Multi-Objective Evolutionary Algorithms and Pattern Search Methods for Circuit Design Problems

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Abstract: The paper concerns the design of evolutionary algorithms and pattern search methods on two circuit design problems: the multi-objective optimization of an Operational Transconductance Amplifier and of a fifth-order leapfrog filter. The experimental results obtained show that evolutionary algorithms are more robust and effective in terms of the quality of the solutions and computational effort than classical methods. In particular, the observed Pareto fronts determined by evolutionary algorithms has a better spread of solutions with a larger number of nondominated solutions when compared to the classical multi-objective techniques.

Key Words: Evolutionary electronics, multi-objective optimization, circuit design problems, evolutionary algorithms, genetic algorithms, classical optimization methods, pattern search methods, operational transconductance amplifier, leapfrog filter.

Category: I.2.8, J.2, B.7.1, B.7.2

1 Introduction

In this research work we experimentally compare the effectiveness of multi-objective evolutionary algorithms and standard optimization techniques applied to problems arising from Analog Circuit Design: the optimization of a Operational Transconductance Amplifier and of a fifth-order leapfrog filter. The former problem is dealt with multi-objective optimization, the latter with multi-objective optimization for constraint satisfaction based on a simulation-based synthesis of circuit [1]. Both problems are real world applications which have been supplied by STMicroelectronics.

Classical methods are characterized as being approximation-based, or better by the fact that they find an individual solution in a single run. On the contrary,
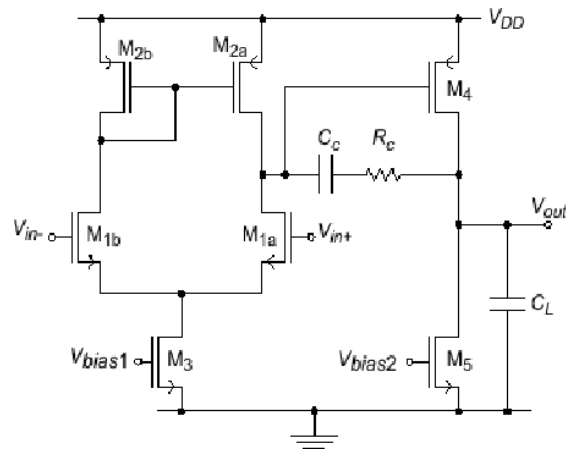
Figure 1: OTA circuit model.

evolutionary algorithms are stochastic population-based, optimization methods, inspired by natural selection and able to find several solutions in a single run. Thus they are considered a good alternative to standard methods [2]. To assess the quality of experimental results a class of metrics has been used to characterize the various approaches and to make proper comparisons.

In section 2, we present the design of an analog circuit, the *Operational Transconductance Amplifier*, modeled as multi-objective optimization problem. In this section we describe the algorithms and metrics used, and report comparisons among the various algorithms. In section 3, the circuit design problem of *pass-band filter* will be introduced and a comparison with an industrial design tool will be made. Finally, section 4 concludes the article with some general remarks.

2 Two-Stage Operational Transconductance Amplifier

In microelectronic design a considerably time is spent on device sizing of the analog circuit to satisfy the performance requirements. The main reason is due to the non-linear relation between device size and performances [3, 4]. In order to improve the efficiency of the device design in the analog circuits, a multi-objective approach has been proposed as alternative to the functional cost approach. The multi-objective formulation avoids the necessity of weighing different objectives in a single cost function which cannot phase completely the analysis of the problem. In this study, the optimization process is coupled to a circuit simulator (SPICE) in order to evaluate the circuit's performances [1].

The SPICE simulator version used in our experiments is NGSPICE [5], which

Parameters	Ranges	Unit
W1b = W1a	7 - 20	μm
W3	7 - 20	μm
W5	7 - 20	μm
L	0.525 - 0.875	μm
C	3 - 5	pF
R	20 - 40	KΩ
W4	7 - 20	μm
W2b = W2a	7 - 20	μm
I	1 - 15	μA

Table 1: Parameters.

Objectives	Specifications
Power Consumption	minimize
Total Width	minimize
Unity Gain Frequency	maximize
Gain at 100 Hz	maximize
Phase Margin	maximize

Table 2: Conflicting objectives for the problem.

implements the BSIM3 MOSFET model for the I-V characterization [6],[7]. The nominal temperature of the simulations was 27°C. The parameters of the model card for PMOS and NMOS transistors are the "default" parameters.

This case study proposes the MOS device sizing and the circuit net setting of an two-stage Operational Transconductance Amplifier (OTA). The OTA is a useful device and it is used with few other devices to realize, for instance, filters, comparators, wave generators, converters.

The parameters and their ranges are shown in Table 1. The "W" parameters refers to the MOS channel width, L is referred to the MOS channel length, R (resistance) and C (capacity) are referred to the circuit net parameters. Minimum performance specifications are formulated with the constraints in Table 3. Finally, the conflicting objectives are showed in Table 2.

Many important performance metrics are considered in the OTA design. Those used in this study are the following. *Low frequency gain* is the gain at 100 Hz, which is the base of the amplification range. *Unity Gain Frequency* is defined as the frequency range where the amplifier has at least the unity gain. *Phase Margin* is a quality measure for the circuit because it is related to the parasitics effects, like cross coupling, which cause the failure in the attainment of the performance. The *circuit area* is an important specification for the design

Objectives	Specifications	Unit
Unity Gain Frequency	> 31.221	Mhz
Gain at 100 Hz	> 64.118	dB
Phase Margin	> 60	Degree

Table 3: Constraints of the objective performances.

because it is related to the yield of the manufacturing process. Generally, if it is possible to increase the number of circuits per unit area then the yield of the manufacturing process increases and the ratio between the number of functional failures and total number of circuits per unit area decreases. In our case we used an underestimation given by the sum of the MOS widths. *Power Consumption* is today an important performance for all system in which the power is supplied by a battery.

2.1 Multi-objective DIRECT

The multi-objective DIRECT (MODIRECT) method is an extension to the multi-objective case of the DIRECT algorithm [8]. The method is based on three operations: Lipschitz constant estimation, choice for potential optimality of domain subregions, and domain subdivision. The choice for potential optimality is based on the estimation of Lipschitz constant for the objective function in a partition of the domain. This partition is built by hyperrectangles, which are sampled in their centers in order to evaluate the value of the objective function. Therefore the estimation of Lipschitz constant leads to a possible choice of the hyperrectangles in the partition for a further sampling. In the main loop of the algorithm, hyperrectangles are selected for sampling if they have a large area, an high Lipschitz constant estimation, and a good value of the function in their center. Formally it is possible to give the following definition for the single objective problem in one variable:

Definition 1 *[**Potential optimality relative to the objective** i] Let $\mathcal{S}$ be the set of hyperrectangles generated by the algorithm after k iterations, and let f^{min} and f^{max} be respectively the ideal and nadir points of the cone centered in $f(c_{\tilde{R}})$. An hyperrectangle $\tilde{R} \in \mathcal{S}$ with center $c_{\tilde{R}}$ and measure $\alpha(\tilde{R})$ is said potentially partial optimal relative to the i-th objective if there exists at least a Lipschitz constant $K_i^{lower} > 0$ such that*

$$f_i(\mathbf{c}_{\tilde{R}}) - K_i^{lower}\alpha(\tilde{R}) \leq f_i(\mathbf{c}_R) - K_i^{lower}\alpha(R) \tag{1}$$

$$f_i(\mathbf{c}_{\tilde{R}}) - K_i^{lower}\alpha(\tilde{R}) \leq f_i^{min} - \varepsilon|f_i^{min}|. \ \forall R \in \mathcal{S} \tag{2}$$

or a constant $K_i^{upper} > 0$ such that

$$f_i(\mathbf{c}_{\tilde{R}}) + K_i^{upper}\alpha(\tilde{R}) \leq f_i(\mathbf{c}_R) + K_i^{upper}\alpha(R) \tag{3}$$

$$f_i(\mathbf{c}_{\tilde{R}}) + K_i^{upper}\alpha(\tilde{R}) \leq f_i^{max} - \varepsilon|f_i^{max}|. \; \forall R \in \mathcal{S} \tag{4}$$

where $\varepsilon \sim 10^{-4}$ is a constant to control the clustering *during the search [8].*

This definition is easily extendible to the case of n variables.

In order to obtain the heuristic which extends the above definition to the multi-objective case, let us redefine the Pareto optimality in general terms of efficiency.

Definition 2 *[**Efficiency criterion**] A decision vector $x^* \in X$ is* efficient *with respect to the convex cone D if there does not exist another decision vector $x \in X$ such that*

$$f(x^*) - f(x) \in D \tag{5}$$

The cone D is called *ordering cone* and if $D = R_+^n$ the efficiency criterion produces a partial ordering for the Pareto optimality criterion. This ordering is used by the algorithm as surrogate of linear ordering.

Remark 2.1 *[**Multiple estimation of the Lipschitz constants**] Starting from the conditions 1 and 3 in 1 it is possible to define the multi-objective optimality in terms of expected efficiency. For every objective i, from the above conditions we obtain estimates for K_i^{lower} in the form of an upper bound $\overline{K}_i^{lower} \geq 0$ and a lower bound $\underline{K}_i^{lower} \geq 0$ for K_i^{lower}. Analogously, for K_i^{upper} there will be an upper bound $\overline{K}_i^{upper} \geq 0$ and a lower bound $\underline{K}_i^{upper} \geq 0$.*

The heuristic criterion leading to the choice of the optimal hyperrectangles in the multi-objective is motivated by the potential increase of the expected efficiency.

Definition 3 *[**Multi-objective Potential optimality**] Given the estimations of the upper bounds and the lower bounds for the Lipschitz constant of every objective i in the cone centered in $f(c_{\tilde{R}})$, the hyperrectangle $\tilde{R}$ is said potentially optimal if*

$$\sqrt{\sum_{i=1}^{k}[\underline{K}_i^{lower}]^2} \leq \sqrt{\sum_{i=1}^{k}[\overline{K}_i^{lower}]^2} \tag{6}$$

or

$$\sqrt{\sum_{i=1}^{k}[\underline{K}_i^{upper}]^2} \leq \sqrt{\sum_{i=1}^{k}[\overline{K}_i^{upper}]^2} \tag{7}$$

Moreover, let f^{min} and f^{max} be respectively the ideal and nadir points of the cone centered in $f(c_{\tilde{R}})$. The choice of hyperrectangle $\tilde{R}$ leads to a non trivial improvement of objective functions

$$\sum_{i=1}^{k}[f_i(\mathbf{c}_{\tilde{R}}) - K_i^{lower}\alpha(\tilde{R})]^2 \leq \sum_{i=1}^{k}[f_i^{min} - \varepsilon|f_i^{min}|]^2 \qquad (8)$$

or

$$\sum_{i=1}^{k}[f_i(\mathbf{c}_{\tilde{R}}) + K_i^{upper}\alpha(\tilde{R})]^2 \leq \sum_{i=1}^{k}[f_i^{max} - \varepsilon|f_i^{max}|]^2 \qquad (9)$$

The above definition gives a heuristic rule to choose hyperrectangles, which are potentially optimal in the sense of either increasing the efficiency of the objective vector or taking into account possible trade-off (the latter arises from considering both lower and upper bounds for the Lipschitz constant). Equations 8 and 9 can be interpreted as controlling the clustering nearby the optimal points.

If an hyperrectangle is potential optimal then it will be sampled in the points $\mathbf{c} \pm \delta\mathbf{e}_i$, $i = 1 \ldots N$, where $\mathbf{c}$ is the center point of the hyperrectangle, δ is one-third the side length of the hyperrectangle, and $\mathbf{e}_i$ is the ith unit vector. It must be remarked that the criterion for multi-objective optimality uses the l_2 norm to synthesize the choice from several estimations (see Eqns. 6, 7, 8, 9). The l_α norm used in this synthesis characterizes the expectation of the Pareto front in terms of convexity. Possibly, the magnitude of α that is required can be related to a measure of the nonconvexity of the Pareto front [9]. Afterwards the hyperrectangle will be subdivided in thirds along its widest sides based on a dominance sorting of function values $f(\mathbf{c} \pm \delta\mathbf{e}_i)$ with respect their efficiency. This strategy increases the attractiveness of searching near points with good function values into the large hyperrectangles.

2.2   Evolutionary Algorithms for Constrained Multi-Objective Optimization

Evolutionary algorithms (EAs) have become one of the main methods for exploring the Pareto-optimal front in multi-objective optimization problems that are too complex to be solved by exact methods, such as linear programming and gradient search [10, 11]. This is not only because there are few alternatives for searching intractably large spaces for multiple Pareto-optimal solutions. Due to their inherent parallelism and their capability to exploit similarities of solutions by recombination, they are able to approximate the Pareto-optimal front in a single optimization run. The numerous applications and the rapidly growing interest in the area of multi-objective EAs take this fact into account.

Two well-known multi-objective evolutionary algorithms (MOEAs) [10, 12] are compared, NSGA2 and SPEA2 with the multi-objective DIRECT algorithm in terms of hypervolume metric (see subsection 2.3). Both evolutionary algorithms use the constrained tournament selection to deal with unfeasible solution during the optimization process.

NSGA2 [13] is a elitist evolutionary algorithm with a fast nondominated sorting procedure and a density estimation of the solutions provided by the crowding distance. SPEA2 has a fitness assignment scheme based on the Pareto dominance relation with a density estimation technique based on the "k-th nearest neighbour" [14].

One difficult matter in constrained optimization problems is find a feasible set. In the first steps it could represent a true challenge. One of the possible reasons is that feasible region could be a very small subset in the search region. NSGA2 and SPEA2 use a technique called "constrained tournament selection" proposed by Deb [12] to solve this problem.

This method use binary tournament selection, that is, two individual of the population are chosen and compared and the fitter in copied in the next population. When a problem presents constraints two solutions can be feasible or unfeasible, and then just one of these case is possible: (i) both are feasible; (ii) one solution is feasible and the other is not; (iii) both are infeasible.

In multi-objective problems case (i) is solved using a dominance relation which takes in account the constraint violation. In case (ii) only the feasible solution is chosen and in case (iii) is used a penalty function (see equation 11)

Let $g_j(\mathbf{x}) \geq 0, j = 1 \ldots m$ be the normalized constraints of the given problem. The constraints violation is defined as follow:

$$\omega_j = \begin{cases} |g_j(\mathbf{x})|, & \text{if } g_j(\mathbf{x}) < 0; \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

The overall violation Ω is defined as:

$$\Omega = \sum_{j=1}^{m} \omega_j(\mathbf{x}) \tag{11}$$

A solution $\mathbf{x}_i$ is said to "constrain dominate" a solution $\mathbf{x}_j$ if one of these conditions is true:

1. Solution $\mathbf{x}_i$ is feasible and $\mathbf{x}_j$ is not.

2. Solutions $\mathbf{x}_i$ and $\mathbf{x}_j$ are infeasible but $\mathbf{x}_i$ has a lesser Ω value.

3. Solutions $\mathbf{x}_i$ and $\mathbf{x}_j$ are both feasible, but $\mathbf{x}_i$ dominate $\mathbf{x}_j$.

In a constrained tournament selection, the individual having a lesser Ω value win the tournament.
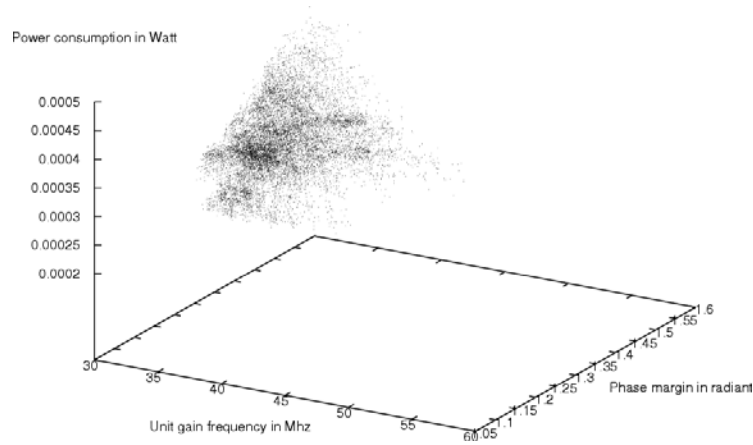
Figure 2: Trade-off among unity gain frequency, phase margin and power of MODirect.

2.3 Hypercubes and Hypervolume metric

The *Hypervolume metric* [15] compute the volume covered by a set of non dominated elements Q obtained by the algorithm at the end of the optimization process. It is computed as follow: for each solution $i \in Q$ an hypercube v_i is built between a reference point W (in this study W is the axis origin) and the solution i. The union of all the hypercubes is computed and its hypervolume (HV) is: $HV = volume(\cup_{i=1}^{|Q|} v_i)$. A large value of HV is expected from a good algorithm, however, Veldhuizen [16], report that a such metric is not useful if the optimal Pareto front is not convex.

The hypervolume metric is not a general rule for circuit design problems. Moreover, the metric implicitly assume the objective space origin coordinates of the problem are $(0, 0, \ldots, 0)$, but this is not always the general case.

The hyperarea metric do not use the true Pareto Front (as Error Ratio, Generational Distance metric, Maximum Pareto Front Error and Average Pareto Front Error) [10]. The drawback is that compute the area of objective value space covered by PF_{known} is time consuming. Recently has been published an improved procedure to compute the metric with lower computational effort [17].

2.4 Results

In this section, using the hypervolume metric, we provide a comparison of NSGA2, SPEA2 and MODirect to multi-objective optimization of the Operational Transconductance Amplifier. Goal of our experimental protocol is to
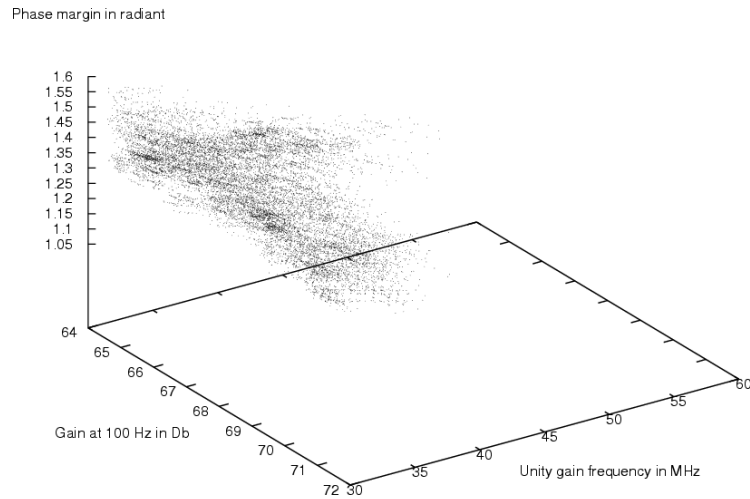
Phase margin in radiant

Figure 3: Trade-off among gain, unity gain frequency and phase margin of MODirect.

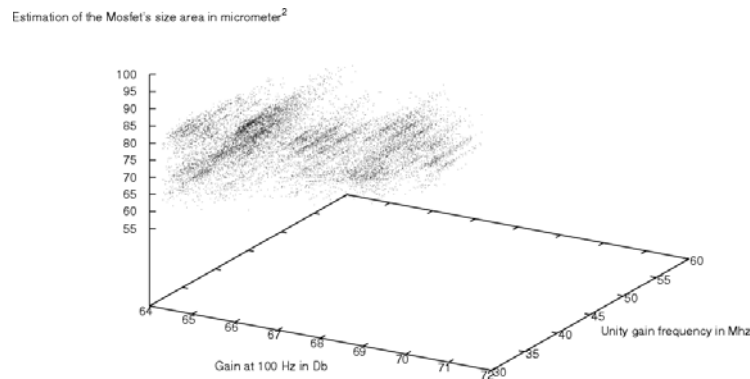Estimation of the Mosfet's size area in micrometer^2

Figure 4: Trade-off among gain, unity gain frequency and Mosfet's area of MODirect.

determine the algorithms that obtain large value of HV, that is good algorithms in term of a well-known metric.

The optimization drives a flow of simulations which is based on an interaction of the optimization algorithm with the circuit simulator (Spice). Every interaction is a circuit evaluation which consist of one (or more) circuit simulations. The stopping criterion used by MOEAs and Lipschitzian pattern search strategies is the maximum number of function evaluations T_{max}, also called fitness function evaluations.
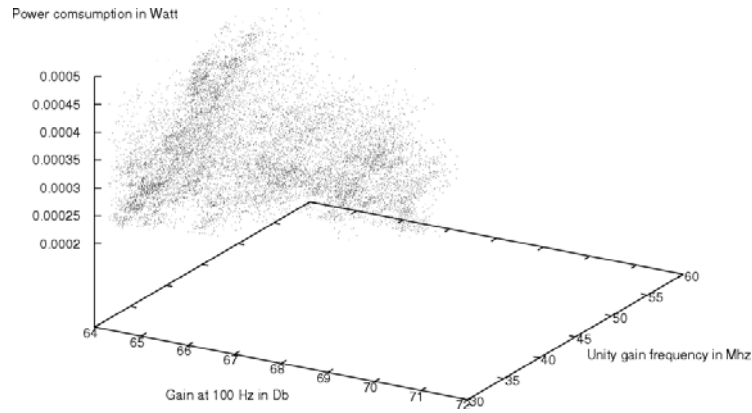
Figure 5: Trade-off among gain, unity gain frequency and power of MODIRECT.
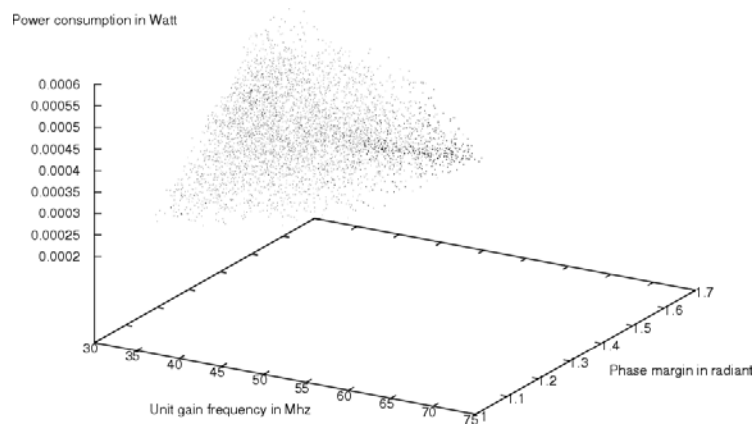

Figure 6: Trade-off among unity gain frequency, phase margin and power of NSGA2.

NSGA2 and SPEA2 parameters are: crossover probability $p_c = 0.9$, mutation probability $p_m = 1$. Both MOEAs use SBX crossover with index $\eta_c = 2$ and gaussian mutation with $\sigma \in \{0.01, 0.1, 0.4\}$. These values have been obtained by a preliminary parameter tuning (not shown due to space limit), the gaussian mutation parameter, σ, seems to be the most relevant parameter. Each algorithm is stopped after 6×10^5 function evaluations. The hypervolume values obtained by the tested algorithms are the following: $HV = 6.402e + 07$ (MODIRECT), $HV = 1.048e + 08$ (SPEA2), $HV = \mathbf{1.051e + 08}$ (NSGA2). The results clearly shows as the MOEAs obtain a larger value of HV with respect MODIRECT.

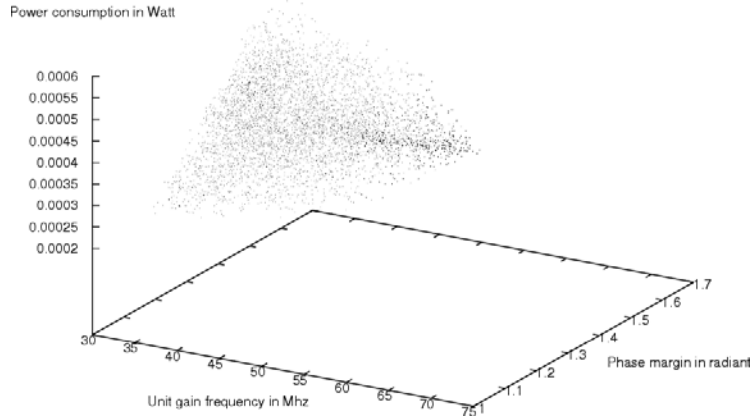The figures 2–9 show the trade-off among triples of objectives. These plots

Figure 7: Trade-off among gain, unity gain frequency and phase margin of NSGA2.
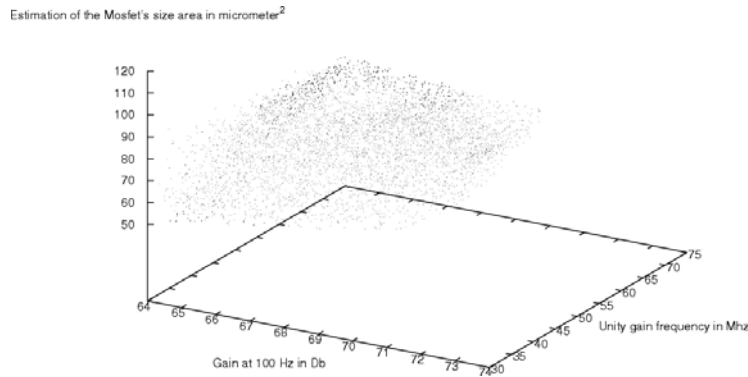

Figure 8: Trade-off among gain, unity gain frequency and Mosfet's area of NSGA2.

characterize the different compromises realized to the solutions discovered by the algorithms. It is possible to see the increasing/decreasing effects of a performance on the others and the dependencies among different specifications. For instance we may assume that the power consumption will increase when the unity-gain frequency increases and the phase margin increase as it can be see in 2D projection of figure 10.

It is easy to note as both MOEAs obtain wider Pareto fronts with respect MODIRECT. In particular, for the Frequency objective function, NSGA2 (analogously SPEA2, not shown) reached non dominated solutions in the ranges from 30 to 75 Mhz, where MODIRECT obtain frequencies in the domain from
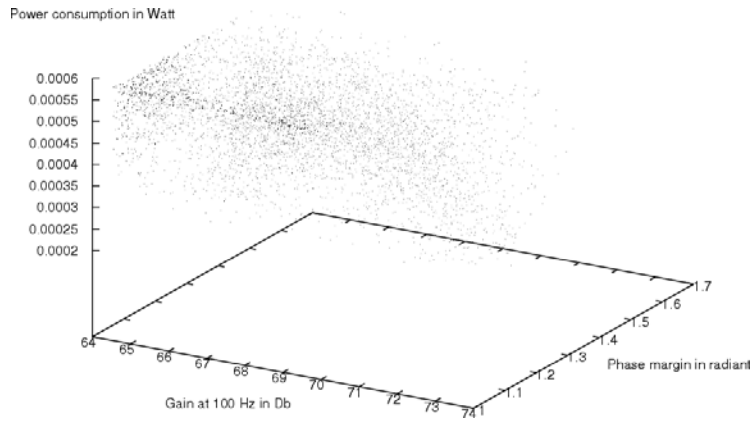
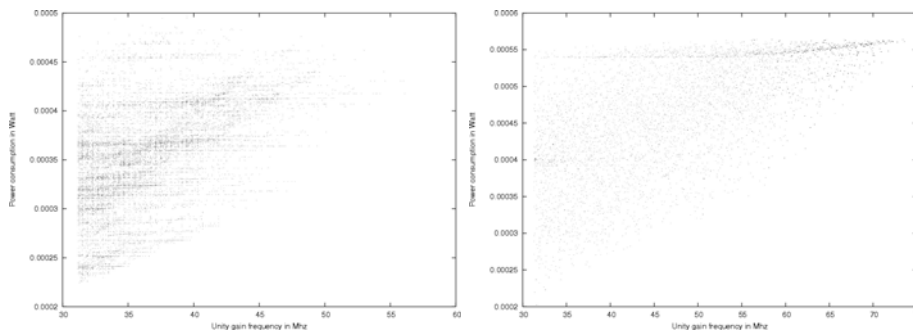**Figure 9:** Trade-off among gain, phase margin and power of NSGA2.

Figure 10: Pareto Fronts between unity gain frequency and power of MODirect (left plot), and NSGA2.

30 to 55 Mhz. Analogously, for the gain objective function, the multi-objective evolutionary algorithms outperform the deterministic optimization algorithm, MODirect. Same conclusions can be reached plotting any other pair of objective functions.

It must be also notice the different point distributions of the proposed algorithms, particularly the spread of the Pareto Front. This phenomenon is related to the convexity or nonconvexity of the multi-objective problem in different regions of the domain which affects the optimization strategies of the algorithms. It can be see a comparable result for NSGA2 and SPEA2 and a totally unlike result of MODirect. In fact, the potentially optimal criterion of MODirect (3) favours convex region of the domain and gives a low priority to the nonconvex regions. This property can explain away the reduce value of the hypervolume.
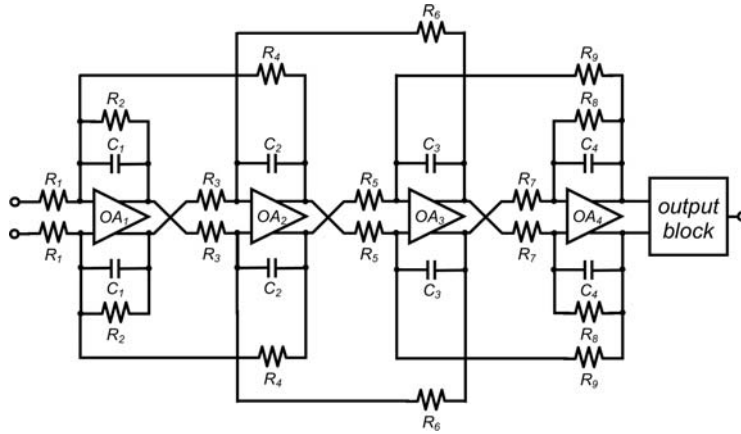
Figure 11: Leapfrog filter schematic view.

The results could be useful to characterize the particular solution sets of the Pareto optimal front which identifies a convex region for the multi-objective problem.

3 Leapfrog Filter

The circuit block employed as test case, i.e. a fifth-order leapfrog filter, is depicted in figure 11. Four leapfrog loops are realized through 4 (equal) differential operational amplifiers (OA_1–OA_4), 18 resistances (two times R_1–R_9), and 8 capacitances (two times C_1–C_4). Differential-to-single conversion is performed in the output block which also implements an additional time constant via a real pole.

Design of the filter involves finding the resistance and capacitance values that satisfy the constraints imposed by a given circuit application. Resistance and capacitance values are actually calculated on the base of other quantities that will be treated as input variables of the optimization problem. Expressions relating resistance and capacitance values to these quantities are quite complex and will not be reported in the following. The 20 variables used as input of the optimization problem are: $k1$, $k2$, $k3$, $k4$, $m1$, $m2$, $m3$, $m4$, $Vn1$, $Vn2$, $Vn3$, $Vn4$, $w0$, wrp, Ra, C, $C1$, $L2$, $C3$, $L4$. The first 4 parameters ($k1-k4$) influence the output dynamic of each operational amplifier, $m1-m4$ allow scaling of the resistances and capacitances leaving unchanged the leapfrog time constants, $Vn1-Vn4$ impose the equivalent input noise of each operational amplifier, $w0$ provides a frequency shift of the filter transfer function, wrp is the frequency of the real pole in the output block, and Ra through $L4$ determine the filter time

constants.

The set of target functions and related constraints employed in the formulation of the optimization problem are reported in table 4 (first two columns).

The first five target functions (i.e., *DC gain, pass-band ripples* and *stop-bands*) are directly related to the frequency mask of the filter. They are measured in dB (decibel), dB_{PP} (peak-to-peak dB) or dB_C (dB with respect to the carrier frequency). *Group delay* is a measure of how long it takes a signal to traverse a network. It is expressed in units of time. *Equivalent input resistance* is measured as the ratio between the AC voltage and current at the input terminals of the filter. *Output dynamic* is measured as the maximum AC voltage at the output of each operational amplifier. *Equivalent input noise* is a measure of the total noise generated by the circuit. It is due to both active components of the operational amplifiers and external resistances ($R_1 - -R_9$). Although noise should in general be minimized in any integrated circuit, providing an upper bound allows other (usually conflicting) constraints to be more easily satisfied. *Inverse squared noise* is computed as the sum of $(1/Vni)^2$ for $i = 1 \ldots 4$. This target was introduced to prevent $Vn1 - -Vn4$ from assuming too small values during the optimization process. Indeed, this would cause an extremely large DC current consumption in the operational amplifiers. *Silicon area* is mainly computed on the base of per-unit-area capacitance and resistance data and length and width of MOS devices within the operational amplifiers.

3.1   Optimization using NEOCIRCUIT, MODIRECT and NSGA2

NEOCIRCUIT is a software package delivered by CADENCE DESIGN SOLUTIONS, INC. for the optimization and design centering of analog, digital and mixed-signal circuits. It is perfectly integrated within the CADENCE environment and allows its optimization engines to be easily interfaced with virtually all commercial circuit simulators officially supported by the CADENCE environment.

Circuit variables to be used in the optimization process can be edited from the user interface and/or copied directly from the circuit schematic. Additional variables can be defined and combined with the original ones to provide the highest flexibility in the definition of the optimization problem. Matching conditions among active devices (i.e., two MOSFETs of a coupled pair) and statistical variables can also be defined.

Information about the simulation setup, i.e. simulator type, corner models, library paths, default parameter values, kind of analysis, expressions of the output quantities, etc., are retrieved from a CADENCE state. Multiple states can be loaded in a single NEOCIRCUIT session to allow different simulation setups during an optimization process. Parallel simulations can be run by properly setting a variable in the configuration file.

Output expressions are automatically loaded from the specified CADENCE state to allow objective functions and constraints to set by the user. Additional computing capabilities are offered via an integrated calculator. Area constraints are defined in a separate section of the interface. Expressions that relate design parameters of a given component to its area must be defined in a proper configuration. Optimization of a given circuit can be carried out at different corners and simulation temperatures simultaneously. This also allows verification of the performance in the worst case conditions. The operating region of each active device in the circuit can be monitored and constrained to prevent the optimizer from searching unfeasible solutions.

NEOCIRCUIT was employed to optimize the fifth-order leapfrog filter described in section 3. The optimization problem had 20 variables and 16 goals (objective functions). To completely explore the investigation domain, a global optimization was first carried out to determine the region where optimal solutions can be found. Afterward, a local optimization was performed to descend toward the absolute minimum. Table4 compares the solution found by NEOCIRCUIT with the original constraints. It can be observed that only one constraint is not strictly satisfied. However, the relative error is below the threshold of 10% which can in general be tolerated.

In order to phrase a multi-objective constrained optimization in terms of multi-objective problem a suitable translation of the constraints in objectives must be done. Generally it is satisfactory to carry out the following mapping

$$g(x) < 0 \quad \mapsto \quad \tilde{g}(x) = \begin{cases} g(x) & g(x) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

where $\tilde{g}(x)$ is a well known *penalty function*. But for computational tasks it must be take in to account a balancing for the different order of magnitude among the constraints and objectives. So a suitable balancing could be

$$g(x) < 0 \quad \mapsto \quad g(x) < \varepsilon_g \quad \mapsto \quad \tilde{g}(x) = \begin{cases} \frac{g(x)}{\varepsilon_g} & g(x) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

where $\varepsilon_g > 0$ gives a magnitude for relational comparison in order to scale different quantities.

Analogously the balancing for the objectives could be

$$f(x) \quad \mapsto \quad \tilde{f}(x) = \frac{f(x) - f_{\text{target}}}{\varepsilon_f} \tag{14}$$

where f_{target} is a reference for the optimization and $\varepsilon_f < |f_{\text{target}}|$ is again a magnitude for relational comparison.

The choice of different ε and f_{target} for constraints and objectives is heavily dependent on the problem, so some preliminary analysis must be done.

Target function	Constraint	MODirect	NSGA2	NeoCircuit
DC gain	≥ -0.01 dB	-0.003 dB	-0.0031 dB	-0.003 dB
Pass-band ripple at 9.1 MHz	$\leq 0.8\ dB_{PP}$	**0.861** dB_{PP}	$0.799\ dB_{PP}$	$0.798\ dB_{PP}$
Pass-band ripple at 9.7 MHz	$\leq 1.8\ dB_{PP}$	$1.332\ dB_{PP}$	$1.141\ dB_{PP}$	$0.798\ dB_{PP}$
Stop-band at 22.5 MHz	$\geq 25\ dB_C$	$36.569\ dB_C$	$30.22\ dB_C$	$36.1\ dB_C$
Stop-band at 34.2 MHz	$\geq 56\ dB_C$	**55.082** dB_C	**50.06** dB_C	**54.9** dB_C
Group-delay ripple at 9.1 MHz	≤ 20 ns	2.07 ns	1.475 ns	19.9 ns
Group-delay ripple at 9.7 MHz	≤ 40 ns	3.96 ns	1.619 ns	30.5 ns
Group-delay slope at 6.0 MHz	≤ 3 fs/Hz	1.28 fs/Hz	2.981 fs/Hz	1.4 fs/Hz
Equivalent input resistance	$\geq 12.2\ k\omega$	$31.549\ k\omega$	$14.04\ k\omega$	$12.8\ k\omega$
Output dynamic of stage 1	≤ 2.8 V	2.232 V	2.799 V	2.79 V
Output dynamic of stage 2	≤ 2.8 V	1.365 V	2.153 V	2.59 V
Output dynamic of stage 3	≤ 2.8 V	2.604 V	2.490 V	2.79 V
Output dynamic of stage 4	≤ 2.8 V	1.322 V	1.529 V	1.25 V
Equivalent input noise	$\leq 44\ nV/Hz^{\frac{1}{2}}$	**139** $nV/Hz^{\frac{1}{2}}$	$43.99\ nV/Hz^{\frac{1}{2}}$	$43.7\ nV/Hz^{\frac{1}{2}}$
Inverse squared noise	$\leq 40m$	30 m	39 m	39 m
Silicon area	minimize $(18.0\mu m^2)$	**29.5** μm^2	$13.45\ \mu m^2$	$12.95\ \mu m^2$

Table 4: Comparisons among MODirect, NSGA2 and NeoCircuit.

From various trials it turn out that the targets for the ideal design requested were hard to satisfy. In fact no method has achieved all targets couched by the constraints. However some results has been accepted as good solution for the designed circuit. As we can see in the table 4 MODirect has been found a promising solution but no improvement has been made after this preliminary result. Most likely the algorithm has focused on global optimization rather than local optimization. The NSGA2 has found a good solution which is comparable with the NeoCircuit tool. But the best solution in term of satisfaction seems to be that of NeoCircuit. Similar results as been obtained by SPEA2 algorithm (not shown).

4   Conclusions

In this article we compared evolutionary algorithms and standard optimization methods facing multi-objective optimization for circuit design problems.

The multi-objective optimization refers to determination of the approximate Pareto Front of a Operational Transconductance Amplifier and the design of a leapfrog filter. Two real world applications supplied by STMicroelectronics. Based only on these circuit design problems and on the above reported statistical analysis, we can claim what follows.

For the multi-objective problem, the observed Pareto fronts determined by evolutionary algorithms have a better spread of solutions with a larger number of nondominated solutions with respect to the standard multi-objective techniques, Multi-objective DIRECT.

MODirect is effective optimization method to face circuits, and numerical optimization in general, with few parameters. When one increases the number of circuit parameters (more than 10 parameters) the evolutionary algorithms out-

performs the Lipschitzian pattern search strategy in terms of solution accuracy and CPU time. In fact, DIRECT algorithm is exponential in time and space.

Currently a combination of the two methodologies is under investigation because we believe there could be an improved robustness and overall speed-up. As future work, we are considering the hybrid approach, that is, seeding the initial population of the MOEAs with the output of Lipschitzian pattern search strategies. Moreover, we are trying to extend our experiments to large scale circuits. Preliminary results are now compared with standard design tools focusing our research on alternatives design solutions.

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