = 3.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.52. In Figure (2.20), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a

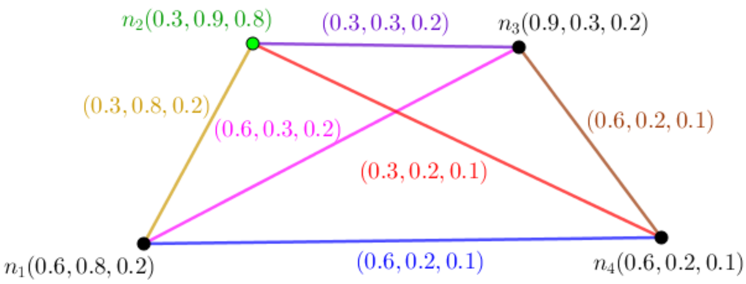

Figure 1.21: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG2

sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

Definition 1.5.53. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞;

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞.

63thm

Theorem 1.5.54. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ∎

Proposition 1.5.55. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 3.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth is three. Thus

$$\mathcal{G}(NTG) = 3.$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.56. In Figure (2.21), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

Definition 1.5.57. (Girth Polynomial and Neutrosophic Girth Polynomial). Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **girth polynomial** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1x^{m_1} + n_2x^{m_2} + \cdots + n_sx^3$ where n_i is the number of cycle with m_i as its crisp cardinality of the set of vertices of cycle;

(ii) **neutrosophic girth polynomial** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1x^{m_1} + n_2x^{m_2} + \cdots + n_sx^{m_s}$ where n_i is the number of cycle with m_i as its neutrosophic cardinality of the set of vertices of cycle.

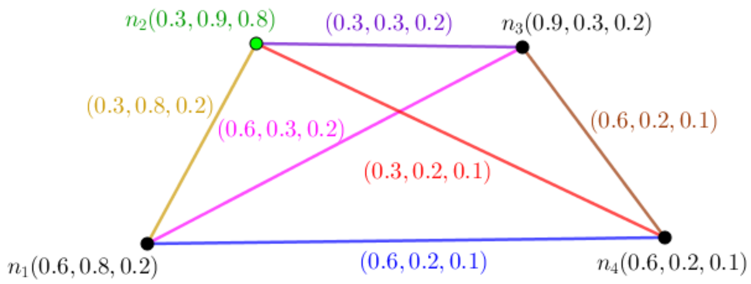

Figure 1.22: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG2

63thm **Theorem 1.5.58.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 1.5.59. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = x^{\mathcal{O}(NTG)} + \mathcal{O}(NTG)x^{\mathcal{O}(NTG)-1} + \cdots + \binom{\mathcal{O}(NTG)}{3}x^3.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth polynomial is three. Thus

$$\mathcal{G}(NTG) = x^{\mathcal{O}(NTG)} + \mathcal{O}(NTG)x^{\mathcal{O}(NTG)-1} + \cdots + \binom{\mathcal{O}(NTG)}{3}x^3.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.60. In Figure (2.22), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most

2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

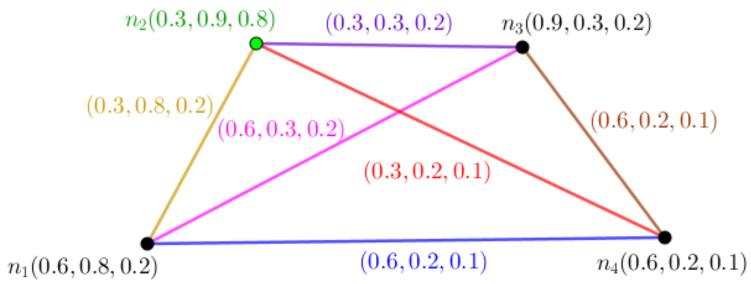

Figure 1.23: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG2

(v) $x^4 + 3x^3$ is girth polynomial and its corresponded sets, for coefficient of smallest term, are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) $x^{5.9} + x^5 + x^{4.5} + x^{4.3} + x^{3.9}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is $\{n_1, n_3, n_4\}$.

Definition 1.5.61. (Hamiltonian Neutrosophic Cycle).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **hamiltonian neutrosophic cycle** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive vertices $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ which is neutrosophic cycle;

(ii) **n-hamiltonian neutrosophic cycle** $\mathcal{N}(HNC)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive vertices $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ which are neutrosophic cycles.

If we use the notion of neutrosophic cardinality in strong type of neutrosophic graphs, then the next result holds. If not, the situation is complicated since it's possible to have all edges in the way that, there's no value of a vertex for an edge.

66thm **Theorem 1.5.62.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ∎

Proposition 1.5.63. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph with two weakest edges. Then*

$$\mathcal{M}(CMT_\sigma) : x_1, x_2, \cdots, x_{\mathcal{O}(CMT_\sigma)-1}, x_{\mathcal{O}(CMT_\sigma)}, x_1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. In other hand, there's a cycle if and only if $\mathcal{O}(CMT_\sigma) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(CMT_\sigma) = 3$. By longest cycle is on demand, the n-hamiltonian neutrosophic cycle is four. The length of longest cycle is $\mathcal{O}(CMT_{\sigma})$. Thus it's hamiltonian neutrosophic cycle. Thus

$$\mathcal{M}(CMT_{\sigma}) : x_1, x_2, \cdots, x_{\mathcal{O}(CMT_{\sigma})-1}, x_{\mathcal{O}(CMT_{\sigma})}, x_1.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.64. In Figure (2.23), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither hamiltonian neutrosophic cycle $\mathcal{M}(CMT_{\sigma})$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CMT_{\sigma})$;

(*ii*) if n_1, n_2, n_3, n_1 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3, n_1$$

isn't corresponded to hamiltonian neutrosophic cycle $\mathcal{M}(CMT_{\sigma})$ and as its consequences, length of this crisp cycle implies

$$n_1, n_2, n_3, n_1$$

isn't corresponded to n-hamiltonian neutrosophic cycle $\mathcal{N}(CMT_{\sigma})$;

(*iii*) if n_1, n_2, n_3, n_4, n_1 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then

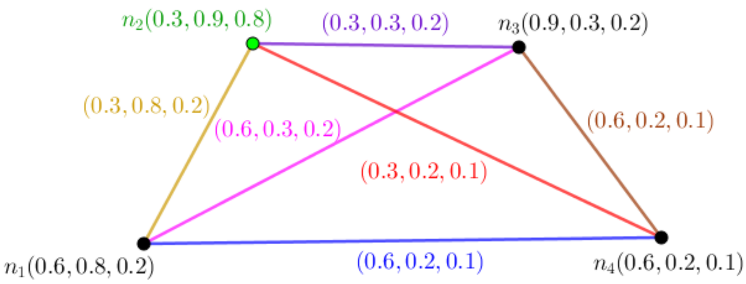

Figure 1.24: A Neutrosophic Graph in the Viewpoint of its hamiltonian neutrosophic cycle. 66NTG2

it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path, four, implies

$$n_1, n_2, n_3, n_4, n_1$$

is corresponded to hamiltonian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ and it's effective to construct n-hamiltonian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(iv) if n_1, n_3, n_4, n_1 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle, three, implies

$$n_1, n_3, n_4, n_1$$

isn't corresponded to hamiltonian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$. The vertex, n_2, isn't in sequence related to this neutrosophic cycle. Thus it implies

$$n_1, n_3, n_4, n_1$$

isn't corresponded to n-hamiltonian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(v) $\mathcal{M}(CMT_\sigma) : n_1, n_2, n_3, n_4, n_1$ is hamiltonian neutrosophic cycle and its corresponded sets. are the sequences which have both the edges n_1n_4 and n_3n_4. Since these edges are two weakest edges in this complete-neutrosophic graph. Other sequences even if they're cycles having all vertices, once, are hamiltonian cycles and not hamiltonian neutrosophic cycles;

(vi) $\mathcal{N}(CMT_\sigma) = 1$ is n-hamiltonian neutrosophic cycle.

Definition 1.5.65. (Eulerian Neutrosophic Cycle).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **Eulerian neutrosophic cycle** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive edges $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}, x_1$ which is neutrosophic cycle;

(*ii*) **n-Eulerian neutrosophic cycle** $\mathcal{N}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive edges $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}, x_1$ which are neutrosophic cycles.

If we use the notion of neutrosophic cardinality in strong type of neutrosophic graphs, then the next result holds. If not, the situation is complicated since it's possible to have all edges in the way that, there's no value of a vertex for an edge.

66thm **Theorem 1.5.66.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ∎

Proposition 1.5.67. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph with two weakest edges. Then*

$$\mathcal{M}(CMT_\sigma) : \textit{Not Existed.}$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. So there's a cycle if and only if $\mathcal{O}(CMT_\sigma) \geq 3$. It's complete. Hence there's only one neutrosophic cycle which its length is $\mathcal{S}(CMT_\sigma) = 3$ where $\mathcal{O}(CMT_\sigma) = 3$. By longest cycle is on demand in the way that all edges are used and there's no repetition of edges, the n-Eulerian neutrosophic cycle doesn't exist. The length of longest cycle isn't $\mathcal{S}(CMT_\sigma)$. Thus it isn't an Eulerian neutrosophic cycle. Thus

$$\mathcal{M}(CMT_\sigma) : \text{Not Existed.}$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.68. In Figure (2.24), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's only a path and it's only two edges but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive edges is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1n_2, n_2n_3$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(*ii*) if n_1n_2, n_2n_3, n_3n_1 is a sequence of consecutive edges, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive edges is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1n_2, n_2n_3, n_3n_1$$

isn't corresponded to Eulerian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ and as its consequences, length of this crisp cycle implies

$$n_1n_2, n_2n_3, n_3n_1$$

isn't corresponded to n-Eulerian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(*iii*) if $n_1n_2, n_2n_3, n_3n_4, n_4n_1$ is a sequence of consecutive edges, then it's obvious that there are two crisp cycles with length three and four. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1n_3, n_3n_4, n_4n_1, and with length four, $n_1n_2, n_2n_3, n_3n_4, n_4n_1$. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive edges is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different lengths three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. Lack of having all edges, for instance n_1n_3, implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_1$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(*iv*) if n_1n_3, n_3n_4, n_4n_1 is a sequence of consecutive edges, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive edges is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle, three, and lack of having all edges, for instance n_1n_2, implies

$$n_1n_3, n_3n_4, n_4n_1$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

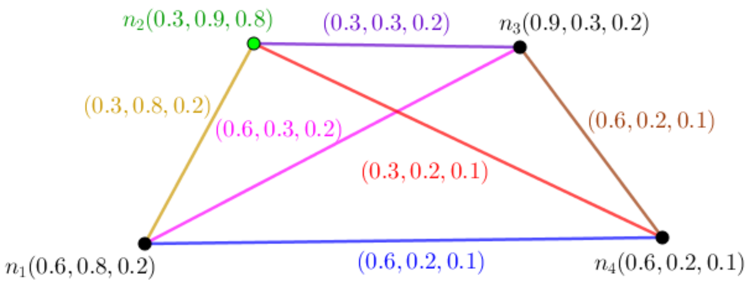

Figure 1.25: A Neutrosophic Graph in the Viewpoint of its Eulerian neutrosophic cycle. 67NTG2

(v) $\mathcal{M}(CMT_\sigma)$: Not Existed. There is no Eulerian neutrosophic cycle and there are no corresponded sets and sequences;

(vi) $\mathcal{N}(CMT_\sigma) = 0$ is n-Eulerian neutrosophic cycle and there are no corresponded sets and sequences.

Definition 1.5.69. (Eulerian(Hamiltonian) Neutrosophic Path).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Eulerian(Hamiltonian) neutrosophic path** $\mathcal{M}_e(NTG)(\mathcal{M}_h(NTG))$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}(x_1, x_2, \cdots, x_{\mathcal{O}(NTG)})$ which is neutrosophic path;

(ii) **n-Eulerian(Hamiltonian) neutrosophic path** $\mathcal{N}_e(NTG)(\mathcal{N}_h(NTG))$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}(x_1, x_2, \cdots, x_{\mathcal{O}(NTG)})$ which is neutrosophic path.

Proposition 1.5.70. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph with two weakest edges. Then*

$$\mathcal{M}_e(CMT_\sigma) : \textit{Not Existed;}$$
$$\mathcal{M}_h(CMT_\sigma) : v_{\tau(1)}, v_{\tau(2)}, \cdots, v_{\tau(\mathcal{O}(CMT_\sigma)-1)}, v_{\tau(\mathcal{O}(CMT_\sigma))}$$

where τ is a permutation on $\mathcal{O}(CMT_\sigma)$.

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By longest path is on demand in the way that all edges are used and there's no repetition of edges, the Eulerian neutrosophic path doesn't exist. The length of longest path isn't $\mathcal{S}(CMT_\sigma)$. Thus it isn't an Eulerian neutrosophic path. By longest path is on demand in the way that all vertices are used and there's no repetition of vertices, the Hamiltonian neutrosophic path doesn't exist. The length of longest path isn't $\mathcal{O}(CMT_\sigma)$. Thus it isn't a Hamiltonian neutrosophic path. Thus

$$\mathcal{M}_e(CMT_\sigma) : \text{Not Existed;}$$
$$\mathcal{M}_h(CMT_\sigma) : v_{\tau(1)}, v_{\tau(2)}, \cdots, v_{\tau(\mathcal{O}(CMT_\sigma)-1)}, v_{\tau(\mathcal{O}(CMT_\sigma))}$$

where τ is a permutation on $\mathcal{O}(CMT_\sigma)$. ∎

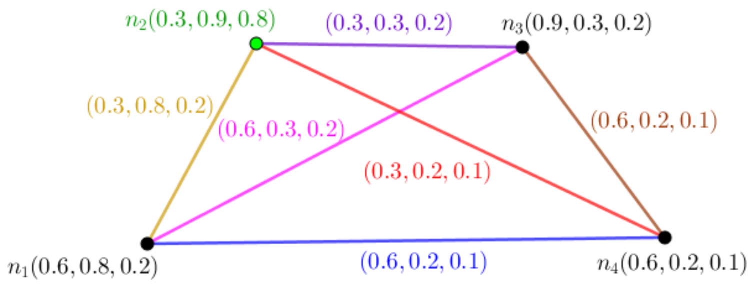

Figure 1.26: A Neutrosophic Graph in the Viewpoint of its Eulerian(Hamiltonian) neutrosophic path. 68NTG2

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.71. In Figure (2.25), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_2n_3) > 0$. But the number of edges isn't $\mathcal{S}(CMT_\sigma)$ and the number of vertices isn't $\mathcal{O}(CMT_\sigma)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CMT_\sigma)(\mathcal{M}_h(CMT_\sigma))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CMT_\sigma)(\mathcal{N}_h(CMT_\sigma))$ isn't corresponded to these sequences n_1, n_2, n_3 and n_1n_2, n_2n_3;

(ii) if n_1n_2, n_2n_3, n_3n_4 is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0,\ \mu(n_2n_3) > 0$ and $\mu(n_3n_4) > 0$. But the number of edges isn't $\mathcal{S}(CMT_\sigma)$. The number of vertices isn't $\mathcal{O}(CMT_\sigma)$. Thus Eulerian neutrosophic path $\mathcal{M}_e(CMT_\sigma)$ doesn't exist but Hamiltonian neutrosophic path $\mathcal{M}_h(CMT_\sigma)$ is n_1, n_2, n_3, n_4. Also, n-Eulerian neutrosophic path $\mathcal{N}_e(CMT_\sigma)$ equals to zero and n-Hamiltonian neutrosophic path $\mathcal{N}_h(CMT_\sigma))$ is greater than six.

Definition 1.5.72. (Neutrosophic Path Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a path from x to y is called **weakest path** if its length is maximum. This length is called **weakest number** amid x and y. The maximum number amid all vertices is called **weakest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{W}(NTG)$;

(ii) a path from x to y is called **neutrosophic weakest path** if its strength is $\mu(uv)$ which is less than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called **neutrosophic weakest number** amid x and y. The maximum number amid all vertices is called **neutrosophic weakest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{W}_n(NTG)$.

Proposition 1.5.73. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{W}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Longest path is on demand. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's a path containing all vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. But the length of the path forms weakest number. Thus

$$\mathcal{W}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.74. In Figure (2.26), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it's weakest path and weakest number amid n_1 and n_4 is three. Also, $\mathcal{W}(CMT_\sigma) = 3$;

(ii) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't weakest path and weakest number amid n_1 and n_3 isn't two. Also, $\mathcal{W}(CMT_\sigma) \neq 2$;

(iii) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't weakest path and weakest number amid n_1 and n_3 isn't two. Also, $\mathcal{W}(CMT_\sigma) \neq 2$. For every given couple of vertices x and y, weakest path is existed, weakest number is three and $\mathcal{W}(CMT_\sigma) = 3$;

(iv) if n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it isn't a neutrosophic weakest path since neutrosophic weakest number amid n_1 and n_4 is $(0.3, 0.2, 0.1)$. Also, $\mathcal{W}_n(CMT_\sigma) = (0.3, 0.2, 0.1)$;

(v) if n_1, n_2, n_4 is a path from n_1 to n_4, then it's a neutrosophic weakest path and neutrosophic weakest number amid n_1 and n_4 is $(0.3, 0.2, 0.1)$. Also, $\mathcal{W}_n(CMT_\sigma) = (0.3, 0.2, 0.1)$;

(vi) for every given couple of vertices x and y, neutrosophic weakest path is existed, neutrosophic weakest number is $(0.3, 0.2, 0.1)$ and $\mathcal{W}_n(CMT_\sigma) = (0.3, 0.2, 0.1)$.

Definition 1.5.75. (Neutrosophic Path Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

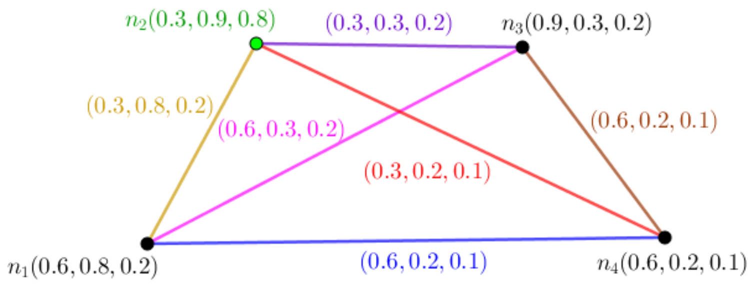

Figure 1.27: A Neutrosophic Graph in the Viewpoint of its Weakest Number and its Neutrosophic Weakest Number. 70NTG2

(*i*) a path from x to y is called **strongest path** if its length is minimum. This length is called **strongest number** amid x and y. The maximum number amid all vertices is called **strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}(NTG)$;

(*ii*) a path from x to y is called **neutrosophic strongest path** if its strength is $\mu(uv)$ which is greater than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called **neutrosophic strongest number** amid x and y. The minimum number amid all vertices is called **neutrosophic strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}_n(NTG)$.

Proposition 1.5.76. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{S}(CMT_\sigma) = 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Minimum path is on demand. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's a path containing all vertices and there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. But the length of the path forms strongest number. Thus

$$\mathcal{S}(CMT_\sigma) = 1.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.77. In Figure (2.27), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is one. Also, $\mathcal{S}(CMT_\sigma) = 1$;

(*ii*) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_{\sigma}) \neq 2$;

(*iii*) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_{\sigma}) \neq 2$. For every given couple of vertices x and y, strongest path is existed, strongest number is one and $\mathcal{S}(CMT_{\sigma}) = 1$;

(*iv*) if n_1, n_4, n_3, n_2 is a path from n_1 to n_2, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_1, n_4, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2) \\ P_2 : n_1, n_4, n_2 &\Rightarrow (0.3, 0.2, 0.1) \\ P_3 : n_1, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2) \\ P_4 : n_1, n_2 &\Rightarrow (0.3, 0.8, 0.2) \\ \text{Maximum is } &(0.3, 0.8, 0.2) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_{\sigma}) = (0.6, 0.2, 0.1)$;

(*v*) if n_2, n_1, n_4, n_3 is a path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.6, 0.3, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_2, n_1, n_4, n_3 &\Rightarrow (0.6, 0.2, 0.1) \\ P_2 : n_2, n_4, n_3 &\Rightarrow (0.3, 0.2, 0.1) \\ P_3 : n_2, n_1, n_3 &\Rightarrow (0.6, 0.3, 0.2) \\ P_4 : n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2) \\ \text{Maximum is } &(0.6, 0.3, 0.2) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_{\sigma}) = (0.6, 0.2, 0.1)$;

(*vi*) if n_3, n_2, n_1, n_4 is a path from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_3, n_2, n_1, n_4 &\Rightarrow (0.3, 0.3, 0.2) \\ P_2 : n_3, n_1, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ P_3 : n_3, n_2, n_4 &\Rightarrow (0.3, 0.2, 0.1) \\ P_4 : n_3, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ \text{Maximum is } &(0.6, 0.2, 0.1) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_{\sigma}) = (0.6, 0.2, 0.1)$.

Definition 1.5.78. (Neutrosophic Cycle Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

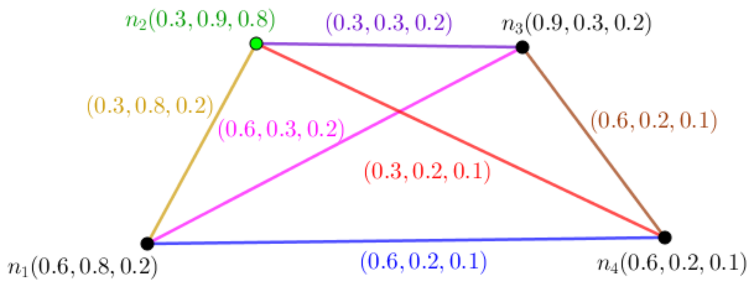

Figure 1.28: A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number. 71NTG2

(*i*) a cycle based on x is called **cyclic connectivity** if its length is minimum. This length is called **connectivity number** based on x. The maximum number amid all vertices is called **connectivity number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{C}(NTG)$;

(*ii*) a cycle based on x is called **neutrosophic cyclic connectivity** if its strength is is greater than all strengths of all cycles based on x. This strength is called **neutrosophic connectivity number** based on x. The minimum number amid all vertices is called **neutrosophic connectivity number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{C}_n(NTG)$.

Proposition 1.5.79. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph where $\mathcal{O}(CMT_\sigma) \geq 3$. Then*

$$\mathcal{C}(CMT_\sigma) = 3.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Minimum cycle is on demand. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. By $\mathcal{O}(CMT_\sigma) \geq 3$, there's a cycle. But the length of the cycle forms connectivity number. Thus

$$\mathcal{C}(CMT_\sigma) = 3.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.80. In Figure (2.28), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2, n_3, n_4, n_1 is a cycle based on n_1, then it isn't cyclic connectivity and connectivity number based on n_1 is three. Also, $\mathcal{C}(CMT_\sigma) = 3$;

(*ii*) if n_1, n_2, n_3, n_1 is a cycle based on n_1, then it's cyclic connectivity and connectivity number based on n_1 is three. Also, $\mathcal{C}(CMT_\sigma) = 3$;

(*iii*) Consider n_1, n_2, n_1. Then it isn't a cycle based on n_1, since the length of consecutive vertices has to be at least three. Then it isn't cyclic connectivity and connectivity number based on n_1 isn't two. Also, $\mathcal{C}(CMT_\sigma) \neq 2$. For every given vertex x, cyclic connectivity is existed, connectivity number is three and $\mathcal{C}(CMT_\sigma) = 3$;

(*iv*) if n_1, n_4, n_3, n_2, n_1 is a cycle based on n_1, then it isn't a neutrosophic cyclic connectivity since neutrosophic connectivity number based on n_2 is $(0.3, 0.3, 0.2)$ where there are six paths as follows.

$$\begin{aligned}
P_1 : n_1, n_4, n_3, n_1 &\Rightarrow (0.6, 0.2, 0.1)\\
P_2 : n_1, n_2, n_3, n_1 &\Rightarrow (0.3, 0.3, 0.2)\\
P_3 : n_1, n_2, n_4, n_1 &\Rightarrow (0.3, 0.2, 0.1)\\
P_4 : n_1, n_4, n_3, n_2, n_1 &\Rightarrow (0.3, 0.3, 0.2)\\
P_5 : n_1, n_3, n_4, n_2, n_1 &\Rightarrow (0.3, 0.2, 0.1)\\
P_6 : n_1, n_4, n_2, n_3, n_1 &\Rightarrow (0.3, 0.2, 0.1)\\
\text{Maximum is } &(0.6, 0.2, 0.1)
\end{aligned}$$

Also, $\mathcal{C}_n(CMT_\sigma) = (0.3, 0.3, 0.2)$ corresponded to cycle n_2, n_1, n_3, n_2 based on n_2;

(*v*) if n_2, n_1, n_4, n_3, n_2 is a cycle based on n_2, then it isn't a neutrosophic cyclic connectivity since neutrosophic connectivity number based on n_2 is $(0.3, 0.3, 0.2)$ where there are six paths as follows.

$$\begin{aligned}
P_1 : n_2, n_4, n_3, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
P_2 : n_2, n_1, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2)\\
P_3 : n_2, n_1, n_4, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
P_4 : n_2, n_4, n_3, n_1, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
P_5 : n_2, n_3, n_4, n_1, n_2 &\Rightarrow (0.3, 0.3, 0.2)\\
P_6 : n_2, n_4, n_1, n_3, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
\text{Maximum is } &(0.3, 0.3, 0.2)
\end{aligned}$$

Also, $\mathcal{C}_n(CMT_\sigma) = (0.3, 0.3, 0.2)$ corresponded to cycle n_2, n_1, n_3, n_2 based on n_2;

(*vi*) if n_3, n_2, n_1, n_4, n_3 is a cycle based on n_3, then it's a neutrosophic cyclic connectivity and neutrosophic connectivity number based on n_2 is $(0.3, 0.3, 0.2)$ where there are six paths as follows.

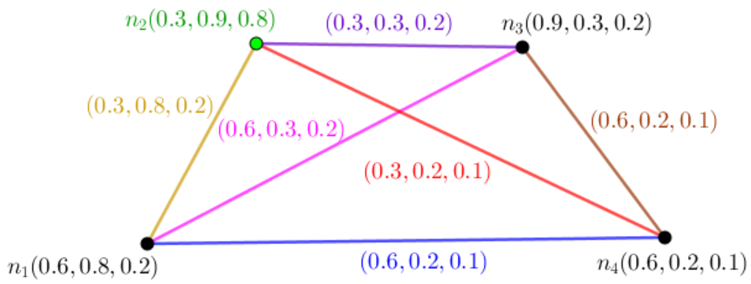

Figure 1.29: A Neutrosophic Graph in the Viewpoint of its connectivity number and its neutrosophic connectivity number. 72NTG2

$$\begin{aligned}
P_1 : n_3, n_4, n_2, n_3 &\Rightarrow (0.3, 0.2, 0.1)\\
P_2 : n_3, n_1, n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2)\\
P_3 : n_3, n_1, n_4, n_3 &\Rightarrow (0.6, 0.2, 0.1)\\
P_4 : n_3, n_4, n_2, n_1, n_3 &\Rightarrow (0.3, 0.2, 0.1)\\
P_5 : n_3, n_2, n_4, n_1, n_3 &\Rightarrow (0.3, 0.2, 0.1)\\
P_6 : n_3, n_4, n_1, n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2)\\
\text{Maximum is } &(0.6, 0.2, 0.1)
\end{aligned}$$

Also, $\mathcal{C}_n(CMT_\sigma) = (0.3, 0.3, 0.2)$ corresponded to cycle n_2, n_1, n_3, n_2 based on n_2.

Definition 1.5.81. (Dense Numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set of vertices is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum cardinality between all dense sets is called **dense number** and it's denoted by $\mathcal{D}(NTG)$;

(ii) a set of vertices S is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{s\in S}\sum_{i=1}^{3}\sigma_i(s)$ between all dense sets is called **neutrosophic dense number** and it's denoted by $\mathcal{D}_n(NTG)$.

Proposition 1.5.82. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{D}(CMT_\sigma) = \lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor + 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected

to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. Sets of vertices with cardinality $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor+1$ are dense sets since every vertex inside has $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor$ neighbors inside and $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor-1$ neighbors outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor+1$. Thus

$$\mathcal{D}(CMT_\sigma)=\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor+1.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.83. In Figure (2.29), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S=\{n_1,n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(ii) if $S=\{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(iii) if $S_1=\{n_1,n_2,n_3\},S_2=\{n_1,n_2,n_4\},S_3=\{n_2,n_3,n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is 3. Thus $\mathcal{D}(CMT_\sigma)=3$;

(iv) if $S=\{n_1,n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(v) if $S=\{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(vi) if $S_1=\{n_1,n_2,n_3\},S_2=\{n_1,n_2,n_4\},S_3=\{n_2,n_3,n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum neutrosophic cardinality $\sum_{s\in S}\sum_{i=1}^{3}\sigma_i(s)$ between all dense sets is 3.9. Thus $\mathcal{D}_n(CMT_\sigma)=3.9$.

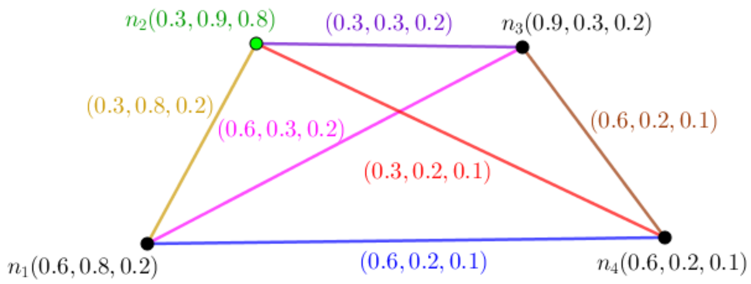

Figure 1.30: A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number. 73NTG2

Definition 1.5.84. (bulky numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set of edges S is called **bulky set** if for every edge e' outside, there's at least one edge e inside such that they've common vertex and the number of edges such that they've common vertex with e is greater than the number of edges such that they've common vertex with e'. The minimum cardinality between all bulky sets is called **bulky number** and it's denoted by $\mathcal{B}(NTG)$;

(ii) a set of edges S is called **bulky set** if for every edge e' outside, there's at least one edge e inside such that they've common vertex and the number of edges such that they've common vertex with e is greater than the number of edges such that they've common vertex with e'. The minimum neutrosophic cardinality $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(s)$ between all bulky sets is called **neutrosophic bulky number** and it's denoted by $\mathcal{B}_n(NTG)$.

Proposition 1.5.85. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{B}(CMT_\sigma) = \lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. If $S = \{e_1, e_2, \cdots, e_{\lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor}\}$ is a set of edges, then it's a bulky set since for every edge e'_j, outside, there's at least one edge e_i inside such that they've common vertex and the number of edges such that they've common vertex with e_i is $\mathcal{O}(CMT_\sigma) - 2$ which is equal to [greater than] $\mathcal{O}(CMT_\sigma) - 2$ which is the number of edges such that they've common vertex with e'_j. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all bulky sets is $\lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor$. Thus

$$\mathcal{B}(CMT_\sigma) = \lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.86. In Figure (2.30), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_2n_4, n_3n_1\}$ is a set of edges, then it's a bulky set since for every edge n_in_j, outside, there's at least one edge n_2n_4 inside such that they've common vertex and the number of edges such that they've common vertex with n_2n_4 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j;

(ii) if $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with n_1n_2 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j;

(iii) All sets [2-sets] of edges containing two edges are bulky sets. Since for every edge n_in_j, outside, there's at least one edge n_tn_s inside such that they've common vertex and the number of edges such that they've common vertex with n_tn_s is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j. Thus $\mathcal{B}(CMT_\sigma) = 2$;

(iv) if $S = \{n_2n_4, n_3n_1\}$ is a set of edges, then it's a bulky set since for every edge n_in_j, outside, there's at least one edge n_2n_4 inside such that they've common vertex and the number of edges such that they've common vertex with n_2n_4 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j;

(v) if $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with n_1n_2 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j;

(vi) if $S = \{n_2n_3, n_2n_4\}$ is set of edges, then they're bulky sets since for every edge n_in_j, outside, there's at least one edge n_2n_3 inside such that they've common vertex and the number of edges such that they've common vertex with n_2n_3 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j. The minimum neutrosophic cardinality $\sum_{s\in S}\sum_{i=1}^{3}\sigma_i(s)$ between all bulky sets is 3.9. Thus $\mathcal{B}_n(CMT_\sigma) = 1.4$.

Definition 1.5.87. (collapsed numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

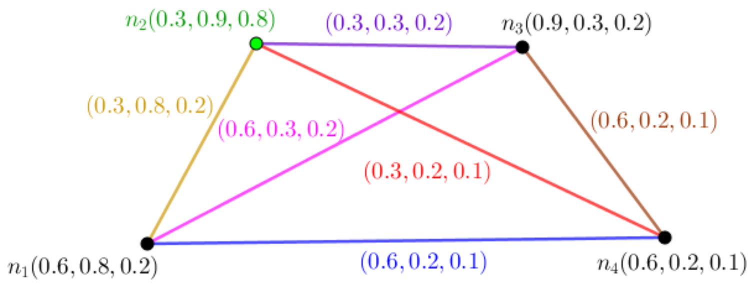

Figure 1.31: A Neutrosophic Graph in the Viewpoint of its bulky number and its neutrosophic bulky number. 74NTG2

(i) a set of vertices S is called **collapsed set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is less than [equal to] the number of neighbors of y. The minimum cardinality between all collapsed sets is called **collapsed number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) a set of vertices S is called **collapsed set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is less than [equal to] the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x)$ between all collapsed sets is called **neutrosophic collapsed number** and it's denoted by $\mathcal{P}_n(NTG)$.

Proposition 1.5.88. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{P}(CMT_\sigma) = 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. All sets [1-set] of vertices containing one vertex $\{x\}$, are called collapsed sets since for every vertex y outside, there's [at least] only one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum cardinality $|S|$, 1, between all collapsed sets is called collapsed number and it's denoted by $\mathcal{P}(CMT_\sigma) = 1$. Thus

$$\mathcal{P}(CMT_\sigma) = 1.$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.89. In Figure (2.31), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1\}$ is a set of vertices, then a set of vertices S is called collapsed set since for every vertex n_i outside, there's only one vertex n_1 inside such that they're endpoints $n_1 n_i \in E$ and the number of neighbors of n_1 is [less than] equal to the number of neighbors of n_i;

(ii) if $S = \{n_1, n_2\}$ is a set of vertices, then a set of vertices S is called collapsed set since for every vertex n_i outside, there are two vertices n_1 and n_2 inside such that they're endpoints $n_1 n_i, n_2 n_i \in E$ and the number of neighbors of n_1 and n_2 is [less than] equal to the number of neighbors of n_i;

(iii) all sets [1-set] of vertices containing one vertex, are called collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum cardinality $|S|$, 1, between all collapsed sets is called collapsed number and it's denoted by $\mathcal{P}(CMT_\sigma) = 1$;

(iv) if $S = \{n_1\}$ is a set of vertices, then a set of vertices S is called collapsed set since for every vertex n_i outside, there's only one vertex n_1 inside such that they're endpoints $n_1 n_i \in E$ and the number of neighbors of n_1 is [less than] equal to the number of neighbors of n_i;

(v) if $S = \{n_1, n_2\}$ is a set of vertices, then a set of vertices S is called collapsed set since for every vertex n_i outside, there are two vertices n_1 and n_2 inside such that they're endpoints $n_1 n_i, n_2 n_i \in E$ and the number of neighbors of n_1 and n_2 is [less than] equal to the number of neighbors of n_i;

(vi) all sets [1-set] of vertices containing one vertex, are called collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum neutrosophic cardinality, $\sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x)$, 0.9, between all collapsed sets is called neutrosophic collapsed number and it's denoted by $\mathcal{P}_n(CMT_\sigma) = 0.9$.

Definition 1.5.90. (path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors in this process is called **path-coloring set** from x to y. The minimum cardinality between all path-coloring sets from two given vertices is called **path-coloring number** and it's denoted by $\mathcal{L}(NTG)$;

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set S of shared edges in this process is called **path-coloring set** from x to y. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$,

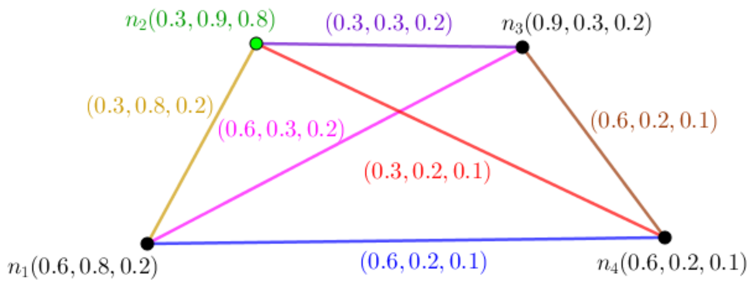

Figure 1.32: A Neutrosophic Graph in the Viewpoint of its collapsed number and its neutrosophic collapsed number. 75NTG2

between all path-coloring sets, Ss, is called **neutrosophic path-coloring number** and it's denoted by $\mathcal{L}_n(NTG)$.

Proposition 1.5.91. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{L}(CMT_\sigma) = \min_S |S|.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. For given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors in this process is called path-coloring set from x to y. The minimum cardinality between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{L}(CMT_\sigma)$. Thus

$$\mathcal{L}(CMT_\sigma) = \min_S |S|.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.92. In Figure (2.32), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(i) Consider two vertices n_1 and n_2. All paths are as follow:

$$P_1 : n_1, n_2 \to \text{red}$$
$$P_2 : n_1, n_3, n_2 \to \text{red}$$
$$P_3 : n_1, n_4, n_2 \to \text{red}$$

$$P_4 : n_1, n_3, n_4, n_2 \to \text{blue}$$
$$P_5 : n_1, n_4, n_3, n_2 \to \text{yellow}$$

The paths P_1, P_2 and P_3 has no shared edge so they've been colored the same as red. The path P_4 has shared edge n_1n_3 with P_2 and shared edge n_4n_2 with P_3 thus it's been colored the different color as blue in comparison to them. The path P_5 has shared edge n_1n_4 with P_3 and shared edge n_3n_4 with P_4 thus it's been colored the different color as yellow in comparison to different paths in the terms of different colors. Thus $S = \{\text{red}, \text{blue}, \text{yellow}\}$ is path-coloring set and its cardinality, 3, is path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors, $S = \{\text{red}, \text{blue}, \text{yellow}\}$, in this process is called path-coloring set from x to y. The minimum cardinality between all path-coloring sets from two given vertices, 3, is called path-coloring number and it's denoted by $\mathcal{L}(CMT_\sigma) = 3$;

(*ii*) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(*iii*) there are three different paths which have no shared edges. So they've been assigned to same color;

(*iv*) shared edges form a set of representatives of colors. Each color is corresponded to an edge which has minimum neutrosophic cardinality;

(*v*) every color in S is corresponded to an edge has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared edges to form S and after that minimum neutrosophic cardinality is optimal;

(*vi*) two edges n_1n_3 and n_4n_2 are shared with P_4 by P_3 and P_2. The minimum neutrosophic cardinality is 0.6 corresponded to n_4n_2. Other corresponded color has only one shared edge n_3n_4 and minimum neutrosophic cardinality is 0.9. Thus minimum neutrosophic cardinality is 1.5. And corresponded set is $S = \{n_4n_2, n_3n_4\}$. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set $S = \{n_4n_2, n_3n_4\}$ of shared edges in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(CMT_\sigma) = 1.5$.

Definition 1.5.93. (dominating path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of different colors, S, in this process is called **dominating path-coloring set** from x to y if for every edge outside there's at least

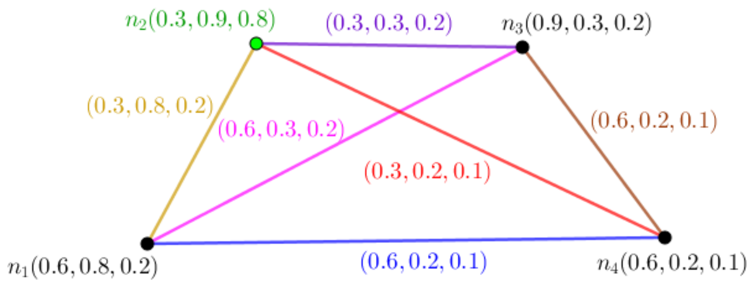

Figure 1.33: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number. 76NTG2

one edge inside which they've common vertex. The minimum cardinality between all dominating path-coloring sets from two given vertices is called **dominating path-coloring number** and it's denoted by $\mathcal{Q}(NTG)$;

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set S of different colors in this process is called **dominating path-coloring set** from x to y if for every edge outside there's at least one edge inside which they've common vertex. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all dominating path-coloring sets, Ss, is called **neutrosophic dominating path-coloring number** and it's denoted by $\mathcal{Q}_n(NTG)$.

Proposition 1.5.94. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{Q}(CMT_\sigma) = \min_S |S|.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. For given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors in this process is called dominating path-coloring set from x to y. The minimum cardinality between all dominating path-coloring sets from two given vertices is called dominating path-coloring number and it's denoted by $\mathcal{Q}(CMT_\sigma)$. Thus

$$\mathcal{Q}(CMT_\sigma) = \min_S |S|.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.95. In Figure (2.33), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(i) Consider two vertices n_1 and n_2. All paths are as follow:

$$\begin{aligned} P_1 &: n_1, n_2 \to \text{red} \\ P_2 &: n_1, n_3, n_2 \to \text{red} \\ P_3 &: n_1, n_4, n_2 \to \text{red} \\ P_4 &: n_1, n_3, n_4, n_2 \to \text{blue} \\ P_5 &: n_1, n_4, n_3, n_2 \to \text{yellow} \end{aligned}$$

The paths P_1, P_2 and P_3 has no shared edge so they've been colored the same as red. The path P_4 has shared edge n_1n_3 with P_2 and shared edge n_4n_2 with P_3 thus it's been colored the different color as blue in comparison to them. The path P_5 has shared edge n_1n_4 with P_3 and shared edge n_3n_4 with P_4 thus it's been colored the different color as yellow in comparison to different paths in the terms of different colors. Thus $S = \{\text{red}, \text{blue}, \text{yellow}\}$ is dominating path-coloring set and its cardinality, 3, is dominating path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors, $S = \{\text{red}, \text{blue}, \text{yellow}\}$, in this process is called dominating path-coloring set from x to y. The minimum cardinality between all dominating path-coloring sets from two given vertices, 3, is called dominating path-coloring number and it's denoted by $\mathcal{Q}(CMT_\sigma) = 3$;

(ii) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(iii) there are three different paths which have no shared edges. So they've been assigned to same color;

(iv) shared edges form a set of representatives of colors. Each color is corresponded to an edge which has minimum neutrosophic cardinality;

(v) every color in S is corresponded to an edge has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared edges to form S and after that minimum neutrosophic cardinality is optimal;

(vi) two edges n_1n_3 and n_4n_2 are shared with P_4 by P_3 and P_2. The minimum neutrosophic cardinality is 0.6 corresponded to n_4n_2. Other corresponded color has only one shared edge n_3n_4 and minimum neutrosophic cardinality is 0.9. Thus minimum neutrosophic cardinality is 1.5. And corresponded set is $S = \{n_4n_2, n_3n_4\}$. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set $S = \{n_4n_2, n_3n_4\}$ of shared edges in this process is called dominating path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between

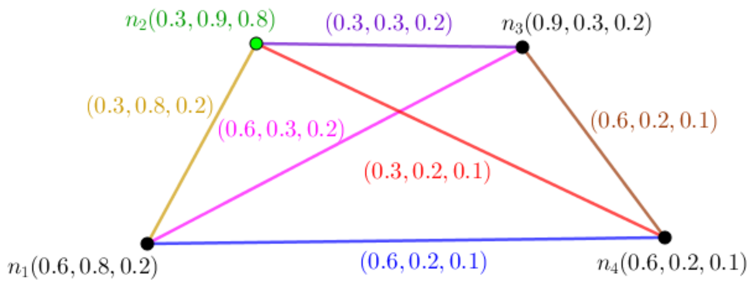

Figure 1.34: A Neutrosophic Graph in the Viewpoint of its dominating path-coloring number and its neutrosophic dominating path-coloring number. 77NTG2

all dominating path-coloring sets, Ss, is called neutrosophic dominating path-coloring number and it's denoted by $\mathcal{Q}_n(CMT_\sigma) = 1.5$.

Definition 1.5.96. (path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, S, in this process is called **path-coloring set** from x to y. The minimum cardinality between all path-coloring sets from two given vertices is called **path-coloring number** and it's denoted by $\mathcal{V}(NTG)$;

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set S of different colors in this process is called **path-coloring set** from x to y. The minimum neutrosophic cardinality, $\sum_{x \in Z} \sum_{i=1}^{3} \sigma_i(x)$, between all sets Zs including the latter endpoints corresponded to path-coloring set Ss, is called **neutrosophic path-coloring number** and it's denoted by $\mathcal{V}_n(NTG)$.

Proposition 1.5.97. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{V}(CMT_\sigma) = (\mathcal{O}(CMT_\sigma) - 1)!.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. If $P : v_i, v_s, v_{s+1}, \cdots, v_{s+z}, v_j$ is a path from v_i to v_j, then all permutations of internal vertices, it means all vertices on the path excluding v_i and v_j, is a path from v_i to v_j, too. Furthermore, all permutations of vertices make a new path. The number of vertices is $\mathcal{O}(CMT_\sigma)$. For given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, S, in this process is called path-coloring set from x to y. The minimum cardinality, $|S|$, between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by

$$\mathcal{V}(CMT_\sigma) = (\mathcal{O}(CMT_\sigma) - 1)!.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.98. In Figure (2.34), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(*i*) Consider two vertices n_1 and n_2. All paths are as follow:

$$\begin{aligned} P_1 &: n_1, n_2 \to \text{red} \\ P_2 &: n_1, n_3, n_2 \to \text{blue} \\ P_3 &: n_1, n_4, n_2 \to \text{yellow} \\ P_4 &: n_1, n_3, n_4, n_2 \to \text{white} \\ P_5 &: n_1, n_4, n_3, n_2 \to \text{black} \end{aligned}$$

Thus $\cup_{i=1}^{3} S_i = \{\text{red}_i, \text{blue}_i, \text{yellow}_i, \text{white}_i, \text{black}_i\}$, is path-coloring set and its cardinality, 15, is path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, $\cup_{i=1}^{3} S_i = \{\text{red}_i, \text{blue}_i, \text{yellow}_i, \text{white}_i, \text{black}_i\}$, in this process is called path-coloring set from x to y. The minimum cardinality, 15, between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{V}(CMT_\sigma) = 15$;

(*ii*) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(*iii*) there are some different paths which have no shared endpoints. So they could been assigned to same color;

(*iv*) shared endpoints form a set of representatives of colors. Each color is corresponded to a vertex which has minimum neutrosophic cardinality;

(*v*) every color in S is corresponded to a vertex has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared endpoints to form S and after that minimum neutrosophic cardinality is optimal;

(*vi*) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, $\cup_{i=1}^{3} S_i =$

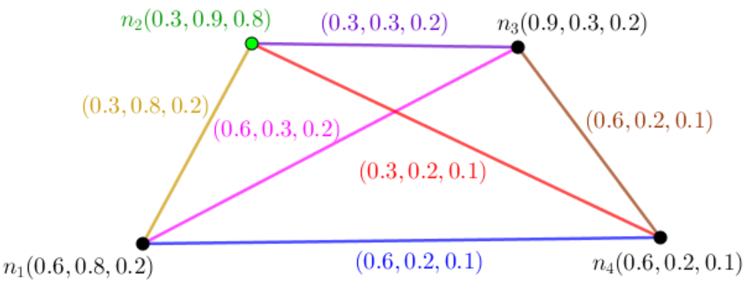

Figure 1.35: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number. 78NTG2

$\{\text{red}_i, \text{blue}_i, \text{yellow}_i, \text{white}_i, \text{black}_i\}$, in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x) = \mathcal{O}_n(CMT_\sigma) - \sum_{i=1}^{3}\sigma_i(n_2) = 3.9$, between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by

$$\mathcal{V}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \sum_{i=1}^{3}\sigma_i(n_2) = 3.9.$$

Definition 1.5.99. (Dual-Dominating Numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum cardinality between all dual-dominating sets is called **dual-dominating number** and it's denoted by $\mathcal{D}(NTG)$;

(ii) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum neutrosophic cardinality between all dual-dominating sets is called **neutrosophic dual-dominating number** and it's denoted by $\mathcal{D}_n(NTG)$.

Proposition 1.5.100. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{D}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected

to each other. So there's one edge between two vertices. For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(NTG) = \mathcal{O}(NTG) - 1$. Thus

$$\mathcal{D}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.101. In Figure (2.35), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$. Thus s dominates n and n dominates s;
- (ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum dominating set and minimal dominating set;
- (iii) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1$;
- (iv) the corresponded set doesn't have to be dominated by the set;
- (v) V is exception when the set is considered in this notion;
- (vi) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \sum_{i=1}^{3} \sigma_i(n_4) = 5$.

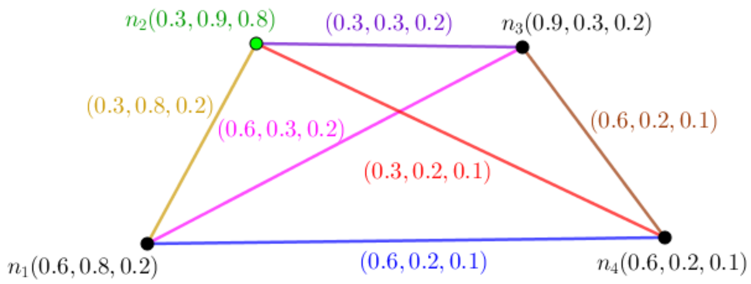

Figure 1.36: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number. 79NTG2

Definition 1.5.102. (dual-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, s and s' if $d(s, n) \neq d(s', n)$, then n resolves s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices s, s' in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, S is called **dual-resolving set**. The maximum cardinality between all dual-resolving sets is called **dual-resolving number** and it's denoted by $\mathcal{R}(NTG)$;

(ii) for given two vertices, s and s' if $d(s, n) \neq d(s', n)$, then n resolves s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices s, s' in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, S is called **dual-resolving set**. The maximum neutrosophic cardinality between all dual-resolving sets is called **dual-resolving number** and it's denoted by $\mathcal{R}_n(NTG)$.

Proposition 1.5.103. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{R}(CMT_\sigma) = 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. For given two vertices, s and s' if $d(s, n) = 1 = d(s', n)$, then n doesn't resolve s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices s, s' in S, there's no neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(NTG) = 1$. Thus

$$\mathcal{R}(CMT_\sigma) = 1.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.104. In Figure (2.36), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s, s', $d(s,n) = 1 = d(s',n)$. Thus n doesn't resolve s and s';

(ii) the existence of one neutrosophic vertex to do this function, resolving, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum resolving set and minimal resolving set as if it seems there's no neutrosophic vertex to resolve so as to choose one vertex outside resolving set so as the function of resolving is impossible;

(iii) for given two vertices, s and s' if $d(s,n) = 1 = d(s',n)$, then n doesn't resolve s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices s, s' in S, there's no neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(NTG) = 1$;

(iv) the corresponded set doesn't have to be resolved by the set;

(v) V isn't used when the set is considered in this notion since $V \setminus \{v\}$ always works;

(vi) for given two vertices, s and s' if $d(s,n) = 1 = d(s',n)$, then n doesn't resolve s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices s, s' in S, there's no neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}_n(NTG) = 2$;

Definition 1.5.105. (joint-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

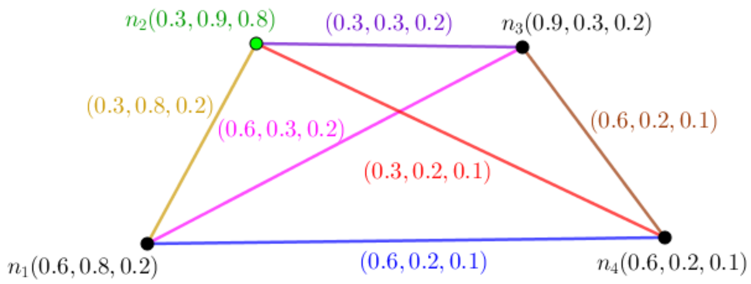

Figure 1.37: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number. 80NTG2

(i) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called **joint-dominating number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called **neutrosophic joint-dominating number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

81prp9 **Proposition 1.5.106.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S dominates if and only if it joint-dominates.*

Proposition 1.5.107. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S is corresponded to joint-dominating number. Then $V \setminus D$ is S-like.*

Proposition 1.5.108. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-dominating number if and only if for all s in S, there's a vertex n in $V \setminus S$, such that $\{n' \mid n'n \in E\} \cap S = \{s\}$.*

Proposition 1.5.109. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{J}(CMT_\sigma) = 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected

to each other. So there's one edge between two vertices. For given vertex n, $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMT_\sigma) = 1$. Thus

$$\mathcal{J}(CMT_\sigma) = 1.$$

■

Proposition 1.5.110. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then joint-dominating number is equal to dominating number.*

Proof. S has one member thus by Proposition (2.5.103), the result holds. ■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.111. In Figure (2.37), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices, s and s', there's an edge between them;
- (ii) one vertex dominates all other vertices thus by there's only one member for S and Proposition (2.5.103), this vertex joint-dominates other vertices;
- (iii) all joint-dominating sets corresponded to joint-dominating number are $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ For given vertex n, $sn \in E$, thus by Proposition (2.5.103), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (2.5.103). The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMT_\sigma) = 1$;
- (iv) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

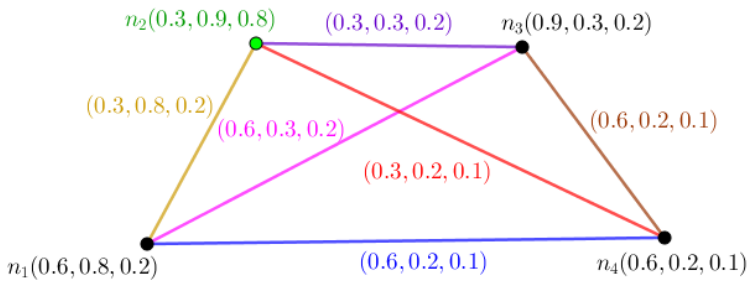

Figure 1.38: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number. 81NTG2

(v) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ corresponded to joint-dominating number as if there are one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4\}$. For given vertex n, $sn \in E$, thus by Proposition (2.5.103), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (2.5.103). The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 0.9$.

Definition 1.5.112. (joint-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called **joint-resolving number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called **neutrosophic joint-resolving number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

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Proposition 1.5.113. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S resolves if and only if it joint-resolves.*

Proposition 1.5.114. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-resolving number if and only if for all s in S, either there are vertices n and n' in $V \setminus S$, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$ or there's vertex s' in S, such that are s and s' twin vertices.*

Proposition 1.5.115. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{J}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. All joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\},$$

For given two vertices n and n', $d(s, n) = 1 = 1 = d(s, n')$, then s doesn't joint-resolve n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}.$$

For every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1$. Thus

$$\mathcal{J}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

■

Proposition 1.5.116. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then joint-resolving number is equal to dominating number.*

Proposition 1.5.117. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$ choose $\mathcal{O}(CMT_\sigma) - 1$. Thus the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$.*

Proposition 1.5.118. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_{\sigma})$ choose $\mathcal{O}(CMT_{\sigma}) - 1$ then minus one. Thus the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_{\sigma}) - 1$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.119. In Figure (2.38), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) Every given two vertices are twin since for all given two vertices, every of them has one edge from every given vertex thus minimum number of edges amid all paths from a vertex to another vertex is forever one;

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CMT_{\sigma}) = 3$;

(iv) there are four joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_1, n_2, n_3, n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are three joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ corresponded to joint-resolving number as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to neutrosophic joint-resolving number are $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and

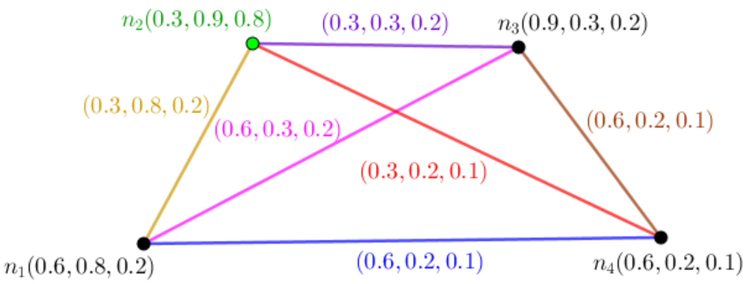

Figure 1.39: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG2

$\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 3.9$.

Definition 1.5.120. (perfect-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called **perfect-dominating set**. The minimum cardinality between all perfect-dominating sets is called **perfect-dominating number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called **perfect-dominating set**. The minimum neutrosophic cardinality between all perfect-dominating sets is called **neutrosophic perfect-dominating number** and it's denoted by $\mathcal{P}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

83prp9 **Proposition 1.5.121.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S dominates if and only if it perfect-dominates.*

Proposition 1.5.122. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and dominating set has one member. Then a vertex of dominating set corresponded to dominating number dominates if and only if it perfect-dominates.*

Proposition 1.5.123. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to perfect-dominating number if and only if for all s in S, there's a vertex n in $V \setminus S$, such that $\{s' \mid s'n \in E\} \cap S = \{s\}$.*

Proposition 1.5.124. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{P}(CMT_{\sigma}) = 1.$$

Proof. Suppose $CMT_{\sigma} : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_{\sigma} : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. All perfect-dominating sets corresponded to perfect-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\}, \ldots \{n_{\mathcal{O}(CMT_{\sigma}-1)}\}, \{n_{\mathcal{O}(CMT_{\sigma})}\}.$$

For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum cardinality between all perfect-dominating sets is called perfect-dominating number and it's denoted by $\mathcal{P}(CMT_{\sigma}) = 1$. Thus

$$\mathcal{P}(CMT_{\sigma}) = 1.$$

■

Proposition 1.5.125. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then perfect-dominating number is equal to dominating number.*

Proposition 1.5.126. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of perfect-dominating sets corresponded to perfect-dominating number is equal to $\mathcal{O}(CMT_{\sigma})$.*

Proposition 1.5.127. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of perfect-dominating sets is equal to $2^{\mathcal{O}(CMT_{\sigma})} - 1$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.128. In Figure (2.39), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates if and only if it perfect-dominates, by Proposition (2.5.118) and S has one member;

(*iii*) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1\}, \{n_2\}, \{n_3\}$, and $\{n_4\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum cardinality between all perfect-dominating sets is called perfect-dominating number and it's denoted by $\mathcal{P}(CMT_\sigma) = 1$ and corresponded to perfect-dominating sets are $\{n_1\}, \{n_2\}, \{n_3\}$, and $\{n_4\}$;

(*iv*) there are five perfect-dominating sets

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\}, \{n_1, n_2, n_3, n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is characteristic;

(*v*) there are five perfect-dominating sets

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\}, \{n_1, n_2, n_3, n_4\},$$

corresponded to perfect-dominating number as if there's one perfect-dominating set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is the determiner;

(*vi*) all perfect-dominating sets corresponded to perfect-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\},$$

For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum neutrosophic cardinality between all perfect-dominating sets is called neutrosophic perfect-dominating number and it's denoted by $\mathcal{P}_n(CMT_\sigma) = 0.9$ and corresponded to perfect-dominating sets $\{n_4\}$.

Definition 1.5.129. (perfect-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex

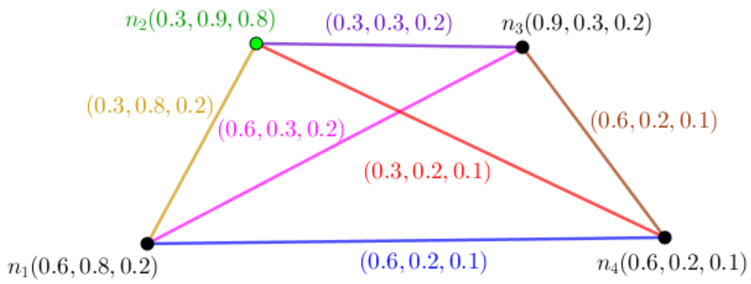

Figure 1.40: A Neutrosophic Graph in the Viewpoint of its perfect-dominating number and its neutrosophic perfect-dominating number. 83NTG2

s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called **perfect-resolving set**. The minimum cardinality between all perfect-resolving sets is called **perfect-resolving number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) for given vertices n and n' if $d(s,n) \neq d(s,n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called **perfect-resolving set**. The minimum neutrosophic cardinality between all perfect-resolving sets is called **neutrosophic perfect-resolving number** and it's denoted by $\mathcal{P}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

84prp9 **Proposition 1.5.130.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S resolves if and only if it perfect-resolves.*

Proposition 1.5.131. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and resolving set has one member. Then a vertex of resolving set corresponded to resolving number resolves if and only if it perfect-resolves.*

Proposition 1.5.132. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to perfect-resolving number if and only if for all s in S, there are neutrosophic vertices n and n' in $V \setminus S$, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$ and for all neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$.*

84prp12 **Proposition 1.5.133.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V and $V \setminus \{x\}$ are S.*

Proposition 1.5.134. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{P}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of resolving set corresponded to resolving number resolves if and only if it perfect-resolves, by no vertices could be resolved in both settings of resolving and perfect-resolving. Thus, by Proposition (2.5.130), S has either $\mathcal{O}(CMT_\sigma) - 1$ or $\mathcal{O}(CMT_\sigma)$. All perfect-resolving sets corresponded to perfect-resolving number are

$$\begin{aligned}
&\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\},\\
&\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)}\},\\
&\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\},\\
&\ldots\\
&\{n_2, n_3, n_4, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\},
\end{aligned}$$

For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum cardinality between all perfect-resolving sets is called perfect-resolving number and it's denoted by

$$\mathcal{P}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1$$

and corresponded to perfect-resolving sets are

$$\begin{aligned}
&\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\},\\
&\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)}\},\\
&\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\},\\
&\ldots\\
&\{n_2, n_3, n_4, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\}.
\end{aligned}$$

Thus

$$\mathcal{P}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

■

Proposition 1.5.135. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then perfect-resolving number is equal to resolving number.*

Proposition 1.5.136. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of perfect-resolving sets corresponded to perfect-resolving number is equal to $\mathcal{O}(CMT_\sigma)$.*

Proposition 1.5.137. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of perfect-resolving sets is equal to $\mathcal{O}(CMT_\sigma) + 1$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.138. In Figure (2.40), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of resolving set corresponded to resolving number resolves if and only if it perfect-resolves, by no vertices could be resolved in both settings of resolving and perfect-resolving. Thus, by Proposition (2.5.130), S has either $\mathcal{O}(CMT_\sigma) - 1$ or $\mathcal{O}(CMT_\sigma)$;

(iii) all perfect-resolving sets corresponded to perfect-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\}$, and $\{n_2, n_3, n_4\}$. For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum cardinality between all perfect-resolving sets is called perfect-resolving number and it's denoted by $\mathcal{P}(CMT_\sigma) = 3$ and corresponded to perfect-resolving sets are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\}$, and $\{n_2, n_3, n_4\}$;

(iv) there are five perfect-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is characteristic;

(v) there are four perfect-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\},$$
$$\{n_2, n_3, n_4\},$$

corresponded to perfect-resolving number as if there's one perfect-resolving set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is the determiner;

(vi) all perfect-resolving sets corresponded to perfect-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\}$, and $\{n_2, n_3, n_4\}$. For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called

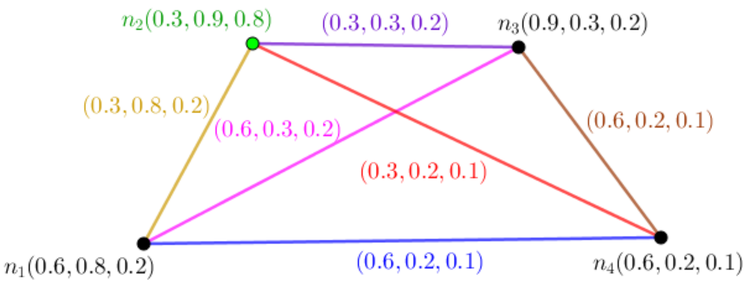

Figure 1.41: A Neutrosophic Graph in the Viewpoint of its perfect-resolving number and its neutrosophic perfect-resolving number. 84NTG2

perfect-resolving set. The minimum neutrosophic cardinality between all perfect-resolving sets is called neutrosophic perfect-resolving number and it's denoted by $\mathcal{P}_n(CMT_\sigma) = 3.9$ and corresponded to perfect-resolving sets are $\{n_1, n_3, n_4\}$.

Definition 1.5.139. (total-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called **total-dominating set**. The minimum cardinality between all total-dominating sets is called **total-dominating number** and it's denoted by $\mathcal{T}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called **total-dominating set**. The minimum neutrosophic cardinality between all total-dominating sets is called **neutrosophic total-dominating number** and it's denoted by $\mathcal{T}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

Proposition 1.5.140. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then* $|S| \geq 2$.

Proposition 1.5.141. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{T}(CMT_\sigma) = 2.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected

to each other. So there's one edge between two vertices. In the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself. All total-dominating sets corresponded to total-dominating number are

$$\{n_1,n_2\},\{n_1,n_3\},\{n_1,n_4\},\ldots,\{n_1,n_{\mathcal{O}(CMT_\sigma)-2}\},\{n_1,n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_1,n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_2,n_3\},\{n_2,n_4\},\{n_2,n_5\},\ldots,\{n_2,n_{\mathcal{O}(CMT_\sigma)-2}\},\{n_2,n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_2,n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_3,n_4\},\{n_3,n_5\},\{n_3,n_6\},\ldots,\{n_3,n_{\mathcal{O}(CMT_\sigma)-2}\},\{n_3,n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_3,n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\ldots$$
$$\{n_{\mathcal{O}(CMT_\sigma)-3},n_{\mathcal{O}(CMT_\sigma)-2}\},\{n_{\mathcal{O}(CMT_\sigma)-3},n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_{\mathcal{O}(CMT_\sigma)-3},n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_{\mathcal{O}(CMT_\sigma)-2},n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_{\mathcal{O}(CMT_\sigma)-2},n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_{\mathcal{O}(CMT_\sigma)-1},n_{\mathcal{O}(CMT_\sigma)}\}$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum cardinality between all total-dominating sets is called total-dominating number and it's denoted by

$$\mathcal{T}(NTG) = 2$$

and corresponded to total-dominating sets are

$$\{n_1,n_2\},\{n_1,n_3\},\{n_1,n_4\},\ldots,\{n_1,n_{\mathcal{O}(CMT_\sigma)-2}\},\{n_1,n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_1,n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_2,n_3\},\{n_2,n_4\},\{n_2,n_5\},\ldots,\{n_2,n_{\mathcal{O}(CMT_\sigma)-2}\},\{n_2,n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_2,n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_3,n_4\},\{n_3,n_5\},\{n_3,n_6\},\ldots,\{n_3,n_{\mathcal{O}(CMT_\sigma)-2}\},\{n_3,n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_3,n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\ldots$$
$$\{n_{\mathcal{O}(CMT_\sigma)-3},n_{\mathcal{O}(CMT_\sigma)-2}\},\{n_{\mathcal{O}(CMT_\sigma)-3},n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_{\mathcal{O}(CMT_\sigma)-3},n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_{\mathcal{O}(CMT_\sigma)-2},n_{\mathcal{O}(CMT_\sigma)-1}\},\{n_{\mathcal{O}(CMT_\sigma)-2},n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_{\mathcal{O}(CMT_\sigma)-1},n_{\mathcal{O}(CMT_\sigma)}\}$$

Thus

$$\mathcal{T}(CMT_\sigma) = 2.$$

■

Proposition 1.5.142. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then total-dominating number isn't equal to dominating number.*

Proposition 1.5.143. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of total-dominating sets corresponded to total-dominating number is equal to $\mathcal{O}(CMT_\sigma)$ choose two.*

Proposition 1.5.144. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of total-dominating sets is equal to $\mathcal{O}(CMT_\sigma)$ choose two plus $\mathcal{O}(CMT_\sigma)$ choose three plus one.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.145. In Figure (2.41), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself;

(iii) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\}.$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum cardinality between all total-dominating sets is called total-dominating number and it's denoted by $\mathcal{T}(CMT_\sigma) = 2$ and corresponded to total-dominating sets are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\};$$

(iv) there are eleven total-dominating sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\},$$
$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is characteristic;

(v) there are six total-dominating sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\},$$

corresponded to total-dominating number as if there's one total-dominating set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is the determiner;

(vi) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\}.$$

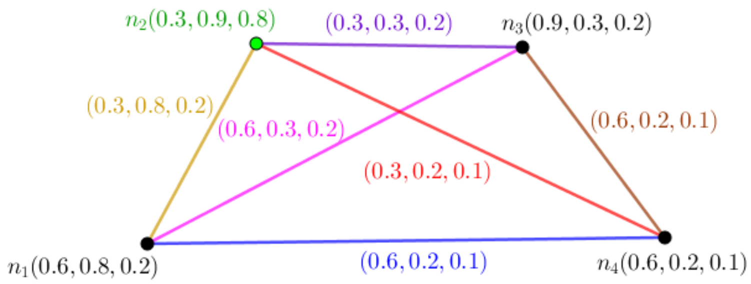

Figure 1.42: A Neutrosophic Graph in the Viewpoint of its total-dominating number and its neutrosophic total-dominating number. 85NTG2

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum neutrosophic cardinality between all total-dominating sets is called neutrosophic total-dominating number and it's denoted by $\mathcal{T}_n(CMT_\sigma) = 2.3$ and corresponded to neutrosophic total-dominating sets are

$$\{n_3, n_4\}.$$

Definition 1.5.146. (total-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices, $d \geq 1$ and all vertices have to be total-resolved otherwise it will be mentioned which is about $d \geq 0$ in some cases but all vertices have to be total-resolved forever. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called **total-resolving set**. The minimum cardinality between all total-resolving sets is called **total-resolving number** and it's denoted by $\mathcal{T}(NTG)$;

(ii) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices, $d \geq 1$ and all vertices have to be total-resolved otherwise it will be mentioned which is about $d \geq 0$ in some cases but all vertices have to be total-resolved forever. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called **total-resolving set**. The minimum neutrosophic cardinality between all total-resolving sets is called **neutrosophic total-resolving number** and it's denoted by $\mathcal{T}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

Proposition 1.5.147. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then $|S| \geq 2$.*

86prp10 **Proposition 1.5.148.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then if there are twin vertices then total-resolving set and total-resolving number are Not Existed.*

Proposition 1.5.149. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{T}(CMT_\sigma) = \textit{Not Existed.}$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve so as resolving is different from total-resolving and by Proposition (2.5.145), total-resolving set and total-resolving number are Not Existed. All total-resolving sets corresponded to total-resolving number are

$$\text{Not Existed.}$$

For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called total-resolving set. The minimum cardinality between all total-resolving sets is called total-resolving number and it's denoted by

$$\mathcal{T}(CMT_\sigma) = \text{Not Existed;}$$

and corresponded to total-resolving sets are

$$\text{Not Existed.}$$

Thus

$$\mathcal{T}(CMT_\sigma) = \text{Not Existed.}$$

∎

Proposition 1.5.150. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then total-resolving number isn't equal to resolving number.*

Proposition 1.5.151. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of total-resolving sets corresponded to total-resolving number is Not Existed.*

Proposition 1.5.152. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of total-resolving sets is Not Existed.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.153. In Figure (2.42), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex, s, there's an edge with other vertices;
- (ii) in the setting of complete, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve so as resolving is different from total-resolving and by Proposition (2.5.145), total-resolving set and total-resolving number are Not Existed;
- (iii) all total-resolving sets corresponded to total-resolving number are

  Not Existed.

  For given vertices n and n' if $d(s,n) \neq d(s,n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called total-resolving set. The minimum cardinality between all total-resolving sets is called total-resolving number and it's denoted by $\mathcal{T}(CMT_\sigma) =$ Not Existed; and corresponded to total-resolving sets are

  Not Existed;
- (iv) there's no total-resolving set

  Not Existed,

  as if it's possible to have one of them as a set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is characteristic;
- (v) there's no total-resolving set

  Not Existed,

  corresponded to total-resolving number as if there's one total-resolving set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is the determiner;
- (vi) all total-resolving sets corresponded to total-resolving number are

  Not Existed.

  For given vertices n and n' if $d(s,n) \neq d(s,n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices. Let S be

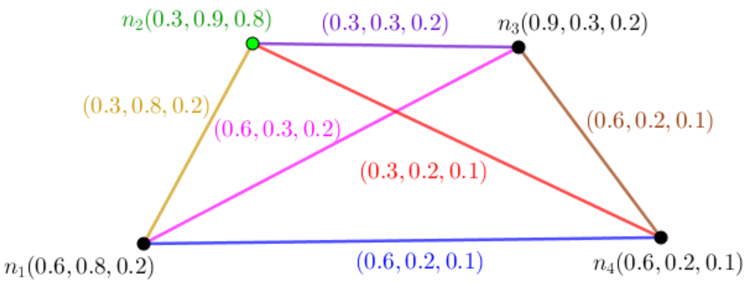

Figure 1.43: A Neutrosophic Graph in the Viewpoint of its total-resolving number and its neutrosophic total-resolving number. 86NTG2

a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called total-resolving set. The minimum neutrosophic cardinality between all total-resolving sets is called neutrosophic total-resolving number and it's denoted by $\mathcal{T}_n(CMT_\sigma) =$ Not Existed; and corresponded to total-resolving sets are

Not Existed.

Definition 1.5.154. (stable-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-dominating set**. The minimum cardinality between all stable-dominating sets is called **stable-dominating number** and it's denoted by $\mathcal{S}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-dominating set**. The minimum neutrosophic cardinality between all stable-dominating sets is called **neutrosophic stable-dominating number** and it's denoted by $\mathcal{S}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

86prp9 **Proposition 1.5.155.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Assume $|S|$ has one member. Then* (*i*) *a vertex dominates if and only if it stable-dominates;*

(*ii*) *S is dominating set if and only if it's stable-dominating set;*

(*iii*) *a number is dominating number if and only if it's stable-dominating number.*

Proposition 1.5.156. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is stable-dominating set corresponded to stable-dominating number if and only if for every neutrosophic vertex s in S, there's at least a neutrosophic vertex n in $V \setminus S$ such that $\{s' \in S \mid s'n \in E\} = \{s\}$.*

Proposition 1.5.157. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V isn't S.*

Proposition 1.5.158. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then stable-dominating number is between one and $\mathcal{O}(NTG) - 1$.*

Proposition 1.5.159. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then stable-dominating number is between one and $\mathcal{O}_n(NTG) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x)$.*

Proposition 1.5.160. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{S}(CMT_\sigma) = 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates so as dominating is the same with stable-dominating, by Proposition (2.5.152), and S has one member. All stable-dominating sets corresponded to stable-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\}, \{n_4\}, \ldots, \{n_{\mathcal{O}(CMT_\sigma)-3}\}, \{n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_{\mathcal{O}(CMT_\sigma)}\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum cardinality between all stable-dominating sets is called stable-dominating number and it's denoted by

$$\mathcal{S}(CMT_\sigma) = 1;$$

and corresponded to stable-dominating sets are

$$\{n_1\}, \{n_2\}, \{n_3\}, \{n_4\}, \ldots, \{n_{\mathcal{O}(CMT_\sigma)-3}\}, \{n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_{\mathcal{O}(CMT_\sigma)}\}.$$

Thus

$$\mathcal{S}(CMT_\sigma) = 1.$$

∎

Proposition 1.5.161. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then stable-dominating number is equal to dominating number.*

Proposition 1.5.162. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of stable-dominating sets corresponded to stable-dominating number is $\mathcal{O}(CMT_{\sigma})$.*

Proposition 1.5.163. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of stable-dominating sets is $\mathcal{O}(CMT_{\sigma})$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.164. In Figure (2.43), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates so as dominating is the same with stable-dominating, by Proposition (2.5.152), and S has one member;

(iii) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum cardinality between all stable-dominating sets is called stable-dominating number and it's denoted by $\mathcal{S}(CMT_{\sigma}) = 1$; and corresponded to stable-dominating sets are

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\};$$

(iv) there are four stable-dominating sets

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is characteristic;

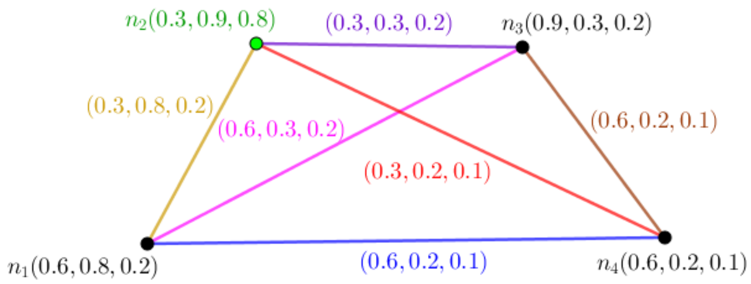

Figure 1.44: A Neutrosophic Graph in the Viewpoint of its stable-dominating number and its neutrosophic stable-dominating number. 87NTG2

(v) there are four stable-dominating sets

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\},$$

corresponded to stable-dominating number as if there's one stable-dominating set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is the determiner;

(vi) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum neutrosophic cardinality between all stable-dominating sets is called neutrosophic stable-dominating number and it's denoted by $\mathcal{S}_n(CMT_\sigma) = 0.9$; and corresponded to stable-dominating sets are

$$\{n_4\}.$$

Definition 1.5.165. (stable-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-resolving set**. The minimum cardinality between all stable-resolving sets is called **stable-resolving number** and it's denoted by $\mathcal{S}(NTG)$;

(ii) for given vertices n and n', if $d(s,n) \neq d(s,n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **neutrosophic stable-resolving set**. The minimum neutrosophic cardinality between all stable-resolving sets is called **neutrosophic stable-resolving number** and it's denoted by $\mathcal{S}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

88prp9

Proposition 1.5.166. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Assume $|S|$ has one member. Then*

(i) *a vertex resolves if and only if it stable-resolves;*

(ii) *S is resolving set if and only if it's stable-resolving set;*

(iii) *a number is resolving number if and only if it's stable-resolving number.*

Proposition 1.5.167. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is stable-resolving set corresponded to stable-resolving number if and only if for every neutrosophic vertex s in S, there are at least neutrosophic vertices n and n' in $V \setminus S$ such that $\{s' \in S \mid d(s', n) \neq d(s', n')\} = \{s\}$.*

Proposition 1.5.168. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V isn't S.*

Proposition 1.5.169. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{S}(CMT_\sigma) = \textit{Not Existed.}$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve so as resolving is different from stable-resolving. Stable-resolving set and stable-resolving number are Not Existed. All stable-resolving sets corresponded to stable-resolving number are

$$\text{Not Existed.}$$

For given vertices n and n', if $d(s,n) \neq d(s,n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum cardinality between all stable-resolving sets is called stable-resolving number and it's denoted by

$$\mathcal{S}(CMT_\sigma) = \text{Not Existed}$$

and corresponded to stable-resolving sets are

Not Existed.

Thus

$$\mathcal{S}(CMT_{\sigma}) = \text{Not Existed}.$$

■

Proposition 1.5.170. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then stable-resolving number isn't equal to resolving number.*

Proposition 1.5.171. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of stable-resolving sets corresponded to stable-resolving number is Not Existed.*

Proposition 1.5.172. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of stable-resolving sets is Not Existed.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.173. In Figure (2.44), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve so as resolving is different from stable-resolving. Stable-resolving set and stable-resolving number are Not Existed;

(iii) all stable-resolving sets corresponded to stable-resolving number are

Not Existed.

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum cardinality between all stable-resolving sets is called stable-resolving number and it's denoted by $\mathcal{S}(CMT_{\sigma}) =$ Not Existed; and corresponded to stable-resolving sets are

Not Existed;

(iv) there's no stable-resolving set

Not Existed,

so as it's possible to have nothing as a set corresponded to neutrosophic stable-resolving number as if neutrosophic cardinality is characteristic;

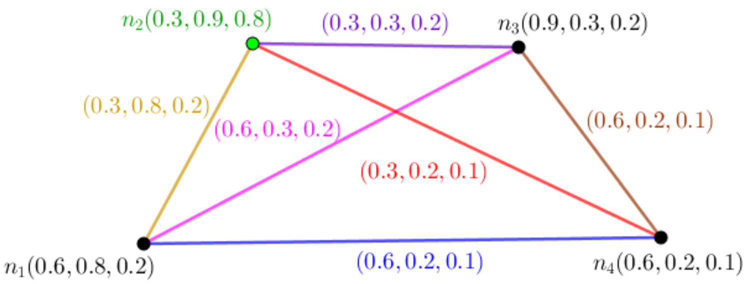

Figure 1.45: A Neutrosophic Graph in the Viewpoint of its stable-resolving number and its neutrosophic stable-resolving number.

88NTG2

(v) there's no stable-resolving set

Not Existed,

corresponded to stable-resolving number so as there's no stable-resolving set corresponded to neutrosophic stable-resolving number as if neutrosophic cardinality is the determiner;

(vi) all stable-resolving sets corresponded to stable-resolving number are

Not Existed.

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum neutrosophic cardinality between all stable-resolving sets is called neutrosophic stable-resolving number and it's denoted by $\mathcal{S}_n(CMT_\sigma)$ = Not Existed; and corresponded to stable-resolving sets are

Not Existed.

1.6 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

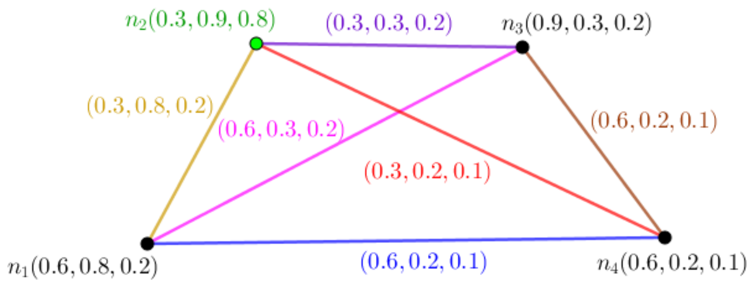

Figure 1.46: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number 88NTG20

1.7 Modelling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (2.1), clarifies about the assigned numbers to these situations.

Table 1.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model. 88tbl1

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

1.8 Case 1: Complete-Model

Step 4. (Solution) The neutrosophic graph alongside its stable-resolving number and its neutrosophic stable-resolving number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its stable-resolving number and its neutrosophic stable-resolving number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are five subjects which are represented as Figure (2.45). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its stable-resolving number and its neutrosophic stable-resolving number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (2.45). In Figure (2.45), an complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) Every given two vertices are twin since for all given two vertices, every of them has one edge from every given vertex thus minimum number of edges amid all paths from a vertex to another vertex is forever one;

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CMT_\sigma) = 3$;

(iv) there are four joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_1, n_2, n_3, n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are three joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ corresponded to joint-resolving number as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

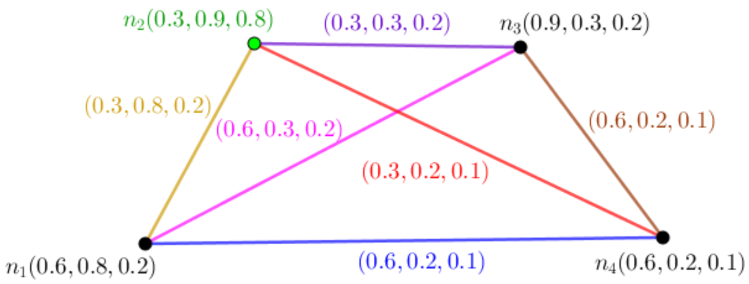

Figure 1.47: A Neutrosophic Graph

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(vi) all joint-resolving sets corresponded to neutrosophic joint-resolving number are $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 3.9$.

1.9 Case 2: Complete Model alongside its Neutrosophic Graph

Step 4. (Solution) The neutrosophic graph alongside its stable-resolving number and its neutrosophic stable-resolving number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its stable-resolving number and its neutrosophic stable-resolving number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are four subjects which are represented in the formation of one model as Figure (2.46). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its stable-resolving number and its neutrosophic stable-resolving number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (2.46). There is one section for clarifications.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(*v*) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(*vi*) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

1.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are defined in complete-neutrosophic graphs. Thus,

Question 1.10.1. *Is it possible to use other types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable?*

Question 1.10.2. *Are existed some connections amid different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs?*

Question 1.10.3. *Is it possible to construct some classes of complete-neutrosophic graphs which have "nice" behavior?*

Question 1.10.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 1.10.5. *Which parameters are related to this parameter?*

Problem 1.10.6. *Which approaches do work to construct applications to create independent study?*

Problem 1.10.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

1.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.
This study uses some definitions concerning different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs. Further studies

Table 1.2: A Brief Overview about Advantages and Limitations of this Study 88tbl

Advantages	Limitations
1. Neutrosophic Numbers of Model	1. Connections amid Classes
2. Acting on All Edges	
3. Minimal Sets	2. Study on Families
4. Maximal Sets	
5. Acting on All Vertices	3. Same Models in Family

could be about changes in the settings to compare these notions amid different settings of complete-neutrosophic graphs. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2.2), some limitations and advantages of this study are pointed out.

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CHAPTER 2

Neutrosophic Tools

2.1 Abstract

New setting is introduced to study different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs. Minimum number and maximum number of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, is a number which is representative based on those vertices or edges. Minimum or maximum neutrosophic number or polynomial of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are called neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number or polynomial. Forming sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable to figure out different types of number of vertices in the sets from different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable sets in the terms of minimum (maximum) number of vertices to get minimum (maximum) number to assign in complete-neutrosophic graphs assigned to complete-neutrosophic

graphs, is key type of approach to have these notions namely different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets. As concluding results, there are some statements, remarks, examples and clarifications about complete-neutrosophic graphs. The clarifications are also presented in both sections "Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable
AMS Subject Classification: 05C17, 05C22, 05E45

2.2 Background

Different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable are addressed in Bibliography. Specially, properties of SuperHyperGraph and neutrosophic SuperHyperGraph by Henry Garrett (2022), is studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book by Henry Garrett (2022).
In this section, I use two sections to illustrate a perspective about the background of this study.

2.3 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 2.3.1. *Is it possible to use mixed versions of ideas concerning "different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial", "neutrosophic different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable* *number and polynomial" and "complete-neutrosophic graphs" to define some notions which are applied to complete-neutrosophic graphs?*

It's motivation to find notions to use in complete-neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. In both settings, corresponded numbers or polynomials conclude the discussion. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In section "Preliminaries", new notions of different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable number and polynomial' in complete-neutrosophic graphs assigned to complete-neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. As concluding results, there are some statements, remarks, examples and clarifications about complete-neutrosophic graphs. The clarifications are also presented in both sections 'Setting of neutrosophic notion number," and " Setting of notion neutrosophic-number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for complete notions, namely complete-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

2.4 Preliminaries

In this section, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 2.4.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 2.4.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v\in V}\sum_{i=1}^{3}\sigma_i(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e\in E}\sum_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 2.4.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1}\mu(x_ix_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t}\ \bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1});$$

(iv) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1}\mu(v_iv_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

Definition 2.4.4. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 2.4.5. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_σ if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** and denoted by CYC where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1,\ x_{\mathcal{O}(NTG)} x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1, σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1, σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1, σ_2}.

Remark 2.4.6. Using notations which is mixed with literatures, are reviewed.

2.4.6.1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$, $\mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;

2.4.6.2. $CMT_\sigma, PTH, CYC, STR_{1, \sigma_2}, CMT_{\sigma_1, \sigma_2}, CMT_{\sigma_1, \sigma_2, \cdots, \sigma_t}$, and WHL_{1, σ_2}.

2.5 Setting of notion neutrosophic-number

In this section, I provide some results in the setting of neutrosophic number.

Definition 2.5.1. (Zero Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Zero forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(ii) **Zero forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

In next result, a complete-neutrosophic graph is considered in the way that, its neutrosophic zero forcing number and its zero forcing neutrosophic-number this model are computed. A complete-neutrosophic graph has specific attribute which implies every vertex is neighbor to all other vertices in the way that, two given vertices have edge is incident to these endpoints.

Proposition 2.5.2. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{Z}_n(NTG) = \mathcal{O}_n(NTG) - \max\{\Sigma_{i=1}^3 \sigma_i(x)\}_{x \in V}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. If S is a set of black vertices and $S < \mathcal{O}(NTG) - 1$, then there are x and y such that they've more than one neighbor in S. Thus the color-change rule doesn't imply these vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}_n(NTG) = \mathcal{O}_n(NTG) - \max\{\Sigma_{i=1}^3 \sigma_i(x)\}_{x \in V}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.3. In Figure (2.1), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_2 is black vertex. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex. Thus n_1 and n_2 aren't black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_2, n_3, n_4\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_1, n_2, n_4\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. Thus n_3 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iv) if $S = \{n_1, n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_2 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 3 is zero forcing number and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_2, n_3, n_4\}$;

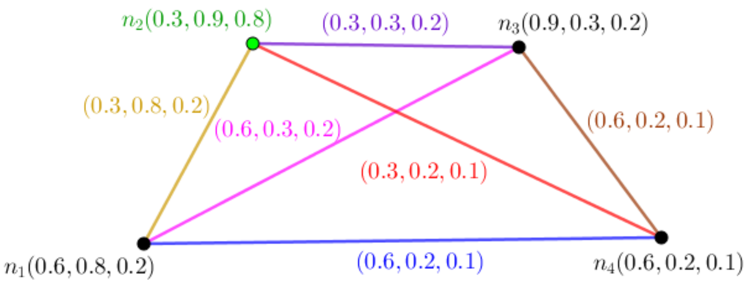

Figure 2.1: A Neutrosophic Graph in the Viewpoint of its Zero Forcing Number. 47NTG2

(vi) 3.9 is zero forcing neutrosophic-number and its corresponded set is $\{n_1, n_3, n_4\}$.

The main definition is presented in next section. The notions of failed zero-forcing number and failed zero-forcing neutrosophic-number facilitate the ground to introduce new results. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types. New setting is introduced to study failed zero-forcing number and failed zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex but now reverse approach is on demand which is finding biggest set which doesn't force.

Definition 2.5.4. (Failed Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Failed zero-forcing number** $\mathcal{Z}'(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(ii) **Failed zero-forcing neutrosophic-number** $\mathcal{Z}'_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

In next result, a complete-neutrosophic graph is considered in the way that, its neutrosophic failed zero-forcing number and its failed zero-forcing neutrosophic-number this model are computed. A complete-neutrosophic graph has specific attribute which implies every vertex is neighbor to all other vertices in the way that, two given vertices have edge is incident to these endpoints.

Proposition 2.5.5. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y)\}_{x,y \in V}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. If S is a set of black vertices and $S < \mathcal{O}(NTG) - 1$, then there are x and y such that they've more than one neighbor in S. Thus the color-change rule doesn't imply these vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y)\}_{x,y \in V}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.6. In Figure (2.2), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_2 is black vertex. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex. Thus n_1 and n_2 aren't black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_3, n_4\}$ could form failed zero-forcing number;

(ii) if $S = \{n_2, n_3, n_4\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_1, n_2, n_4\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. Thus n_3 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iv) if $S = \{n_1, n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_2 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_1, n_2\}, \{n_1, n_3\}$,
$\{n_1, n_4\}, \{n_2, n_3\}$,$\{n_2, n_4\}$, and $\{n_3, n_4\}$;

(vi) 3.6 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_2\}$.

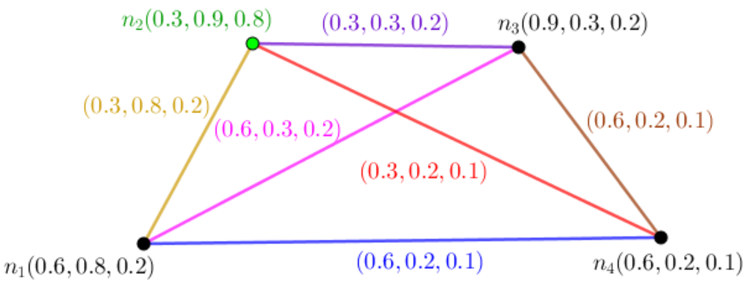

Figure 2.2: A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

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The main definition is presented in next section. The notions of 1-zero-forcing number and 1-zero-forcing neutrosophic-number facilitate the ground to introduce new results. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types. New setting is introduced to study 1-zero-forcing number and 1-zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex and now approach is on demand which is finding smallest set which forces.

Definition 2.5.7. (1-Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-zero-forcing number** $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

(ii) **1-zero-forcing neutrosophic-number** $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

In next result, a complete-neutrosophic graph is considered in the way that, its neutrosophic 1-zero-forcing number and its 1-zero-forcing neutrosophic-number these models are computed. A complete-neutrosophic graph has specific attribute which implies every vertex is neighbor to all other vertices in the way that, two given vertices have edge is incident to these endpoints.

Proposition 2.5.8. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{Z}_n(NTG) = \mathcal{O}_n(NTG) - \max\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y)\}_{x,y \in V}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. If S is a set of black vertices and $|S| < \mathcal{O}(NTG) - 1$, then there are x and y such that they've more than one neighbor in S. Thus the color-change rule doesn't imply these vertices are black vertices but extra condition implies where $|S| = \mathcal{O}(NTG) - 2$. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition. So

$$\mathcal{Z}_n(NTG) = \mathcal{O}_n(NTG) - \max\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y)\}_{x,y \in V}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.9. In Figure (2.3), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) if $S = \{n_1, n_4\}$ is a set of black vertices, then n_2 and n_3 are white neighbors of n_1 and n_4. Thus the color-change rule doesn't imply n_2 is black vertex but extra condition implies. n_2 is white neighbor of n_1 and n_4. Thus the color-change rule implies n_3 is black vertex. Thus n_2 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;
- (ii) if $S = \{n_2, n_4\}$ is a set of black vertices, then n_1 and n_3 are white neighbors of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex but extra condition implies. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule implies n_3 is black vertex. Thus n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;
- (iii) if $S = \{n_1\}$ is a set of black vertices, then n_2, n_3 and n_4 are white neighbors of n_2. Thus the color-change rule doesn't imply neither of n_2, n_3 and n_4 are black vertices and extra condition doesn't imply, too. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule" and extra condition;
- (iv) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_1 and n_2 are white neighbors of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex but extra condition implies. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule implies n_2 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule" and extra condition;

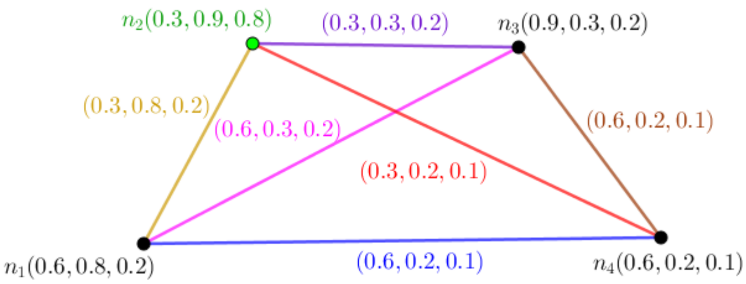

Figure 2.3: A Neutrosophic Graph in the Viewpoint of its 1-Zero-Forcing Number.

49NTG2

(v) 3 is 1-zero-forcing number and its corresponded sets are $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, and $\{n_3, n_4\}$;

(vi) 2.3 is 1-zero-forcing neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Definition 2.5.10. (Independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously;

(ii) **independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously.

Proposition 2.5.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to independent number $\mathcal{I}(NTG)$. It induces if $S = \{n\}$ is a set of vertices, then there's no vertex in S but n. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n\}$ is corresponded to independent number. Thus

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.12. In Figure (2.4), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_2\}$ is corresponded to independent number $\mathcal{I}(NTG)$ but not independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$ isn't corresponded to both independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n_4\}$ is corresponded to independent number $\mathcal{I}(NTG)$ and independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 1 is independent number and its corresponded sets are $\{n_1\}, \{n_2\}$, $\{n_3\}$, and $\{n_4\}$;

(vi) 0.9 is independent neutrosophic-number and its corresponded set is $\{n_4\}$.

The natural way proposes us to use the restriction "minimum" instead of "maximum."

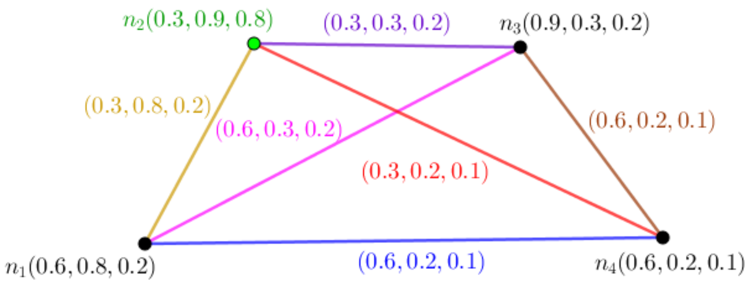

Figure 2.4: A Neutrosophic Graph in the Viewpoint of its Independent Number. 50NTG2

Definition 2.5.13. (Failed independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(ii) **failed independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Example 2.5.14. In Figure (2.5), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1, n_2\}$ is corresponded to failed independent number $\mathcal{I}(NTG)$ but not failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. It implies that $S = \{n_2, n_4\}$ is corresponded to failed independent number $\mathcal{I}(NTG)$ but not failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. But it implies that $S = \{n_1\}$ isn't corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_3 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of

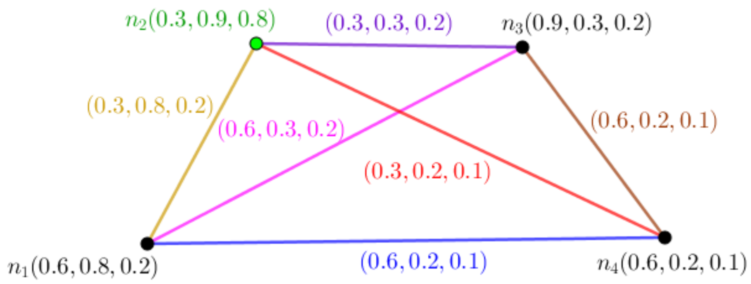

Figure 2.5: A Neutrosophic Graph in the Viewpoint of its Failed independent Number and its Failed Independent Neutrosophic-Number. 51NTG1

an edge. There's one edge to have exclusive endpoints from S. It implies that $S = \{n_2, n_4\}$ is corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 2 is failed independent number and its corresponded sets are $\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_3\}, \{n_2, n_4\}$, and $\{n_3 n_4\}$;

(vi) 2.3 is failed independent neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Proposition 2.5.15. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \mathcal{O}_n(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x, y and z in S such that they're endpoints of an edge, simultaneously, and they form a triangle. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are all possible edges to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S|=\mathcal{O}(NTG)}$ is corresponded to failed independent number. Thus

$$\mathcal{I}_n(NTG) = \mathcal{O}_n(NTG).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.16. In Figure (2.6), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

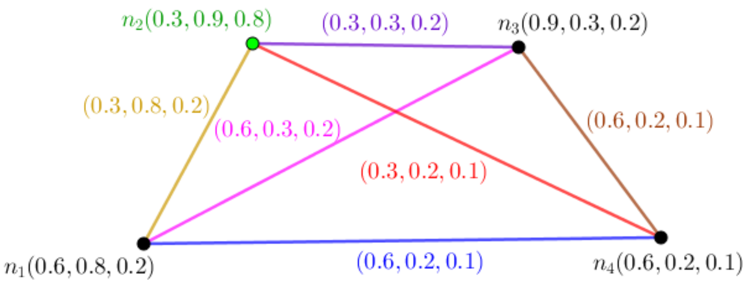

Figure 2.6: A Neutrosophic Graph in the Viewpoint of its Failed Independent Number. 51NTG2

(*i*) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*ii*) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_2, n_4\}$ is corresponded to neither failed independent number $\mathcal{I}(NTG)$ nor failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iii*) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1\}$ is corresponded to neither failed independent number $\mathcal{I}(NTG)$ nor failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iv*) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. $S = \{n_i\}_{|S| = \mathcal{O}(NTG)}$. Thus there are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both failed independent number $\mathcal{I}(NTG)$ and failed independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*v*) 4 is failed independent number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(*vi*) $\mathcal{O}_n(NTG) = 5.9$ is failed independent neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Definition 2.5.17. (1-independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **1-independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously For one time, one vertex is allowed to be endpoint;

(*ii*) **1-independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously. For one time, one vertex is allowed to be endpoint.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 2.5.18. In Figure (2.8), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. Extra condition implies that $S = \{n_1\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*ii*) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. Extra condition implies that $S = \{n_2\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iii*) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S but extra condition implies that $S = \{n_1, n_2\}$ is corresponded to both 1-independent number $\mathcal{I}(NTG)$ and 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*iv*) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S but extra condition implies that $S = \{n_4\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(*v*) 2 is 1-independent number and its corresponded sets are $\{n_1, n_2\}$,$\{n_1, n_3\}$,$\{n_1, n_4\}$,$\{n_2, n_3\}$,$\{n_2, n_4$ and $\{n_3, n_4\}$;

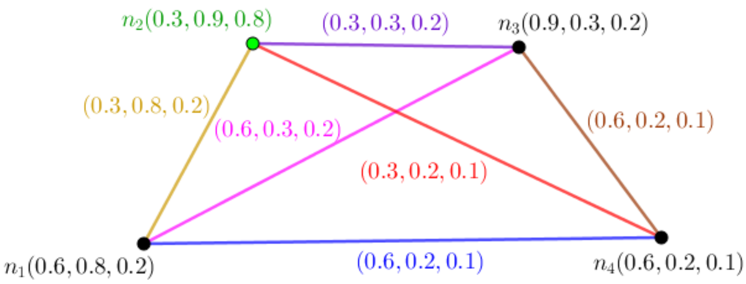

Figure 2.7: A Neutrosophic Graph in the Viewpoint of its 1-Independent Number and its 1-Independent Neutrosophic-Number. 52NTG1

(vi) 3.6 is 1-independent neutrosophic-number and its corresponded set is $\{n_1, n_2\}$.

Definition 2.5.19. (Failed 1-independent Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed 1-independent number** $\mathcal{I}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. For one time, one vertex is allowed not to be endpoint;

(ii) **failed 1-independent neutrosophic-number** $\mathcal{I}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. For one time, one vertex is allowed not to be endpoint.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 2.5.20. In Figure (2.8), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of failed 1-independent number $\mathcal{I}(NTG)$ and failed 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that

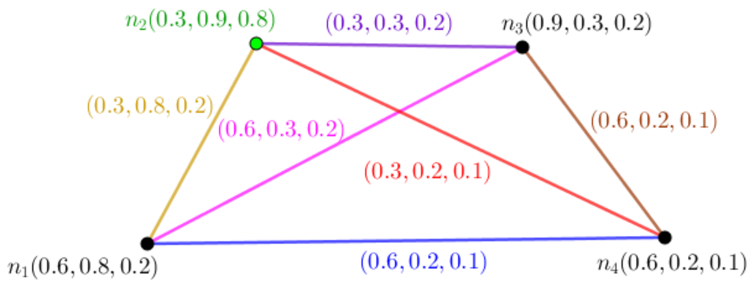

Figure 2.8: A Neutrosophic Graph in the Viewpoint of its Failed 1-Independent Number and its Failed 1-Independent Neutrosophic-Number. 52NTG1

$S = \{n_2, n_4\}$ isn't corresponded to both of failed 1-independent number $\mathcal{I}(NTG)$ and failed 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. But it implies that $S = \{n_1\}$ isn't corresponded to both of failed 1-independent number $\mathcal{I}(NTG)$ and failed 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both failed 1-independent number $\mathcal{I}(NTG)$ and failed 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 4 is failed 1-independent number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(vi) $\mathcal{O}_n(NTG) = 5.9$ is failed 1-independent neutrosophic-number and its corresponded set is $\{n_3, n_4\}$.

Proposition 2.5.21. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3} \sigma_i(x) + \sigma_i(y)\}_{x,y \in V}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x and y in S such that they're endpoints of an edge, simultaneously. If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. It implies that $S = \{n_1\}$

isn't corresponded to 1-independent number $\mathcal{I}(NTG)$. It induces if $S = \{n\}$ is a set of vertices, then there's no vertex in S but n. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. It implies that $S = \{n\}$ is corresponded to 1-independent number. But extra condition implies

$$\mathcal{I}_n(NTG) = \max\{\sum_{i=1}^{3} \sigma_i(x) + \sigma_i(y)\}_{x,y \in V}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.22. In Figure (2.9), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_1\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(ii) if $S = \{n_2\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_2\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iii) if $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. Furthermore, There's one edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_1, n_2\}$ is corresponded to both of 1-independent number $\mathcal{I}(NTG)$ and 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(iv) if $S = \{n_4\}$ is a set of vertices, then there's no vertex in S but n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. There's no edge to have exclusive endpoints from S. But extra condition implies that $S = \{n_4\}$ is corresponded to neither 1-independent number $\mathcal{I}(NTG)$ nor 1-independent neutrosophic-number $\mathcal{I}_n(NTG)$;

(v) 2 is 1-independent number and its corresponded sets are $\{n_1, n_2\}, \{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_3\}$, $\{n_2, n_4\}$, and $\{n_3, n_4\}$;

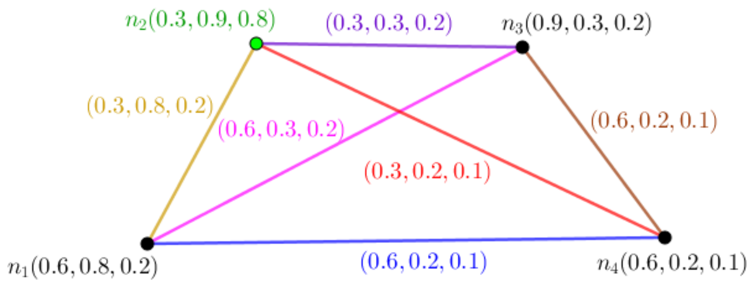

Figure 2.9: A Neutrosophic Graph in the Viewpoint of its 1-Independent Number. 52NTG2

(vi) 3.6 is 1-independent neutrosophic-number and its corresponded set is $\{n_1, n_2\}$.

The natural way proposes us to use the restriction "maximum" instead of "minimum."

Definition 2.5.23. (Clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **clique number** $\mathcal{C}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(ii) **clique neutrosophic-number** $\mathcal{C}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Proposition 2.5.24. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{C}_n(NTG) = \mathcal{O}_n(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x, y and z in S such that they're endpoints of an edge, simultaneously, and they form a triangle. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are all possible edges to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S| = \mathcal{O}(NTG)}$ is corresponded to clique number. Thus

$$\mathcal{C}_n(NTG) = \mathcal{O}_n(NTG).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.25. In Figure (2.10), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of clique number $\mathcal{C}(NTG)$ and clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*ii*) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_2, n_4\}$ is corresponded to neither clique number $\mathcal{C}(NTG)$ nor clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iii*) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1\}$ is corresponded to neither clique number $\mathcal{C}(NTG)$ nor clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*iv*) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. $S = \{n_i\}_{|S| = \mathcal{O}(NTG)}$. Thus there are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both clique number $\mathcal{C}(NTG)$ and clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(*v*) 4 is clique number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(*vi*) $\mathcal{O}_n(NTG) = 5.9$ is clique neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4\}$.

The natural way proposes us to use the restriction "minimum" instead of "maximum."

Definition 2.5.26. (Failed Clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **failed clique number** $\mathcal{C}^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously;

(*ii*) **failed clique neutrosophic-number** $\mathcal{C}_n^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously.

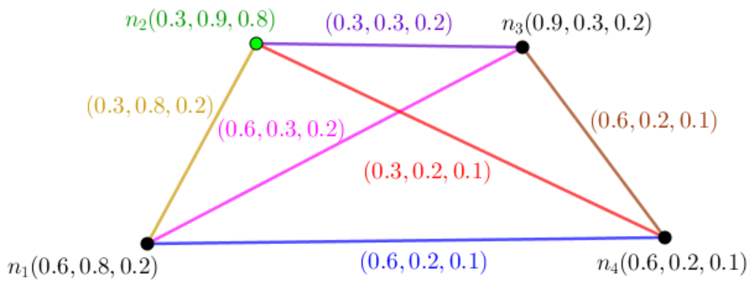

Figure 2.10: A Neutrosophic Graph in the Viewpoint of its clique Number. 53NTG2

Proposition 2.5.27. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x, y and z in S such that they're endpoints of an edge, simultaneously, and they form a triangle. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are all possible edges to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S|=0}$ is corresponded to clique number. Thus

$$\mathcal{C}_n^{\mathcal{F}}(NTG) = 0.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.28. In Figure (2.11), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 0}$. Thus it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S|\neq 0}$. Thus it implies that $S = \{n_2, n_4\}$ is corresponded to neither failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ nor failed clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

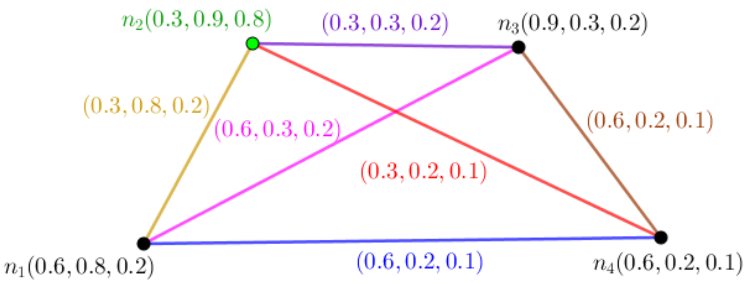

Figure 2.11: A Neutrosophic Graph in the Viewpoint of its Failed Clique Number.

54NTG2

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq 0}$. Thus it implies that $S = \{n_1\}$ is corresponded to neither failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ nor failed clique neutrosophic-number $\mathcal{C}^{\mathcal{F}}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. $S = \{n_i\}_{|S| \neq 0}$. Thus there are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ isn't corresponded to both failed clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed clique neutrosophic-number $\mathcal{C}^{\mathcal{F}}_n(NTG)$;

(v) 0 is failed clique number and its corresponded sets is $\{\}$;

(vi) $\mathcal{O}_n(NTG) = 0$ is failed clique neutrosophic-number and its corresponded set is $\{\}$.

Definition 2.5.29. (1-clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **1-clique number** $\mathcal{C}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time;

(ii) **1-clique neutrosophic-number** $\mathcal{C}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 2.5.30. In Figure (2.13), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_2, n_4\}$ isn't corresponded to both of 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. But it implies that $S = \{n_1\}$ isn't corresponded to both of 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 4 is 1-clique number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(vi) $\mathcal{O}_n(NTG) = 5.9$ is 1-clique neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4\}$.

Definition 2.5.31. (Failed 1-clique Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **failed 1-clique number** $\mathcal{C}^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time;

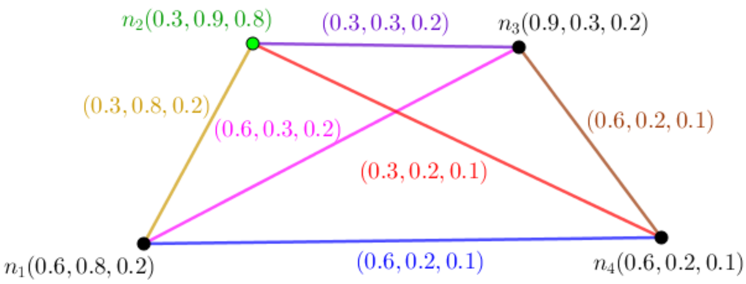

Figure 2.12: A Neutrosophic Graph in the Viewpoint of its 1-Clique Number and its 1-Clique Neutrosophic-Number.

55NTG1

(*ii*) **failed 1-clique neutrosophic-number** $\mathcal{C}_n^{\mathcal{F}}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of a set S of vertices such that there are two vertices in S aren't endpoints for an edge, simultaneously. It holds extra condition which is as follows: two vertices have no edge in common are considered as exception but only for one time.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 2.5.32. In Figure (2.13), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of failed 1-clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed 1-clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(*ii*) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. But it implies that $S = \{n_2, n_4\}$ isn't corresponded to both of failed 1-clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed 1-clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(*iii*) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. But it implies that $S = \{n_1\}$ isn't corresponded to both of failed 1-clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed 1-clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

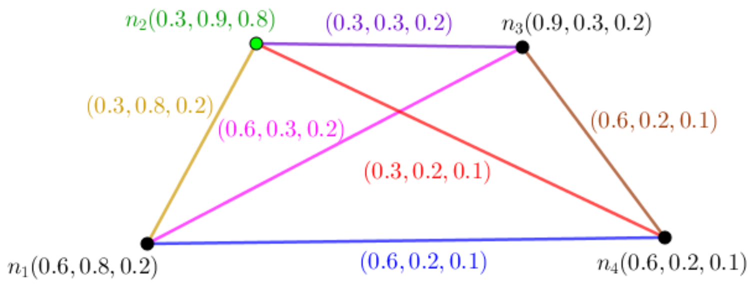

Figure 2.13: A Neutrosophic Graph in the Viewpoint of its Failed 1-Clique number and its Failed 1-Clique neutrosophic-number. 55NTG1

(*iv*) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ isn't corresponded to both failed 1-clique number $\mathcal{C}^{\mathcal{F}}(NTG)$ and failed 1-clique neutrosophic-number $\mathcal{C}_n^{\mathcal{F}}(NTG)$;

(*v*) 0 is failed 1-clique number and its corresponded sets is $\{\}$;

(*vi*) $\mathcal{O}_n(NTG) = 0$ is failed 1-clique neutrosophic-number and its corresponded set is $\{\}$.

Proposition 2.5.33. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{C}_n(NTG) = \mathcal{O}_n(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. Then there are x, y and z in S such that they're endpoints of an edge, simultaneously, and they form a triangle. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There are all possible edges to have exclusive endpoints from S. It implies that $S = \{n_i\}_{|S| = \mathcal{O}(NTG)}$ is corresponded to 1-clique number. Thus

$$\mathcal{C}_n(NTG) = \mathcal{O}_n(NTG).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

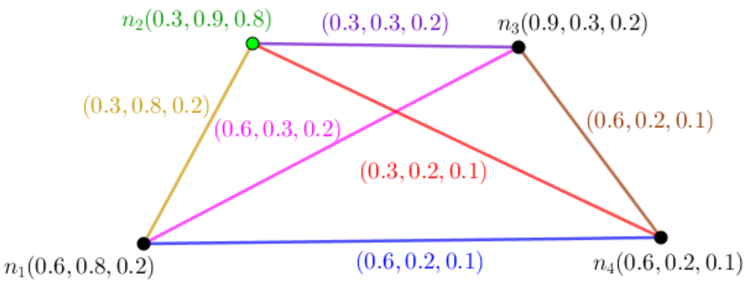

Figure 2.14: A Neutrosophic Graph in the Viewpoint of its 1-Clique Number. 55NTG2

Example 2.5.34. In Figure (2.14), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then there's no vertex in S but n_1 and n_2. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1, n_2\}$ isn't corresponded to both of 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(ii) if $S = \{n_2, n_4\}$ is a set of vertices, then there's no vertex in S but n_2 and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. There's one edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_2, n_4\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iii) if $S = \{n_1\}$ is a set of vertices, then there's no vertex in S but n_1. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's impossible to have endpoints of an edge. Furthermore, There's no edge to have exclusive endpoints from S. $S = \{n_i\}_{|S| \neq \mathcal{O}(NTG)}$. Thus it implies that $S = \{n_1\}$ is corresponded to neither 1-clique number $\mathcal{C}(NTG)$ nor 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(iv) if $S = \{n_1, n_2, n_3, n_4\}$ is a set of vertices, then there's no vertex in S but n_1, n_2, n_3, and n_4. In other side, for having an edge, there's a need to have two vertices. So by using the members of S, it's possible to have endpoints of an edge. $S = \{n_i\}_{|S| = \mathcal{O}(NTG)}$. Thus there are twelve edges to have exclusive endpoints from S. It implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to both 1-clique number $\mathcal{C}(NTG)$ and 1-clique neutrosophic-number $\mathcal{C}_n(NTG)$;

(v) 4 is 1-clique number and its corresponded sets is $\{n_1, n_2, n_3, n_4\}$;

(vi) $\mathcal{O}_n(NTG) = 5.9$ is 1-clique neutrosophic-number and its corresponded set is $\{n_1, n_2, n_3, n_4\}$.

Definition 2.5.35. (Matching Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **matching number** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of edges such that every two edges of S don't have any vertex in common;

(ii) **matching neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of edges such that every two edges of S don't have any vertex in common.

Proposition 2.5.36. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_1x_2) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{j=\lfloor \frac{n}{2} \rfloor}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. For every given vertex, there's one option to choose an edge. Thus a set S, referred to a set of edges with a maximal cardinality, has the cardinality $\lfloor \frac{n}{2} \rfloor$. This number is maximum so

$$\mathcal{M}_n(NTG) = \max\{\sum_{i=1}^{3} \mu_i(x_0x_1) + \sum_{i=1}^{3} \mu_i(x_1x_2) + \cdots + \sum_{i=1}^{3} \mu_i(x_{j-1}x_j)\}_{j=\lfloor \frac{n}{2} \rfloor}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.37. In Figure (2.15), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_1n_3, n_2n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_2n_3, n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

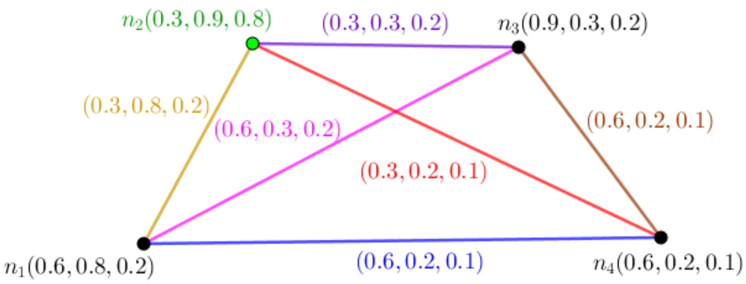

Figure 2.15: A Neutrosophic Graph in the Viewpoint of its matching Number. 58NTG2

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_4\}$ isn't corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 0.9, of S implies $S = \{n_1n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.2, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) 2 is matching number and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$;

(vi) 2.2 is matching neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$.

Definition 2.5.38. (Matching Polynomial).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **matching polynomial** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the matching polynomial represent the number of sets of independent edges of various cardinalities in G.

(ii) **matching polynomial neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the matching polynomial represent the number of sets of independent edges of various neutrosophic cardinalities in G.

Proposition 2.5.39. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = cx^{\max\{\sum_{s\in S}\sum_{i=1}^{3}\mu_i(s)\}\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}} + \cdots + c'x^{\min\{\sum_{s\in E}\sum_{i=1}^{3}\mu_i(s)\}}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. For every given vertex, there's one option to choose an edge. Thus a set S, referred to a set of edges with a maximal cardinality, has the cardinality $\lfloor \frac{n}{2} \rfloor$. This number is maximum so

$$\mathcal{M}_n(NTG) = cx^{\max\{\sum_{s\in S}\sum_{i=1}^{3}\mu_i(s)\}\}_{|S|=\lfloor\frac{\mathcal{O}(NTG)}{2}\rfloor}} + \cdots + c'x^{\min\{\sum_{s\in E}\sum_{i=1}^{3}\mu_i(s)\}}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.40. In Figure (2.16), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_3, n_2n_4\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_1n_3, n_2n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $S = \{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_2n_3, n_1n_4\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ but neutrosophic cardinality, 1.7, of S implies $S = \{n_2n_3, n_1n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $S = \{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_4\}$ isn't corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 0.9, of S implies $S = \{n_1n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to

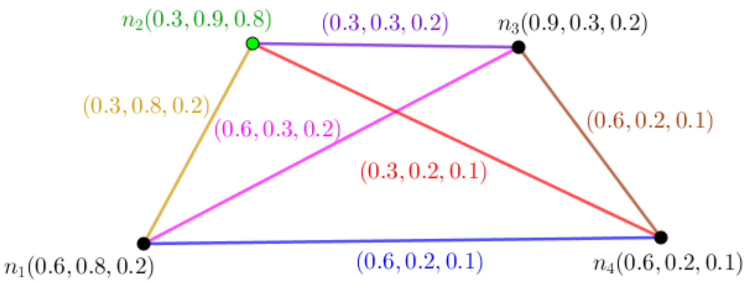

Figure 2.16: A Neutrosophic Graph in the Viewpoint of its Matching Polynomial. 60NTG2

have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge. There are two edges from S. Cardinality of S implies that $S = \{n_1n_2, n_3n_4\}$ is corresponded to matching polynomial $\mathcal{M}(NTG)$ and neutrosophic cardinality, 2.2, of S implies $S = \{n_1n_2, n_3n_4\}$ isn't corresponded to matching polynomial neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $3x^2 + 6x + 1$ is matching polynomial and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$ for coefficient of biggest term;

(vi) $x^{2.2} + x^{1.1}$ is matching polynomial neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$ for coefficient of biggest term.

Definition 2.5.41. (e-Matching Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **e-matching number** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S containing endpoints of edges such that every two edges of S don't have any vertex in common;

(ii) **e-matching neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S containing endpoints of edges such that every two edges of S don't have any vertex in common.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 2.5.42. In Figure (2.17), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $\{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two distinct edges which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $\{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of at least two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_4\}$ isn't corresponded to e-matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 2.5, of S implies $S = \{n_1, n_4\}$ isn't corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iv) if $\{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_2, n_3, n_4\} = V$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $\{n_1, n_2, n_3, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $4 = \mathcal{O}(NTG)$ is e-matching number and its corresponded set is $S = \{n_1, n_2, n_3, n_4\} = V$;

(vi) $5.9 = \mathcal{O}_n(NTG)$ is e-matching neutrosophic-number and its corresponded set is $S = \{n_1, n_2, n_3, n_4\} = V$.

Definition 2.5.43. (e-Matching Polynomial).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **e-matching polynomial** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the e-matching polynomial represent the number of sets of endpoints of independent edges of various cardinalities in G.

(ii) **e-matching polynomial neutrosophic-number** $\mathcal{M}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a polynomial where the coefficients of the terms of the e-matching polynomial represent the number of sets of endpoints of independent edges of various neutrosophic cardinalities in G.

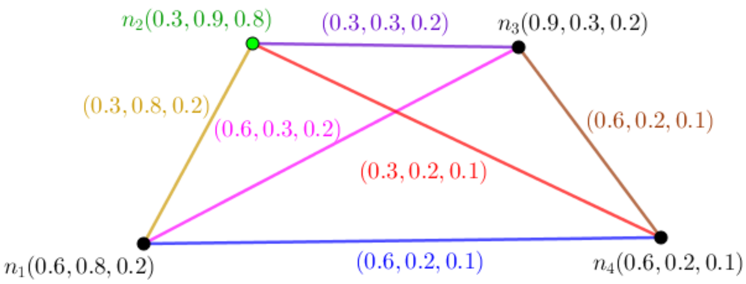

Figure 2.17: A Neutrosophic Graph in the Viewpoint of its e-Matching Number and its e-Matching Neutrosophic-Number. 61NTG1

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 2.5.44. In Figure (2.18), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $\{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(ii) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two distinct edges which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(iii) if $\{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of at least two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_4\}$ isn't corresponded to e-matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 2.5, of S implies $S = \{n_1, n_4\}$ isn't corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

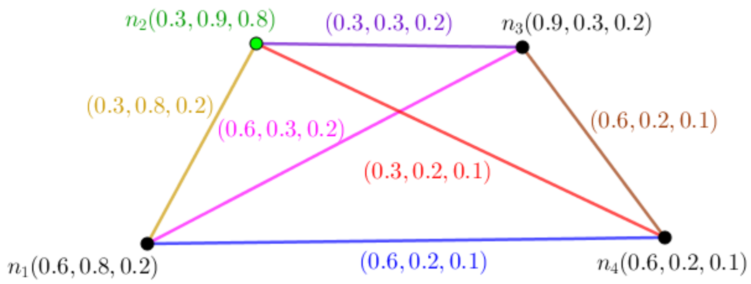

Figure 2.18: A Neutrosophic Graph in the Viewpoint of its e-Matching Polynomial and its e-Matching Polynomial Neutrosophic-Number. 61NTG2

(iv) if $S = \{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_1, n_2, n_3, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(v) $x^4 + 3x^2$ is e-matching polynomial and its corresponded sets are $\{n_1n_2, n_3n_4\}$, $\{n_2n_3, n_1n_4\}$, and $\{n_1n_3, n_2n_4\}$ for coefficient of biggest term; also $S = \{n_1, n_2, n_3, n_4\}$;

(vi) $x^{5.9} + x^{3.4}$ is e-matching polynomial neutrosophic-number and its corresponded set is $\{n_1n_2, n_3n_4\}$ for coefficient of biggest term; also $S = \{n_1, n_2, n_3, n_4\}$.

Proposition 2.5.45. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{M}_n(NTG) = \mathcal{O}_n(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. For every given vertex, there's one option to choose an edge. Thus a set S, referred to a set of edges with a maximal cardinality, has the cardinality $\lfloor \frac{n}{2} \rfloor$. This number is maximum so

$$\mathcal{M}_n(NTG) = \mathcal{O}_n(NTG).$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.46. In Figure (2.19), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $\{n_1n_3, n_2n_4\}$ is a set of edges, then there's no edge in S but n_1n_3 and n_2n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of an edge which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_1, n_3, n_2, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*ii*) if $\{n_2n_3, n_1n_4\}$ is a set of edges, then there's no edge in S but n_2n_3 and n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two distinct edges which is impossible. So by using the members of S, it's impossible to have endpoints of an edge more than one time. There are two edges from S. Cardinality of S implies that $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $S = \{n_2, n_3, n_1, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iii*) if $\{n_1n_4\}$ is a set of edges, then there's no edge in S but n_1n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of at least two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_4\}$ isn't corresponded to e-matching number $\mathcal{M}(NTG)$ but neutrosophic cardinality, 2.5, of S implies $S = \{n_1, n_4\}$ isn't corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*iv*) if $\{n_1n_2, n_3n_4\}$ is a set of edges, then there's no edge in S but n_1n_2 and n_3n_4. In other side, for having a common vertex, there's a need to have one vertex as endpoint of two edges which is impossible. So by using the members of S, it's impossible to have endpoints of two edges. There are two edges from S. Cardinality of S implies that $S = \{n_1, n_2, n_3, n_4\} = V$ is corresponded to e-matching number $\mathcal{M}(NTG)$ and neutrosophic cardinality, 5.9, of S implies $\{n_1, n_2, n_3, n_4\}$ is corresponded to e-matching neutrosophic-number $\mathcal{M}_n(NTG)$;

(*v*) $4 = \mathcal{O}(NTG)$ is e-matching number and its corresponded set is $S = \{n_1, n_2, n_3, n_4\} = V$;

(*vi*) $5.9 = \mathcal{O}_n(NTG)$ is e-matching neutrosophic-number and its corresponded set is $S = \{n_1, n_2, n_3, n_4\} = V$.

Definition 2.5.47. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞;

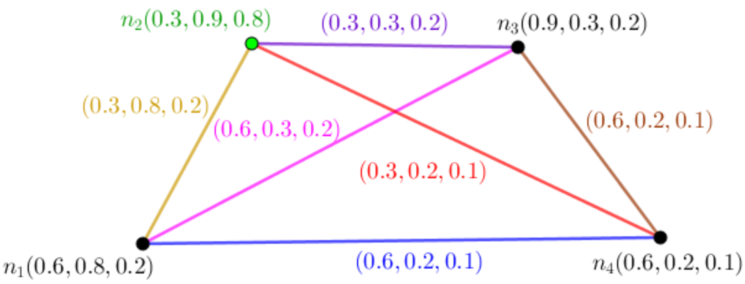

Figure 2.19: A Neutrosophic Graph in the Viewpoint of its e-Matching Number. 61NTG3

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞.

Proposition 2.5.48. *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth is three. Thus

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.49. In Figure (2.20), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

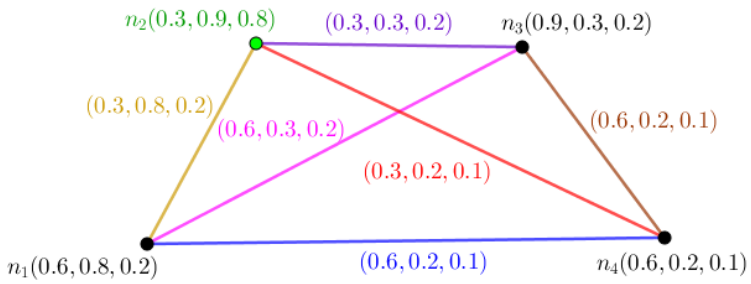

Figure 2.20: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG2

Definition 2.5.50. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞;

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞.

63thm **Theorem 2.5.51.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 2.5.52. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth is three. Thus

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.53. In Figure (2.21), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp

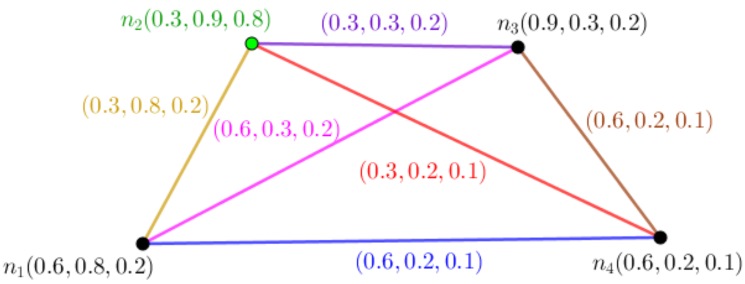

Figure 2.21: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG2

cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

Definition 2.5.54. (Girth Polynomial and Neutrosophic Girth Polynomial). Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **girth polynomial** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1 x^{m_1} + n_2 x^{m_2} + \cdots + n_s x^3$ where n_i is the number of cycle with m_i as its crisp cardinality of the set of vertices of cycle;

(ii) **neutrosophic girth polynomial** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1 x^{m_1} + n_2 x^{m_2} + \cdots + n_s x^{m_s}$ where n_i is the number of cycle with m_i as its neutrosophic cardinality of the set of vertices of cycle.

63thm **Theorem 2.5.55.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 2.5.56. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)} + \mathcal{O}(NTG) x^{\mathcal{O}(NTG) - \sum_{i=1}^{3} \sigma_i(x)} + \cdots + \binom{\mathcal{O}(NTG)}{3} x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth polynomial is three. Thus

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)} + \mathcal{O}(NTG)x^{\mathcal{O}(NTG)-\sum_{i=1}^{3}\sigma_i(x)} + \cdots + \binom{\mathcal{O}(NTG)}{3} x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z))\}}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.57. In Figure (2.22), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only

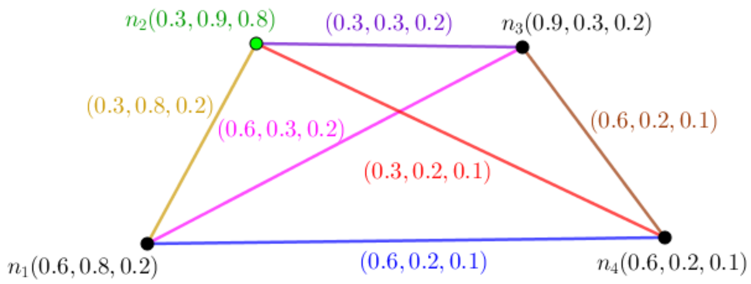

Figure 2.22: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG2

two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^4 + 3x^3$ is girth polynomial and its corresponded sets, for coefficient of smallest term, are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) $x^{5.9} + x^5 + x^{4.5} + x^{4.3} + x^{3.9}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is $\{n_1, n_3, n_4\}$.

Definition 2.5.58. (Hamiltonian Neutrosophic Cycle).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) **hamiltonian neutrosophic cycle** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive vertices $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ which is neutrosophic cycle;

(*ii*) **n-hamiltonian neutrosophic cycle** $\mathcal{N}(HNC)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive vertices $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ which are neutrosophic cycles.

If we use the notion of neutrosophic cardinality in strong type of neutrosophic graphs, then the next result holds. If not, the situation is complicated since it's possible to have all edges in the way that, there's no value of a vertex for an edge.

66thm

Theorem 2.5.59. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ∎

Proposition 2.5.60. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph with two weakest edges. Then*

$$\mathcal{N}(CMT_\sigma) = 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. In other hand, there's a cycle if and only if $\mathcal{O}(CMT_\sigma) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(CMT_\sigma) = 3$. By longest cycle is on demand, the n-hamiltonian neutrosophic cycle is four. The length of longest cycle is $\mathcal{O}(CMT_\sigma)$. Thus it's hamiltonian neutrosophic cycle. Thus

$$\mathcal{N}(CMT_\sigma) = 1.$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.61. In Figure (2.23), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither hamiltonian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ nor n-hamiltonian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(ii) if n_1, n_2, n_3, n_1 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3, n_1$$

isn't corresponded to hamiltonian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ and as its consequences, length of this crisp cycle implies

$$n_1, n_2, n_3, n_1$$

isn't corresponded to n-hamiltonian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(iii) if n_1, n_2, n_3, n_4, n_1 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path, four, implies

$$n_1, n_2, n_3, n_4, n_1$$

is corresponded to hamiltonian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ and it's effective to construct n-hamiltonian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(iv) if n_1, n_3, n_4, n_1 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle, three, implies

$$n_1, n_3, n_4, n_1$$

isn't corresponded to hamiltonian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$. The vertex, n_2, isn't in sequence related to this neutrosophic cycle. Thus it implies

$$n_1, n_3, n_4, n_1$$

isn't corresponded to n-hamiltonian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

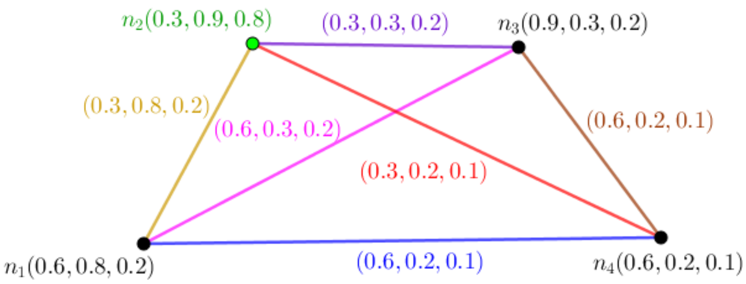

Figure 2.23: A Neutrosophic Graph in the Viewpoint of its hamiltonian neutrosophic cycle. 66NTG2

(v) $\mathcal{M}(CMT_\sigma) : n_1, n_2, n_3, n_4, n_1$ is hamiltonian neutrosophic cycle and its corresponded sets. are the sequences which have both the edges n_1n_4 and n_3n_4. Since these edges are two weakest edges in this complete-neutrosophic graph. Other sequences even if they're cycles having all vertices, once, are hamiltonian cycles and not hamiltonian neutrosophic cycles;

(vi) $\mathcal{N}(CMT_\sigma) = 1$ is n-hamiltonian neutrosophic cycle.

Definition 2.5.62. (Eulerian Neutrosophic Cycle).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Eulerian neutrosophic cycle** $\mathcal{M}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive edges $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}, x_1$ which is neutrosophic cycle;

(ii) **n-Eulerian neutrosophic cycle** $\mathcal{N}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive edges $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}, x_1$ which are neutrosophic cycles.

If we use the notion of neutrosophic cardinality in strong type of neutrosophic graphs, then the next result holds. If not, the situation is complicated since it's possible to have all edges in the way that, there's no value of a vertex for an edge.

66thm **Theorem 2.5.63.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

Proposition 2.5.64. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph with two weakest edges. Then*

$$\mathcal{N}(CMT_\sigma) = 0.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. So there's a cycle if and only if $\mathcal{O}(CMT_\sigma) \geq 3$. It's complete. Hence there's only one neutrosophic cycle which its length is $\mathcal{S}(CMT_\sigma) = 3$ where $\mathcal{O}(CMT_\sigma) = 3$. By longest cycle is on demand in the way that all edges are used and there's no repetition of edges, the n-Eulerian neutrosophic cycle doesn't exist. The length of longest cycle isn't $\mathcal{S}(CMT_\sigma)$. Thus it isn't an Eulerian neutrosophic cycle. Thus

$$\mathcal{N}(CMT_\sigma) = 0.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.65. In Figure (2.24), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's obvious that there's no crisp cycle. It's only a path and it's only two edges but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive edges is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1n_2, n_2n_3$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(*ii*) if n_1n_2, n_2n_3, n_3n_1 is a sequence of consecutive edges, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive edges is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1n_2, n_2n_3, n_3n_1$$

isn't corresponded to Eulerian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ and as its consequences, length of this crisp cycle implies

$$n_1n_2, n_2n_3, n_3n_1$$

isn't corresponded to n-Eulerian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(*iii*) if $n_1n_2, n_2n_3, n_3n_4, n_4n_1$ is a sequence of consecutive edges, then it's obvious that there are two crisp cycles with length three and four. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1n_3, n_3n_4, n_4n_1,

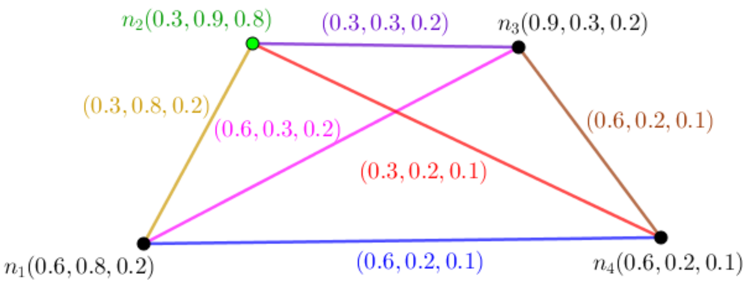

Figure 2.24: A Neutrosophic Graph in the Viewpoint of its Eulerian neutrosophic cycle. 67NTG2

and with length four, $n_1n_2, n_2n_3, n_3n_4, n_4n_1$. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive edges is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different lengths three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. Lack of having all edges, for instance n_1n_3, implies

$$n_1n_2, n_2n_3, n_3n_4, n_4n_1$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(iv) if n_1n_3, n_3n_4, n_4n_1 is a sequence of consecutive edges, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive edges is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle, three, and lack of having all edges, for instance n_1n_2, implies

$$n_1n_3, n_3n_4, n_4n_1$$

is corresponded to neither Eulerian neutrosophic cycle $\mathcal{M}(CMT_\sigma)$ nor n-Eulerian neutrosophic cycle $\mathcal{N}(CMT_\sigma)$;

(v) $\mathcal{M}(CMT_\sigma)$: Not Existed. There is no Eulerian neutrosophic cycle and there are no corresponded sets and sequences;

(vi) $\mathcal{N}(CMT_\sigma) = 0$ is n-Eulerian neutrosophic cycle and there are no corresponded sets and sequences.

Definition 2.5.66. (Eulerian(Hamiltonian) Neutrosophic Path).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Eulerian(Hamiltonian) neutrosophic path** $\mathcal{M}_e(NTG)(\mathcal{M}_h(NTG))$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is a sequence of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}(x_1, x_2, \cdots, x_{\mathcal{O}(NTG)})$ which is neutrosophic path;

(ii) **n-Eulerian(Hamiltonian) neutrosophic path** $\mathcal{N}_e(NTG)(\mathcal{N}_h(NTG))$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is the number of sequences of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(NTG)}(x_1, x_2, \cdots, x_{\mathcal{O}(NTG)})$ which is neutrosophic path.

Proposition 2.5.67. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph with two weakest edges. Then*

$$\mathcal{N}_e(CMT_\sigma) = 0;$$
$$\mathcal{N}_h(CMT_\sigma) = \mathcal{O}(CMT_\sigma)!.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By longest path is on demand in the way that all edges are used and there's no repetition of edges, the Eulerian neutrosophic path doesn't exist. The length of longest path isn't $\mathcal{S}(CMT_\sigma)$. Thus it isn't an Eulerian neutrosophic path. By longest path is on demand in the way that all vertices are used and there's no repetition of vertices, the Hamiltonian neutrosophic path doesn't exist. The length of longest path isn't $\mathcal{O}(CMT_\sigma)$. Thus it isn't a Hamiltonian neutrosophic path. Thus

$$\mathcal{N}_e(CMT_\sigma) = 0;$$
$$\mathcal{N}_h(CMT_\sigma) = \mathcal{O}(CMT_\sigma)!.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.68. In Figure (2.25), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1n_2, n_2n_3 is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$ and $\mu(n_2n_3) > 0$. But the number of edges isn't $\mathcal{S}(CMT_\sigma)$ and the number of vertices isn't $\mathcal{O}(CMT_\sigma)$. Thus Eulerian(Hamiltonian) neutrosophic path $\mathcal{M}_e(CMT_\sigma)(\mathcal{M}_h(CMT_\sigma))$ doesn't exist. Also, n-Eulerian(Hamiltonian) neutrosophic path $\mathcal{N}_e(CMT_\sigma)(\mathcal{N}_h(CMT_\sigma))$ isn't corresponded to these sequences n_1, n_2, n_3 and n_1n_2, n_2n_3;

(ii) if n_1n_2, n_2n_3, n_3n_4 is a sequence of consecutive edges, then it's neutrosophic path since $\mu(n_1n_2) > 0$, $\mu(n_2n_3) > 0$ and $\mu(n_3n_4) > 0$. But the number of edges isn't $\mathcal{S}(CMT_\sigma)$. The number of vertices isn't $\mathcal{O}(CMT_\sigma)$. Thus Eulerian neutrosophic path $\mathcal{M}_e(CMT_\sigma)$ doesn't exist but Hamiltonian neutrosophic path $\mathcal{M}_h(CMT_\sigma)$ is n_1, n_2, n_3, n_4. Also, n-Eulerian neutrosophic path $\mathcal{N}_e(CMT_\sigma)$ equals to zero and n-Hamiltonian neutrosophic path $\mathcal{N}_h(CMT_\sigma))$ is greater than six.

Definition 2.5.69. (Neutrosophic Path Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

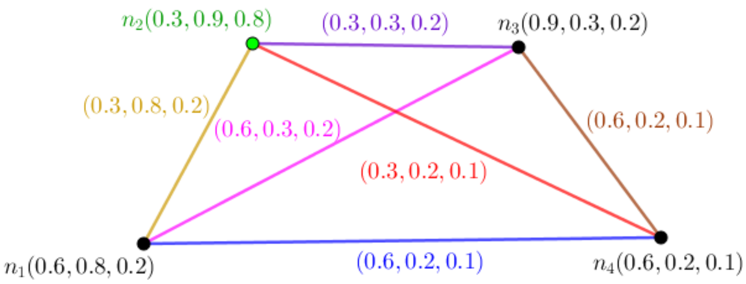

Figure 2.25: A Neutrosophic Graph in the Viewpoint of its Eulerian(Hamiltonian) neutrosophic path. 68NTG2

(*i*) a path from x to y is called **weakest path** if its length is maximum. This length is called **weakest number** amid x and y. The maximum number amid all vertices is called **weakest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{W}(NTG)$;

(*ii*) a path from x to y is called **neutrosophic weakest path** if its strength is $\mu(uv)$ which is less than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called **neutrosophic weakest number** amid x and y. The maximum number amid all vertices is called **neutrosophic weakest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{W}_n(NTG)$.

Proposition 2.5.70. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{W}_n(CMT_\sigma) = \max\{\mu(xy) \mid \mu(xy) = \bigwedge_{i=1,2,\cdots,s-1} \mu(v_iv_{i+1}),\ P : v_1, v_2, \cdots, v_s\}.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Longest path is on demand. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's a path containing all vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. But the length of the path forms weakest number. Thus

$$\mathcal{W}_n(CMT_\sigma) = \max\{\mu(xy) \mid \mu(xy) = \bigwedge_{i=1,2,\cdots,s-1} \mu(v_iv_{i+1}),\ P : v_1, v_2, \cdots, v_s\}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.71. In Figure (2.26), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

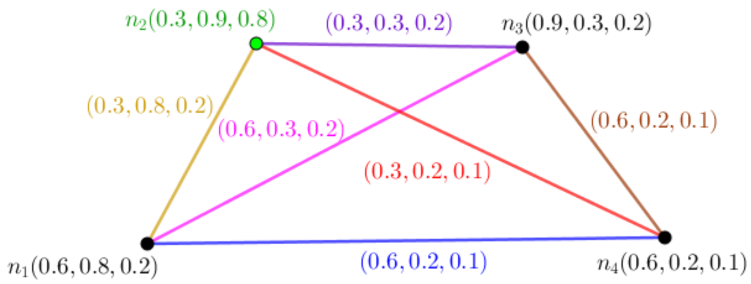

Figure 2.26: A Neutrosophic Graph in the Viewpoint of its Weakest Number and its Neutrosophic Weakest Number. 70NTG2

(i) If n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it's weakest path and weakest number amid n_1 and n_4 is three. Also, $\mathcal{W}(CMT_\sigma) = 3$;

(ii) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't weakest path and weakest number amid n_1 and n_3 isn't two. Also, $\mathcal{W}(CMT_\sigma) \neq 2$;

(iii) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't weakest path and weakest number amid n_1 and n_3 isn't two. Also, $\mathcal{W}(CMT_\sigma) \neq 2$. For every given couple of vertices x and y, weakest path is existed, weakest number is three and $\mathcal{W}(CMT_\sigma) = 3$;

(iv) if n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it isn't a neutrosophic weakest path since neutrosophic weakest number amid n_1 and n_4 is $(0.3, 0.2, 0.1)$. Also, $\mathcal{W}_n(CMT_\sigma) = (0.3, 0.2, 0.1)$;

(v) if n_1, n_2, n_4 is a path from n_1 to n_4, then it's a neutrosophic weakest path and neutrosophic weakest number amid n_1 and n_4 is $(0.3, 0.2, 0.1)$. Also, $\mathcal{W}_n(CMT_\sigma) = (0.3, 0.2, 0.1)$;

(vi) for every given couple of vertices x and y, neutrosophic weakest path is existed, neutrosophic weakest number is $(0.3, 0.2, 0.1)$ and $\mathcal{W}_n(CMT_\sigma) = (0.3, 0.2, 0.1)$.

Definition 2.5.72. (Neutrosophic Path Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a path from x to y is called **strongest path** if its length is minimum. This length is called **strongest number** amid x and y. The maximum number amid all vertices is called **strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}(NTG)$;

(ii) a path from x to y is called **neutrosophic strongest path** if its strength is $\mu(uv)$ which is greater than all strengths of all paths from x to y where $x, \cdots, u, v, \cdots, y$ is a path. This strength is called **neutrosophic strongest number** amid x and y. The minimum number amid all vertices is called **neutrosophic strongest number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{S}_n(NTG)$.

Proposition 2.5.73. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{S}_n(CMT_\sigma) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Minimum path is on demand. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's a path containing all vertices and there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. But the length of the path forms strongest number. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All paths involving s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of path from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. Consider the maximum number assigning to couple of vertices arising from their paths as the start and the end. Thus the maximum strengths of paths from s to a given vertex is $\sigma(s) = \min_{v \in V} \sigma(v)$. It implies the minimum number amid these intended numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$. Thus

$$\mathcal{S}_n(CMT_\sigma) = \min_{v \in V} \sigma(v).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.74. In Figure (2.27), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If n_1, n_2, n_3, n_4 is a path from n_1 to n_4, then it isn't strongest path and strongest number amid n_1 and n_4 is one. Also, $\mathcal{S}(CMT_\sigma) = 1$;
- (ii) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_\sigma) \neq 2$;
- (iii) if n_1, n_2, n_3 is a path from n_1 to n_3, then it isn't strongest path and strongest number amid n_1 and n_3 isn't two. Also, $\mathcal{S}(CMT_\sigma) \neq 2$. For every given couple of vertices x and y, strongest path is existed, strongest number is one and $\mathcal{S}(CMT_\sigma) = 1$;
- (iv) if n_1, n_4, n_3, n_2 is a path from n_1 to n_2, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned}
P_1 : n_1, n_4, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2) \\
P_2 : n_1, n_4, n_2 &\Rightarrow (0.3, 0.2, 0.1) \\
P_3 : n_1, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2) \\
P_4 : n_1, n_2 &\Rightarrow (0.3, 0.8, 0.2) \\
\text{Maximum is } &(0.3, 0.8, 0.2)
\end{aligned}$$

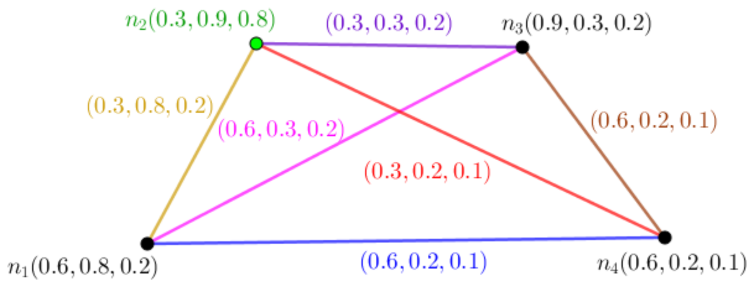

Figure 2.27: A Neutrosophic Graph in the Viewpoint of its strongest Number and its Neutrosophic strongest Number. 71NTG2

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

(v) if n_2, n_1, n_4, n_3 is a path from n_2 to n_3, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_1 and n_2 is $(0.6, 0.3, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_2, n_1, n_4, n_3 &\Rightarrow (0.6, 0.2, 0.1) \\ P_2 : n_2, n_4, n_3 &\Rightarrow (0.3, 0.2, 0.1) \\ P_3 : n_2, n_1, n_3 &\Rightarrow (0.6, 0.3, 0.2) \\ P_4 : n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2) \\ \text{Maximum is } &(0.6, 0.3, 0.2) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$;

(vi) if n_3, n_2, n_1, n_4 is a path from n_3 to n_4, then it isn't a neutrosophic strongest path since neutrosophic strongest number amid n_3 and n_4 is $(0.3, 0.8, 0.2)$ where there are four paths as follows.

$$\begin{aligned} P_1 : n_3, n_2, n_1, n_4 &\Rightarrow (0.3, 0.3, 0.2) \\ P_2 : n_3, n_1, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ P_3 : n_3, n_2, n_4 &\Rightarrow (0.3, 0.2, 0.1) \\ P_4 : n_3, n_4 &\Rightarrow (0.6, 0.2, 0.1) \\ \text{Maximum is } &(0.6, 0.2, 0.1) \end{aligned}$$

Also, $\mathcal{S}_n(CMT_\sigma) = (0.6, 0.2, 0.1)$.

Definition 2.5.75. (Neutrosophic Cycle Connectivity).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a cycle based on x is called **cyclic connectivity** if its length is minimum. This length is called **connectivity number** based on x. The maximum number amid all vertices is called **connectivity number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{C}(NTG)$;

(*ii*) a cycle based on x is called **neutrosophic cyclic connectivity** if its strength is is greater than all strengths of all cycles based on x. This strength is called **neutrosophic connectivity number** based on x. The minimum number amid all vertices is called **neutrosophic connectivity number** of $NTG : (V, E, \sigma, \mu)$ and it's denoted by $\mathcal{C}_n(NTG)$.

Proposition 2.5.76. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{C}_n(CMT_\sigma) = \min_{v \in V} \sigma(v).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Minimum cycle is on demand. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. Consider $s \in S$ such that $\sigma(s) = \min_{v \in V} \sigma(v)$. All cycles based on s has the strength $\sigma(s) = \min_{v \in V} \sigma(v)$. So the maximum strengths of all cycles based on s is $\sigma(s) = \min_{v \in V} \sigma(v)$ which is representative strength based on s. It implies the minimum number amid all representative numbers is $\sigma(s) = \min_{v \in V} \sigma(v)$, too. Thus

$$\mathcal{C}_n(CMT_\sigma) = \min_{v \in V} \sigma(v).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.77. In Figure (2.28), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2, n_3, n_4, n_1 is a cycle based on n_1, then it isn't cyclic connectivity and connectivity number based on n_1 is three. Also, $\mathcal{C}(CMT_\sigma) = 3$;

(*ii*) if n_1, n_2, n_3, n_1 is a cycle based on n_1, then it's cyclic connectivity and connectivity number based on n_1 is three. Also, $\mathcal{C}(CMT_\sigma) = 3$;

(*iii*) Consider n_1, n_2, n_1. Then it isn't a cycle based on n_1, since the length of consecutive vertices has to be at least three. Then it isn't cyclic connectivity and connectivity number based on n_1 isn't two. Also, $\mathcal{C}(CMT_\sigma) \neq 2$. For every given vertex x, cyclic connectivity is existed, connectivity number is three and $\mathcal{C}(CMT_\sigma) = 3$;

(*iv*) if n_1, n_4, n_3, n_2, n_1 is a cycle based on n_1, then it isn't a neutrosophic cyclic connectivity since neutrosophic connectivity number based on n_2 is $(0.3, 0.3, 0.2)$ where there are six paths as follows.

$$
\begin{aligned}
P_1 : n_1, n_4, n_3, n_1 &\Rightarrow (0.6, 0.2, 0.1)\\
P_2 : n_1, n_2, n_3, n_1 &\Rightarrow (0.3, 0.3, 0.2)\\
P_3 : n_1, n_2, n_4, n_1 &\Rightarrow (0.3, 0.2, 0.1)\\
P_4 : n_1, n_4, n_3, n_2, n_1 &\Rightarrow (0.3, 0.3, 0.2)\\
P_5 : n_1, n_3, n_4, n_2, n_1 &\Rightarrow (0.3, 0.2, 0.1)\\
P_6 : n_1, n_4, n_2, n_3, n_1 &\Rightarrow (0.3, 0.2, 0.1)\\
\text{Maximum is } &(0.6, 0.2, 0.1)
\end{aligned}
$$

Also, $\mathcal{C}_n(CMT_\sigma) = (0.3, 0.3, 0.2)$ corresponded to cycle n_2, n_1, n_3, n_2 based on n_2;

(v) if n_2, n_1, n_4, n_3, n_2 is a cycle based on n_2, then it isn't a neutrosophic cyclic connectivity since neutrosophic connectivity number based on n_2 is $(0.3, 0.3, 0.2)$ where there are six paths as follows.

$$
\begin{aligned}
P_1 : n_2, n_4, n_3, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
P_2 : n_2, n_1, n_3, n_2 &\Rightarrow (0.3, 0.3, 0.2)\\
P_3 : n_2, n_1, n_4, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
P_4 : n_2, n_4, n_3, n_1, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
P_5 : n_2, n_3, n_4, n_1, n_2 &\Rightarrow (0.3, 0.3, 0.2)\\
P_6 : n_2, n_4, n_1, n_3, n_2 &\Rightarrow (0.3, 0.2, 0.1)\\
\text{Maximum is } &(0.3, 0.3, 0.2)
\end{aligned}
$$

Also, $\mathcal{C}_n(CMT_\sigma) = (0.3, 0.3, 0.2)$ corresponded to cycle n_2, n_1, n_3, n_2 based on n_2;

(vi) if n_3, n_2, n_1, n_4, n_3 is a cycle based on n_3, then it's a neutrosophic cyclic connectivity and neutrosophic connectivity number based on n_2 is $(0.3, 0.3, 0.2)$ where there are six paths as follows.

$$
\begin{aligned}
P_1 : n_3, n_4, n_2, n_3 &\Rightarrow (0.3, 0.2, 0.1)\\
P_2 : n_3, n_1, n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2)\\
P_3 : n_3, n_1, n_4, n_3 &\Rightarrow (0.6, 0.2, 0.1)\\
P_4 : n_3, n_4, n_2, n_1, n_3 &\Rightarrow (0.3, 0.2, 0.1)\\
P_5 : n_3, n_2, n_4, n_1, n_3 &\Rightarrow (0.3, 0.2, 0.1)\\
P_6 : n_3, n_4, n_1, n_2, n_3 &\Rightarrow (0.3, 0.3, 0.2)\\
\text{Maximum is } &(0.6, 0.2, 0.1)
\end{aligned}
$$

Also, $\mathcal{C}_n(CMT_\sigma) = (0.3, 0.3, 0.2)$ corresponded to cycle n_2, n_1, n_3, n_2 based on n_2.

Definition 2.5.78. (Dense Numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

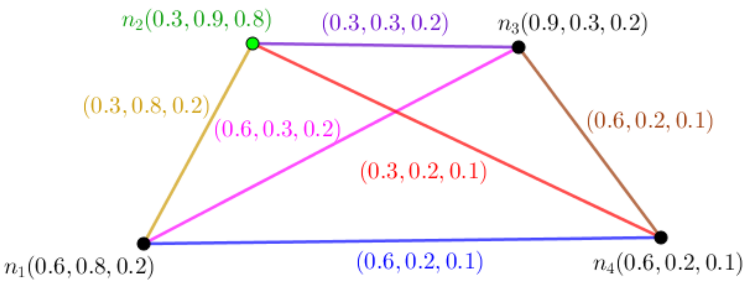

Figure 2.28: A Neutrosophic Graph in the Viewpoint of its connectivity number and its neutrosophic connectivity number. 72NTG2

(i) a set of vertices is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum cardinality between all dense sets is called **dense number** and it's denoted by $\mathcal{D}(NTG)$;

(ii) a set of vertices S is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{s\in S}\sum_{i=1}^{3}\sigma_i(s)$ between all dense sets is called **neutrosophic dense number** and it's denoted by $\mathcal{D}_n(NTG)$.

Proposition 2.5.79. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{D}_n(CMT_\sigma) = \min\{\sum_{i=1}^{\lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor + 1} \sigma(x_i)\}.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. Sets of vertices with cardinality $\lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor + 1$ are dense sets since every vertex inside has $\lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor$ neighbors inside and $\lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor - 1$ neighbors outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is $\lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor + 1$. Thus

$$\mathcal{D}_n(CMT_\sigma) = \min\{\sum_{i=1}^{\lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor + 1} \sigma(x_i)\}.$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.80. In Figure (2.29), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(iii) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is 3. Thus $\mathcal{D}(CMT_\sigma) = 3$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(vi) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum neutrosophic cardinality $\sum_{s\in S}\sum_{i=1}^{3}\sigma_i(s)$ between all dense sets is 3.9. Thus $\mathcal{D}_n(CMT_\sigma) = 3.9$.

Definition 2.5.81. (bulky numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set of edges S is called **bulky set** if for every edge e' outside, there's at least one edge e inside such that they've common vertex and the number of edges such that they've common vertex with e is greater than the number of edges such that they've common vertex with e'. The minimum cardinality between all bulky sets is called **bulky number** and it's denoted by $\mathcal{B}(NTG)$;

(ii) a set of edges S is called **bulky set** if for every edge e' outside, there's at least one edge e inside such that they've common vertex and the number of edges such that they've common vertex with e is greater than the

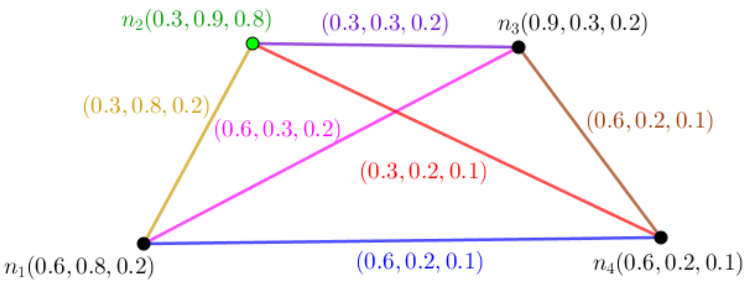

Figure 2.29: A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

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number of edges such that they've common vertex with e'. The minimum neutrosophic cardinality $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(s)$ between all bulky sets is called **neutrosophic bulky number** and it's denoted by $\mathcal{B}_n(NTG)$.

Proposition 2.5.82. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{B}_n(CMT_\sigma) = \min\{\sum_{i=1}^{\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor}\mu(e_i)\}.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. If $S = \{e_1, e_2, \cdots, e_{\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor}\}$ is a set of edges, then it's a bulky set since for every edge e'_j, outside, there's at least one edge e_i inside such that they've common vertex and the number of edges such that they've common vertex with e_i is $\mathcal{O}(CMT_\sigma) - 2$ which is equal to [greater than] $\mathcal{O}(CMT_\sigma) - 2$ which is the number of edges such that they've common vertex with e'_j. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all bulky sets is $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor$. Thus

$$\mathcal{B}_n(CMT_\sigma) = \min\{\sum_{i=1}^{\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor}\mu(e_i)\}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.83. In Figure (2.30), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_2n_4, n_3n_1\}$ is a set of edges, then it's a bulky set since for every edge n_in_j, outside, there's at least one edge n_2n_4 inside such that they've common vertex and the number of edges such that they've common vertex with n_2n_4 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j;

(*ii*) if $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with n_1n_2 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j;

(*iii*) All sets [2-sets] of edges containing two edges are bulky sets. Since for every edge n_in_j, outside, there's at least one edge n_tn_s inside such that they've common vertex and the number of edges such that they've common vertex with n_tn_s is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j. Thus $\mathcal{B}(CMT_\sigma) = 2$;

(*iv*) if $S = \{n_2n_4, n_3n_1\}$ is a set of edges, then it's a bulky set since for every edge n_in_j, outside, there's at least one edge n_2n_4 inside such that they've common vertex and the number of edges such that they've common vertex with n_2n_4 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j;

(*v*) if $S = \{n_1n_2, n_2n_3\}$ is a set of edges, then it's bulky set since for every edge n_in_j, outside, there's at least one edge n_1n_2 inside such that they've common vertex and the number of edges such that they've common vertex with n_1n_2 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j;

(*vi*) if $S = \{n_2n_3, n_2n_4\}$ is set of edges, then they're bulky sets since for every edge n_in_j, outside, there's at least one edge n_2n_3 inside such that they've common vertex and the number of edges such that they've common vertex with n_2n_3 is three which is equal to [greater than] three which is the number of edges such that they've common vertex with n_in_j. The minimum neutrosophic cardinality $\sum_{s\in S}\sum_{i=1}^{3}\sigma_i(s)$ between all bulky sets is 3.9. Thus $\mathcal{B}_n(CMT_\sigma) = 1.4$.

Definition 2.5.84. (collapsed numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) a set of vertices S is called **collapsed set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is less than [equal to] the number of neighbors of y. The minimum cardinality between all collapsed sets is called **collapsed number** and it's denoted by $\mathcal{P}(NTG)$;

(*ii*) a set of vertices S is called **collapsed set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is less than [equal to] the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x)$

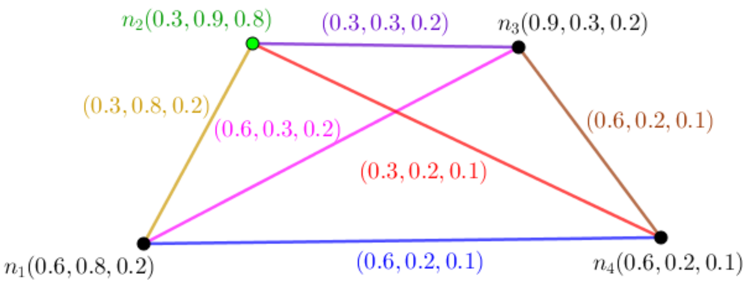

Figure 2.30: A Neutrosophic Graph in the Viewpoint of its bulky number and its neutrosophic bulky number. 74NTG2

between all collapsed sets is called **neutrosophic collapsed number** and it's denoted by $\mathcal{P}_n(NTG)$.

Proposition 2.5.85. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{P}_n(CMT_\sigma) = \min_{x \in V} \sigma(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. All sets [1-set] of vertices containing one vertex $\{x\}$, are called collapsed sets since for every vertex y outside, there's [at least] only one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum neutrosophic cardinality, $\sum_{x \in S} \sum_{i=1}^{3} \sigma_i(x)$, $\min_{x \in V} \sigma(x)$, between all collapsed sets is called neutrosophic collapsed number and it's denoted by $\mathcal{P}_n(CMT_\sigma) = \min_{x \in V} \sigma(x)$. Thus

$$\mathcal{P}_n(CMT_\sigma) = \min_{x \in V} \sigma(x).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.86. In Figure (2.31), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1\}$ is a set of vertices, then a set of vertices S is called collapsed set since for every vertex n_i outside, there's only one vertex n_1 inside such that they're endpoints $n_1 n_i \in E$ and the number of neighbors of n_1 is [less than] equal to the number of neighbors of n_i;

(ii) if $S = \{n_1, n_2\}$ is a set of vertices, then a set of vertices S is called collapsed set since for every vertex n_i outside, there are two vertices n_1

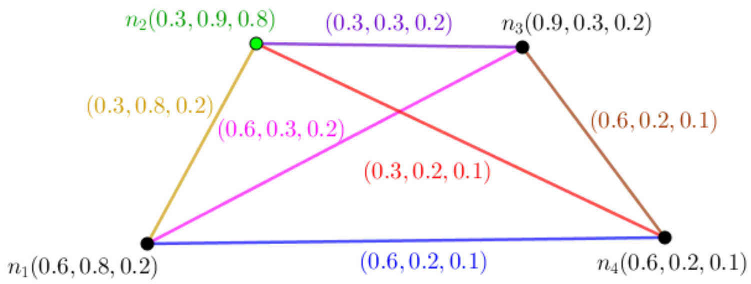

Figure 2.31: A Neutrosophic Graph in the Viewpoint of its collapsed number and its neutrosophic collapsed number. 75NTG2

and n_2 inside such that they're endpoints $n_1n_i, n_2n_i \in E$ and the number of neighbors of n_1 and n_2 is [less than] equal to the number of neighbors of n_i;

(iii) all sets [1-set] of vertices containing one vertex, are called collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum cardinality $|S|$, 1, between all collapsed sets is called collapsed number and it's denoted by $\mathcal{P}(CMT_\sigma) = 1$;

(iv) if $S = \{n_1\}$ is a set of vertices, then a set of vertices S is called collapsed set since for every vertex n_i outside, there's only one vertex n_1 inside such that they're endpoints $n_1n_i \in E$ and the number of neighbors of n_1 is [less than] equal to the number of neighbors of n_i;

(v) if $S = \{n_1, n_2\}$ is a set of vertices, then a set of vertices S is called collapsed set since for every vertex n_i outside, there are two vertices n_1 and n_2 inside such that they're endpoints $n_1n_i, n_2n_i \in E$ and the number of neighbors of n_1 and n_2 is [less than] equal to the number of neighbors of n_i;

(vi) all sets [1-set] of vertices containing one vertex, are called collapsed sets since for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is [less than] equal to the number of neighbors of y. The minimum neutrosophic cardinality, $\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x)$, 0.9, between all collapsed sets is called neutrosophic collapsed number and it's denoted by $\mathcal{P}_n(CMT_\sigma) = 0.9$.

Definition 2.5.87. (path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors in this process is called **path-coloring set** from x to y. The minimum cardinality between all path-coloring sets from two given vertices is called **path-coloring number** and it's denoted by $\mathcal{L}(NTG)$;

(*ii*) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set S of shared edges in this process is called **path-coloring set** from x to y. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all path-coloring sets, Ss, is called **neutrosophic path-coloring number** and it's denoted by $\mathcal{L}_n(NTG)$.

Proposition 2.5.88. *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-neutrosophic graph. Then*

$$\mathcal{L}_n(CMT_\sigma) = \min_S \sum_{e\in S}\sum_{i=1}^{3}\mu_i(e).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. For given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set S of shared edges in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(CMT_\sigma)$.Thus

$$\mathcal{L}_n(CMT_\sigma) = \min_S \sum_{e\in S}\sum_{i=1}^{3}\mu_i(e).$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.89. In Figure (2.32), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(*i*) Consider two vertices n_1 and n_2. All paths are as follow:

$$P_1 : n_1, n_2 \to \text{red}$$
$$P_2 : n_1, n_3, n_2 \to \text{red}$$
$$P_3 : n_1, n_4, n_2 \to \text{red}$$
$$P_4 : n_1, n_3, n_4, n_2 \to \text{blue}$$
$$P_5 : n_1, n_4, n_3, n_2 \to \text{yellow}$$

The paths P_1, P_2 and P_3 has no shared edge so they've been colored the same as red. The path P_4 has shared edge n_1n_3 with P_2 and shared edge n_4n_2 with P_3 thus it's been colored the different color as blue in comparison to them. The path P_5 has shared edge n_1n_4 with P_3 and shared edge n_3n_4 with P_4 thus it's been colored the different color as yellow in comparison to different paths in the terms of different colors. Thus $S = \{\text{red}, \text{blue}, \text{yellow}\}$ is path-coloring set and its cardinality, 3, is path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors, $S = \{\text{red}, \text{blue}, \text{yellow}\}$, in this process is called path-coloring set from x to y. The minimum cardinality between all path-coloring sets from two given vertices, 3, is called path-coloring number and it's denoted by $\mathcal{L}(CMT_\sigma) = 3$;

(ii) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(iii) there are three different paths which have no shared edges. So they've been assigned to same color;

(iv) shared edges form a set of representatives of colors. Each color is corresponded to an edge which has minimum neutrosophic cardinality;

(v) every color in S is corresponded to an edge has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared edges to form S and after that minimum neutrosophic cardinality is optimal;

(vi) two edges n_1n_3 and n_4n_2 are shared with P_4 by P_3 and P_2. The minimum neutrosophic cardinality is 0.6 corresponded to n_4n_2. Other corresponded color has only one shared edge n_3n_4 and minimum neutrosophic cardinality is 0.9. Thus minimum neutrosophic cardinality is 1.5. And corresponded set is $S = \{n_4n_2, n_3n_4\}$. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set $S = \{n_4n_2, n_3n_4\}$ of shared edges in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(CMT_\sigma) = 1.5$.

Definition 2.5.90. (dominating path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of different colors, S, in this process is called **dominating path-coloring set** from x to y if for every edge outside there's at least one edge inside which they've common vertex. The minimum cardinality between all dominating path-coloring sets from two given vertices is called **dominating path-coloring number** and it's denoted by $\mathcal{Q}(NTG)$;

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different

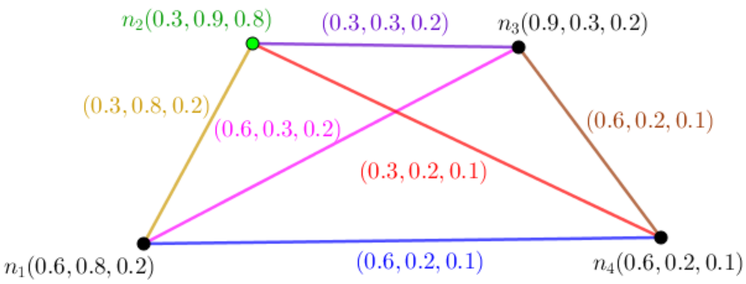

Figure 2.32: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number. 76NTG2

colors. The set S of different colors in this process is called **dominating path-coloring set** from x to y if for every edge outside there's at least one edge inside which they've common vertex. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all dominating path-coloring sets, Ss, is called **neutrosophic dominating path-coloring number** and it's denoted by $\mathcal{Q}_n(NTG)$.

Proposition 2.5.91. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{Q}_n(CMT_\sigma) = \min_S \sum_{e\in S}\sum_{i=1}^{3}\mu_i(e).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. For given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set S of shared edges in this process is called dominating path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all dominating path-coloring sets, Ss, is called neutrosophic dominating path-coloring number and it's denoted by $\mathcal{Q}_n(CMT_\sigma)$.Thus

$$\mathcal{Q}_n(CMT_\sigma) = \min_S \sum_{e\in S}\sum_{i=1}^{3}\mu_i(e).$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.92. In Figure (2.33), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(*i*) Consider two vertices n_1 and n_2. All paths are as follow:

$$P_1 : n_1, n_2 \to \text{red}$$
$$P_2 : n_1, n_3, n_2 \to \text{red}$$
$$P_3 : n_1, n_4, n_2 \to \text{red}$$
$$P_4 : n_1, n_3, n_4, n_2 \to \text{blue}$$
$$P_5 : n_1, n_4, n_3, n_2 \to \text{yellow}$$

The paths P_1, P_2 and P_3 has no shared edge so they've been colored the same as red. The path P_4 has shared edge n_1n_3 with P_2 and shared edge n_4n_2 with P_3 thus it's been colored the different color as blue in comparison to them. The path P_5 has shared edge n_1n_4 with P_3 and shared edge n_3n_4 with P_4 thus it's been colored the different color as yellow in comparison to different paths in the terms of different colors. Thus $S = \{\text{red}, \text{blue}, \text{yellow}\}$ is dominating path-coloring set and its cardinality, 3, is dominating path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set of colors, $S = \{\text{red}, \text{blue}, \text{yellow}\}$, in this process is called dominating path-coloring set from x to y. The minimum cardinality between all dominating path-coloring sets from two given vertices, 3, is called dominating path-coloring number and it's denoted by $\mathcal{Q}(CMT_\sigma) = 3$;

(*ii*) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(*iii*) there are three different paths which have no shared edges. So they've been assigned to same color;

(*iv*) shared edges form a set of representatives of colors. Each color is corresponded to an edge which has minimum neutrosophic cardinality;

(*v*) every color in S is corresponded to an edge has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared edges to form S and after that minimum neutrosophic cardinality is optimal;

(*vi*) two edges n_1n_3 and n_4n_2 are shared with P_4 by P_3 and P_2. The minimum neutrosophic cardinality is 0.6 corresponded to n_4n_2. Other corresponded color has only one shared edge n_3n_4 and minimum neutrosophic cardinality is 0.9. Thus minimum neutrosophic cardinality is 1.5. And corresponded set is $S = \{n_4n_2, n_3n_4\}$. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share one edge, then they're assigned to different colors. The set $S = \{n_4n_2, n_3n_4\}$ of shared edges in this process is called dominating path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all dominating path-coloring sets, Ss, is called neutrosophic dominating path-coloring number and it's denoted by $\mathcal{Q}_n(CMT_\sigma) = 1.5$.

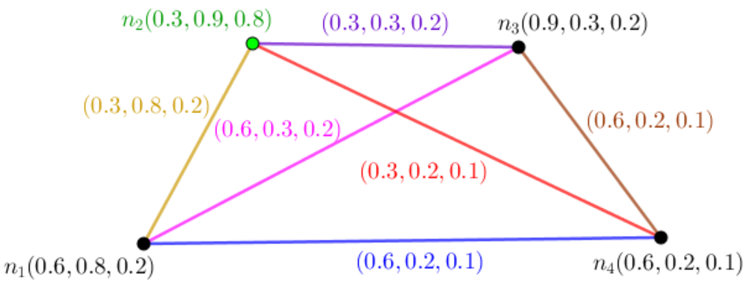

Figure 2.33: A Neutrosophic Graph in the Viewpoint of its dominating path-coloring number and its neutrosophic dominating path-coloring number.

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Definition 2.5.93. (path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, S, in this process is called **path-coloring set** from x to y. The minimum cardinality between all path-coloring sets from two given vertices is called **path-coloring number** and it's denoted by $\mathcal{V}(NTG)$;

(ii) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set S of different colors in this process is called **path-coloring set** from x to y. The minimum neutrosophic cardinality, $\sum_{x\in Z}\sum_{i=1}^{3}\sigma_i(x)$, between all sets Zs including the latter endpoints corresponded to path-coloring set Ss, is called **neutrosophic path-coloring number** and it's denoted by $\mathcal{V}_n(NTG)$.

Proposition 2.5.94. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{V}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max_{x\in S}\sum_{i=1}^{3}\sigma_i(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. If $P : v_i, v_s, v_{s+1}, \cdots, v_{s+z}, v_j$ is a path from v_i to v_j, then all permutations of internal vertices, it means all vertices on the path excluding v_i and v_j, is a path from v_i to v_j, too. Furthermore, all permutations of vertices make a new path. The number of vertices is $\mathcal{O}(CMT_\sigma)$. For given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set S of different colors in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{x\in Z}\sum_{i=1}^{3}\sigma_i(x)$, between all sets Zs including the latter endpoints corresponded to path-coloring set Ss, is called neutrosophic path-coloring number and it's denoted by $\mathcal{V}_n(CMT_\sigma)$. Thus

$$\mathcal{V}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max_{x \in S} \sum_{i=1}^{3} \sigma_i(x).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.95. In Figure (2.34), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(i) Consider two vertices n_1 and n_2. All paths are as follow:

$$\begin{aligned} P_1 &: n_1, n_2 \to \text{red} \\ P_2 &: n_1, n_3, n_2 \to \text{blue} \\ P_3 &: n_1, n_4, n_2 \to \text{yellow} \\ P_4 &: n_1, n_3, n_4, n_2 \to \text{white} \\ P_5 &: n_1, n_4, n_3, n_2 \to \text{black} \end{aligned}$$

Thus $\cup_{i=1}^{3} S_i = \{\text{red}_i, \text{blue}_i, \text{yellow}_i, \text{white}_i, \text{black}_i\}$, is path-coloring set and its cardinality, 15, is path-coloring number. To sum them up, for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, $\cup_{i=1}^{3} S_i = \{\text{red}_i, \text{blue}_i, \text{yellow}_i, \text{white}_i, \text{black}_i\}$, in this process is called path-coloring set from x to y. The minimum cardinality, 15, between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{V}(CMT_\sigma) = 15$;

(ii) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

(iii) there are some different paths which have no shared endpoints. So they could been assigned to same color;

(iv) shared endpoints form a set of representatives of colors. Each color is corresponded to a vertex which has minimum neutrosophic cardinality;

(v) every color in S is corresponded to a vertex has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared endpoints to form S and after that minimum neutrosophic cardinality is optimal;

(vi) for given two vertices, x and y, there are some paths from x to y. If two paths from x to y share an endpoint, then they're assigned to different colors. The set of different colors, $\cup_{i=1}^{3} S_i =$

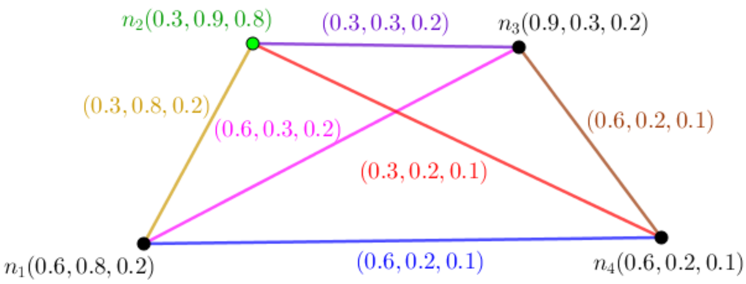

Figure 2.34: A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

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$\{\text{red}_i, \text{blue}_i, \text{yellow}_i, \text{white}_i, \text{black}_i\}$, in this process is called path-coloring set from x to y. The minimum neutrosophic cardinality, $\sum_{x\in S}\sum_{i=1}^{3}\sigma_i(x) = \mathcal{O}_n(CMT_\sigma) - \sum_{i=1}^{3}\sigma_i(n_2) = 3.9$, between all path-coloring sets, Ss, is called neutrosophic path-coloring number and it's denoted by

$$\mathcal{V}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \sum_{i=1}^{3}\sigma_i(n_2) = 3.9.$$

Definition 2.5.96. (Dual-Dominating Numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum cardinality between all dual-dominating sets is called **dual-dominating number** and it's denoted by $\mathcal{D}(NTG)$;

(ii) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum neutrosophic cardinality between all dual-dominating sets is called **neutrosophic dual-dominating number** and it's denoted by $\mathcal{D}_n(NTG)$.

Proposition 2.5.97. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{D}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \min_{x\in V}\sum_{i=1}^{3}\sigma_i(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(NTG) = \mathcal{O}(NTG) - 1$. Thus

$$\mathcal{D}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.98. In Figure (2.35), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$. Thus s dominates n and n dominates s;

(ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum dominating set and minimal dominating set;

(iii) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1$;

(iv) the corresponded set doesn't have to be dominated by the set;

(v) V is exception when the set is considered in this notion;

(vi) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating

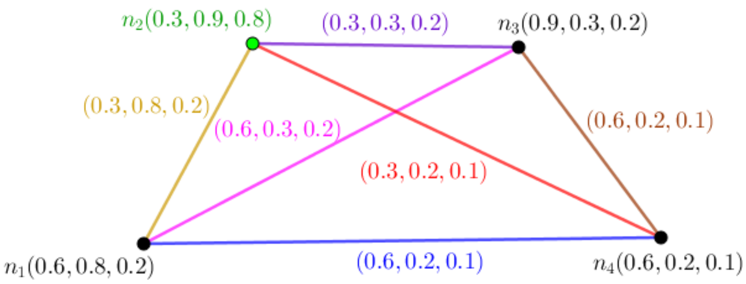

Figure 2.35: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

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sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \sum_{i=1}^{3} \sigma_i(n_4) = 5$.

Definition 2.5.99. (dual-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, s and s' if $d(s, n) \neq d(s', n)$, then n resolves s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices s, s' in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, S is called **dual-resolving set**. The maximum cardinality between all dual-resolving sets is called **dual-resolving number** and it's denoted by $\mathcal{R}(NTG)$;

(ii) for given two vertices, s and s' if $d(s, n) \neq d(s', n)$, then n resolves s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices s, s' in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, S is called **dual-resolving set**. The maximum neutrosophic cardinality between all dual-resolving sets is called **dual-resolving number** and it's denoted by $\mathcal{R}_n(NTG)$.

Proposition 2.5.100. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{R}_n(CMT_\sigma) = \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. For given two vertices, s and s' if $d(s, n) = 1 = d(s', n)$, then n doesn't resolve s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices s, s' in S, there's no neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by

$$\mathcal{R}_n(CMT_\sigma) = \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Thus

$$\mathcal{R}_n(CMT_\sigma) = \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.101. In Figure (2.36), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s, s', $d(s, n) = 1 = d(s', n)$. Thus n doesn't resolve s and s';

(ii) the existence of one neutrosophic vertex to do this function, resolving, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum resolving set and minimal resolving set as if it seems there's no neutrosophic vertex to resolve so as to choose one vertex outside resolving set so as the function of resolving is impossible;

(iii) for given two vertices, s and s' if $d(s, n) = 1 = d(s', n)$, then n doesn't resolve s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices s, s' in S, there's no neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}(NTG) = 1$;

(iv) the corresponded set doesn't have to be resolved by the set;

(v) V isn't used when the set is considered in this notion since $V \setminus \{v\}$ always works;

(vi) for given two vertices, s and s' if $d(s, n) = 1 = d(s', n)$, then n doesn't resolve s and s' where d is the minimum number of edges amid all paths from s to s'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two

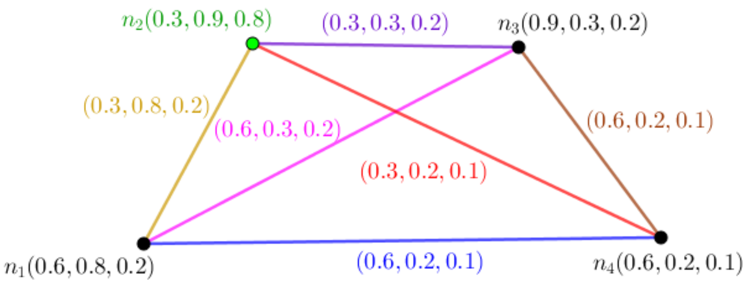

Figure 2.36: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number. 80NTG2

neutrosophic vertices s, s' in S, there's no neutrosophic vertex n in $V \setminus S$ such that n resolves s, s', then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it's denoted by $\mathcal{R}_n(NTG) = 2$;

Definition 2.5.102. (joint-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-dominating sets is called **joint-dominating number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given vertex n if $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called **joint-dominating set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called **neutrosophic joint-dominating number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

81prp9 **Proposition 2.5.103.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S dominates if and only if it joint-dominates.*

Proposition 2.5.104. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S is corresponded to joint-dominating number. Then $V \setminus D$ is S-like.*

Proposition 2.5.105. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-dominating number if and only if for all s in S, there's a vertex n in $V \setminus S$, such that $\{n' \mid n'n \in E\} \cap S = \{s\}$.*

Proposition 2.5.106. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{J}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. For given vertex n, $sn \in E$, then s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, S is called joint-dominating set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by

$$\mathcal{J}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Thus

$$\mathcal{J}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

∎

Proposition 2.5.107. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then joint-dominating number is equal to dominating number.*

Proof. S has one member thus by Proposition (2.5.103), the result holds. ∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.108. In Figure (2.37), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) For given two neutrosophic vertices, s and s', there's an edge between them;

(*ii*) one vertex dominates all other vertices thus by there's only one member for S and Proposition (2.5.103), this vertex joint-dominates other vertices;

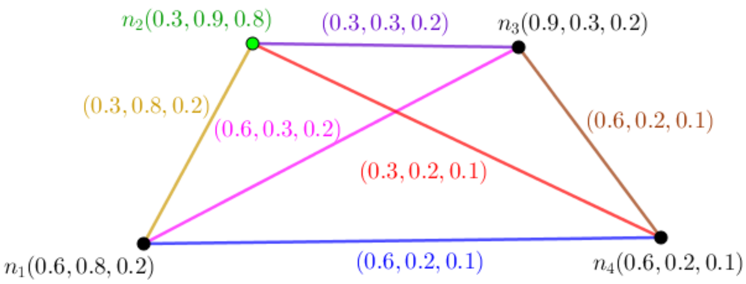

Figure 2.37: A Neutrosophic Graph in the Viewpoint of its joint-dominating number and its neutrosophic joint-dominating number.

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(iii) all joint-dominating sets corresponded to joint-dominating number are $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ For given vertex n, $sn \in E$, thus by Proposition (2.5.103), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (2.5.103). The minimum cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}(CMT_\sigma) = 1$;

(iv) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four joint-dominating sets $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$ corresponded to joint-dominating number as if there are one joint-dominating set corresponded to neutrosophic joint-dominating number so as neutrosophic cardinality is the determiner;

(vi) there's only one joint-dominating set corresponded to joint-dominating number is $\{n_4\}$. For given vertex n, $sn \in E$, thus by Proposition (2.5.103), s joint-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like $\{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. For every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s joint-dominates n, then the set of neutrosophic vertices, $S = \{n_1\}, \{n_2\}, \{n_3\}$ and $\{n_4\}$. is called joint-dominating set where for every two vertices in S, there's no need to have a path in S amid them or we could refer this case holds by Proposition (2.5.103). The minimum neutrosophic cardinality between all joint-dominating sets is called joint-dominating number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 0.9$.

Definition 2.5.109. (joint-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called **joint-resolving number** and it's denoted by $\mathcal{J}(NTG)$;

(ii) for given two vertices n and n', if $d(s,n) \neq d(s,n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is called **joint-resolving set** where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called **neutrosophic joint-resolving number** and it's denoted by $\mathcal{J}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

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Proposition 2.5.110. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S resolves if and only if it joint-resolves.*

Proposition 2.5.111. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to joint-resolving number if and only if for all s in S, either there are vertices n and n' in $V \setminus S$, such that $\{s' \mid d(s',n) \neq d(s',n')\} \cap S = \{s\}$ or there's vertex s' in S, such that are s and s' twin vertices.*

Proposition 2.5.112. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{J}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. All joint-resolving sets corresponded to joint-resolving number are

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\},$$

For given two vertices n and n', $d(s,n) = 1 = 1 = d(s,n')$, then s doesn't joint-resolve n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}.$$

For every neutrosophic vertices n and n' in $V\backslash S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}$$

is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by

$$\mathcal{J}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

Thus

$$\mathcal{J}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max\{\sum_{i=1}^{3} \sigma_i(x)\}_{x \in V}.$$

■

Proposition 2.5.113. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then joint-resolving number is equal to dominating number.*

Proposition 2.5.114. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$ choose $\mathcal{O}(CMT_\sigma) - 1$. Thus the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$.*

Proposition 2.5.115. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma)$ choose $\mathcal{O}(CMT_\sigma) - 1$ then minus one. Thus the number of joint-resolving number corresponded to joint-resolving number is equal to $\mathcal{O}(CMT_\sigma) - 1$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.116. In Figure (2.38), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) Every given two vertices are twin since for all given two vertices, every of them has one edge from every given vertex thus minimum number of edges amid all paths from a vertex to another vertex is forever one;

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex

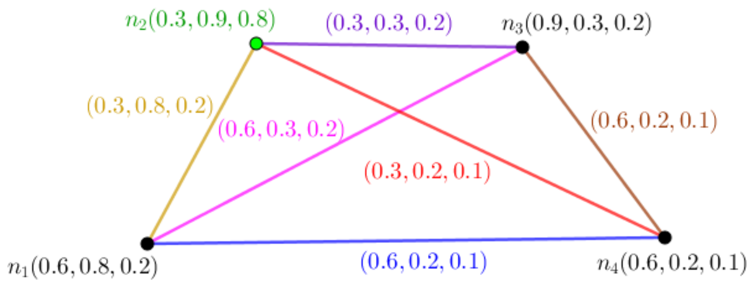

Figure 2.38: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number. 82NTG2

alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CMT_\sigma) = 3$;

(*iv*) there are four joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_1, n_2, n_3, n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(*v*) there are three joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ corresponded to joint-resolving number as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(*vi*) all joint-resolving sets corresponded to neutrosophic joint-resolving number are $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 3.9$.

Definition 2.5.117. (perfect-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(*i*) for given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called **perfect-dominating set**. The minimum cardinality between all perfect-dominating sets is called **perfect-dominating number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called **perfect-dominating set**. The minimum neutrosophic cardinality between all perfect-dominating sets is called **neutrosophic perfect-dominating number** and it's denoted by $\mathcal{P}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

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Proposition 2.5.118. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S dominates if and only if it perfect-dominates.*

Proposition 2.5.119. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and dominating set has one member. Then a vertex of dominating set corresponded to dominating number dominates if and only if it perfect-dominates.*

Proposition 2.5.120. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to perfect-dominating number if and only if for all s in S, there's a vertex n in $V \setminus S$, such that $\{s' \mid s'n \in E\} \cap S = \{s\}$.*

Proposition 2.5.121. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{P}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. All perfect-dominating sets corresponded to perfect-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\}, \ldots \{n_{\mathcal{O}(CMT_\sigma - 1)}\}, \{n_{\mathcal{O}(CMT_\sigma)}\}.$$

For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum neutrosophic cardinality between all perfect-dominating sets is called neutrosophic perfect-dominating number and it's denoted by

$$\mathcal{P}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Thus

$$\mathcal{P}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

■

Proposition 2.5.122. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then perfect-dominating number is equal to dominating number.*

Proposition 2.5.123. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of perfect-dominating sets corresponded to perfect-dominating number is equal to $\mathcal{O}(CMT_\sigma)$.*

Proposition 2.5.124. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of perfect-dominating sets is equal to $2^{\mathcal{O}(CMT_\sigma)} - 1$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.125. In Figure (2.39), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates if and only if it perfect-dominates, by Proposition (2.5.118) and S has one member;

(iii) all perfect-dominating sets corresponded to perfect-dominating number are $\{n_1\}, \{n_2\}, \{n_3\}$, and $\{n_4\}$. For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum cardinality between all perfect-dominating sets is called perfect-dominating number and it's denoted by $\mathcal{P}(CMT_\sigma) = 1$ and corresponded to perfect-dominating sets are $\{n_1\}, \{n_2\}, \{n_3\}$, and $\{n_4\}$;

(iv) there are five perfect-dominating sets

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\}, \{n_1, n_2, n_3, n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is characteristic;

(v) there are five perfect-dominating sets

$$\{n_1\}, \{n_2\}, \{n_3\},$$

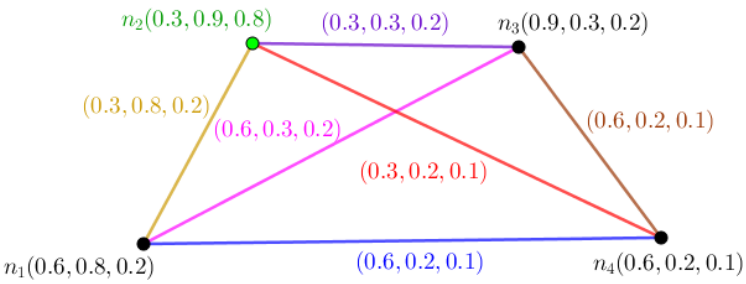

Figure 2.39: A Neutrosophic Graph in the Viewpoint of its perfect-dominating number and its neutrosophic perfect-dominating number.

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$$\{n_4\}, \{n_1, n_2, n_3, n_4\},$$

corresponded to perfect-dominating number as if there's one perfect-dominating set corresponded to neutrosophic perfect-dominating number so as neutrosophic cardinality is the determiner;

(vi) all perfect-dominating sets corresponded to perfect-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\},$$

For given vertex n, if $sn \in E$, then s perfect-dominates n where s is the unique vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-dominates n, then the set of neutrosophic vertices, S is called perfect-dominating set. The minimum neutrosophic cardinality between all perfect-dominating sets is called neutrosophic perfect-dominating number and it's denoted by $\mathcal{P}_n(CMT_\sigma) = 0.9$ and corresponded to perfect-dominating sets $\{n_4\}$.

Definition 2.5.126. (perfect-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called **perfect-resolving set**. The minimum cardinality between all perfect-resolving sets is called **perfect-resolving number** and it's denoted by $\mathcal{P}(NTG)$;

(ii) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called **perfect-resolving set**. The minimum neutrosophic cardinality between all perfect-resolving sets is called **neutrosophic perfect-resolving number** and it's denoted by $\mathcal{P}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

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Proposition 2.5.127. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and S has one member. Then a vertex of S resolves if and only if it perfect-resolves.*

Proposition 2.5.128. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and resolving set has one member. Then a vertex of resolving set corresponded to resolving number resolves if and only if it perfect-resolves.*

Proposition 2.5.129. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is corresponded to perfect-resolving number if and only if for all s in S, there are neutrosophic vertices n and n' in $V \setminus S$, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$ and for all neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S, such that $\{s' \mid d(s', n) \neq d(s', n')\} \cap S = \{s\}$.*

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Proposition 2.5.130. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V and $V \setminus \{x\}$ are S.*

Proposition 2.5.131. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{P}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of resolving set corresponded to resolving number resolves if and only if it perfect-resolves, by no vertices could be resolved in both settings of resolving and perfect-resolving. Thus, by Proposition (2.5.130), S has either $\mathcal{O}(CMT_\sigma) - 1$ or $\mathcal{O}(CMT_\sigma)$. All perfect-resolving sets corresponded to perfect-resolving number are

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\},$$
$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)}\},$$
$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\},$$
$$\ldots$$
$$\{n_2, n_3, n_4, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\},$$

For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum neutrosophic cardinality between all perfect-resolving sets is called neutrosophic perfect-resolving number and it's denoted by

$$\mathcal{P}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x)$$

and corresponded to perfect-resolving sets are

$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\},$$
$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)}\},$$
$$\{n_1, n_2, n_3, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\},$$
$$\ldots$$
$$\{n_2, n_3, n_4, \ldots, n_{\mathcal{O}(CMT_\sigma)-4}, n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\}.$$

Thus

$$\mathcal{P}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

■

Proposition 2.5.132. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then perfect-resolving number is equal to resolving number.*

Proposition 2.5.133. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of perfect-resolving sets corresponded to perfect-resolving number is equal to $\mathcal{O}(CMT_\sigma)$.*

Proposition 2.5.134. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of perfect-resolving sets is equal to $\mathcal{O}(CMT_\sigma) + 1$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.135. In Figure (2.40), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex, s, there's an edge with other vertices;
- (ii) in the setting of complete, a vertex of resolving set corresponded to resolving number resolves if and only if it perfect-resolves, by no vertices could be resolved in both settings of resolving and perfect-resolving. Thus, by Proposition (2.5.130), S has either $\mathcal{O}(CMT_\sigma) - 1$ or $\mathcal{O}(CMT_\sigma)$;
- (iii) all perfect-resolving sets corresponded to perfect-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\}$, and $\{n_2, n_3, n_4\}$. For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is

the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum cardinality between all perfect-resolving sets is called perfect-resolving number and it's denoted by $\mathcal{P}(CMT_\sigma) = 3$ and corresponded to perfect-resolving sets are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\}$, and $\{n_2, n_3, n_4\}$;

(iv) there are five perfect-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is characteristic;

(v) there are four perfect-resolving sets

$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\},$$
$$\{n_2, n_3, n_4\},$$

corresponded to perfect-resolving number as if there's one perfect-resolving set corresponded to neutrosophic perfect-resolving number so as neutrosophic cardinality is the determiner;

(vi) all perfect-resolving sets corresponded to perfect-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\}$, and $\{n_2, n_3, n_4\}$. For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s perfect-resolves n and n' where s is the unique vertex and d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in $V \setminus S$, there's only one neutrosophic vertex s in S such that s perfect-resolves n and n', then the set of neutrosophic vertices, S is called perfect-resolving set. The minimum neutrosophic cardinality between all perfect-resolving sets is called neutrosophic perfect-resolving number and it's denoted by $\mathcal{P}_n(CMT_\sigma) = 3.9$ and corresponded to perfect-resolving sets are $\{n_1, n_3, n_4\}$.

Definition 2.5.136. (total-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called **total-dominating set**. The minimum cardinality between all total-dominating sets is called **total-dominating number** and it's denoted by $\mathcal{T}(NTG)$;

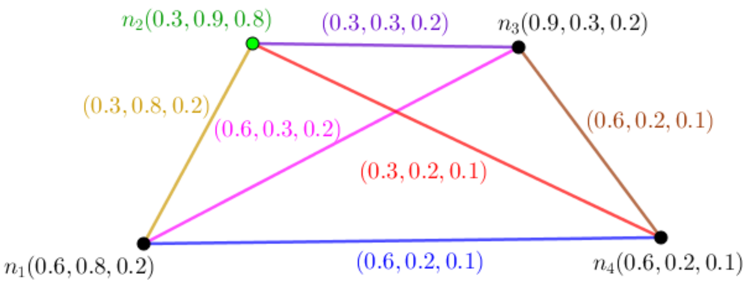

Figure 2.40: A Neutrosophic Graph in the Viewpoint of its perfect-resolving number and its neutrosophic perfect-resolving number. 84NTG2

(ii) for given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called **total-dominating set**. The minimum neutrosophic cardinality between all total-dominating sets is called **neutrosophic total-dominating number** and it's denoted by $\mathcal{T}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

Proposition 2.5.137. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then $|S| \geq 2$.*

Proposition 2.5.138. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{T}_n(CMT_\sigma) = \min_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself. All total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_1, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_1, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_2, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_2, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_2, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_3, n_4\}, \{n_3, n_5\}, \{n_3, n_6\}, \ldots, \{n_3, n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_3, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_3, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\ldots$$
$$\{n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\}$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum neutrosophic cardinality between all total-dominating sets is called neutrosophic total-dominating number and it's denoted by

$$\mathcal{T}_n(CMT_\sigma) = \min_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y))$$

and corresponded to total-dominating sets are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\}, \ldots, \{n_1, n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_1, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_1, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_2, n_5\}, \ldots, \{n_2, n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_2, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_2, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_3, n_4\}, \{n_3, n_5\}, \{n_3, n_6\}, \ldots, \{n_3, n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_3, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_3, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\ldots$$
$$\{n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_{\mathcal{O}(CMT_\sigma)-3}, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)-1}\}, \{n_{\mathcal{O}(CMT_\sigma)-2}, n_{\mathcal{O}(CMT_\sigma)}\}$$
$$\{n_{\mathcal{O}(CMT_\sigma)-1}, n_{\mathcal{O}(CMT_\sigma)}\}$$

Thus

$$\mathcal{T}_n(CMT_\sigma) = \min_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

■

Proposition 2.5.139. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then total-dominating number isn't equal to dominating number.*

Proposition 2.5.140. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of total-dominating sets corresponded to total-dominating number is equal to $\mathcal{O}(CMT_\sigma)$ choose two.*

Proposition 2.5.141. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of total-dominating sets is equal to $\mathcal{O}(CMT_\sigma)$ choose two plus $\mathcal{O}(CMT_\sigma)$ choose three plus one.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.142. In Figure (2.41), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't total-dominate since a vertex couldn't dominate itself;

(iii) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\}.$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum cardinality between all total-dominating sets is called total-dominating number and it's denoted by $\mathcal{T}(CMT_\sigma) = 2$ and corresponded to total-dominating sets are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\};$$

(iv) there are eleven total-dominating sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\},$$
$$\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}, \{n_1, n_3, n_4\},$$
$$\{n_2, n_3, n_4\}, \{n_1, n_2, n_3, n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is characteristic;

(v) there are six total-dominating sets

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\},$$

corresponded to total-dominating number as if there's one total-dominating set corresponded to neutrosophic total-dominating number so as neutrosophic cardinality is the determiner;

(vi) all total-dominating sets corresponded to total-dominating number are

$$\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\},$$
$$\{n_2, n_3\}, \{n_2, n_4\}, \{n_3, n_4\}.$$

For given vertex n, if $sn \in E$, then s total-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in V, there's at least a neutrosophic vertex s in S such that s total-dominates n, then the set of neutrosophic vertices, S is called total-dominating set. The minimum neutrosophic cardinality between all total-dominating sets is called neutrosophic total-dominating number and it's denoted by $\mathcal{T}_n(CMT_\sigma) = 2.3$ and corresponded to neutrosophic total-dominating sets are

$$\{n_3, n_4\}.$$

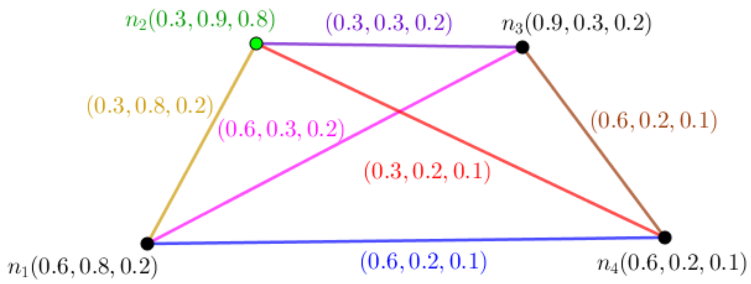

Figure 2.41: A Neutrosophic Graph in the Viewpoint of its total-dominating number and its neutrosophic total-dominating number. 85NTG2

Definition 2.5.143. (total-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices, $d \geq 1$ and all vertices have to be total-resolved otherwise it will be mentioned which is about $d \geq 0$ in some cases but all vertices have to be total-resolved forever. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called **total-resolving set**. The minimum cardinality between all total-resolving sets is called **total-resolving number** and it's denoted by $\mathcal{T}(NTG)$;

(ii) for given vertices n and n' if $d(s, n) \neq d(s, n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices, $d \geq 1$ and all vertices have to be total-resolved otherwise it will be mentioned which is about $d \geq 0$ in some cases but all vertices have to be total-resolved forever. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called **total-resolving set**. The minimum neutrosophic cardinality between all total-resolving sets is called **neutrosophic total-resolving number** and it's denoted by $\mathcal{T}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

Proposition 2.5.144. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then* $|S| \geq 2$.

86prp10 **Proposition 2.5.145.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then if there are twin vertices then total-resolving set and total-resolving number are Not Existed.*

Proposition 2.5.146. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{T}_n(CMT_\sigma) = \textit{Not Existed}.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve so as resolving is different from total-resolving and by Proposition (2.5.145), total-resolving set and total-resolving number are Not Existed. All total-resolving sets corresponded to total-resolving number are

$$\text{Not Existed.}$$

For given vertices n and n' if $d(s, n) \neq d(s, n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called total-resolving set. The minimum neutrosophic cardinality between all total-resolving sets is called neutrosophic total-resolving number and it's denoted by

$$\mathcal{T}_n(CMT_\sigma) = \text{Not Existed.}$$

and corresponded to total-resolving sets are

$$\text{Not Existed.}$$

Thus

$$\mathcal{T}_n(CMT_\sigma) = \text{Not Existed.}$$

∎

Proposition 2.5.147. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then total-resolving number isn't equal to resolving number.*

Proposition 2.5.148. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of total-resolving sets corresponded to total-resolving number is Not Existed.*

Proposition 2.5.149. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of total-resolving sets is Not Existed.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.150. In Figure (2.42), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of resolving set corresponded to resolving number resolves as if it doesn't total-resolve so as resolving is different from total-resolving and by Proposition (2.5.145), total-resolving set and total-resolving number are Not Existed;

(iii) all total-resolving sets corresponded to total-resolving number are

Not Existed.

For given vertices n and n' if $d(s,n) \neq d(s,n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called total-resolving set. The minimum cardinality between all total-resolving sets is called total-resolving number and it's denoted by $\mathcal{T}(CMT_\sigma) =$ Not Existed; and corresponded to total-resolving sets are

Not Existed;

(iv) there's no total-resolving set

Not Existed,

as if it's possible to have one of them as a set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is characteristic;

(v) there's no total-resolving set

Not Existed,

corresponded to total-resolving number as if there's one total-resolving set corresponded to neutrosophic total-resolving number so as neutrosophic cardinality is the determiner;

(vi) all total-resolving sets corresponded to total-resolving number are

Not Existed.

For given vertices n and n' if $d(s,n) \neq d(s,n')$, then s total-resolves n and n' where d is minimum number of edges amid two vertices. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n' in V, there's at least a neutrosophic vertex s in S such that s total-resolves n and n', then the set of neutrosophic vertices, S is called total-resolving set. The minimum neutrosophic cardinality between all total-resolving sets is called neutrosophic total-resolving number and it's denoted by $\mathcal{T}_n(CMT_\sigma) =$ Not Existed; and corresponded to total-resolving sets are

Not Existed.

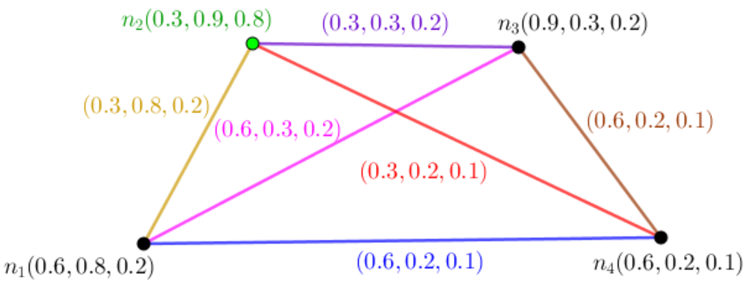

Figure 2.42: A Neutrosophic Graph in the Viewpoint of its total-resolving number and its neutrosophic total-resolving number. 86NTG2

Definition 2.5.151. (stable-dominating numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-dominating set**. The minimum cardinality between all stable-dominating sets is called **stable-dominating number** and it's denoted by $\mathcal{S}(NTG)$;

(ii) for given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-dominating set**. The minimum neutrosophic cardinality between all stable-dominating sets is called **neutrosophic stable-dominating number** and it's denoted by $\mathcal{S}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

86prp9 **Proposition 2.5.152.** *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Assume $|S|$ has one member. Then*

(i) *a vertex dominates if and only if it stable-dominates;*

(ii) *S is dominating set if and only if it's stable-dominating set;*

(iii) *a number is dominating number if and only if it's stable-dominating number.*

Proposition 2.5.153. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is stable-dominating set corresponded to stable-dominating number if and only if for every neutrosophic vertex s in S, there's at least a neutrosophic vertex n in $V \setminus S$ such that $\{s' \in S \mid s'n \in E\} = \{s\}$.*

Proposition 2.5.154. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V isn't S.*

Proposition 2.5.155. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then stable-dominating number is between one and $\mathcal{O}(NTG) - 1$.*

Proposition 2.5.156. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then stable-dominating number is between one and $\mathcal{O}_n(NTG) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x)$.*

Proposition 2.5.157. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{S}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates so as dominating is the same with stable-dominating, by Proposition (2.5.152), and S has one member. All stable-dominating sets corresponded to stable-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\}, \{n_4\}, \ldots, \{n_{\mathcal{O}(CMT_\sigma)-3}\}, \{n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_{\mathcal{O}(CMT_\sigma)-1\}}, \{n_{\mathcal{O}(CMT_\sigma)}\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum neutrosophic cardinality between all stable-dominating sets is called neutrosophic stable-dominating number and it's denoted by

$$\mathcal{S}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x)$$

and corresponded to stable-dominating sets are

$$\{n_1\}, \{n_2\}, \{n_3\}, \{n_4\}, \ldots, \{n_{\mathcal{O}(CMT_\sigma)-3}\}, \{n_{\mathcal{O}(CMT_\sigma)-2}\}, \{n_{\mathcal{O}(CMT_\sigma)-1\}}, \{n_{\mathcal{O}(CMT_\sigma)}\}.$$

Thus

$$\mathcal{S}_n(CMT_\sigma) = \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

■

Proposition 2.5.158. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then stable-dominating number is equal to dominating number.*

Proposition 2.5.159. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of stable-dominating sets corresponded to stable-dominating number is $\mathcal{O}(CMT_\sigma)$.*

Proposition 2.5.160. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of stable-dominating sets is $\mathcal{O}(CMT_{\sigma})$.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.161. In Figure (2.43), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates if and only if it stable-dominates so as dominating is the same with stable-dominating, by Proposition (2.5.152), and S has one member;

(iii) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum cardinality between all stable-dominating sets is called stable-dominating number and it's denoted by $\mathcal{S}(CMT_{\sigma}) = 1$; and corresponded to stable-dominating sets are

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\};$$

(iv) there are four stable-dominating sets

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\},$$

as if it's possible to have one of them as a set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four stable-dominating sets

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\},$$

corresponded to stable-dominating number as if there's one stable-dominating set corresponded to neutrosophic stable-dominating number so as neutrosophic cardinality is the determiner;

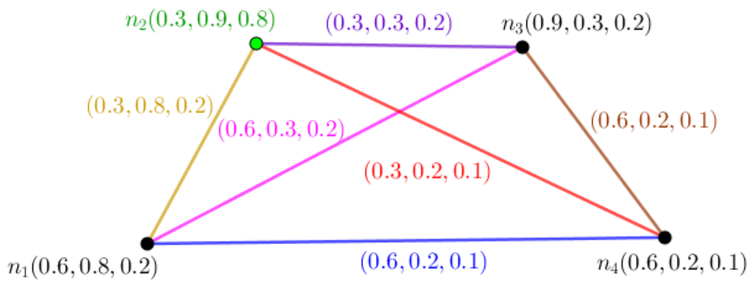

Figure 2.43: A Neutrosophic Graph in the Viewpoint of its stable-dominating number and its neutrosophic stable-dominating number.

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(vi) all stable-dominating sets corresponded to stable-dominating number are

$$\{n_1\}, \{n_2\}, \{n_3\},$$
$$\{n_4\}.$$

For given vertex n, if $sn \in E$, then s stable-dominates n. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex n in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-dominates n where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-dominating set. The minimum neutrosophic cardinality between all stable-dominating sets is called neutrosophic stable-dominating number and it's denoted by $\mathcal{S}_n(CMT_\sigma) = 0.9$; and corresponded to stable-dominating sets are

$$\{n_4\}.$$

Definition 2.5.162. (stable-resolving numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **stable-resolving set**. The minimum cardinality between all stable-resolving sets is called **stable-resolving number** and it's denoted by $\mathcal{S}(NTG)$;

(ii) for given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called **neutrosophic stable-resolving set**. The minimum neutrosophic cardinality between all stable-resolving sets is called **neutrosophic stable-resolving number** and it's denoted by $\mathcal{S}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

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Proposition 2.5.163. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Assume $|S|$ has one member. Then*

(i) *a vertex resolves if and only if it stable-resolves;*

(ii) *S is resolving set if and only if it's stable-resolving set;*

(iii) *a number is resolving number if and only if it's stable-resolving number.*

Proposition 2.5.164. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then S is stable-resolving set corresponded to stable-resolving number if and only if for every neutrosophic vertex s in S, there are at least neutrosophic vertices n and n' in $V \setminus S$ such that $\{s' \in S \mid d(s', n) \neq d(s', n')\} = \{s\}$.*

Proposition 2.5.165. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V isn't S.*

Proposition 2.5.166. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{S}_n(CMT_\sigma) = \textit{Not Existed.}$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve so as resolving is different from stable-resolving. Stable-resolving set and stable-resolving number are Not Existed. All stable-resolving sets corresponded to stable-resolving number are

$$\text{Not Existed.}$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum neutrosophic cardinality between all stable-resolving sets is called neutrosophic stable-resolving number and it's denoted by

$$\mathcal{S}_n(CMT_\sigma) = \text{Not Existed;}$$

and corresponded to stable-resolving sets are

$$\text{Not Existed.}$$

Thus

$$\mathcal{S}_n(CMT_\sigma) = \text{Not Existed.}$$

■

Proposition 2.5.167. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then stable-resolving number isn't equal to resolving number.*

Proposition 2.5.168. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of stable-resolving sets corresponded to stable-resolving number is Not Existed.*

Proposition 2.5.169. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then the number of stable-resolving sets is Not Existed.*

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.170. In Figure (2.44), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex, s, there's an edge with other vertices;

(ii) in the setting of complete, a vertex of resolving set corresponded to resolving number resolves as if it doesn't stable-resolve so as resolving is different from stable-resolving. Stable-resolving set and stable-resolving number are Not Existed;

(iii) all stable-resolving sets corresponded to stable-resolving number are

Not Existed.

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum cardinality between all stable-resolving sets is called stable-resolving number and it's denoted by $\mathcal{S}(CMT_\sigma) =$ Not Existed; and corresponded to stable-resolving sets are

Not Existed;

(iv) there's no stable-resolving set

Not Existed,

so as it's possible to have nothing as a set corresponded to neutrosophic stable-resolving number as if neutrosophic cardinality is characteristic;

(v) there's no stable-resolving set

Not Existed,

corresponded to stable-resolving number so as there's no stable-resolving set corresponded to neutrosophic stable-resolving number as if neutrosophic cardinality is the determiner;

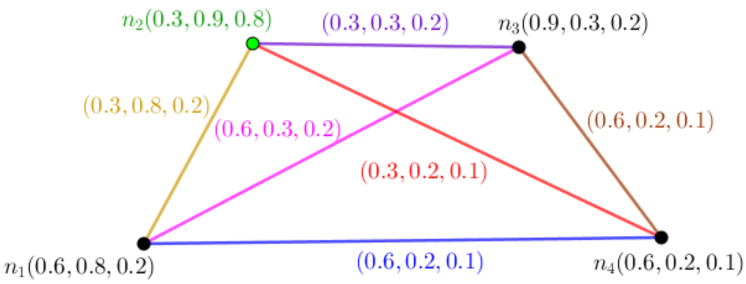

Figure 2.44: A Neutrosophic Graph in the Viewpoint of its stable-resolving number and its neutrosophic stable-resolving number. 88NTG2

(vi) all stable-resolving sets corresponded to stable-resolving number are

$$\text{Not Existed.}$$

For given vertices n and n', if $d(s, n) \neq d(s, n')$, then s stable-resolves n and n'. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertices n and n', in $V \setminus S$, there's at least a neutrosophic vertex s in S such that s stable-resolves n and n' where for all given two vertices in S, there's no edge between them, then the set of neutrosophic vertices, S is called stable-resolving set. The minimum neutrosophic cardinality between all stable-resolving sets is called neutrosophic stable-resolving number and it's denoted by $\mathcal{S}_n(CMT_\sigma) = \text{Not Existed}$; and corresponded to stable-resolving sets are

$$\text{Not Existed.}$$

2.6 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

2.7 Modelling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

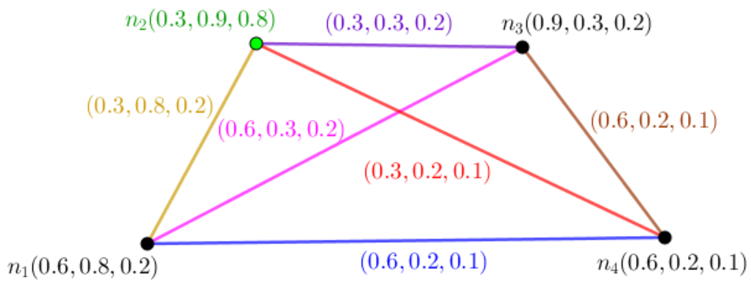

Figure 2.45: A Neutrosophic Graph in the Viewpoint of its joint-resolving number and its neutrosophic joint-resolving number 88NTG20

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (2.1), clarifies about the assigned numbers to these situations.

Table 2.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model. 88tbl1

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

2.8 Case 1: Complete-Model

Step 4. (Solution) The neutrosophic graph alongside its stable-resolving number and its neutrosophic stable-resolving number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its stable-resolving number and its neutrosophic stable-resolving number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are five subjects which are represented as Figure (2.45). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its stable-resolving number and its neutrosophic stable-resolving number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (2.45). In Figure (2.45), an complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, s and s', there's an edge between them;

(ii) Every given two vertices are twin since for all given two vertices, every of them has one edge from every given vertex thus minimum number of edges amid all paths from a vertex to another vertex is forever one;

(iii) all joint-resolving sets corresponded to joint-resolving number are $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's a path in S amid them. The minimum cardinality between all joint-resolving sets is called joint-resolving number and it's denoted by $\mathcal{J}(CMT_\sigma) = 3$;

(iv) there are four joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, $\{n_1, n_3, n_4\}$, and $\{n_1, n_2, n_3, n_4\}$ as if it's possible to have one of them as a set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is characteristic;

(v) there are three joint-resolving sets $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ corresponded to joint-resolving number as if there's one joint-resolving set corresponded to neutrosophic joint-resolving number so as neutrosophic cardinality is the determiner;

(vi) all joint-resolving sets corresponded to neutrosophic joint-resolving number are $\{n_1, n_3, n_4\}$. For given two vertices n and n', if $d(s, n) \neq d(s, n')$, then s joint-resolves n and n' where d is the minimum number of edges amid all paths from the vertex and the another vertex. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] like either of $\{n_1, n_2, n_3\}, \{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$. If for every neutrosophic vertices n and n' in $V \setminus S$, there's at least one neutrosophic vertex s in S such that s joint-resolves n and n', then the set of neutrosophic vertices, S is either of $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_1, n_3, n_4\}$ is called joint-resolving set where for every two vertices in S, there's

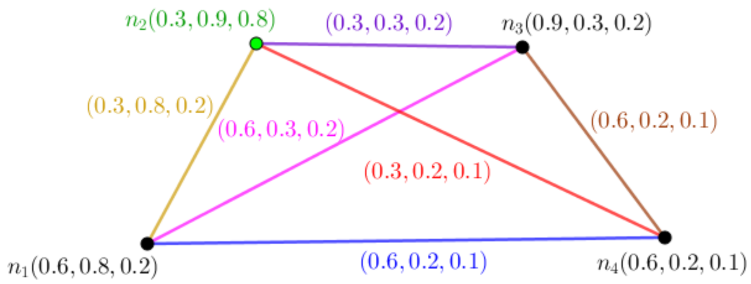

Figure 2.46: A Neutrosophic Graph

88NTG21

a path in S amid them. The minimum neutrosophic cardinality between all joint-resolving sets is called neutrosophic joint-resolving number and it's denoted by $\mathcal{J}_n(CMT_\sigma) = 3.9$.

2.9 Case 2: Complete Model alongside its Neutrosophic Graph

Step 4. (Solution) The neutrosophic graph alongside its stable-resolving number and its neutrosophic stable-resolving number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its stable-resolving number and its neutrosophic stable-resolving number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are four subjects which are represented in the formation of one model as Figure (2.46). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its stable-resolving number and its neutrosophic stable-resolving number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (2.46). There is one section for clarifications.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

2.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, are defined in complete-neutrosophic graphs. Thus,

Question 2.10.1. *Is it possible to use other types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable?*

Question 2.10.2. *Are existed some connections amid different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs?*

Question 2.10.3. *Is it possible to construct some classes of complete-neutrosophic graphs which have "nice" behavior?*

Question 2.10.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 2.10.5. *Which parameters are related to this parameter?*

Problem 2.10.6. *Which approaches do work to construct applications to create independent study?*

Problem 2.10.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

2.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this

study are highlighted.
This study uses some definitions concerning different types of neutrosophic zero-forcing, neutrosophic independence, neutrosophic clique, neutrosophic matching, neutrosophic girth, neutrosophic cycles, neutrosophic connectivity, neutrosophic density, neutrosophic path-coloring, neutrosophic duality, neutrosophic join, neutrosophic perfect, neutrosophic total, neutrosophic stable, in complete-neutrosophic graphs assigned to complete-neutrosophic graphs. Further studies

Table 2.2: A Brief Overview about Advantages and Limitations of this Study 88tbl

Advantages	Limitations
1. Neutrosophic Numbers of Model	1. Connections amid Classes
2. Acting on All Edges	
3. Minimal Sets	2. Study on Families
4. Maximal Sets	
5. Acting on All Vertices	3. Same Models in Family

could be about changes in the settings to compare these notions amid different settings of complete-neutrosophic graphs. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2.2), some limitations and advantages of this study are pointed out.

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